Functional Renormalization Group in perturbative Algebraic Quantum Field Theory

(in an A0-sized nutshell)

Edoardo D'Angelo

Department of Mathematics, University of Genova

Istituto Nazionale di Fisica Matematica (INFN)

Istituto Nazionale di Alta Matematica (INdAM-GNFM)

Motivations

Functional Renormalization Group

- Flow from micro- to macro- physics
- Investigation of fundamental vs effective QFT
- Non-perturbative effects

Perturbative Algebraic QFT

- Lorentzian approach to interacting QFT
- Distinction between observables and states
- Ideal for curved spacetimes, thermal theories

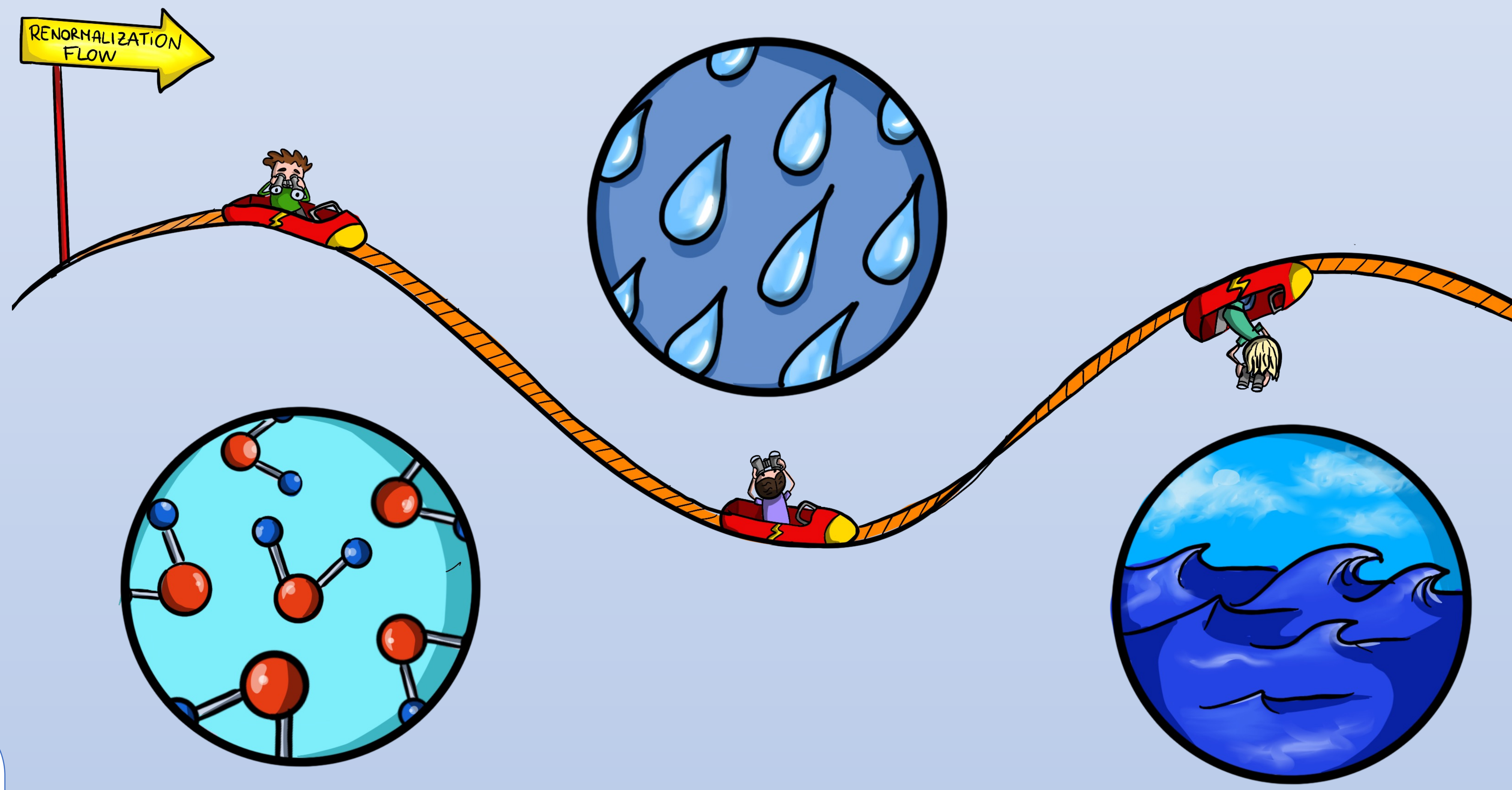
From **Euclidean** to **Lorentzian**: flow from **small** to **large** scales becomes flow in an **external mass**: $R_k(p) \sim k^2$

Functional Renormalization Group flow equations

$$\partial_k \Gamma_k = -\frac{1}{2}\int_{x,y} \partial_k q_k(x) : G_k : (x,x)$$

$$(\Gamma_k^{(2)} - q_k) G_k = -\delta$$

Wetterich equation with a local regulator (functional Callan-Symanzik equation) in Lorentzian signature



Hadamard regularization

RHS: coincidence limit of the **interacting propagator**:

- Starting point: $\left(\Gamma_k^{(2)} - q_k\right) G_k = -1$
- Expansion:

$$\Gamma_k = \phi P_0 \phi + U_k(\phi)$$

- In the free limit $U_k = 0 \Rightarrow G_k = \Delta_F$
- Perturbative expansion: $G_k = \sum \left(-i\Delta_F U_k^{(2)}\right)^n i\Delta_F$
- Hadamard subtraction at each order

NB 1. Non-perturbative in the coupling constant

2. The series converges (in some cases)

$$: G_k : (x,x) = \int_y \left(\Gamma_k^{(2)} - q_k\right)^{-1}(x,y)\, \lambda(y,x)$$

$$= \left(1 + i\,\Delta_F U_k^{(2)}\right)^{-1} (\Delta_F - H_F)$$

Initial data:

- Globally hyperbolic space-time
- Reference state for the free theory $(\Delta_F - H_F)$

Main features:

- **State** dependence
- **Hadamard** regularization

Generalization to gauge theories with the Batalin-Vilkovisky formalism

- Scale-dependent BRST-BV differential $s_k = s_{BRST} + \int_x q_k \frac{\delta}{\delta\eta}$
- New auxiliary field $\eta = s_k q_k,\ s_k q_k = 0 \Rightarrow$
- Quantum Master Equation

$$\mathrm{QME} = (\mathrm{I}, \mathrm{I}) - i\hbar\Delta I = 0 \Rightarrow$$

Gauge-independence of the S-matrix
- $\langle \mathrm{QME} \rangle = 0 \Rightarrow$

$$\int_x \frac{\delta\tilde{\Gamma}_k}{\delta\phi(x)} \frac{\delta\tilde{\Gamma}_k}{\delta\sigma(x)} + q_k \frac{\delta\tilde{\Gamma}_k}{\delta\eta(x)} = 0$$

Applications

β-functions for $\lambda\phi^4$ with the Local Potential Approximation in

- Bunch-Davies in de Sitter background
- High-temperature thermal states in Minkowski:

$$k\partial_k \widetilde{m}_k^2 = -2\widetilde{m}_k^2 - \frac{1}{2\pi^2}\frac{\tilde{\lambda}_k}{(1+\widetilde{m}_k^2)^{\frac{1}{2}}}$$

$$k\partial_k\, \tilde{\lambda}_k = -\, \tilde{\lambda}_k + \frac{3}{8\,\pi^2}\frac{\tilde{\lambda}_k^2}{(1+\widetilde{m}_k^2)^{\frac{3}{2}}} \Rightarrow$$

→ Non-trivial fixed points

Work in progress: Theorem on existence of local solutions within LPA

Applications in curved spacetimes

… Quantum gravity!

SCAN ME

Further reading

[1] *Existence of local solutions of Renormalization Group flow equations and Hadamard regularization*, with N. Drago, N. Pinamonti, K. Rejzner. *In preparation*.

[2] *Lorentzian Wetterich equation for gauge theories,* with K. Rejzner. ArXiv e-Print: 2303.01479 [math-ph] *Ann. Henri Poincaré* **24** (2023) 4, 1211–1243.

[3] *An algebraic QFT approach to the Wetterich equation in Lorentzian manifolds*, with N. Drago, N. Pinamonti, K. Rejzner. Accepted for publication in *Ann. Henri Poincaré,* arXiv e-Print: 2202.07580 [math-ph]

Illustrations by Elena Varoli

Università di Genova