Quantum gravity and cosmological perturbations:

Effects of a modified Friedmann equation on gauge-invariant perturbations

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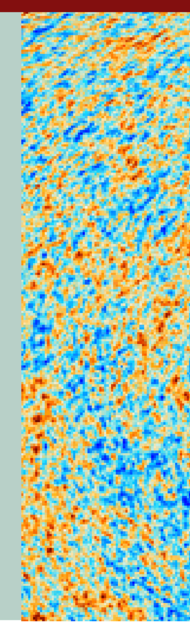
Cosmology: a testing ground for quantum gravity

- Explicit calculations in quantum gravity models are difficult
- Simpler in cosmological scenario due to **isotropy and homogeneity**
- First test for quantum gravitational models: Recover FLRW dynamics at late times

  $\longrightarrow$ *Can we also make statements about perturbations?*

Cosmological perturbation theory

- Include **linear inhomogeneous perturbations** in the
  - **metric**, e.g. $a(t) \to a(t)(1-\psi(t,x^i))$
  - and **matter** content $\rho(t) \to \rho(t) + \delta\rho(t,x^i)$



Modified Friedmann equation

- **General form** of a modified Friedmann equation that encodes the universe dynamics

$$H^2 = \frac{8\pi G}{3} N^2 \rho \mathcal{F}$$

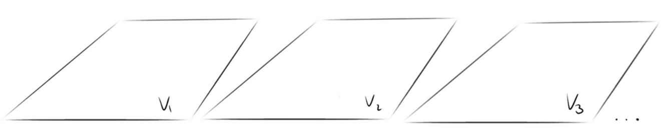
Hubble parameter $H = \frac{a'}{a}$; Lapse; Energy density; Generic function that encodes modification, GR: $\mathcal{F}=1$

- Unchanged matter dynamics

Example: A bouncing universe in Group Field Theory cosmology

- Field theory on a group manifold: **quanta of space**
- Evolution of the universe from the **expectation value of the volume operator over coherent state** $\langle \hat{V} \rangle = a^3$
- Spectrum of $\hat{V}$ bounded from below $\longrightarrow$ **bounce**
- Evolution with respect to matter clock

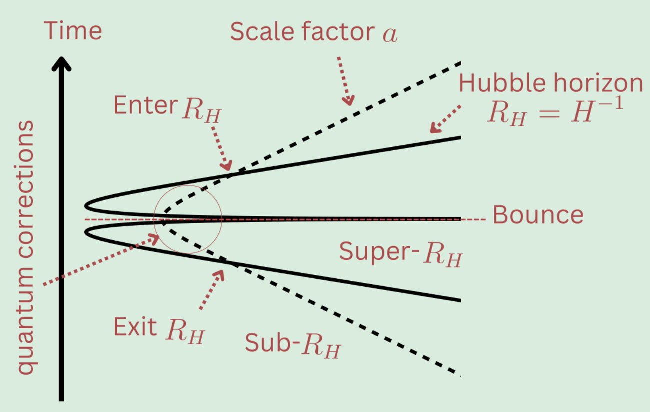
Separate Universe Picture



- Model the universe as an **ensemble of independent homogeneous patches**
- Each patch behaves like an FLRW universe
- Perturbations = deviations from the mean

$$\psi_p = \frac{1}{3}\left(1 - \frac{V_p}{V_{bg}}\right) \qquad V_{bg} := \frac{1}{N_{\text{patches}}}\sum_p V_p$$

Perturbation Equations



- Before and after the bounce modes are **super-horizon** and **approximately homogeneous**
- Work in the SU picture and **perturb the Friedmann equation** at linear order to obtain perturbation equations

$$H\psi' = -H^2\left(\tilde{\Phi} + \frac{\delta\rho}{2\rho} + \frac{\delta\mathcal{F}}{2\mathcal{F}}\right)$$

Relational Clock

- **Massless scalar field** ϕ follows Klein-Gordon equation

$$\phi'' - \frac{N'}{N}\phi' + 3H\phi' = 0$$

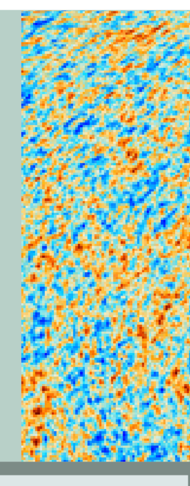
- Assign value of scalar field to each quanta of space

$\longleftarrow \phi$

$\longrightarrow$ **time coordinate**

Gauge invariant perturbation variables

- Invariance under **infinitesimal diffeomorphisms** acting on perturbed coordinate system $\longrightarrow$ relate to **physical quantities** (e.g. CMB)
- **Comoving curvature perturbation** $\mathcal{R} = \psi + \frac{H}{\phi'}\delta\phi$
  - Conserved on super-horizon scales in GR
- In the comoving gauge $\delta\phi = 0 \to \mathcal{R} = \psi$



Special Case $\mathcal{F} = \mathcal{F}(\rho)$

- Modification $\mathcal{F}$ is a function of the energy density only, such that $\delta\mathcal{F} \propto \delta\rho$

$\longrightarrow$ **Analogue to the diffeomorphism constraint** of GR:

$$\psi' + H\tilde{\Phi} = 0$$

$\Longrightarrow$ **Recover GR conservation law for $\mathcal{R}$**

- **Generalises** the LQC case $\mathcal{F}_{\text{LQC}} = 1 - \frac{\rho}{\rho_c}$

General case $\mathcal{F} \neq \mathcal{F}(\rho)$

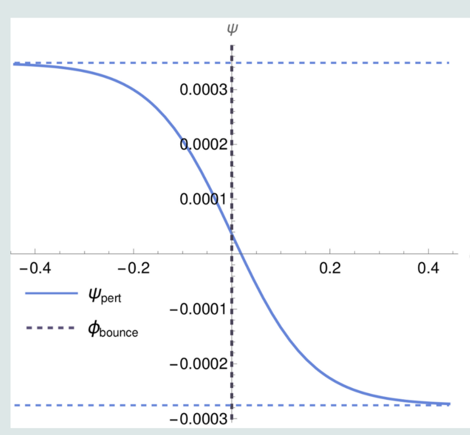
- For a massless scalar field $\mathcal{R}' = \psi' = -H\frac{\delta\mathcal{F}}{2\mathcal{F}}$
- Consider the **GFT example**

Fundamental constant; Initial condition

$$\mathcal{F}_{\text{GFT}} = 1 + \frac{v_0}{a^3} + \frac{\mathcal{Y}}{a^6} \quad \to \quad \delta\mathcal{F} = 3\frac{v_0}{a^3}\psi + 6\frac{\mathcal{Y}}{a^6}\psi + \frac{\delta\mathcal{Y}}{a^6}$$

$\Longrightarrow$ **Dynamics in $\mathcal{R}$ around bounce**

- Evolution from **perturbation equations agree with quantum evolution** up to second order effects



Conclusion

- Statements about **large scale perturbations from a modified Friedmann equation**
  - without specifying inhomogeneities in the quantum framework

Outlook

- Study **inhomogeneities in the quantum framework** (model dependent)
  - GFT: Add additional scalar fields to obtain a **relational framework**