Quantum Spacetime *and* Random Geometry

FROM ASYMPTOTIC SAFETY TO JT GRAVITY

QUANTUM GRAVITY

In the search for a theory of ***Quantum Gravity*** (QG), a straightforward question is What are the geometrical properties of quantum spacetime? The QG Path Integral

$$\mathcal{Z}_{QG} = \sum_{topology} \int_{\frac{Riem}{Diff}} \mathcal{D}g \; e^{\frac{i}{\hbar} S_{grav.}}$$

suggest that we need to sum over configurations far away for the geometries we know from Classical Gravity, namely ***Random Geometries***.

Due to quantum fluctuations, these geometries have fractal properties. In particular, the dimension of spacetime is a classical property for which a quantum counterpart is not yet well understood.

RANDOM GEOMETRY

A random geometry is a geometry sampled from an ensemble with a well-defined probability.

e.g. a map of n vertices decorated by a statistical system

$$Z_n^* = \sum_{\mathfrak{m} \in \mathcal{M}_d} \sum_{\text{decorations of } \mathfrak{m}} 1$$

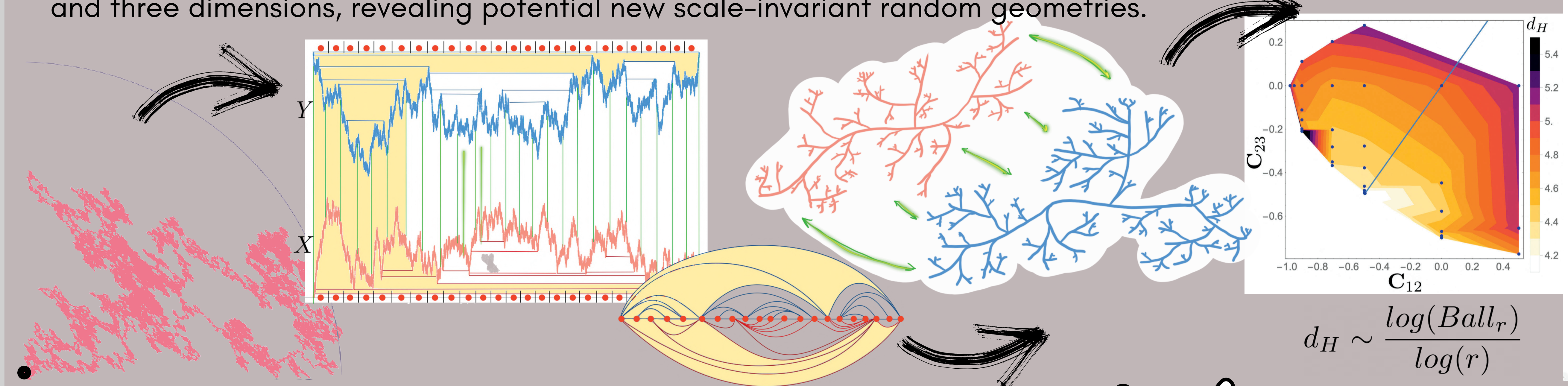
e.g. a hyperbolic surface with any number of decorated boundaries

$$Z_g^{WP}[\mu] = \sum_{k=0}^{\infty} \frac{2^{2-2g-k}}{k!} \prod_i^k \int_0^\infty d\alpha_i \mu(\alpha_i) V_{g,k}^{WP}(\alpha_1 \ldots, \alpha_k)$$

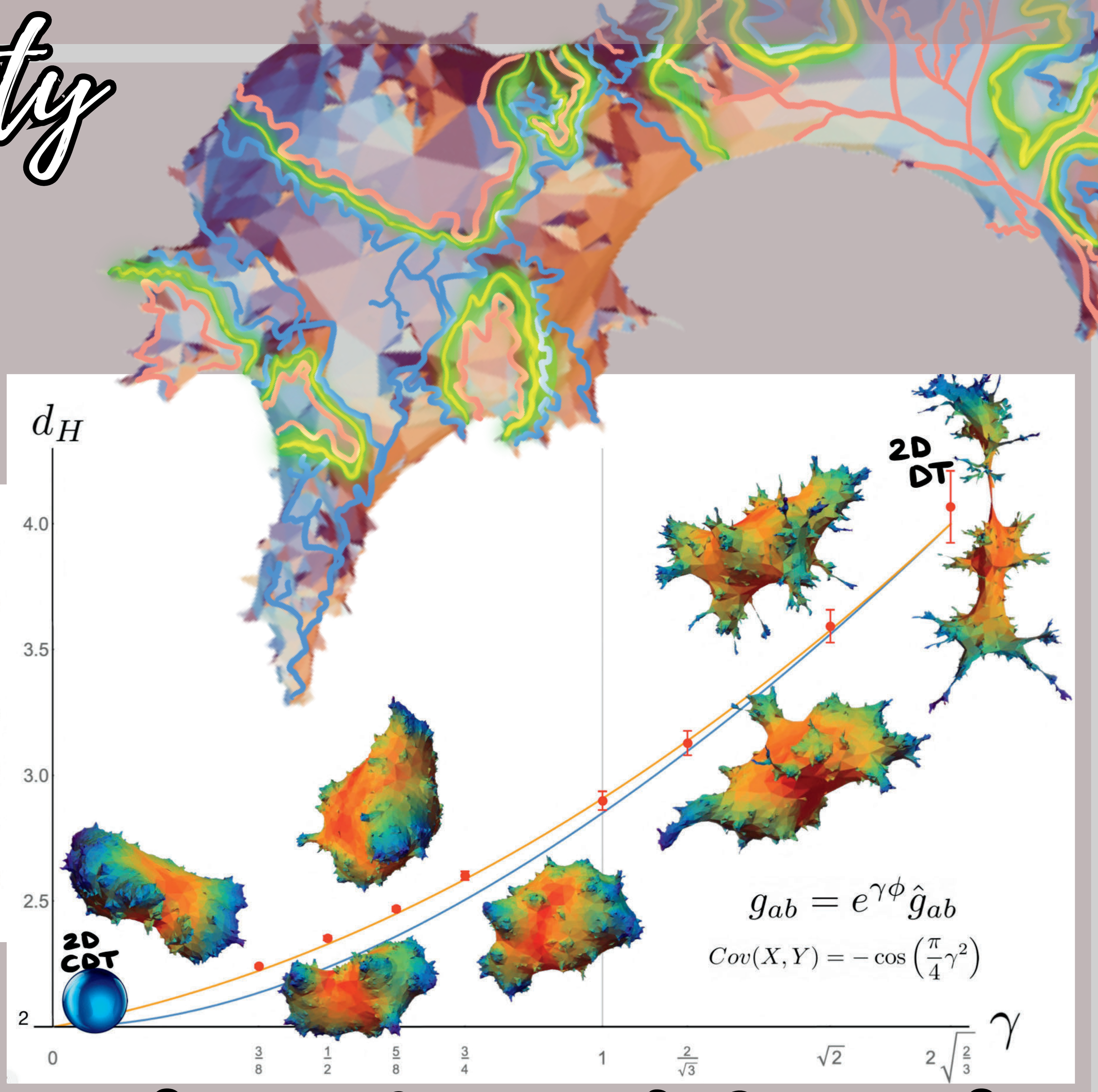
There are a lot of formal results in 2D but very few for higher dimensions.

scale-invariant random geometries for Asymptotic Safety

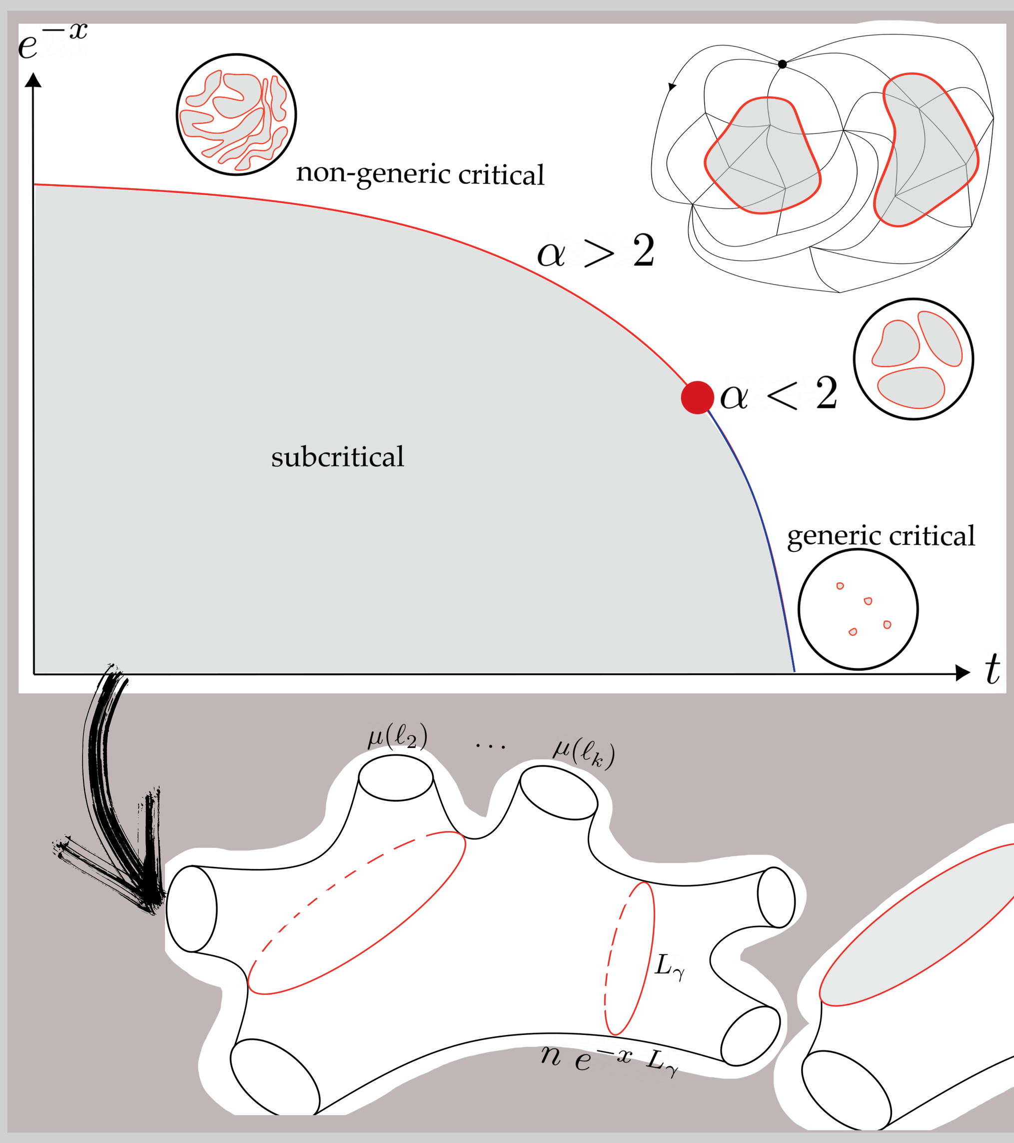
- In **Asymptotic Safety** one looks for an interacting fixed point of QG, which requires scale invariance.
- We use the mathematical framework of **Mating of Trees,** which relates Brownian motion and **Liouville Quantum Gravity** to explore new universality classes of scale-invariant random geometries.
- We run numerical simulations and, through distance measurements, we estimate Hausdorff dimensions in two and three dimensions, revealing potential new scale-invariant random geometries.



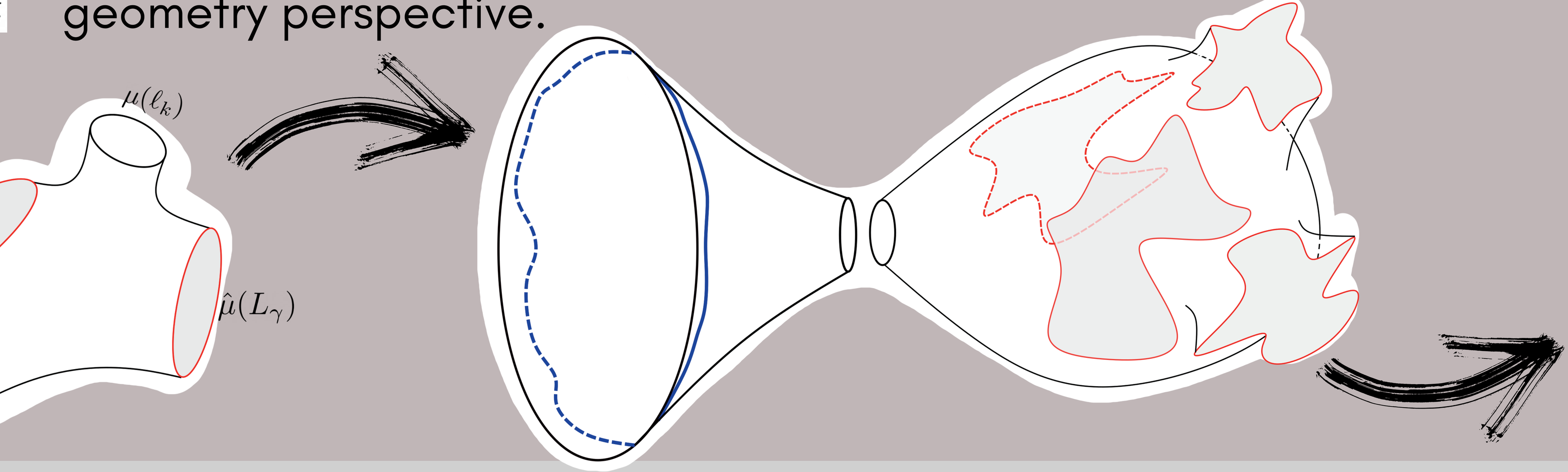
$$d_H \sim \frac{log(Ball_r)}{log(r)}$$

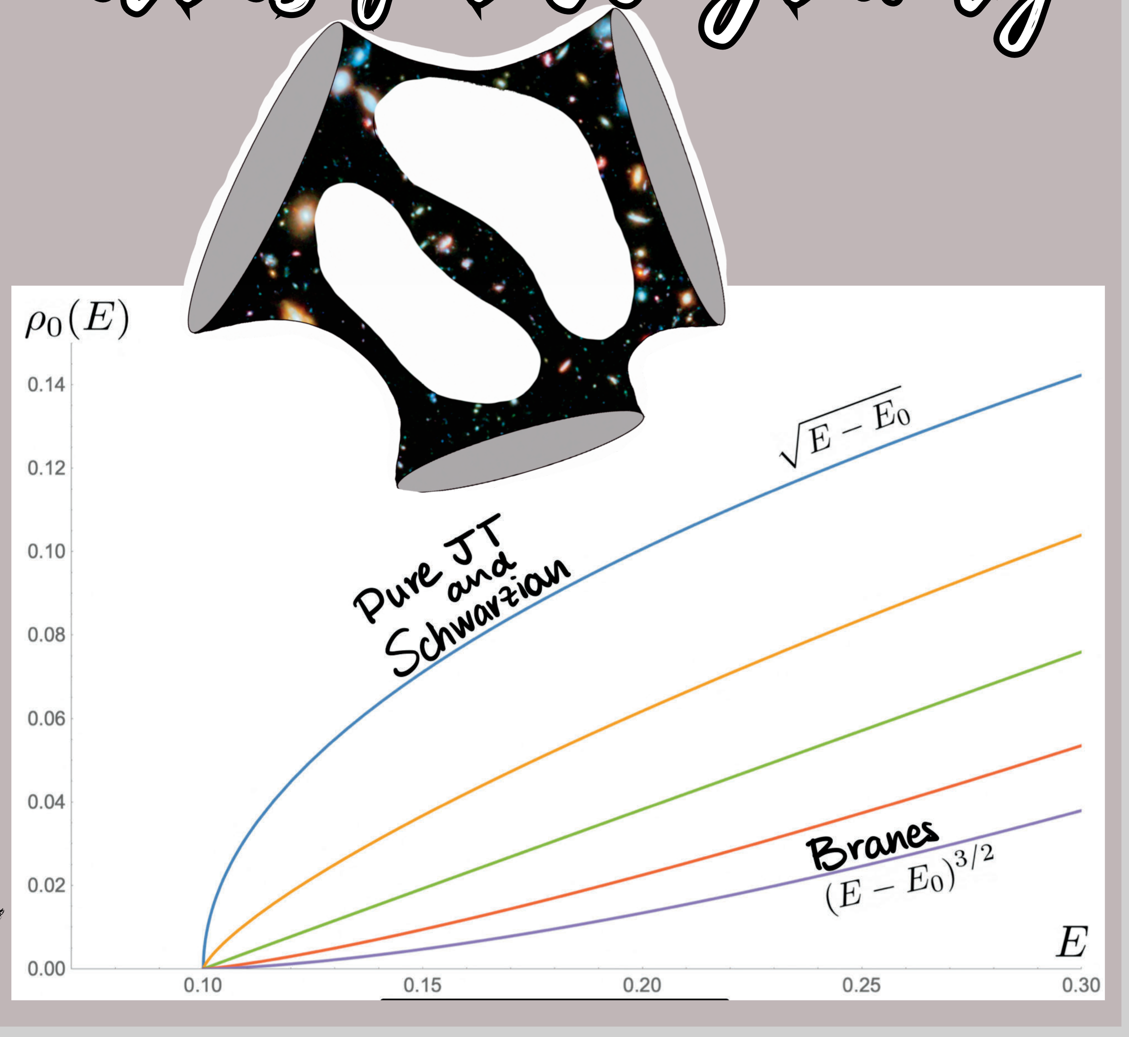


critical random geometries for JT Gravity



- In **JT gravity** one studies two-dimensional QG constant negatively curved spacetimes with boundaries = coupled to branes.
- Utilize techniques from **random maps,** where criticality is well-understood: big holes dominate the surfaces.
- Similar behavior was found for Weil-Petersson volumes, where we analyze the impact of criticality on the density of states **AdS/CFT.**
- Provide a precise definition of the phase transition from a random geometry perspective.





- Universal results allow us to compare characteristics of spacetime in the quantum regime coming from various QG approaches.
- A better understanding of these **random geometries** helps us to better understand the QG path integral and search candidates for **quantum spacetime**.

can you find the random geometry side of your QG research?

Based on
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