The fate of chiral symmetry in Riemann-Cartan geometry

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Motivation

• Existence of light fermions ($m_i << M_{\rm Pl}$) is in accordance with χSB below the Planck scale;

• Gravity changes the spacetime in which fields propagate;
→ The background structure changes the spectra of the Dirac operator.
→ This affect χSB mechanism (*e.g.*, *gravitational catalysis*).
[Buchbinder and Kirillova, '89; Sachs and Wipf, '94, H. Gies and R. Martini, '18]

• Different formulations of GR (*e.g.*, Einstein-Cartan) could lead to richer structures;
→ Fermions source torsion (although suppressed by $1/M_{\rm Pl}$).

Identifying χSB

• **χSB from 4-fermion interactions**:

$$S_{\rm NJL} = \int_x \left(i\bar{\psi}_i \not{\partial} \psi_i - \frac{\bar{\lambda}}{4}(\mathcal{V} + \mathcal{A}) \right)$$

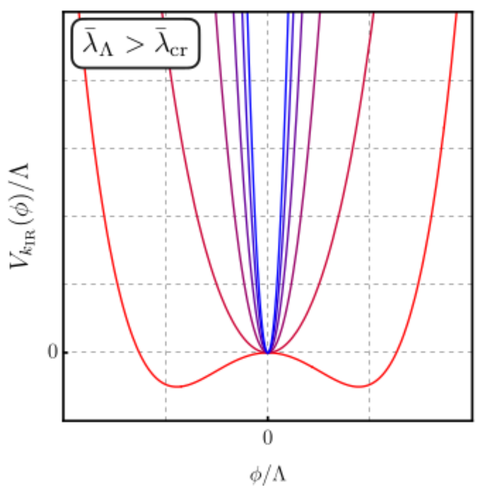
with

$$\mathcal{V} = (\bar{\psi}^i \gamma_\mu \psi^i)(\bar{\psi}^j \gamma^\mu \psi^j),$$
$$\mathcal{A} = (\bar{\psi}^i i\gamma_\mu \gamma_5 \psi^i)(\bar{\psi}^j i\gamma^\mu \gamma_5 \psi^j)$$

→ Bosonized picture:

$$S_{\rm B} = \int_x \left(i\bar{\psi}_i \not{\partial} \psi_i + \frac{1}{2\bar{\lambda}} {\rm tr}(\phi^\dagger \phi) + i\bar{\psi}^i \left[P_{\rm L} (\phi^\dagger)_{ij} + P_{\rm R} \phi_{ij} \right] \psi^j \right)$$

• **From effective potential**:



$$V''_{k_{\rm IR}}(\phi = 0) = N_f \left(\frac{1}{\bar{\lambda}_\Lambda} - \frac{1}{\bar{\lambda}_{\rm cr}} + \frac{1}{8\pi^2} k^2_{\rm IR} \right)$$

• **Curvature-based catalysis**:

→ Coarse-graining now as $-\not{D}^2 = -D^2 + \frac{1}{4}R$

$$V''_{k_{\rm IR}}(0) \approx N_f \left(\frac{1}{\bar{\lambda}_\Lambda} - \frac{1}{\bar{\lambda}_{\rm cr}} + \frac{1}{4\pi^2} k_{\rm IR} + \frac{R}{96\pi k_{\rm IR}} \right)$$

→ If $R < 0$, then $V''_{k_{\rm IR}}(0)$ becomes negative for sufficientely small $k_{\rm IR}$ (3d result).
[H. Gies and R. Martini, '18]

Executive summary

• **Can torsion act in favour of χSB and be responsible for gravitational catalysis?**
→ Our findings suggest that torsion acts in favor of χSB, but not enough to engender gravitational catalysis.

• Toy-model: **Four-fermion interactions in a Riemann-Cartan framework.**

The impact of torsion on the mechanism of χSB

• Toy-model: **Nambu-Jona-Lasinio model in the Einstein-Cartan formalism:**

$$S[\bar{\psi}, \psi; g] = \int_x \left(i\bar{\psi}^i \not{\nabla} \psi^i + \frac{i}{8} \bar{\psi}^i \gamma_5 \not{A} \psi^i - \frac{\bar{\lambda}}{4} \left[(\bar{\psi}^i \gamma_\mu \psi^i)^2 + (\bar{\psi}^i i\gamma_\mu \gamma_5 \psi^i)^2 \right] \right)$$

→ Minimal coupling via $\mathcal{D}_\mu \psi_i = \partial_\mu \psi_i + \frac{1}{8} \omega^{ab}{}_\mu(e) [\gamma_a, \gamma_b] \psi_i - \frac{1}{8} \gamma_5 A_\mu \psi_i$.

→ Axial-vector component: $A^\rho = \epsilon_\lambda{}^{\mu\nu\rho} T^\lambda{}_{\mu\nu}$.

→ After bosonization, define FRG-inspired regularized potential:

$$V_{k_{\rm IR}}(\phi) = \frac{N_{\rm f}}{2\bar{\lambda}_\Lambda} \phi^2 + \frac{N_{\rm f}}{2v_4} \int_{k_{\rm IR}}^{\Lambda} \frac{dk}{k} {\rm Tr} \Big[\Big(-\not{\mathcal{D}}^2 + R_k(-\not{\mathcal{D}}^2) + \phi^2 \Big)^{-1} k \partial_k R_k(-\not{\mathcal{D}}^2) \Big].$$

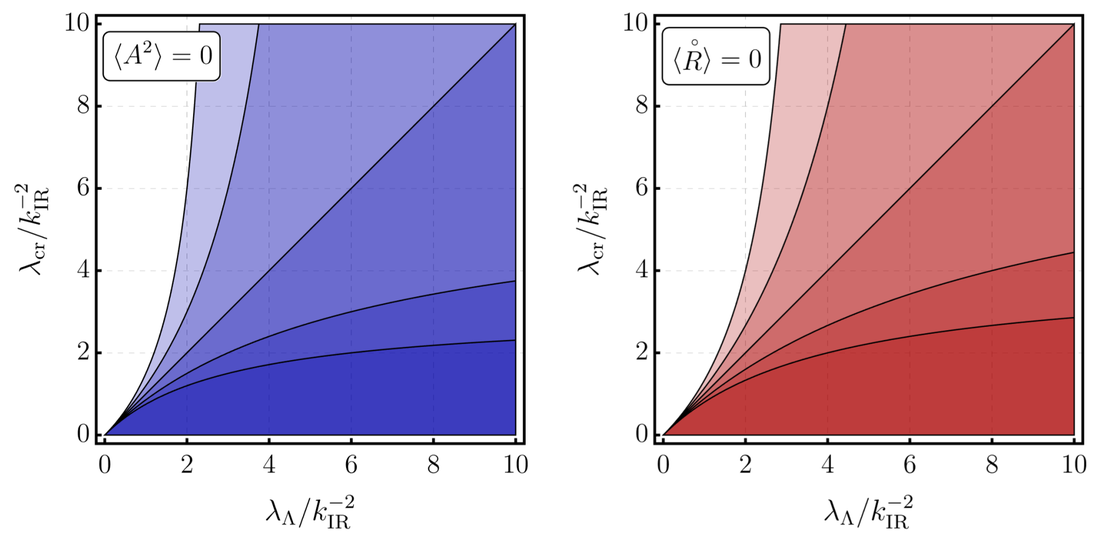
• **$V''_{k_{\rm IR}}(\phi = 0)$ in the presence of torsion**
→ $k^{-2} \mathring{R} << 1$ and $k^{-2} A^2 << 1$ regime in the heat-kernel expansion.
→ χSB for $V''_{k_{\rm IR}}(\phi = 0) < 0$.

$$N_{\rm f}^{-1} V''_{k_{\rm IR}}(\phi = 0) = \frac{1}{\bar{\lambda}_\Lambda} - \frac{1}{\bar{\lambda}_{\rm cr}} + \frac{1}{4\pi^2} k^2_{\rm IR} \mathcal{I}_2[r] + \frac{2}{3} \xi_{\rm IR} \langle \mathring{R} \rangle - \frac{1}{4} \xi_{\rm IR} \langle A^2 \rangle.$$

• Flat space ($\mathring{R} = 0$, $A_\mu = 0$): **χSB for $\bar{\lambda}_\Lambda > \bar{\lambda}_{\rm cr}$;**

• Curvature, but no torsion ($\mathring{R} \neq 0$, $A_\mu = 0$): **negative curvature favours χSB;**

• Torsion, but no curvature ($\mathring{R} = 0$, $A_\mu \neq 0$): **torsion favours χSB.**



→ Regions where χSB is triggered by a background curvature $\langle \mathring{R} \rangle$ or axial-vector torsion squared $\langle A^2 \rangle$. Left panel: $\xi_{\rm IR} k^{-2}_{\rm IR} \langle \mathring{R} \rangle = -1/2, -1/4, 0, 1/4, 1/2$ from lighter to darker regions. Right panel: $\xi_{\rm IR} k^{-2}_{\rm IR} \langle A^2 \rangle = -1, -1/2, 0, 1/2, 1$ from darker to lighter.

• But the heat-kernel expansion has limitations. Higher-order terms in torsion and curvature become more relevant in the deep IR! Essential for a torsion-induced gravitational catalysis → Future work...

• **Outlook:** χSB triggered by quantum Einstein-Cartan gravity (propagating torsion effects). Work in progress.

Acknowlegments

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