Renormalization Group in a higher derivative scalar model

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[1] D. B. and R.Percacci, *Renormalization group flows between Gaussian fixedpoints*, JHEP 10 (2022) 113, e-Print: 2207.10596 [hep-th]
[2] D. B., J. Donoghue, R.Percacci. in preparation

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1. the model

Quadratic Gravity is perturbatively **renormalizable** and **Asymptotically free**, hence it is a possible **UV completion** of **Einstein Gravity**. However it has two **ghost** modes which give instability at classical level and the **RG flow** between asymptotically free and interacting fixed points given by FRG is not clear. In order to get some more insight, we considered another **higher derivative** quantum field theory as a toy model:

$$\Gamma[\phi] = \int d^4x \left[\frac{1}{2}Z_1(\partial\phi)^2 + \frac{1}{2}Z_2(\Box\phi)^2 + \frac{1}{4}g((\partial\phi)^2)^2 + \dots\right] \tag{1}$$

The action describes a scalar field with a four particles interaction invariant respect to **shift symmetry** $\phi \to \phi + c$ and $\mathbb{Z}_2$ symmetry $\phi \to -\phi$. Also this theory is plagued by a **ghost** mode with **mass** $m^2 = Z_1/Z_2$

2. Running definitions

there are different ways to define a running coupling:

Physical running. Define the coupling in terms of the scattering amplitude at some particular momentum $p = \mu_R$.

μ-running. In perturbation theory using dim-reg or cutoff regularization one has to introduce a parameter μ to preserve dimensions, e.g. in $\log(p^2/\mu^2)$. Taking the derivative of the coupling with respect to μ defines another kind of RG.

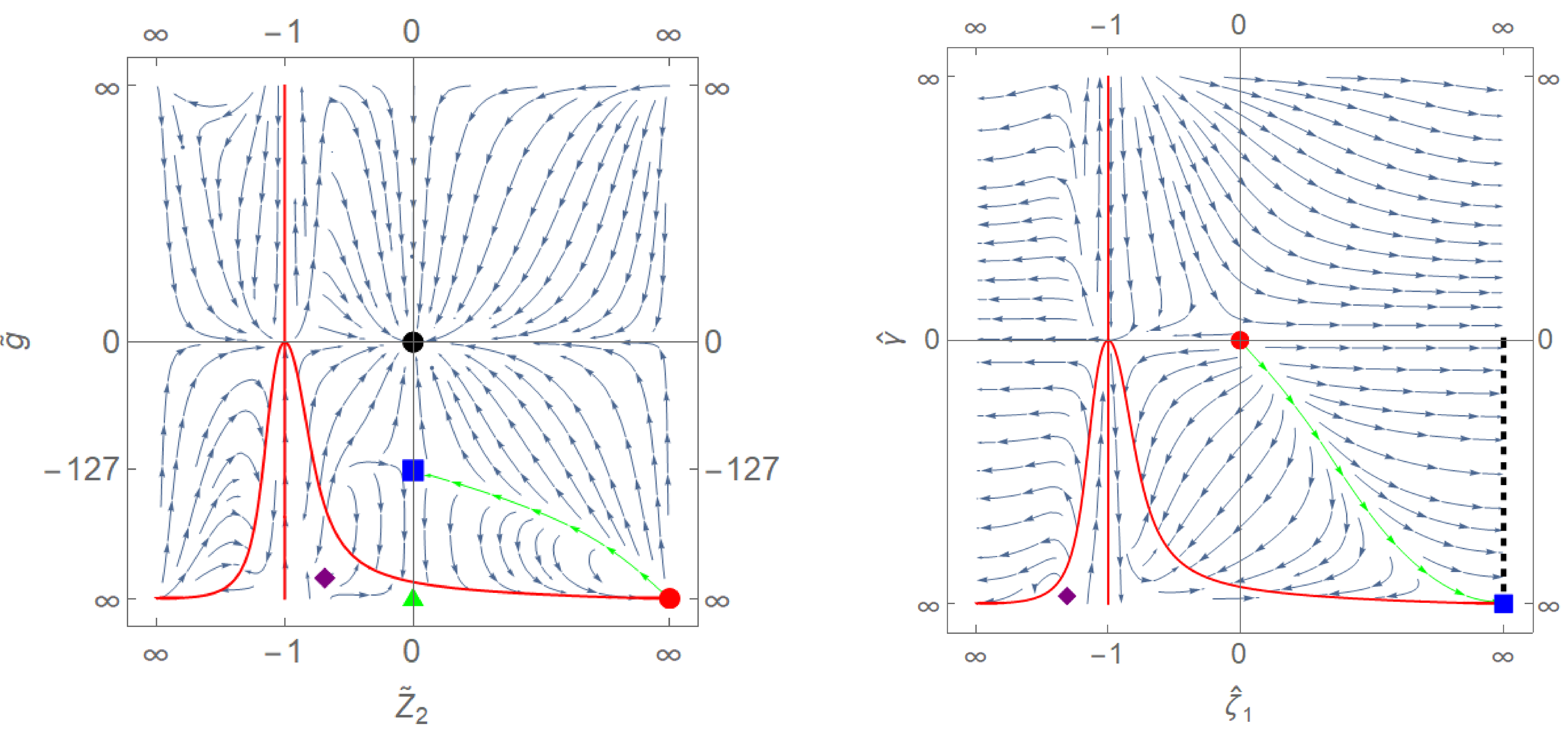
Non-perturbative RG. One studies the dependence of the couplings in the quantum effective action on a UV cutoff(Wilsonian RG) or IR cutoff (FRG).

3. FRG analysis

The theory has two **Gaussian fixed points**:

$$GFP_1: \; S[\phi] = \frac{1}{2}\int d^4x\,(\partial\phi)^2, \; [\phi] = 1 \qquad GFP_2: \; S[\phi] = \frac{1}{2}\int d^4x\,(\Box\phi)^2, \; [\phi] = 0 \tag{2}$$

each one can be chosen in order to fix field dimension and normalization, giving two possible representations of the RG flow. The two GFP's are alternatively in the **origin** of the phase space or at **infinity**.



In the **UV**, the theory is **asymptotically free** if $g < 0$. There is an RG flow between the two GFP's in a region bounded from above by the axis $g = 0$, near which the theory remains weakly interacting, and from below by a **separatrix** joining a non-Gaussian FP and GFP_2. Thanks to the **anomalous dimension** η, the FRG **correctly interpolates** the dimension of the field between its classical values in the two GFP's.

The **mass of the ghost increases** when trajectory becomes less **perturbative**. It goes to infinity for the AS trajectory (in green).

4. Amplitudes

there are three energy domains where one can study the 4-point scattering amplitude:

- $E < m$ **low energy regime**: only massless particles propagate and are weakly coupled
- $m < E < m/4\sqrt{g}$ **intermediate energy regime**; also ghosts propagate and are weakly coupled
- $m/4\sqrt{g} < E$ **high energy regime**: apparently strongly interacting

5. EFT

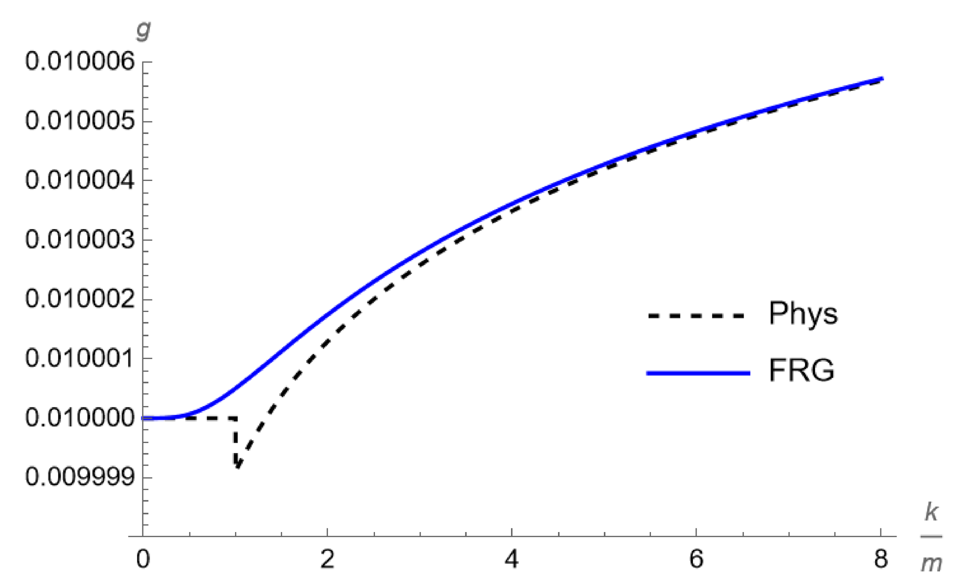
In the **low energy** regime one can adopt an **EFT approach** and neglect the $\Box^2$ kinetic term. We have to consider higher dimension operators which are generated at one-loop

$$\begin{aligned}\mathcal{L}_6 &= \frac{g_6}{4}\partial_\mu\phi\partial^\mu\phi\Box\partial_\nu\phi\partial^\nu\phi + \frac{g_6'}{4}\partial_\mu\phi\partial_\nu\phi\Box\partial^\mu\phi\partial^\nu\phi \\ \mathcal{L}_8 &= -\frac{g_8}{4}\partial_\mu\phi\partial^\mu\phi\Box^2\partial_\nu\phi\partial^\nu\phi - \frac{g_8'}{4}\partial_\mu\phi\partial_\nu\phi\Box^2\partial^\mu\phi\partial^\nu\phi\end{aligned} \tag{3}$$

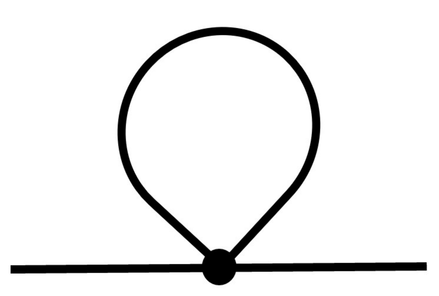
We have $\beta_{g_8} = \frac{41g^2}{480\pi^2}$ and $\beta_{g_8'} = \frac{g^2}{240\pi^2}$.
The running of g is more delicate: it is $\beta_g^\mu = \frac{5g^2m^4}{16\pi^2}$ in the μ-**running** while one finds $\beta_g = 0$ in **physical running**.

6. Above mass threshold

When **ghosts start to propagate**, higher dimension operators do not run anymore. On the other hand, the coupling g now depends on the energy scale of the process, hence also the **physical running** scheme gives $\beta_g = \frac{5g^2m^4}{16\pi^2}$. This running matches the one given by the **FRG**, up to the region $E \sim m$, where there is a strong dependence on the renormalization scheme. At high energy, with $g < 0$, the theory is **AF** and **perturbative** respect to the 4-derivative kinetic term, but also **strongly interacting**.



7. Two-point function



The **two-point function** receives quantum corrections from the **tadpole** diagram, whose integral does not depend on the external momentum. One observes a nonzero **anomalous dimension** of the field in μ scheme, but not in the physical one. In the FRG framework, $\eta \neq 0$ permits to go from GFP_1 to GFP_2.

8. Conclusions and outlooks

1. Physical running is only defined in **asymptotic regions**. The FRG running of g agrees there
2. In the low energy **EFT** at one loop there are **higher order operators** with 6 and 8 derivatives
3. The **disappearance** of **higher dimension operators** above the mass threshold is a new and somehow unexpected phenomenon, joining the EFT regime to a (possibly) AF or AS regime
4. μ-running is **not** always physical. Well established results such as the universal beta functions of **Quadratic Gravity** and **NLSM** seem to contain unphysical terms of this type
5. We want to improve our understanding of the **high energy region**, studying inclusive cross sections in physical processes