Parameterizations of black-hole spacetimes beyond circularity

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Why black holes beyond GR?

- Test of validity of GR from near-horizon observations
[Broderick '21; E.H.T. Collaboration '19, '22]
- Learn about true nature of black-holes
→ Glimpse on quantum gravitational effects?
→ How black-holes beyond GR look like?
[Held '19]
- Usual parameterized deviations from Kerr metric assume symmetries which might not hold beyond GR:
  - Circularity
  [Lewis '32; Papapetrou '66; Konoplya, Rezzolla, Zhidenko '16]
  - Hidden constant of motion
  [Benenti, Francaviglia '79; Vigeland, Yunes, Stein '11; Johannsen '13]

What are circularity & hidden constant of motion?

- Consider axisymmetric, stationary & asymptotically flat spacetimes

  → Two commuting Killing vectors ξ_1, ξ_2

- Circularity holds if [Papapetrou '66]

$$\xi_1^\mu R_\mu^{[\nu} \xi_2^\kappa \xi_1^{\lambda]} = 0 \text{ everywhere,}$$
$$\xi_2^\mu R_\mu^{[\nu} \xi_1^\kappa \xi_2^{\lambda]} = 0 \text{ everywhere.}$$

  → Implies an isometry. In BL coordinates, $t \to -t$ & $\phi_{\mathrm{BL}} \to -\phi_{\mathrm{BL}}$

- In GR circularity holds because $R_{\mu\nu} = 0$. But some non-vacuum GR & beyond GR spacetimes break circularity
[Ioka, Sasaki '03, '04; Birkl, Stergioulas, Muller '11]
[Anson, Babichev, Charmousis, Hassaine '20; Minamitsuji '20]
- Killing vectors K_μ → isometries
Rank-2 Killing tensors $K_{\mu\nu}$ → hidden constants of motion $C = K_{\mu\nu} u^\mu u^\nu$

  → Leads to separability of geodesic equations

Which choice of coordinates?

Boyer-Lindquist	Horizon-penetrating
$(t, r, \theta, \phi_{\mathrm{BL}})$	$(u = t - r^*, r, \chi, \phi)$
Requires fine tuning at horizon [Johannsen '13]	Singularity-free at horizon
Circular spacetime metrics	Non-circular spacetime metrics

Open question

- Minimal, general parameterization of axisymmetric, stationary & asymptotically flat BH spacetimes?

Parameterizations and symmetries

1st key result: Usual parameterizations assume some symmetries

	Symmetries	Metric components	Free functions
BHs:	axisymmetry + stationarity	10 → 6	at least 6
add:	circularity	5	4 (LP form)
add:	hidden constant of motion	5	10 of 1 coord.

- Most general circular metric: [Johannsen '13]

$$ds^2_{RZ} = -g_{tt}dt^2 - 2g_{t\phi_{\mathrm{BL}}}dt d\phi_{\mathrm{BL}} + g_{\phi_{\mathrm{BL}}\phi_{\mathrm{BL}}}d\phi^2_{\mathrm{BL}} + g_{rr}dr^2 + g_{\theta\theta}d\theta^2$$

  → 5 non-zero metric components & 5 free functions ✓

  → 1 free function can be removed by coordinate transformation

  ⟹ 5 metric components & 4 free functions in some coordinates

- Most general circular metric with new Carter-like hidden constant of motion:
[Benenti, Francaviglia '79]

$$g^{\mu\nu}\partial_\mu\partial_\nu = \frac{1}{S_{x_1} + S_{x_2}}\left[\left(G^{ij}_{x_1} + G^{ij}_{x_2}\right)\partial x_i \partial x_j + \Delta_{x_1}\partial x_1^2 + \Delta_{x_2}\partial x_2^2\right]$$

with i, j indices related to Killing coordinates, and $1, 2$ to explicit coordinates

  → Forms a subclass of circular metrics

2nd key result: Non-circular parameterization of deviations from Kerr (HP coordinates)

$$\begin{aligned} ds^2_{\mathrm{NC}} = & -\left(\frac{r^2 - 2Mr + a^2\chi^2}{r^2 + a^2\chi^2}\right)(1 + \Delta_1(r,\chi))\,du^2 + 2\,(1 + \Delta_2(r,\chi))\,du dr \\ & -4\frac{Mar}{r^2 + a^2\chi^2}(1-\chi^2)(1 + \Delta_3(r,\chi))du d\phi - 2a(1-\chi^2)(1 + \Delta_4(r,\chi))dr d\phi \\ & + \frac{r^2 + a^2\chi^2}{1-\chi^2}(1 + \Delta_5(r,\chi))d\chi^2 \\ & + \frac{1-\chi^2}{r^2 + a^2\chi^2}\left(\left(a^2 + r^2\right)^2 - a^2\left(r^2 - 2Mr + a^2\right)(1-\chi^2)\right)(1 + \Delta_6(r,\chi))d\phi^2 \end{aligned}$$

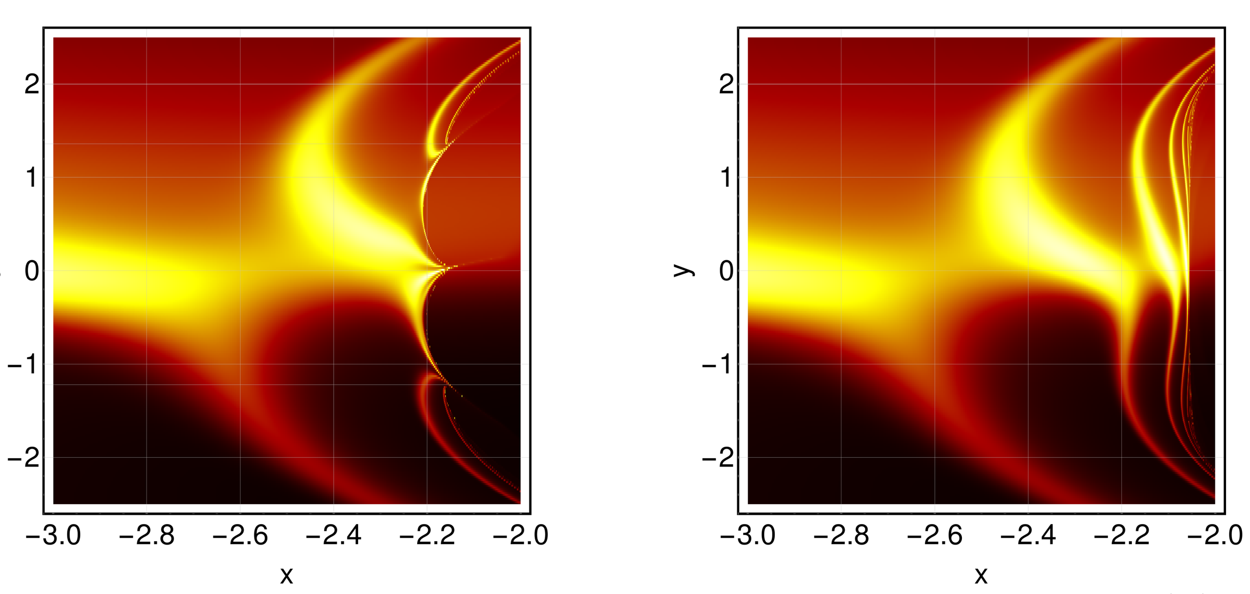
- Asymptotic flatness: $\Delta_{\mathrm{HP,i}}(r,\chi) \xrightarrow{r\to\infty} 0$ ✓
- Correct Newtonian limit: $\Delta_{\mathrm{HP,i}}(r,\chi) \sim \mathcal{O}\left(\frac{1}{r^2}\right)$ ✓
- Flat Minkowski limit: only reached for $M \to 0$ if $\Delta_{\mathrm{HP,i}}(r,\chi) \sim M$ & no extra-hair
- Limit $a \to 0$ breaks spherical symmetry due to χ-dependence of $\Delta_{\mathrm{HP},1/2}$ and $\Delta_{\mathrm{HP},1/2/5/6}$ encoding additional (quantum) hair
- Can reduce to circular spacetimes if e.g. $\Delta_{\mathrm{HP,i}} = \varepsilon$, $\forall i$ or for $\Delta_{\mathrm{HP},5} \neq 0$, $\Delta_{\mathrm{HP,i}\neq 5} = 0$ (non-trivial) ✓

Challenge: directly relating non-circular HP parameterization to general circular BL parameterization

  → Possible via a counting argument: 14 free functions − 9 constraints = 5 free functions ✓

Example included in non-circular HP parameterization

Locality-based regular, non-circular black-hole with $M \to M(r,\chi)$



Non-circular | Circular $M \to M(r)$

3rd key result:
Specific image features:

- Cusps (in shadow boundary and photon rings)
- A dent (in $y = 0$ image axis)
- A broken reflection symmetry (about $y = 0$ image axis)

4th key result:
No equivalent for $a \to a(r,\chi)$

- Can we establish a 1-to-1 map between non-circularity and specific image features?