The University Of Sheffield.
SAPIENZA UNIVERSITÀ DI ROMA
MARIAVERONICA DE ANGELIS
mdeangelis1@sheffield.ac.uk

f(R) THEORIES

These theories arise from a straightforward generalization of the Einstein-Hilbert action which becomes a general function of R

$$S_J = \frac{1}{2\kappa}\int d^4x\sqrt{-g}(\xi R - V(\xi) + 2\kappa\mathcal{L}_m)$$ JORDAN FRAME

INDEPENDENT DEGREES OF FREEDOM

$V(\xi)$ fixed by the specific functional form f(R)

$$V(\xi) \equiv f(R(\xi)) - \xi R(\xi) \qquad \text{where} \quad \xi = \frac{df}{dR}$$

BIANCHI I UNIVERSE

Homogeneous and anisotropic (flat ${}^3R = 0$) model

$$ds^2 = N^2(t)dt^2 - \eta_{ab}(t)\,\omega^a \otimes \omega^b$$ 4D LINE ELEMENT

$$\eta_{ab} = R_0^2 e^{\alpha}(e^{\beta})_{ab}$$

- CHANGE OF VOLUME
- CHANGE OF SHAPE

INITAL RADIUS OF THE UNIVERSE

The matrix β_{ab} satisfies the condition $\mathrm{Tr}\,\beta = 0$, ensuring the volume depends only on the conformal factor α

$$\mathcal{V}_{Uni} \sim R_0^3 e^{3\alpha}$$

ON THE EMERGENCE OF A CLASSICAL UNIVERSE FROM A QUANTUM f(R) COSMOLOGY IN THE JORDAN FRAME

Bianchi I Universe in the Jordan frame of an f(R) theory is associated to the reduced ADM action

$$S_B = \int d\xi\left(p_\alpha\frac{d\alpha}{d\xi} + p_+\frac{d\beta_+}{d\xi} + p_-\frac{d\beta_-}{d\xi} - H_{ADM}\right)$$

TEMPORAL GAUGE

$$\dot{\xi} = 1 \longrightarrow N = \frac{3e^{3\alpha}}{2(\xi H_{ADM} - p_\alpha)}$$

The Hamiltonian constraint is classically solved (RPSQ)

$$H_{ADM} \equiv \frac{1}{\xi}\left(p_\alpha + \sqrt{p_\alpha^2 - p_+^2 - p_-^2 - 6\xi e^{3\alpha}(V(\xi) + \Lambda)}\right)$$

- ASSUMPTION: $\Lambda \gg V(\xi)$

QUANTISING H_{ADM} yields the Schrödinger equation

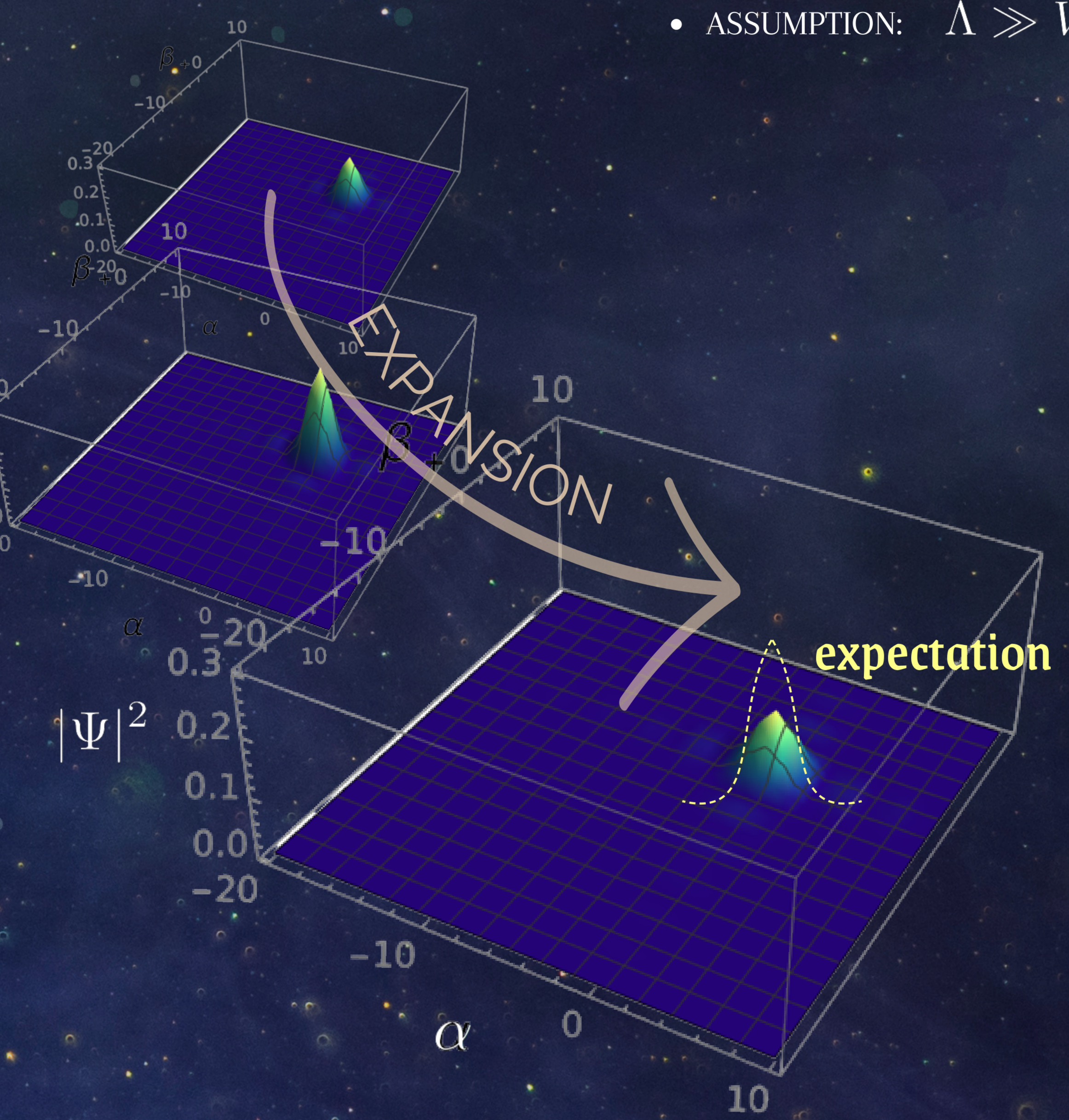
$$i\partial_\xi\psi = -\frac{1}{\xi}\left(-i\partial_\alpha + \sqrt{-\partial_\alpha^2 + \partial_+^2 + \partial_-^2 - 6\xi e^{3\alpha}\Lambda}\right)\psi$$

- WITHOUT Λ

$$\psi(\alpha,\beta_\pm,\xi) = e^{i(k_\alpha\alpha + k_+\beta_+ + k_-\beta_-)}\xi^{i(k_\alpha + \sqrt{k_\alpha^2 - k_+^2 - k_-^2})}$$

Use a Gaussian distribution $A(k_\alpha, k_+, k_-)$ to construct a localized wave packet

$$\Psi(\alpha,\beta_\pm,\xi) = \int_{-\infty}^{+\infty} dk_\alpha dk_+ dk_- A(k_\alpha,k_+,k_-)\psi(\alpha,\beta_\pm,\xi)$$



RESULT

The wave packet becomes localised (classicalisation) but is dynamically unstable

(NOT TRUE FOR FRLW → REACHES THE SINGULARITY IN A QUASI-CLASSICAL WAY)

BIANCHI I *VS* FLRW

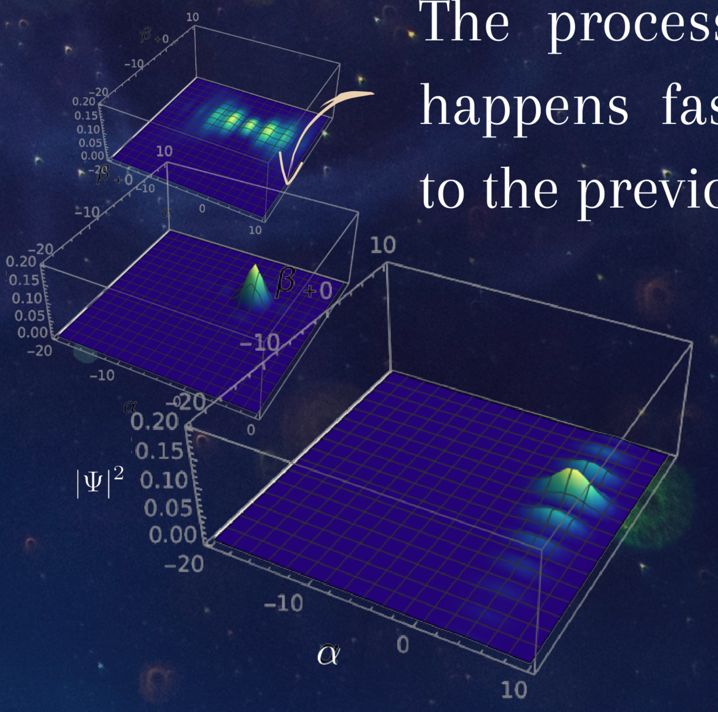
- WITH Λ

TAYLOR EXPANSION FOR SMALL Λ

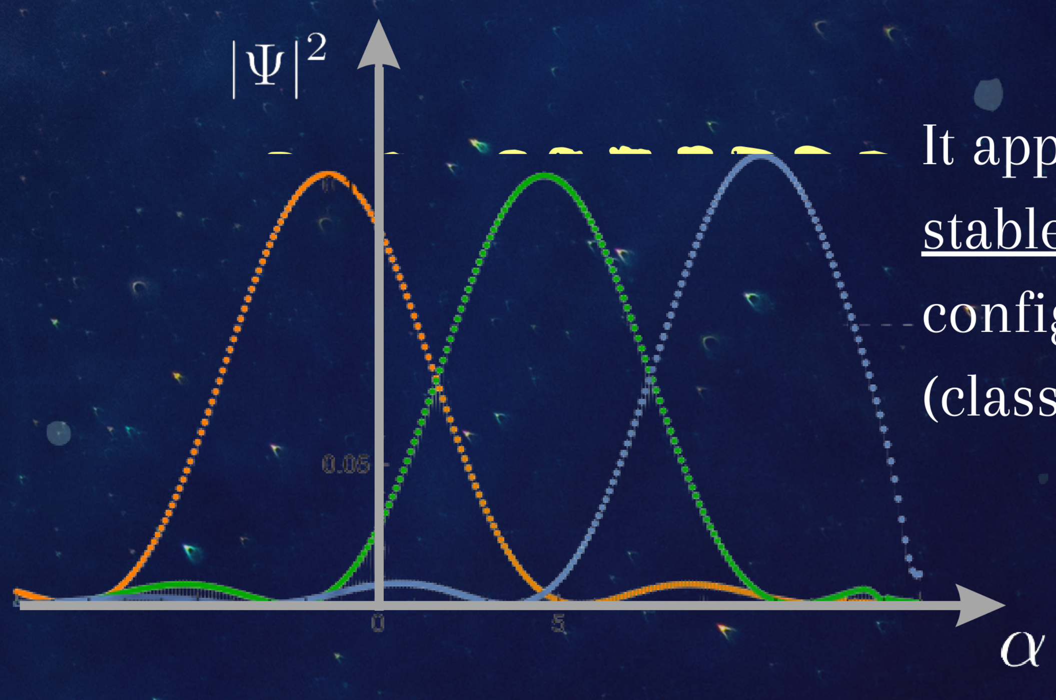
$$\psi(\alpha,\xi) = e^{ik_\alpha\alpha}e^{i\left(\frac{3\xi\Lambda e^{3\alpha}}{k_\alpha} + 2ik_\alpha\log(\xi)\right)}$$

The introduction of Λ seems not to be the mechanism to make the model a classical one

- ASSUMPTION:
  On-set of the inflationary phase (and we neglect $V(\xi)$)
- TO INVESTIGATE:
  At late(r) times $V(\xi)$ could play a significant role



The process of localisation happens faster with respect to the previous case!



It approaches a stable peaked configuration (classicalisation)

REFERENCES
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