Properties of a quantum field and Schwarzschild spacetime can be reconciled from an assumption that a particle has oscillation in time

Hou Y. Yau
San Francisco State University, Department of Physics and Astronomy, USA
hyau@mail.sfsu.edu

Abstract

A particle carries an internal clock, as conjectured by de Broglie. What if this clock runs at a varying rate and not along a 'smooth' timelike geodesic? Here, we investigate this possibility. If there are merits to this assumption, we will obtain properties that are derived from the standard theories. Interestingly, we can reconcile the following results by assuming a particle has varying internal time rate - oscillation in proper time:

- Schwarzschild spacetime field.
- Properties of a spin-zero bosonic field.
- Self-adjoint internal time operator.
- Proper time uncertainty relation.

These properties reconciled can reduce the differences between quantum theory and general relativity, allowing a more symmetrical treatment of space and time in a matter field. This assumption can also be checked experimentally by measuring the uncertainty of a neutrino's arrival time. The information presented in this poster summarizes the results from refs. [1, 2, 3, 4, 5, 6].

Proper Time Oscillator

Assumption: A particle has oscillation in proper time, i.e.

$$\mathring{t}_f = t + \mathring{t}_d = t - \frac{\sin(\omega_0 t)}{\omega_0}. \quad (1)$$

- Coordinate time t measured by an inertial observer at spatial infinity; ω_0 =de Broglie mass-energy angular frequency.
- Internal time $\mathring{t}_f$ is the assumed time of a particle's internal clock that runs at an oscillating rate relative to t with a fixed amplitude $1/\omega_0$.
- A particle will appear to travel along a 'smooth' timelike geodesic if the measuring instrument is not sensitive enough to detect the oscillation.

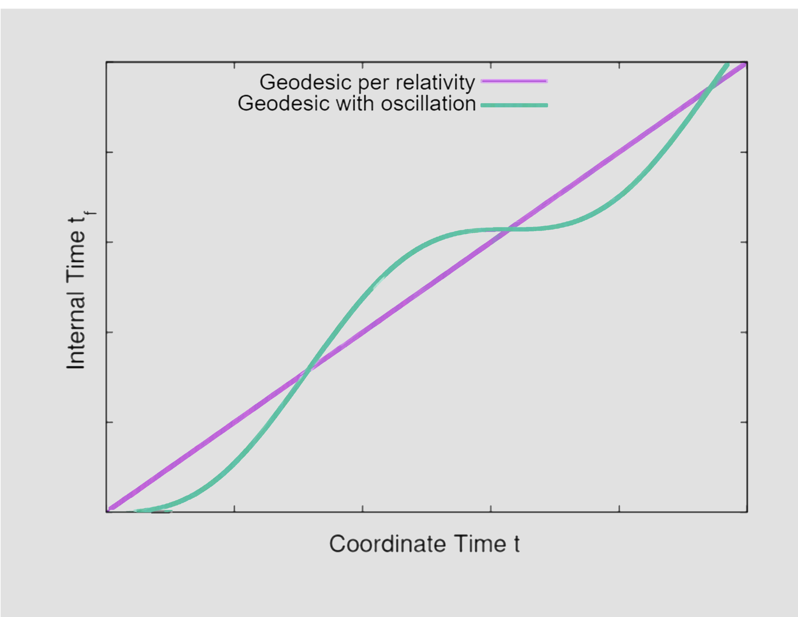

Figure 1: A particle's internal time with and without proper time oscillation.

Bosonic Field [1, 3]

Particles inside a plane wave can also have the assumed oscillation in proper time. Under a Lorentz transformation, matter in the plane wave acquires oscillations in both space and time

$$t'_f = t' + t'_d = t' + \mathrm{Re}(\zeta_{t\mathbf{k}}) = t' + T_{\mathbf{k}} \sin(\mathbf{k}\cdot\mathbf{x}' - \omega t'), \quad (2)$$

$$\mathbf{x}'_f = \mathbf{x}' + \mathbf{x}'_d = \mathbf{x}' + \mathrm{Re}(\zeta_{\mathbf{x}\mathbf{k}}) = \mathbf{x}' + \mathbf{X}_{\mathbf{k}} \sin(\mathbf{k}\cdot\mathbf{x}' - \omega t'), \quad (3)$$

We can further define a plane wave,

$$\zeta_{\mathbf{k}} = \frac{T_{0\mathbf{k}}}{\omega_0} e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}, \quad (4)$$

such that the temporal and spatial oscillation displacements are $\zeta_{t\mathbf{k}} = \partial_0 \zeta_{\mathbf{k}}$ and $\zeta_{\mathbf{x}\mathbf{k}} = -\nabla\zeta_{\mathbf{k}}$. Note that $\zeta_{\mathbf{k}}$ satisfies the Klein Gordon equation:

$$\partial_u \partial^u \zeta_{\mathbf{k}} + \omega_0^2 \zeta_{\mathbf{k}} = 0. \quad (5)$$

A real scalar field can be formed by superposition of $\zeta_{\mathbf{k}}$ and $\zeta^*_{\mathbf{k}}$ that has oscillations of metter in space and time

$$\zeta(x) = \sum_{\mathbf{k}} (2\omega\omega_0)^{-1/2} [T_{0\mathbf{k}} e^{-ikx} + T^*_{0\mathbf{k}} e^{ikx}]. \quad (6)$$

Since the number of particles is discrete, and each has a fixed proper time amplitude, $\zeta(x)$ must be quantized and can be transformed into a quantized field via canonical quantization. $\zeta(x)$ can be related to the bosonic field $\varphi(x)$ in quantum theory

$$\varphi(x) = \zeta(x)\sqrt{\frac{\omega_0^3}{V}} = \sum_{\mathbf{k}} (2\omega V)^{-1/2} [a_{\mathbf{k}} e^{-ikx} + a^\dagger_{\mathbf{k}} e^{ikx}], \quad (7)$$

with annihilation and creation operators

$$a_{\mathbf{k}} = \omega_0 T_{0\mathbf{k}}, \quad a^\dagger_{\mathbf{k}} = \omega_0 T^\dagger_{0\mathbf{k}} \quad (8)$$

Matter field with oscillations in time has the same properties as a bosonic field [1, 3, 5, 6].

Self-Adjoint Internal Time Operator [5]

Conjugate momenta of $\zeta(x)$ is,

$$\eta(x) = \frac{\partial \mathcal{L}}{\partial[\partial_0 \zeta(x)]} = \frac{-i\omega_0^3}{\sqrt{2}V} \sum_{\mathbf{k}} [\tilde{T}_{\mathbf{k}} e^{-ikx} - \tilde{T}^\dagger_{\mathbf{k}} e^{ikx}]. \quad (9)$$

The displaced time is linearly related to $\eta(x)$

$$t_d(x) = \zeta_t(x) = \partial_0 \zeta(x) = \sum_{\mathbf{k}} \frac{-i}{\sqrt{2}} [\tilde{T}_{\mathbf{k}} e^{-ikx} - \tilde{T}^\dagger_{\mathbf{k}} e^{ikx}] = \frac{\eta(x) V}{\omega_0^3}. \quad (10)$$

$t_d(x)$ and $\zeta(x)$ form a conjugate pair and satisfy commutation relations

$$(\omega_0^3 V^{-1})[\zeta(t,\mathbf{x}), t_d(t,\mathbf{x}')] = i\delta(\mathbf{x} - \mathbf{x}'), \quad (11)$$

$$[t_d(t,\mathbf{x}), t_d(t,\mathbf{x}')] = 0. \quad (12)$$

$\zeta(x)$, $\eta(x)$ and $t_d(x)$ are self-adjoint operators. Meanwhile, internal time in a matter field is

$$t_f(t,\mathbf{x}) = t + t_d(t,\mathbf{x})m \quad (13)$$

where t is a parameter, but $t_d(t,\mathbf{x})$ is a self-adjoint operator. Thus, internal time t_f is also a self-adjoint operator with no conflict with Pauli's theorem since t_f, and the Hamiltonian are not a conjugate pair. [5]

Proper Time Uncertainty Relation [6]

Consider a real scalar field that has particles oscillating in proper time only

$$\zeta' = \frac{1}{\sqrt{2}}[\zeta_0 + \zeta_0^\dagger] = \frac{1}{\sqrt{2}\omega_0}[T_0 e^{-i\omega_0 t} + T_0^\dagger e^{i\omega_0 t}]. \quad (14)$$

Displaced time t'_d and displaced time rate u'_d are,

$$t'_d = \frac{-i}{\sqrt{2}}[T_0 e^{-i\omega_0 t} - T_0^\dagger e^{i\omega_0 t}] = \frac{-i}{\sqrt{2\omega_0}}[a e^{-i\omega_0 t} - a^\dagger e^{i\omega_0 t}], \quad (15)$$

$$u'_d = \partial_0 t'_d = \frac{-\omega_0}{\sqrt{2}}[T_0 e^{-i\omega_0 t} + T_0^\dagger e^{i\omega_0 t}] = \frac{-1}{\sqrt{2}}[a e^{-i\omega_0 t} + a^\dagger e^{i\omega_0 t}]. \quad (16)$$

The Hamiltonian density is

$$H' = \frac{1}{2}(m\omega_0^2 {t'_d}^2 + {P'_d}^2/m) = \omega_0(a^\dagger a + \frac{1}{2}), \quad (17)$$

where $P'_d = m u'_d$.

	Proper Time Oscillator	Quantum Harmonic Oscillator
Hamiltonian	$H' = \omega_0(a^\dagger a + \frac{1}{2})$	$H = \omega(a^\dagger a + \frac{1}{2})$
Commutation Relation	$[t'_d, P'_d] = i$	[x,p]=i
Uncertainty Relation	$\Delta t'_d \Delta P'_d \geq \frac{1}{2}$	$\Delta x \Delta p \geq \frac{1}{2}$

Table 1: Comparing the properties of the proper time oscillator and the quantum harmonic oscillator. Displaced time t'_d and the 'temporal momentum' P'_d are analogies of the spatial position and momentum operators. [6]

Neutrino's Arrival Time Uncertainty

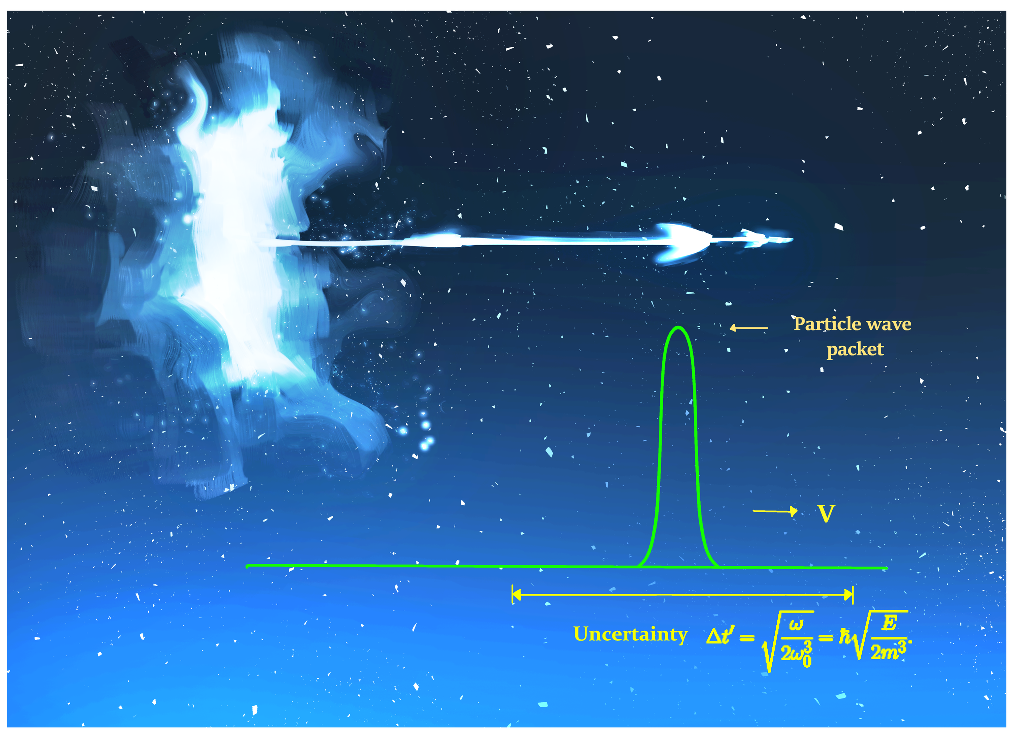

Figure 2: A particle's uncertainty in arrival time

Consider a normalized plane wave

$$\tilde{\zeta} = \frac{e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}}{\sqrt{\omega\omega_0^3}}. \quad (18)$$

Observed particles in this plane wave travel at an average velocity of $\mathbf{v} = \mathbf{k}/\omega$. As the particle propagates, it oscillates in time and space. The spatial oscillation of the particle will result in uncertainty of arrival time when we measure a large collection of particles with the same average velocity, i.e.

$$\Delta t' = \sqrt{\frac{\omega}{2\omega_0^3}} = \hbar\sqrt{\frac{E}{2m^3}}. \quad (19)$$

At a higher energy level, the effects of the particle's oscillations will be easier to detect. With the arrival time uncertainty obtained from experiments, the mass of a neutrino can be derived,

$$m = [\frac{\hbar^2 E}{2(\Delta t')^2}]^{1/3}. \quad (20)$$

The experiments on neutrinos' speed could provide evidence for a particle's temporal oscillation.

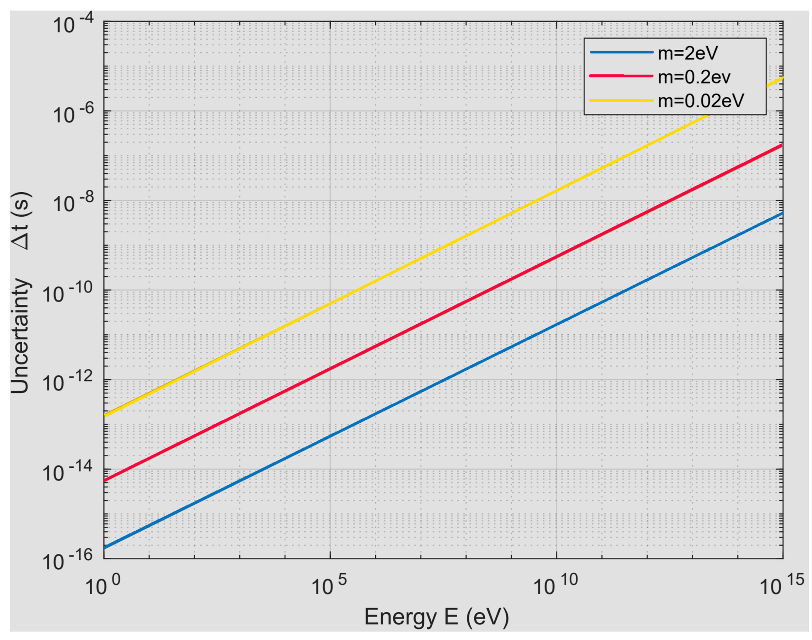

Figure 3: Neutrino's arrival time uncertainty as related to the particle's energy given by Eq. (19). Since the mass of a neutrino is not yet known, three different assumed masses are used in the plot.

Schwarzschild Field [2, 4]

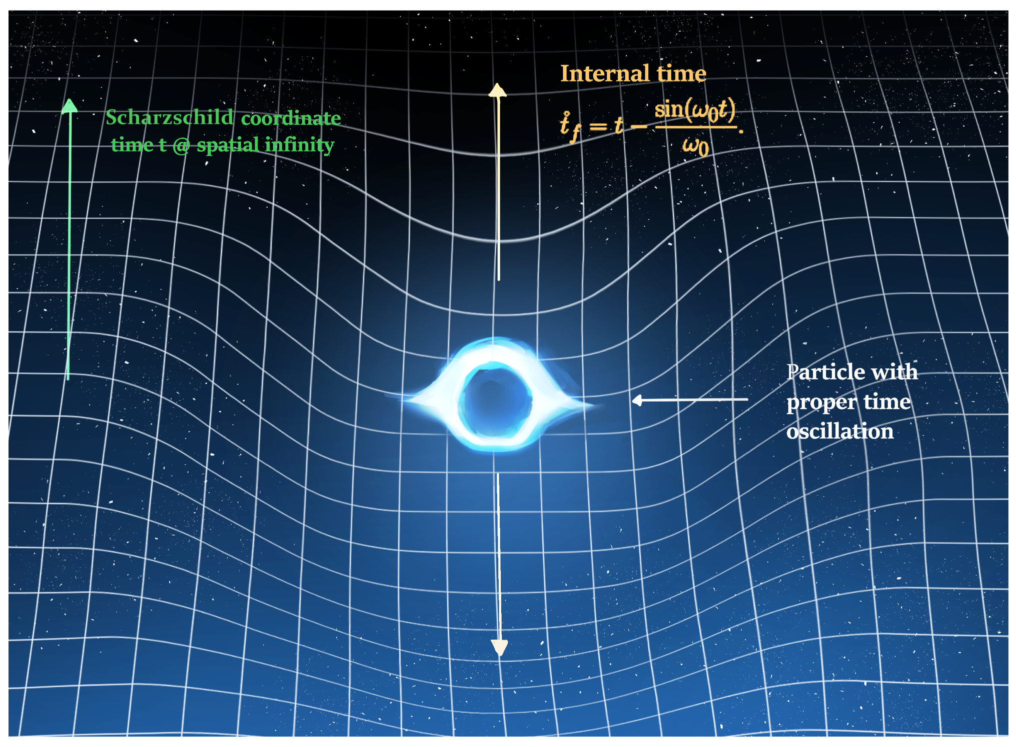

Figure 4: Schwarzschild field of the proper time oscillator

Neglect all the quantum effects, the proper time oscillator can be treated as a 'stationary' classical object at the spatial origin of a coordinate system. The proper time oscillation is a pulse that can be Fourier decomposed using the 0-component of the Lorentz covariant plane waves, i.e.,

$$\begin{bmatrix} \bar{\xi}_{t\mathbf{k}} \\ \bar{\xi}_{\mathbf{x}\mathbf{k}} \end{bmatrix} = -i \begin{bmatrix} \bar{T}_{\mathbf{k}} \\ \bar{\mathbf{X}}_{\mathbf{k}} \end{bmatrix} e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}. \quad (21)$$

However, the spatial components of the plane waves cannot be neglected. Superpose $\bar{\xi}_{t\mathbf{k}}$ to obtain the proper time oscillation will have spatial oscillations associated with the superposition of $\bar{\xi}_{\mathbf{x}\mathbf{k}}$. After carrying out the decomposition, we find that radial oscillations are oscillating about a thin shell Σ_0 with infinitesimal radius ($r = \epsilon/2 \to 0$) centered at the proper time oscillator,

$$\bar{r}_f(t, \epsilon/2) = \epsilon/2 + \Re_\infty \cos(\omega_0 t). \quad (22)$$

The amplitude of the radial oscillation ($\Re_\infty \to \infty$) is not the motions of matter. They shall be considered spacetime geometrical effects acting on an observer stationary on Σ_0.

In refs. [2, 4], we demonstrate that the vacuum space-time v^+ outside a time-like hypersurface with radius $\breve{r}$ and radial amplitude $\breve{\Re}$ is the Schwarzschild spacetime,

$$ds^2 = [1 - \frac{\breve{r}\breve{\Re}^2\omega_0^2}{r}]dt^2 - [1 - \frac{\breve{r}\breve{\Re}^2\omega_0^2}{r}]^{-1}dr^2 - r^2 d\Omega^2. \quad (23)$$

By Birkoff's theorem, the thin shell can be contracted to an infinitesimal radius, the same infinitesimal thin shell Σ_0 around the proper time oscillator. The spacetime outside the proper time oscillator is Schwarzschild [4].

Conclusions

Our analyses support the assumption that a particle has oscillation in proper time (varying internal time rate).

References

[1] Hou Y. Yau. *Temporal vibrations in a quantized field. In: Khrennikov, A., Toni, B. (eds.) Quantum Foundations, Probability and Information.* Springer-Verlag, Heidelberg, 2018.

[2] Hou Y. Yau. *Thin shell with fictitious oscillations", in Spacetime Physics1907 – 2017, Chapter 6.* Minkowski Institute Press, 2019.

[3] Hou Y. Yau. Time and space symmetry in a quantum field. *J. Phys.: Conf. Ser.*, 1194:012116, 2019.

[4] Hou Y. Yau. Schwarzschild field of a proper time oscillator. *Symmetry*, 12(2):312, 2020.

[5] Hou Y. Yau. Self-adjoint time operator in a quantum field. *J. Quant. Info.*, page 1941016, 2020.

[6] Hou Y. Yau. Proper time operator and its uncertainty relation. *J. Phys, Commun.*, page 105001, 2021.