A regular black hole from an effective action for collapsing matter in quantum gravity

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We consider a modified lagrangian for collapsing matter which encodes the feature of a running Newton constant accordingly to the Asymptotic Safety program for quantum gravity. Our aim is to study if it is possible to avoid the formation of the singularity and how this would reflect on the geometry resulting from the collapse.

Markov-Mukhanov Action and Running Newton Coupling

As proprosed by Markov and Mukhanov [1], a consistent way to implement a running Newton constant is to consider the action

$$S = \frac{1}{2\kappa}\int d^4x\,\sqrt{-g}\left[R + 2\,\chi(\epsilon)\,\mathcal{L}^{(m)}\right] \tag{1}$$

where $\chi(\epsilon)$ is a multiplicative gravity-matter coupling depending on the energy density ϵ and $\mathcal{L}^{(m)} = \epsilon$. Varying this action we obtain the field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \partial_\epsilon(\chi\epsilon)T^{(m)}_{\mu\nu} + \partial_\epsilon\chi\epsilon^2 g_{\mu\nu} \tag{2}$$

where we can identify the running Newton constant as $G(\epsilon) = \partial_\epsilon(\chi\epsilon)$, and an induced running cosmological constant $\Lambda(\epsilon) = -\partial_\epsilon\chi\epsilon^2$. To close the system we need to specify a functional form for $G(\epsilon)$. The Asymptotic Safety program for quantum gravity suggests that gravity is renormalizable around a non-gaussian fixed point and that the Newton coupling runs with respect to the momentum k as $G(k) = G_{\rm N}/(1+\omega k^2 G_{\rm N})$, where ω is a positive dimensionless constant related to the scale at which quantum gravity sets in. Now, we assume that a similar scaling holds also in a gravity-matter system for $G(\epsilon)$. In particular, requiring $G = G(\epsilon/\epsilon_0)$ being ϵ_0 the Planck energy density, $\lim_{\epsilon/\epsilon_0\to\infty} G(\epsilon/\epsilon_0) = 0$ and $\lim_{\epsilon/\epsilon_0\to 0} G(\epsilon/\epsilon_0) = G_{\rm N}$, and prescribing the relation $k^2 = \xi G_{\rm N}\epsilon$ [2], where ξ is a positive dimensionless constant, we arrive, considering that $\epsilon_0 = G_{\rm N}^{-2}$ and working in geometrized units, to the form

$$G(\epsilon) = \frac{1}{1+q\cdot\epsilon} \tag{3}$$

where we have set $q = \xi\omega$. Finally, we choose as matter source a perfect fluid.

Semiclassical Dust Collapse

To model a collapsing body [3] we take a spherically symmetric line element in comoving coordinates $\{t, r, \theta, \phi\}$

$$ds_-^2 = -e^{2\nu(t,r)}dt^2 + e^{2\psi(t,r)}dr^2 + C(r,t)d\Omega^2 \tag{4}$$

It can be shown [4], choosing an homogenous dust matter source ($\epsilon = \epsilon(t)$, $p = 0$), and studying the field equations and the conservation equation for the effective energy-momentum tensor, that the solution is given by a Friedmann metric

$$ds_-^2 = -dt^2 + \frac{a(t)^2}{1-Kr^2}dr^2 + a(t)^2r^2d\Omega^2 \tag{5}$$

with $a(t)$ solution of the ordinary differential equation

$$\frac{da}{dt} = -\sqrt{\frac{\ln\left(1+\frac{3m_0q}{a^3}\right)}{3q}a^2 - K} \qquad V(a) = -\frac{a^2}{3q}\ln\left(1+\frac{3m_0q}{a^3}\right) \tag{6}$$

The problem is equivalent to the motion of a particle of energy K in a field with the potential $V(a)$. We underline that for a comoving shell r we have an effective Misner-Sharp mass varying in time, namely $F_{\rm eff}(r,t) = r^3m_{\rm eff}(t)$ with $m_{\rm eff} = a(\dot{a}^2 + K)$.

Marginally Bound Collapse ($K = 0$)

In this case, that corresponds to a contracting cloud of particles having zero initial velocity at spatial infinity, the behaviour of the infinitesimal scale factor at large times

$$a(t) \sim \exp\left(-\frac{t^2}{4q}\right) \tag{7}$$

indicates an *eternal collapse*.

Bound Collapse ($K = 1$)

In this case, that corresponds to a cloud of particles having zero initial velocity at a finite radius, the solution for the scale factor oscillates periodically between a finite minimum and a maximum. The instants of the *bounce* and of the crunch depend on the value of q.

Matching Exterior

We choose a static exterior with generalized mass function $M(R)$ in Schwarzschild coordinates $\{T, R, \theta, \phi\}$

$$ds_+^2 = -\left(1-\frac{2M(R)}{R}\right)dT^2 + \left(1-\frac{2M(R)}{R}\right)^{-1}dR^2 + R^2d\Omega^2 \tag{8}$$

(we define also $f(R) = 1 - 2M(R)/R$) and for the matching on the collapsing boundary $R = R_b(T)$ we follow the formalism of the first γ_{ab} and second K_{ab} fundamental form [3].

First Attempt: Schwarzschild

Fixing $M(R) = M_0$, with the requirement $2M_0 = r_b^3m_0$, we would have an action generating both the interior and the vacuum exterior. However, since in the interior we have a mass varying in time, to pursue this matching we have to introduce a shell Σ at the boundary, with its own surface energy-momentum tensor S_{ab}:

$$\gamma^-_{\theta\theta} = \gamma^+_{\theta\theta} \implies R_b(T(t)) = r_ba(t) \tag{9}$$

$$\gamma^-_{\tau\tau} = \gamma^+_{\tau\tau} \tag{10}$$

$$\implies \frac{dt}{dT} = \frac{f(R_b)}{\sqrt{f(R_b)+\dot{R}_b^2}}$$

$$K^+_{ab} - K^-_{ab} = S_{ab} - \frac{1}{2}\gamma_{ab}S \tag{11}$$

The shell would confine the quantum effects in a finite spacetime region. However, this attempt turned out to be problematic since we have found that the relation between t and T is not well defined.

New Regular Black Hole

We do not need to introduce a shell if we require the continuity of the second fundamental form. This uniquely determines [4] the mass function $M(R)$:

$$K^-_{\theta\theta} = K^+_{\theta\theta}$$

$$\implies M(R) = \frac{R^3}{6q}\ln\left(1+\frac{6M_0q}{R^3}\right) \tag{12}$$

where $2M_0 = r_b^3m_0$. The altered causal structure, with respect to that of a Schwarzschild black hole, can be thought of as a signature of the quantum effects, now affecting the whole spacetime.

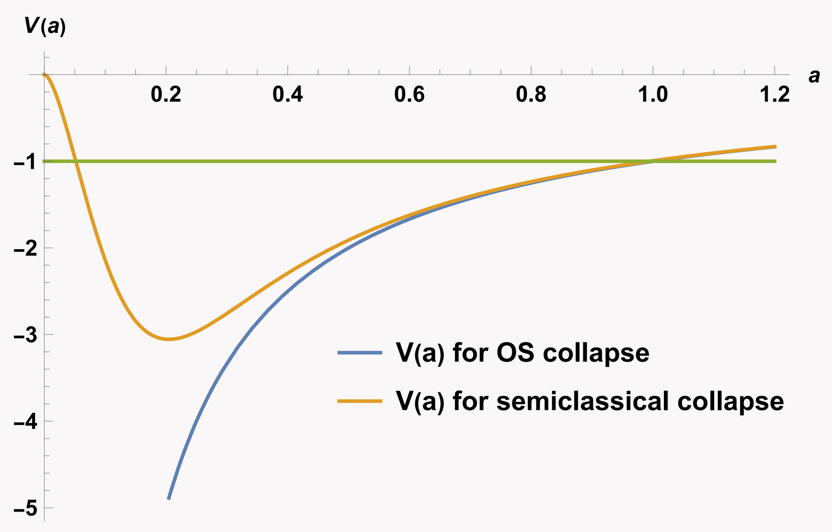

Figure 1: Potential stemming from the Markov-Mukhanov action compared to the divergent one of the usual Oppenheimer-Snyder (OS) collapse. If $K = 1$ the scale factor inverts its motion in two points. The parameters are fixed for illustrative purpose to $m_0 = 1$ and $q = 0.004$.

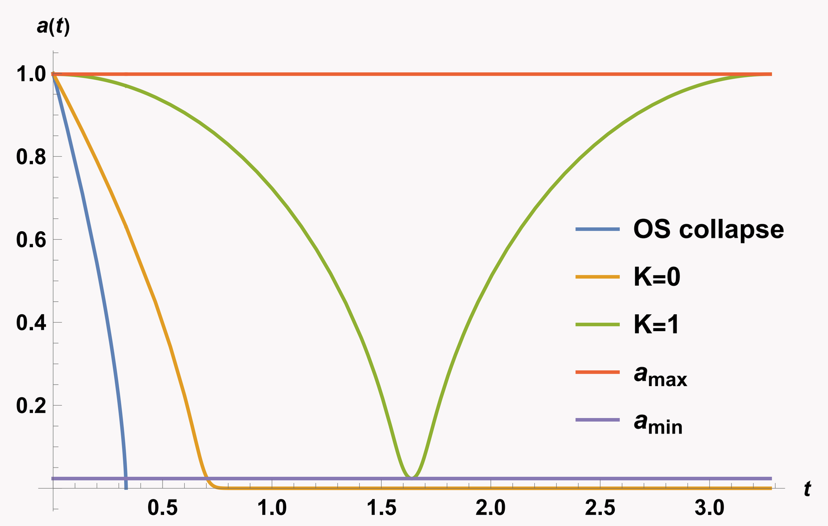

Figure 2: Behaviour of the scale factor in OS collapse compared to our model. In the OS case $a(t)$ becomes null in a finite interval of time, while in our model, for $K = 0$, we have a decreasingly exponential tail. The parameters are fixed for illustrative purpose to $m_0 = 1$ and $q = 0.004$.

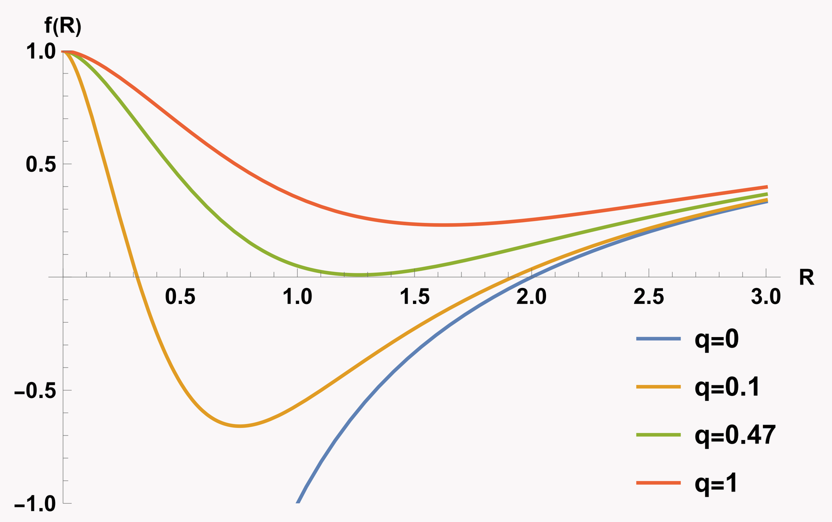

Figure 3: Behaviour of the temporal component of the new metric for different critical values of q, in units of M_0. For $q = 0$ we obtain the Schwarzschild metric, while for other values of q we can have an inner and an outer horizon, an extremal black hole or no horizons.

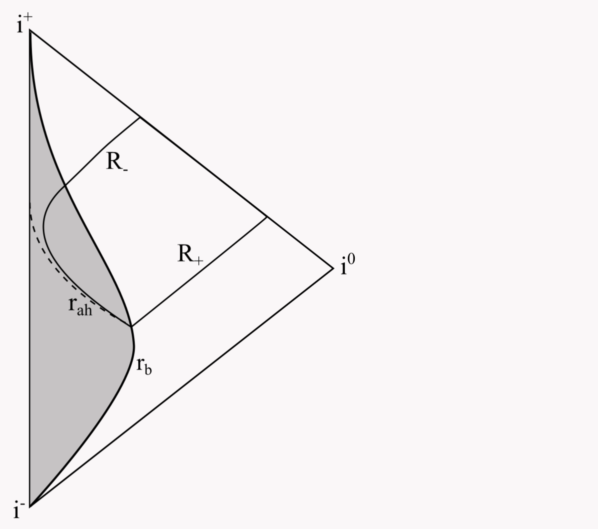

Figure 4: Possible Penrose diagram of the collapse for the case $K = 0$. The line r_b represents the trajectory of the collapsing boundary and the grey region represents the interior. The line $r_{\rm ah}$ refers to the mathematical curve $r_{\rm ah}(t)$, and the dashed line refers to $r_{\rm ah}(t)$ in the OS collapse. R_+ and R_- are the outer and inner horizons.

Conclusions

We discussed a toy model for the gravitational collapse of a dust cloud which implements the idea of an asymptotically safe gravitational interaction. As a consequence, in the matter interior the formation of the singularity is replaced by an eternal collapse or a bounce, while the requirement for regularity in the matching with a static exterior lead us to a novel regular black hole geometry.

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