Discreteness Unravels the Black Hole Information Puzzle : Insights from a Quantum Gravity Toy Model

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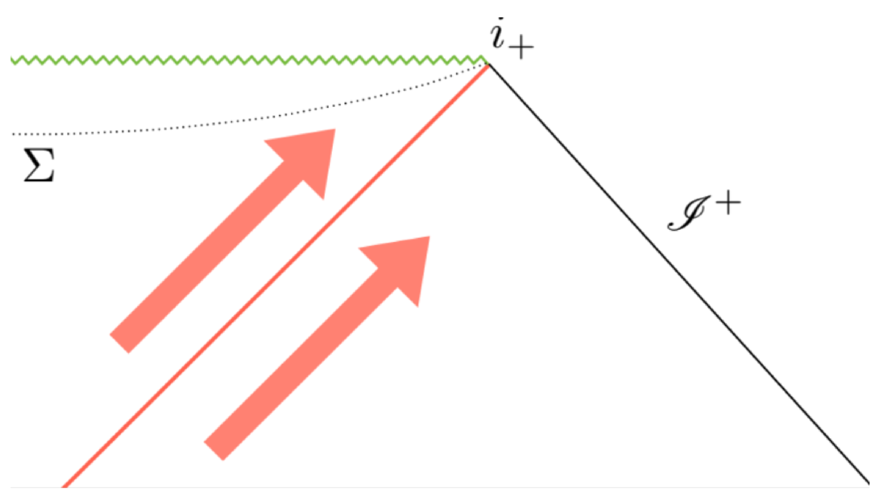
Motivations

- Describe the classical dynamics of a particle falling into a black hole as a KG scalar field excitation.
- Implement the discreteness of geometry suggested in LQG in the quantization of the dynamics of matter and geometry.
- Use this quantum model to investigate the information loss paradox.

Model of an Hawking particle near the singularity

We first study the dynamics of a particle close to the BH singularity, representing the ingoing Hawking particle :



The particle falls with a 4 wave vector

$$k^a = -E(\partial_t)^a + p_r(\partial_r)^a$$

The physical moment in the $(\partial_t)^a$direction is

$$p_t = \frac{k_a(\partial_t)^a}{\sqrt{|\partial_t.\partial_t|}} = -E\sqrt{\frac{r}{2M-r}} \xrightarrow{r\to 0} 0$$

The physical moment of the particle is vanishing in the t, θ and ϕ direction

The KG scalar field excitation describing the particle is homogeneous in the t, θ and direction ϕ

The classical dynamics

On the other hand, the interior of a BH is isometric to an homogenous and anisotrpic spacetime

$$\Downarrow$$

Inside of black hole + particle falling in = Homogenous & Anisotropic S.T. + masslessϕ field

Therefore the system is well described by :

$$ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2 d\Omega^2 \quad \text{and} \quad \phi = \phi(r)$$

The action is $\quad S = \frac{1}{16\pi}\int_R d^4x\sqrt{-g}R + 2\int_{\partial_R} K - \frac{1}{2}\int_R d^4x\sqrt{-g}\partial_a\phi\partial^a\phi$

After reparametrisation (r added to the phase space), the Hamiltonian is :

$$H(f, p_f, h, p_h, r, p_r, \phi, p_\phi) \approx 0$$

The near singularity regime

We work in the limit $r \ll 2M$ (near singularity regime) and we perform the canonical transformation

$$a = 4\pi r^2 \qquad p_a = \frac{p_r}{8\pi r} \qquad m = -fp_f \qquad p_m = -\log(-f)$$

The Hamiltonian constraint simply becomes : $\boxed{H = p_a + \frac{1}{2a}\left(m + \frac{p_\phi^2}{m}\right) \approx 0}$

and $\quad ds^2 = e^{-\mathrm{p}_m}dt^2 - \frac{\ell_0^2}{(4\ell_p)^4\pi^2}\frac{e^{-\mathrm{p}_m}}{m^2}da^2 + \frac{a}{4\pi}d\Omega^2$ with $M = \frac{2\sqrt{4\pi}\ell_p^4}{\ell_0^2\sqrt{a}}m^2e^{\mathrm{p}_m}$.

The quantization scheme

We now implement the discreteness suggested in LQG in the quantization scheme of the model

Schrödinger quantisation : $\left[-i\hbar\frac{\partial}{\partial a} + \frac{1}{2a}\left(m + \frac{\mathrm{p}_\phi^2}{m}\right)\right]|\psi\rangle = 0$

$$\Downarrow$$

Input : no infinitesimal shift in a allowed

$\boxed{\cancel{\lambda \mathrm{p}_a \rightarrow \sin(\lambda\mathrm{p}_a)} \quad \text{BUT} \quad \lambda\mathrm{p}_a \rightarrow e^{i\lambda\mathrm{p}_a}}$

Polymer quantisation : $\boxed{\underbrace{\exp(i\lambda\mathrm{p}_a)}_{\text{finite areatime translation}} |\psi\rangle - \overbrace{\exp\left(\frac{i}{2}\left[\int_a^{a+\lambda}\left[\frac{1}{a}\right]_q da\right]\left(m + \frac{\mathrm{p}_\phi^2}{m}\right)\right)}^{\text{finite areatime unitary evolution operator}} |\psi\rangle = 0,}$

one can obtain the full quantum dynamics from this equation

$$\psi(m, \mathrm{p}_\phi, a + \lambda\ell_p^2) = e^{\frac{i}{2}[\tau(a+\lambda\ell_p^2)-\tau(a)]\left(m+\frac{\mathrm{p}_\phi^2}{16\pi\ell_p^2 m}\right)}\psi(m, \mathrm{p}_\phi, a).$$

Degeneracy of the mass operator

It is possible to define a mass operator and to study its spectrum :

$$M = \frac{2\sqrt{4\pi}\ell_p^4}{\ell_0^2\sqrt{a}}m^2e^{\mathrm{p}_m} \xrightarrow{\hbar} \hat{M} = \frac{2\sqrt{4\pi}\ell_p^4}{\ell_0^2}\widehat{\frac{1}{\sqrt{a}}}\hat{m}\widehat{e^{\mathrm{p}_m}}\hat{m}$$

Remark : $[\hat{M}, \hat{H}] = 0$

Eigenvector : $|M, \epsilon\rangle = \sum_{a\in\Gamma_{\epsilon,\lambda}}\int \phi_M(\mathrm{p}_m, a)|\mathrm{p}_m\rangle|a\rangle d\mathrm{p}_m$ such that $\hat{M}|M,\epsilon\rangle = M|M,\epsilon\rangle$

Bessel function

$\Gamma_{\epsilon,\lambda} =$ 0 ϵ $\lambda\ell_p^2$ a with $\epsilon \in [0, \lambda)$

$\Longrightarrow$ The mass operator is infinitly degenerate due to this ϵ quantity representing a Planckian geometric d.o.f !

Dynamics of an Hawking pair on a discrete quantum geometry

We can decompose the total Hilbert space describing the geometric and the matter d.o.f as follows

$$\mathcal{H}_{total} = \underbrace{\mathcal{H}_m \otimes \mathcal{H}_\epsilon}_{\text{geometry}} \otimes \overbrace{\mathcal{H}_{in} \otimes \mathcal{H}_{out}}^{\text{matter}} .$$

Elements of this Hilbert space are noted $|m, \epsilon, \mathrm{p}_{in}, p_{out}\rangle$.
We consider the following initial state for which the two particles are maximally entangled

$$|\psi_0\rangle = \sum_{\substack{i=\pm\\ \varepsilon=\pm}}\int \psi_0(m)|m, \epsilon_\varepsilon, i, i\rangle dm .$$

The dynamics gives

$$|\psi\rangle = \sum_{\substack{i=\pm\\ \varepsilon=\pm}}\int e^{\frac{i}{2}[\tau(a_0+n\lambda\ell_p^2+\epsilon_\varepsilon)-\tau(a_0+\epsilon_\varepsilon)]\left(m+\frac{\mathrm{p}_i^2}{16\pi\ell_p^2 m}\right)}\psi_0(m)|m, \epsilon_\varepsilon, i, i\rangle dm .$$

Mutual information as a measure of correlations

One start with the relative entropy

$$S(\rho|\sigma) := \mathrm{Tr}(\rho\log\rho - \rho\log\sigma)$$

which quantifies the distinguishability of ρ from σ.

If one consider the case of a system that can be decomposed into three subsystems A, B and C, one can define the mutual information by

$$I_{A,B} := S(\rho_{AB}|\rho_A \otimes \rho_B).$$

It quantifies how much a mixed states is distinguishable from the uncorrelated state that we can have by separating the two subsystems.

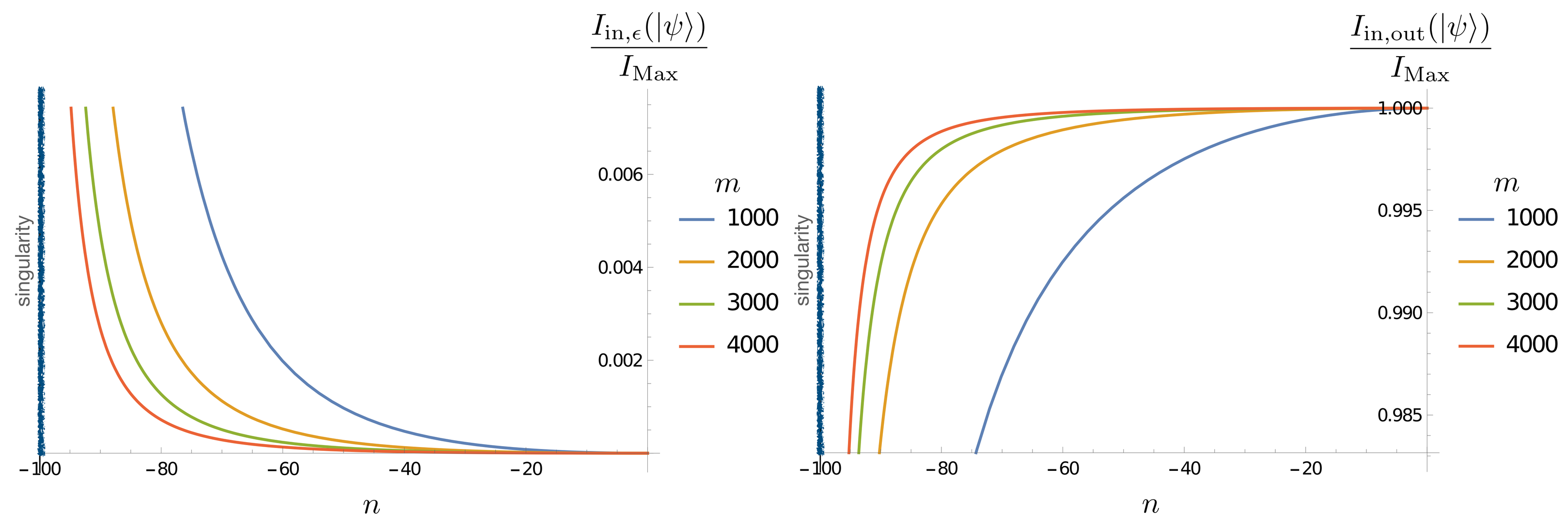
Moreover one has

$$\frac{\left(\psi\hat{O}_A\hat{O}_B\psi - \psi\hat{O}_A\psi\psi\hat{O}_B\psi\right)^2}{2||\hat{O}_A||||\hat{O}_B||} \leq I_{A,B}(\psi)$$

for all $\hat{O} = \hat{O}_A \otimes \hat{O}_B$ such that $\hat{O}_A$ and $\hat{O}_B$ are bounded.
The mutual information is an upper bound of the correlations between A and B.

Evolution of correlations between Hawking particle and the Planckian geometric d.o.f

We can determine the evolution of $I_{in,\epsilon}$ and $I_{in,out}$



The correlations between Hawking's pair disappear in favor of correlations between the particle falling inside and the Planckian geometric d.o.f.

Moreover, one can also show that

$$I_{in,\epsilon} = I_{out,\epsilon}$$

The particle escaping to infinity also becomes entangled with the Planckian geometric d.o.f.

To restore unitarity, the Planckian geometric d.o.f must be taken into account.

References

[1] A. Perez, S. Ribisi, and S. Viollet, "Modeling Quantum Particles Falling into a Black Hole: The Deep Interior Limit," Universe 9 (2023), no. 2, 75, arXiv:2301.03951.
[2] A. Perez and S. Viollet, "Discreteness Unravels the Black Hole Information Puzzle : Insights from a Quantum Gravity Toy Model" *To appear*