Studying the Topological Sector of Quantum Gravity with Branched Covering Spaces

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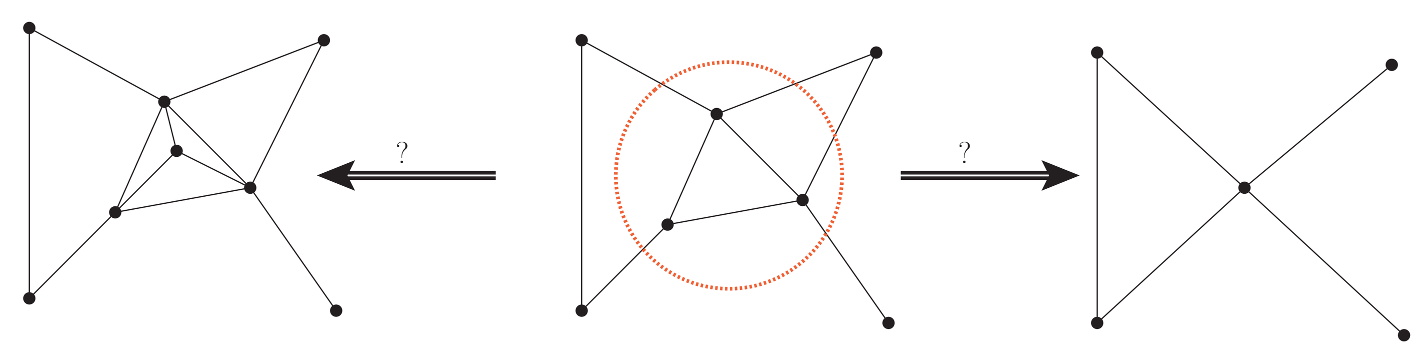
Spatial topology change is generally expected to be prohibited classically, being present only at the quantum scale. However, including topology change into many of the current approaches to quantum gravity (such as Loop Quantum Gravity (LQG) and Causal Dynamical Triangulations (CDT)) has proven difficult for technical reasons. In this work we explore this sector numerically using branched covering spaces (in LQG this corresponds to the utilization of Topspin Networks). To this end, we have implemented a group-theoretic algorithm coming from knot theory to measure the topology of arbitrary spatial sections in a systematic way, and have constructed a statistical ensemble of initial states to demonstrate access to quantities such as the expectation value of the size of the fundamental group. These are the first steps towards the incorporation of topology change into numerical approaches like CDT.

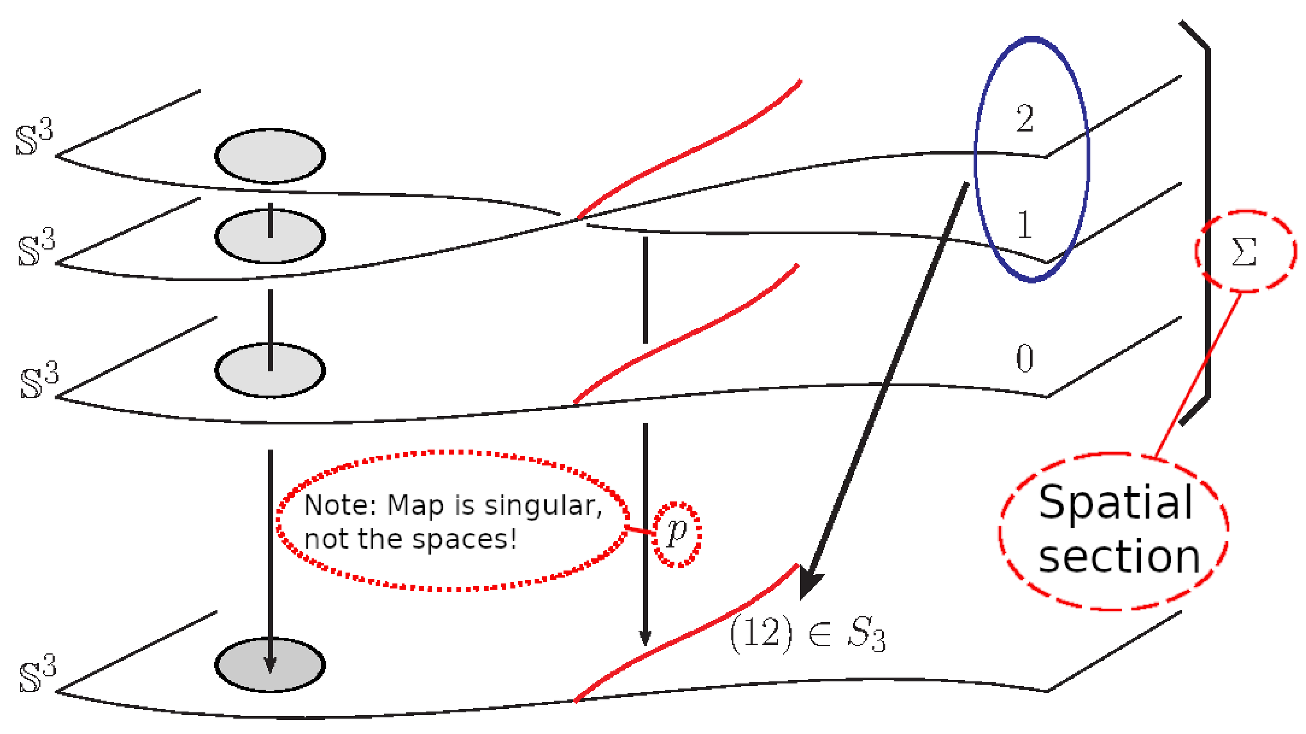
Topology in Gravity

- Unlike geometry, topology is not part of the background independence of GR; we must specify a topology *a priori*. (same is true of dimension and differentiable structure...)
- LQG: Topology (3-sphere $\mathbb{S}^3$) is set ahead of time
- CDT: Moves are chosen so that the topology remains fixed (Pachner moves).
- More generally "The coarse/fine graining problem"[1]:

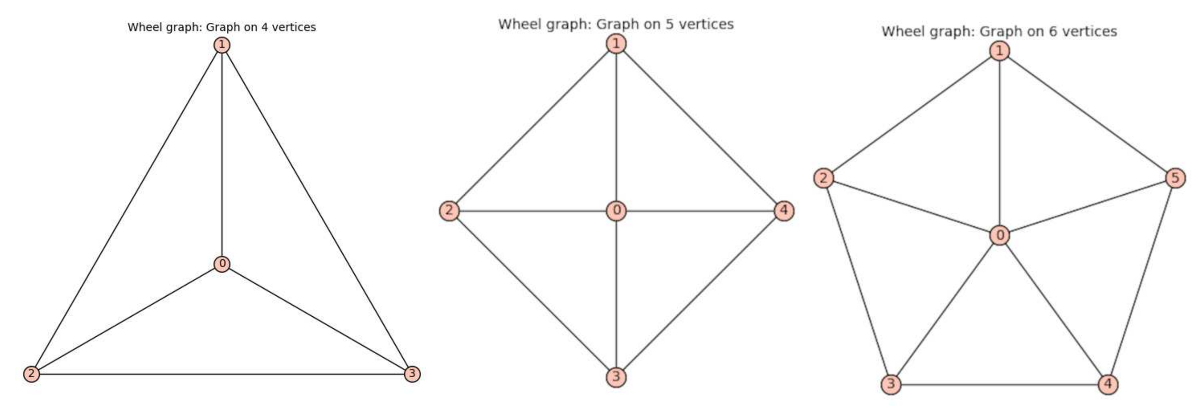


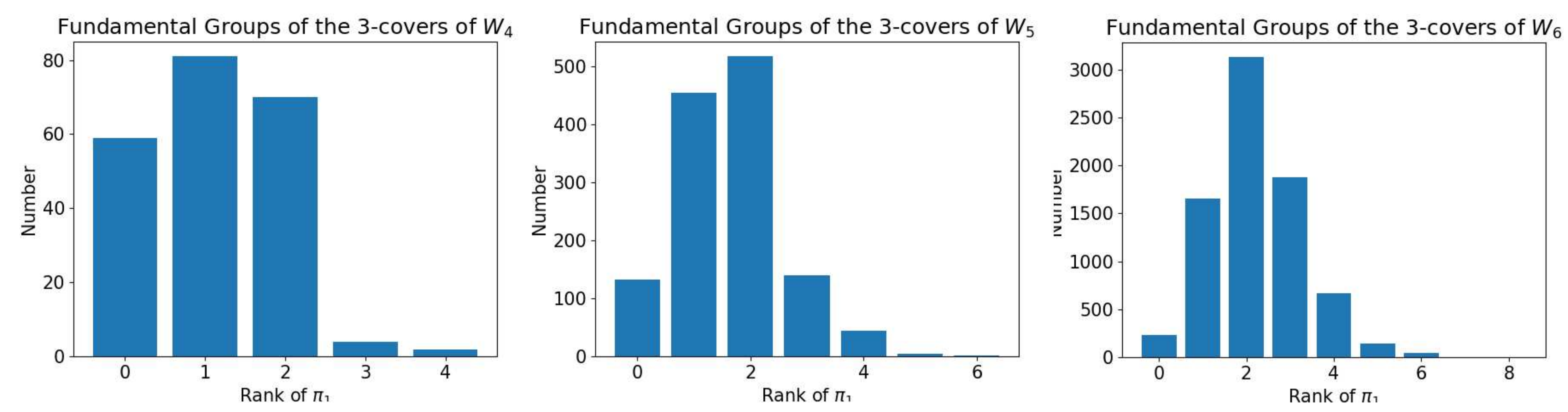
Branched Covering Spaces[2, 3, 4]

Alexander's Theorem: Any compact oriented 3-manifold can be described as a branched covering of $\mathbb{S}^3$, branched along a graph.

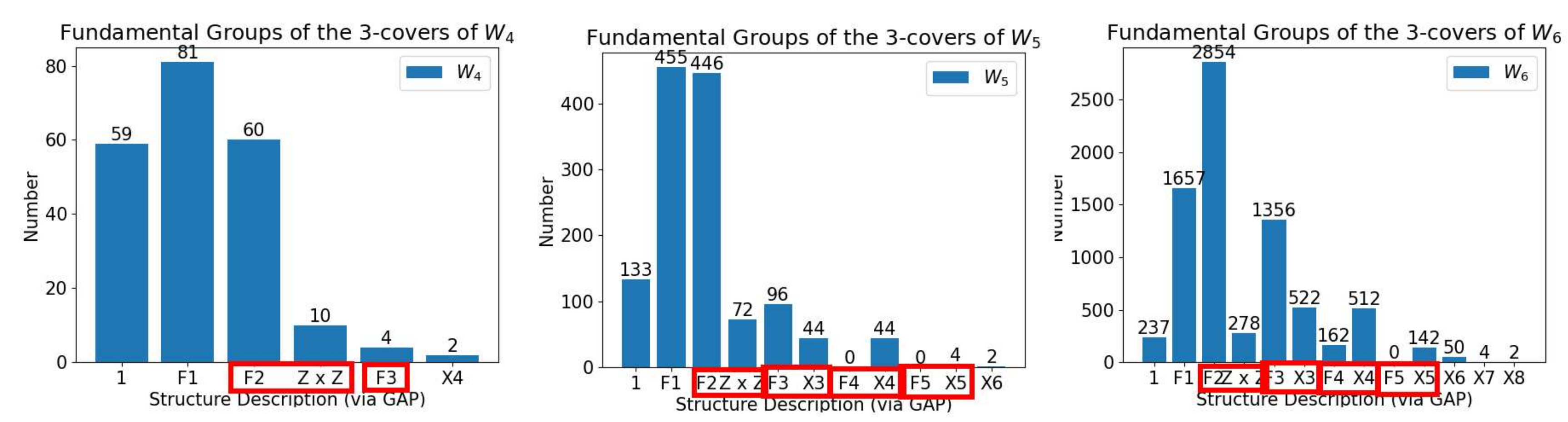


Exact Calculation: Fixed Graphs





The distribution of the order of the fundamental group of *all* 3-manifolds realised over the wheel graphs shown above. This demonstrates the essential task - to explore the topological parameter space of spatial sections.



There can be degeneracies in this system, as different fundamental groups can have the same rank. In some cases, GAP can give us more information to try and split this degeneracy.

Tracking Topological Information

We have implemented the Fox Algorithm[5] in SageMath[7], using routines based heavily on GAP[6].

- Requires a regular projection of the graph along with crossing/intersection data.
- All possible assignments of permutation labels for a particular covering order g are found. (Some are fixed by the graph, others are free.)
- The Fox Algorithm is implemented to find a presentation $\pi_1(\Sigma) = (x_1, ..., x_n : r_1, ..., r_m)$.
- The rank (number of generators) of the fundamental group is found (guaranteed), $|\pi_1(\Sigma)|$.
- If possible, a "structure description" of the fundamental group is found, based on a GAP algorithm. This allows us to "break the degeneracy" in $|\pi_1(\Sigma)|$.

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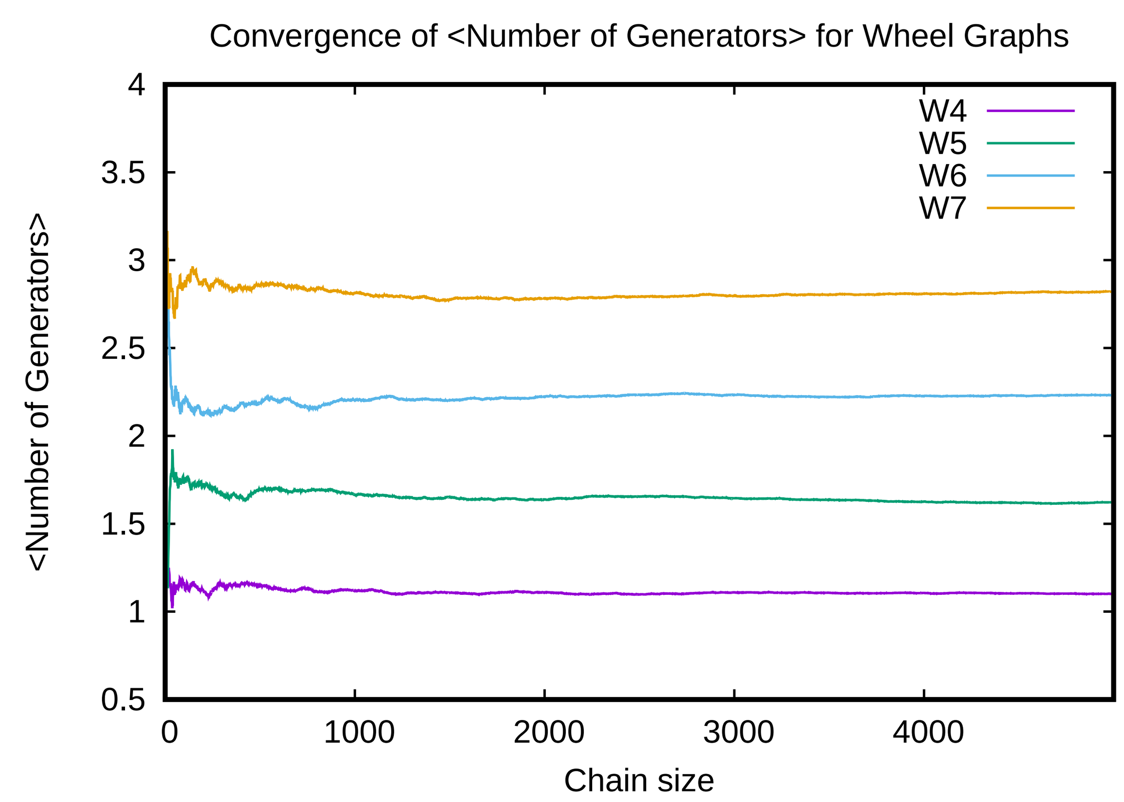
Initial Statistical Model

As a first step towards "topological-CDT", we have implemented a Metropolis algorithm using a branched covering space over a fixed graph, with the Regge action as the probability distribution. In this way we are verifying the essential approach by comparison to the exact calculation, and have shown they rapidly converge (see figure).

The next steps will be to allow more aspects of the topological structure to vary, to increase the topological and geometric parameter space. Eventually, the goal is to have a dedicated graph in a topologically $\mathbb{S}^3$ base, but with geometry given by CDT. The branched cover will then be $\Sigma \times \mathbb{R}$, where the spatial section Σ is allowed to vary in both topology and geometry.



References

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