Hawking temperature of black holes via Rindler transformations

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Summary

1) **Assumptions of finite time formation of trapped region and finiteness of curvature scalars on their boundary are enough to constrain a general spherically symmetric metric to correspond to two classes of solutions of the Einstein equations.**
2) **One class of solution describe an evaporating black hole. In the leading order approximation, evaporating black hole solution takes the form of the Vaidya metric.**
3) **Schwarzschild spacetime behaving Rindler-like near the horizon allows us to use the periodicity time trick to derive the Hawking temperature.**
4) **We obtain the Hawking temperature for the Vaidya metric using the periodicity time trick to show that this trick would be useful not only for stationary spacetimes but also for dynamical spacetimes.**

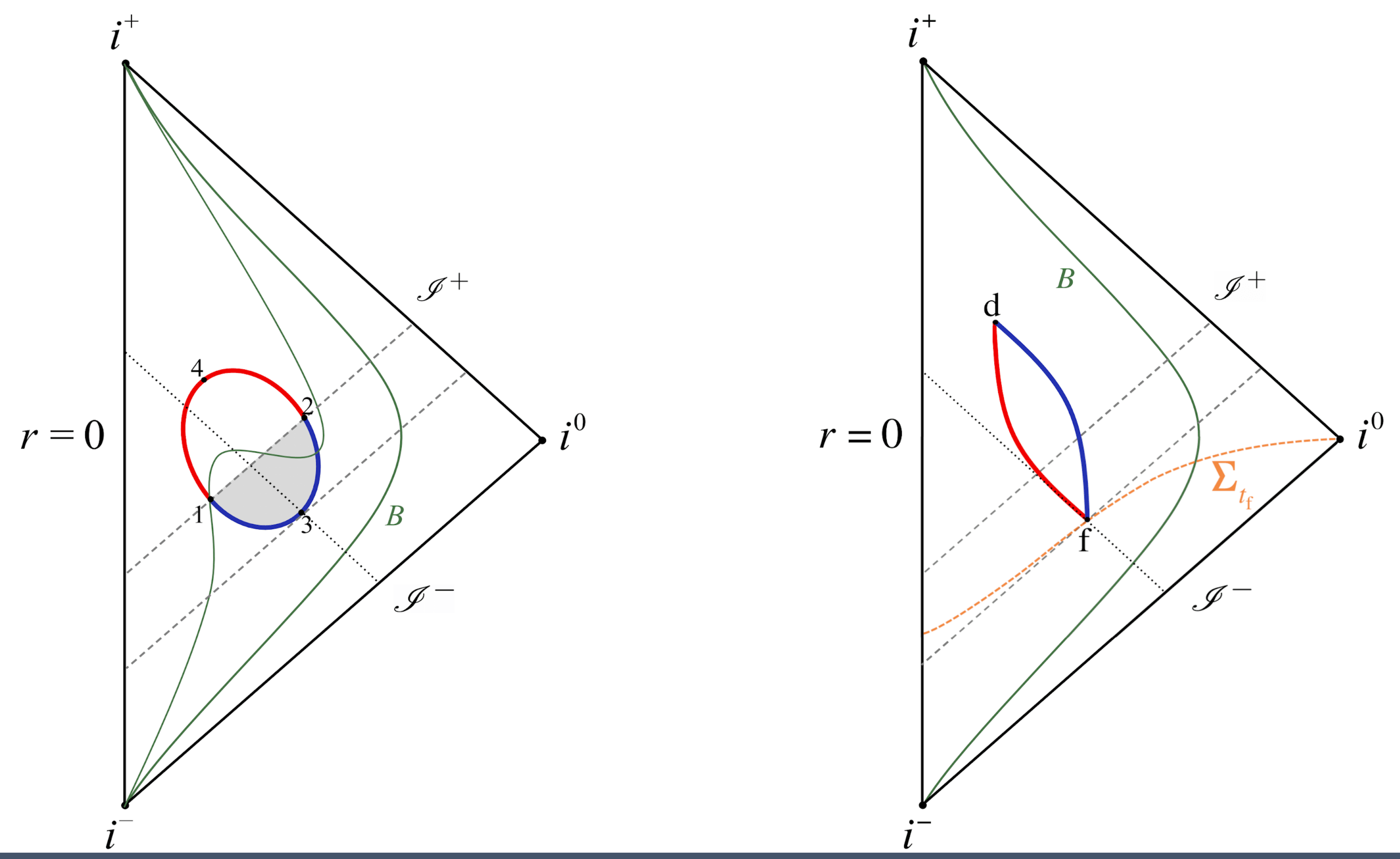

Fig. (a) A conventional RBH (b) RBH treated as a PBH

The trajectory of a distant observer is indicated in green. The dashed grey lines correspond to outgoing radial null geodesics, and the dotted lines represent the ingoing radial null geodesics. The asymptotic structure of a simple RBH spacetime coincides with that of Minkowski spacetime. (a) Dark blue is the outer and dark red is the inner apparent horizons. The quantum ergosphere is indicated by the light grey shading. A curved line is $r =$ const hypersurface, which connects i^- and i^+ and goes through the trapped region. The NEC is satisfied along the segment (413). The segments (14) and (23) are timelike. (b) RBH treated as PBH where the outer (dark blue) and inner (dark red) apparent horizons are timelike (membranes). The points f and d represent the events of formation and disappearance of the trapped region. The equal time hypersurface Σ_{t_f} is shown as a dashed orange line.

Calculations

Take the Vaidya metric in advanced coordinates

$$ds^2 = -\left(1 - \frac{2\,M(\tilde v)}{r}\right)\tilde v^2 + 2\,d\tilde v dr + r^2 d\Omega^2,$$

and apply the following substitutions/coordinate transformations

$$r = 2\,M(\tilde v)\big(1 + \epsilon^2 + 4\,M'(\tilde v)\big), \qquad \epsilon^2 \ll 1,$$

$$du = \frac{d\tilde v}{4M(\tilde v)} - \frac{2}{\epsilon}d\epsilon = dv - \frac{2}{\epsilon}d\epsilon.$$

The resulting metric will be in double null coordinates

$$ds^2 = -16M(v)^2\epsilon^2 du dv + 4M^2 d\Omega^2. \qquad (1)$$

This is the conformal transform of the Rindler metric (in two dimensions u, v). It can be maximally extended by further transformations

$$V = e^v, \qquad U = e^{-u},$$

which gives

$$ds^2 = 16M^2 dUdV + 4M^2 d\Omega^2. \qquad (2)$$

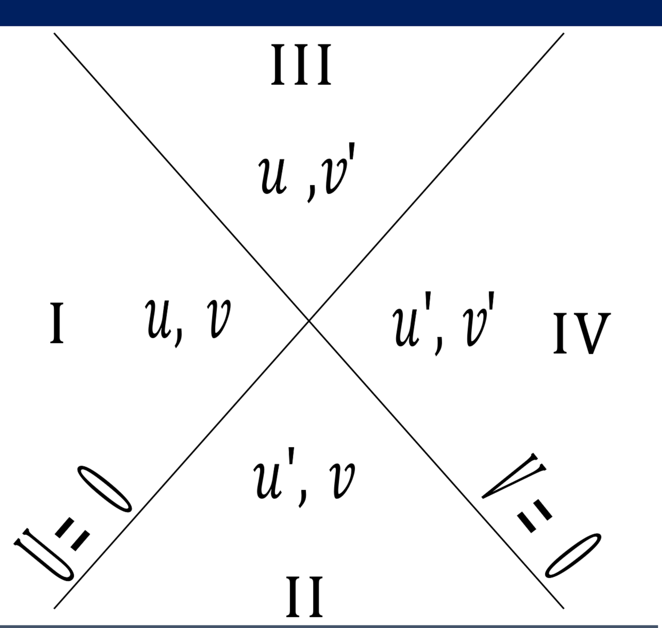

Fig. (c) Rindler spacetime. Rindler coordinates u, v cover only a wedge of the entire Minkowski space. We can extend these coordinates to all four quadrants by adding u', v':

$$V = e^{-v'} \quad for \quad V < 0,$$
$$U = e^{u'} \quad for \quad U < 0.$$

1) Rindler horizon, where the acceleration of the Rindler observer grows infinitely
$$\epsilon^2 = 0 \Longrightarrow r = 2M(v)\big(1 + 4M'(v)\big).$$
2) Also, a solution of outgoing radial null curves for which $d^2r/d\tau^2 = 0$. For the Vaidya black hole, this condition closely approximates the location of the event horizon.
3) Also, the location of the conformal Killing horizon $k^\mu k_\mu = 0$,
where $k^\mu = -\left(\frac{M}{M'}, r, 0, 0\right)$ is the conformal Killing vector.

Acknowledgements

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Motivation

- Equivalence principle implies that uniform acceleration is indistinguishable from the static homogenous gravitational field.
- Assuming that the equivalence principle holds, one could derive black hole radiation from the application of the Unruh effect. The Unruh effect is an important technique for investigating particle emissions from black hole horizons.
- Analysis of classical gravity, including quantum effects like Unruh-Davies temperature of the local Rindler horizon, might imply that classical gravity is indeed an emergent phenomenon (Padmanabhan, 2014).

Set-up

We take the Vaidya metric as a starting point of our calculation as it is the simplest non-static solution of the Einstein equation. It represents evaporating black hole solution in the leading order approximation in the near horizon expansion. We generalize our calculation to the Kerr-Vaidya metric, the simplest evaporating rotating black hole solution.

Results

- The geometric surface gravity corresponding to the conformal Killing vector is
$$\kappa_c = -\frac{1}{4M'(v)} + 3 + 4M'(v) + \mathbf{O}\big(M'(v)\big)^2,$$
where $\nabla_\mu(k^\nu k_\nu) = -2\kappa_c k_\mu$.
- The Killing vector should be normalized to coincide with the four-velocity of the observer via $k^\mu \to \tilde k^\mu = ck^\mu$ with c being constant. The measured surface gravity along the trajectory of the conformal Killing vector will be $1/\Omega$ times the geometric surface gravity, thereby ensuring that the physical predictions of the two conformal frames are equivalent (Flanagan, 2004)
$$\kappa_v = \Omega c \kappa_c = \sqrt{-k^\nu k_\nu}\kappa_c = \frac{1}{4M(v)\sqrt{1 - \frac{2M(v)}{r}}} + \mathbf{O}\big(M'(v)\big),$$
evaluated at the observer's location.
- Ignoring the conformal factors of the conformal Rindler metric of Eq.1 and its its maximal extension of Eq.2, the quantum field theoretic calculation reduces to finding the temperature measured by a uniformly accelerated observer with acceleration $a = 1$, which is $T = 1/2\pi$.
- For a stationary observer, the temperature differs by a redshift factor. The proper time at fixed $r = r_0$ is $d\tau = (4Mdt)^2$, and the temperature actually observed is
$$T_{\text{thermometer}} = \frac{1}{\sqrt{g_{tt}}}\frac{1}{2\pi} = \frac{1}{8\pi M}.$$
- There exists a unique vacuum state called the conformal vacuum state near the Rindler horizon, which is a conformal surface.

The temperature and surface gravity of stationary black holes are invariant under conformal transformations of the metric identity at infinity (Jacobson and Kang, 1993).

Generalization

Kerr-Vaidya metric in advanced coordinates could be taken as a model of rotating evaporating black holes solution in the leading order:

$$ds^2 = -\frac{\Delta\rho^2}{(r^2 + a^2)^2}d\tilde v^2 + 2\frac{\rho^2}{r^2 + a^2}d\tilde v dr + \rho^2 d\theta^2 + \rho^2 \sin^2\theta\, d\tilde\phi^2,$$

where

$$d\,\tilde v = \frac{(r^2 + a^2)}{\rho^2}(dv - a\sin^2\theta\, d\phi), \qquad d\,\tilde\phi = \frac{(r^2 + a^2)}{\rho^2}\left(d\phi - \frac{a}{a^2 + r^2}dv\right).$$

Near the outer horizon $r_+ = M + \sqrt{M^2 - a^2}$, series of coordinate transformations analogous to the Vaidya metric reduces this metric to the form

$$ds^2 = -\frac{4(r_+^2 + a^2)^2\epsilon^2}{\gamma^2}du'dv' + \rho_+^2 d\Omega^2.$$

The temperature corresponding to this metric for the stationary observer (accounting for the redshift factor) is $T = \frac{1}{2\pi}\frac{\gamma}{2(r_+^2 + a^2)} = \frac{\sqrt{M^2 - a^2}}{2\pi(r_+^2 + a^2)}$.

Conclusion

- **Where transformation to the Rindler coordinates is possible, there seems to be strong evidence that Hawking radiation is associated with the Rindler horizon.**
- **Generalization to the more general dynamical axial symmetry allows the study of thermodynamic properties of such black holes, which are generally difficult by other procedures like the tunnelling method.**

References

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