Recursion with defects? Go tight!

A topological recursion for hyperbolic surfaces with defects

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Applications

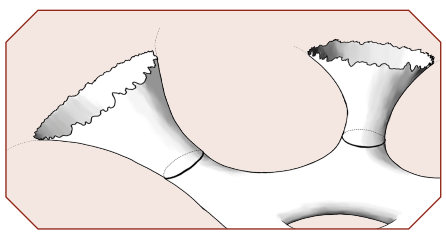
JT gravity is a toy model for 2D Quantum Gravity.

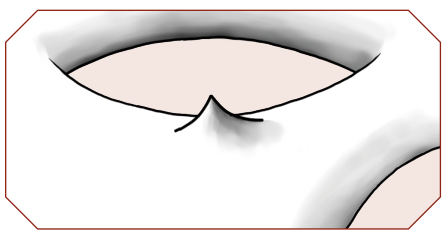
Its action forces the surfaces to have a constant negative curvature.

The dynamics only lives at the boundaries, which are far away.

JT gravity has some natural extensions that introduce an arbirary number of defects to the surfaces.

These defects can be geodesic boundaries or conical defects and have weights asociated with them.





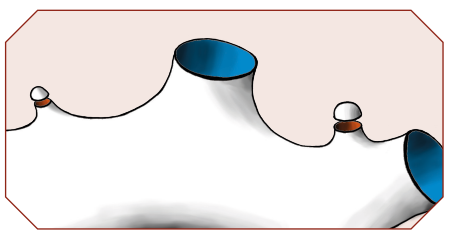
Preparation

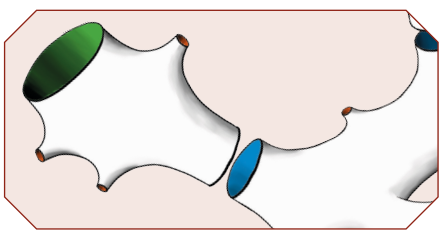
Normally, there is a unique geodesic in each homotopy class.

We want to 'ignore' defects, so we allow homotopies to jump across defects.
You can see this as capping off the defects using topological disks.

Now it is possible to have multiple geodesics in a homotopy class, but in general there is a single shortest. We call this geodesic tight.

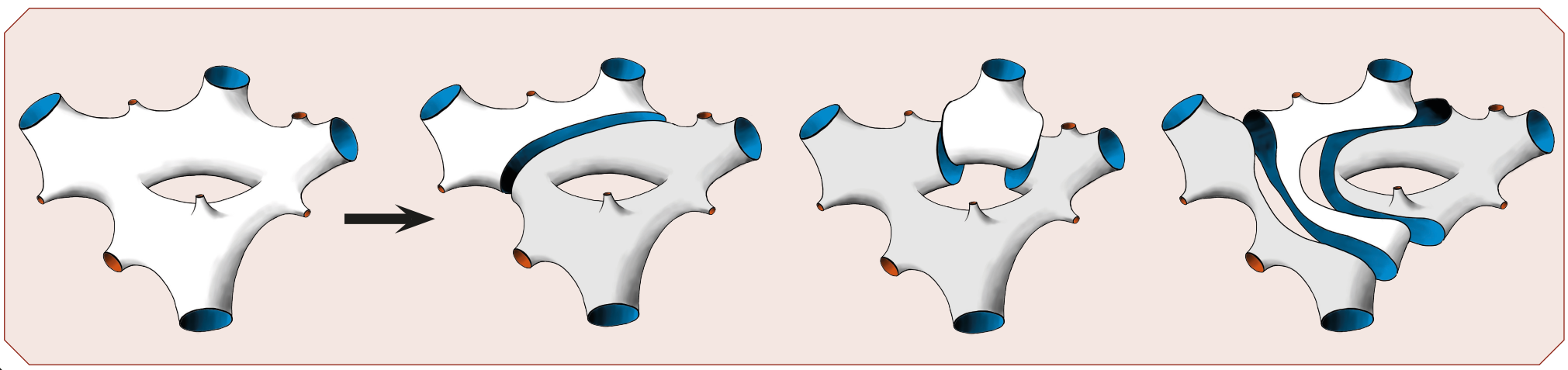
We need our boundaries to be tight. This can be done by removing a half-tight cylinder.



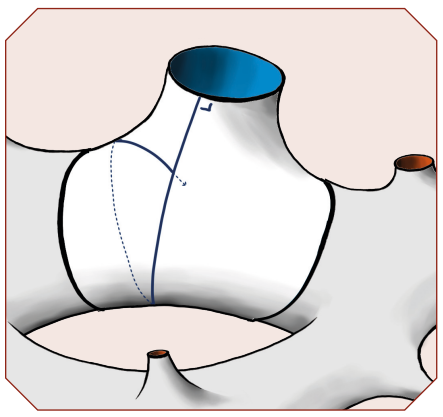


Mirzakhani's topological recursion using tight pairs of pants

Just like Mirzakhani's topological recursion we can compute the Weil-Petersson volumes of our surfaces by removing pairs of pants.
In our case these are tight pairs of pants: surfaces with three tight boundaries and an arbitrary number of defects.
There are 3 possible outcomes which need to be incuded. Each outcome gets its own weight depending on the lengths of the boundaries and the weights of the defects.



Open questions



- The original recursion has a nice geometric proof using 'shooting' of geodesics. The new recursion has only been proven algebraically.
  Can we find a geometric proof?
- Can we find a tree bijection that matches this interpretation?

In fact: we already can for genus 0 surfaces without defects. [BZ, Timothy Budd, Thomas Meeusen; '23+]

Bonus

In this tight framework there is another recursion just in the number of boundaries.

It has an interpretation as removing leaves from a tree.

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Based on work in progress together with Timothy Budd