Curvature from multipartite entanglement in quantum gravity states

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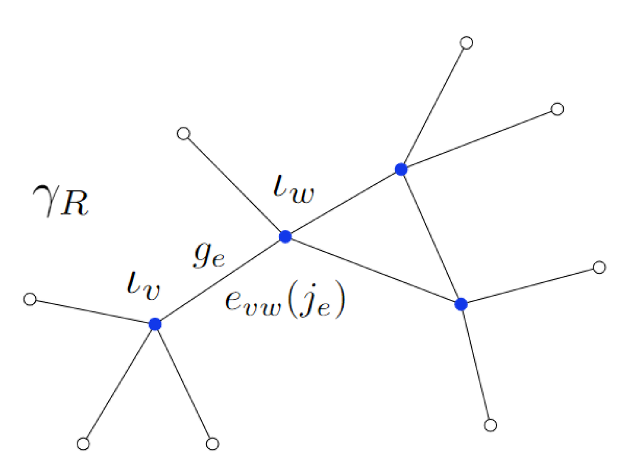
Based on works with: **Goffredo Chirco, Gianluca Cuffaro, Vittorio D'Esposito**

Abstract

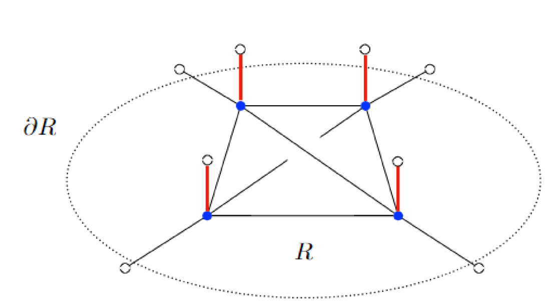
We investigate the multipartite entanglement of a uniformly curved subregion of a $3D$ quantum spacetime slice with boundary, realised in terms of spin networks defined on a graph with non trivial topology. The presence of intrinsic curvature in the region is encoded in topological defects associated with tag-spins attached to the vertices of the graph. We model the generalized boundary mapping on a tripartite random state and compute the logarithm negativity for the bipartite reduced boundary, focusing on the universal typical behavior of the large spin regime. We find three entanglement regimes, depending on the ratio between the number of tags (curvature) and the area of the dual surface at the boundary.

Setting

Imagine to cut a generic bounded region R out of quantum 3D space.



We consider topological defect in the bulk, encoded in additional virtual spin J, or *tag*, attached to the vertex.



We want to study the topology of R via measures of quantum correlations.

Extended boundary tripartition

Consider an observer having access only to the boundary system. Such an observer is precluded from measuring the information on the bulk, Hence he will typically describe the system via a mixed boundary state. Concretely, this amounts to a partial tracing over the bulk.

We shall define a tripartition of the extended boundary system into three subsystems A_1, A_2, B, with A_1, A_2 complementary regions of the external boundary, and B the set of tags in the bulk. This corresponds to the following factorization of the extended boundary Hilbert space:

$$H_\tau = \bigotimes_{e\in A_1} V^{j_e} \otimes \bigotimes_{e\in A_2} V^{j_e} \otimes \bigotimes_{i\in B} V^{J_i}.$$

Tracing out the tag degrees of freedom

$$\rho_{A_1A_2} = \mathrm{Tr}_B[\rho_\tau].$$

we come up with a mixed boundary states.

We study the role of the system B as *environment* and how it affects boundary correlations.

Topologial defects as candidates for curvature

1. Entanglement/area correspondence;
2. Curved areas are proportional to closure defects

   From a classical viewpoint they give us insight about curvature

$$\frac{\text{Area}(\partial B_\varepsilon(p)\subset \mathcal{M})}{\text{Area}(\partial B_\varepsilon(0)\subset \mathbb{R}^n)} = 1 - \frac{\mathcal{R}}{6n}\varepsilon^2 + O(\varepsilon^3).$$

3. Tags represent quantum topological defects.

We expect that tag induced correlations among boundary subsystems contain information about *bulk curvature*.

Recipe

Figure 1. A quantum gravity chef

- **Entanglement negativity**: good witness of quantum correlations for mixed states; entanglement monotone under general PPT preserving operations.

$$E_N(\rho_{A_1A_2}) \equiv \log\|\rho_{A_1A_2}^{T_{A_2}}\|_1.$$

- **Random measurement**: independent Haar random vertex; we compute the Rényi negativity in expected value

$$\mathbb{E}_\mu\left[N_k(\rho_{A_1A_2})\right] \equiv \overline{N_k(\rho_{A_1A_2})}.$$

- **Large spin regime**: log Rényi Negativity is well approximated by its *typical value*

$$N_k(\rho_{A_1A_2}) \simeq \overline{N_k(\rho_{A_1A_2})}.$$

Generalized Ising model with bulk contribution

The large spins regime allows us to approximate the Rènyi negativity as

$$\overline{N_k(\rho_{A_1A_2})} \simeq \frac{\overline{\mathrm{Tr}\left[\left(\rho_{A_1A_2}^{T_{A_2}}\right)^k\right]}}{\overline{(\mathrm{Tr}\left[(\rho_{A_1A_2})\right])^k}} \equiv \frac{\overline{Z_1^{(k)}}}{\overline{Z_0^{(k)}}} = \sum_{\{g_e\}} e^{-A_{1/0}^{(k)}},$$

where $A_{1/0}^{(k)}$ is the action of a classical *generalized* Ising-like model, with spins replaced by elements of permutation group S_k.

Cyclic, anti cyclic and identity permutations play the role of boundary conditions on the boundary subsystems and tags respectively.

The calculation of typical value of Rényi negativity is then mapped to the minimization of a classical Ising action.

Rényi negativity should depend directly on the *area* of such minimal domain walls in the network: thanks to the existence of a well defined loop quantum gravity area operator acting on the edges of the spin network, we can map domain walls areas to actual quantum geometry surfaces in our $3D$ space region R.

Results

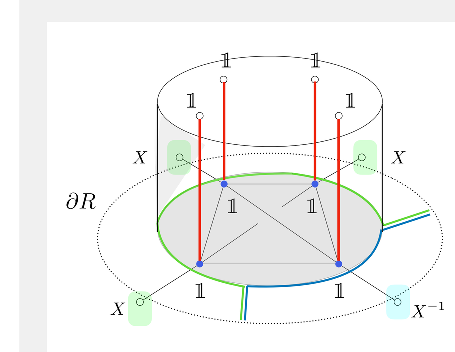
Within the typical regime, the entanglement phases of the random boundary state can be described solely in terms of the relative dimensions of the three subsystems via the parameter

$$q = \beta_t T/\beta E_{\partial R} = \frac{\log[\dim(B)]}{\log[\dim(A_1)\dim(A_2)]},$$

which expresses the ratio of bulk curvature over boundary surface.

Hole regime

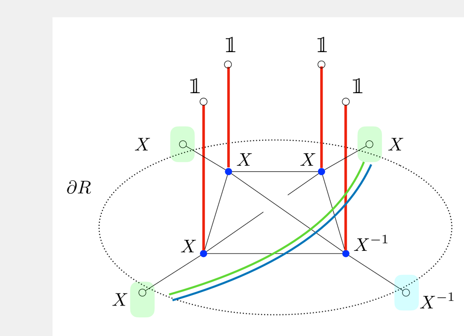
$q > 1$



$$E_N^{(\text{hole})}(\rho_{A_1A_2}) \simeq 0$$

Bipartite regime

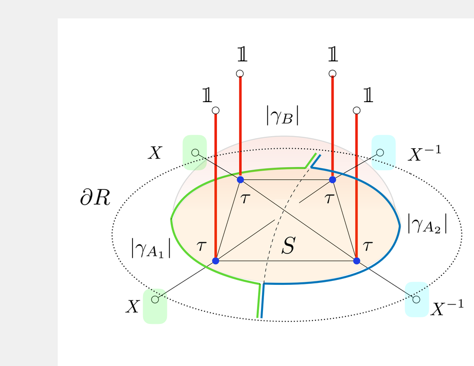
$q < 1 - \frac{2S}{E_{\partial R}}$



$$E_N^{(\text{bipartite})}(\rho_{A_1A_2}) \simeq \beta S$$

Island regime

$1 - \frac{2S}{E_{\partial R}} < q < 1$



$$E_N^{(\text{island})}(\rho_{A_1A_2}) \propto \frac{1}{2}\beta E_{\partial R}(1-q)$$

Conclusions and outlook

The large spin regime is necessarily associated with a *semiclassical limit* for our quantum geometry states: we can tentatively interpret the formulas found for the entanglement negativity in terms of areas of a bounded $3D$ region of a Riemannian manifold and look for a direct relation with its Ricci curvature.

The area of the boundary of a flat three ball region (no tags) is

$$\text{Area}(\partial B_\varepsilon(0)) \simeq \beta E_{\partial R}(\varepsilon)$$

If we assume

$$\text{Area}(\partial B_\varepsilon(p)) \simeq \beta E_{\partial R}(\varepsilon) + \beta_t T(\varepsilon)$$

in the Island regime we obtain

$$\frac{\text{Area}(\partial B_\varepsilon(p))}{\text{Area}(\partial B_\varepsilon(0))} = 1 - q$$

Such relation hints toward a possible characterisation of the curvature in purely information theoretic terms.

$$q(\varepsilon) \simeq -\frac{\mathcal{R}}{18}\varepsilon^2$$

References

[1] Simone Cepollaro, Goffredo Chirco, Gianluca Cuffaro, and Vittorio D'Esposito. *Phys. Rev. D*, 107:086003, Apr 2023.

[2] Simone Cepollaro, Goffredo Chirco, Gianluca Cuffaro, and Vittorio D'Esposito. *arXiv:2305.02670*, 5 2023.

[3] Goffredo Chirco, Eugenia Colafranceschi, and Daniele Oriti. *Phys. Rev. D*, 105(4):046018, 2022.

[4] Xi Dong, Xiao-Liang Qi, and Michael Walter. *JHEP*, 06:024, 2021.

[5] Patrick Hayden, Sepehr Nezami, Xiao-Liang Qi, Nathaniel Thomas, Michael Walter, and Zhao Yang. *JHEP*, 11:009, 2016.

[6] Etera R. Livine. *Class. Quant. Grav.*, 36(18):185009, 2019.

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