Suppression of proton decay in quantum gravity

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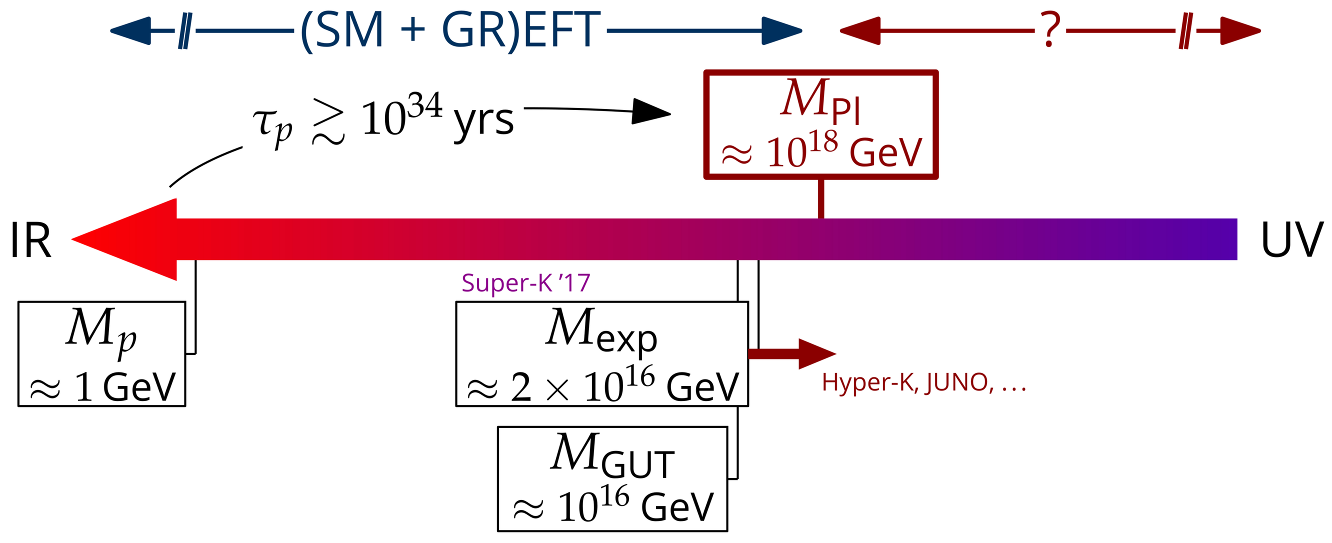
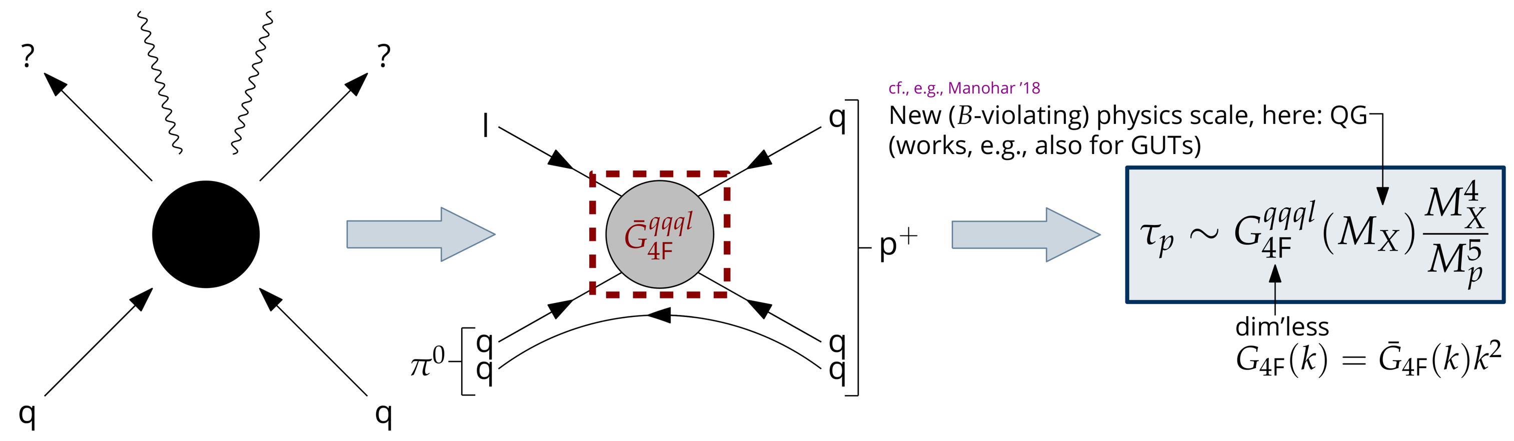
Introduction – Why proton decay?

- Large proton lifetime $\tau_p \gtrsim 10^{34}$ yrs acts as amplification factor
  $\Rightarrow$ Near-Planckian physics from low-energy ($M_p \sim 1$ GeV) measurement!



- Folklore: Quantum gravity (QG) breaks any global (= ungauged) sym – incl. baryon number B, which protects proton in SM.
  cartoon: virtual black holes



- Classic estimate: $\tau_{p,\mathrm{QG}} \sim 10^{41}\,\mathrm{yrs}(M_X/M_{\mathrm{Pl}})^4$ Zel'dovich '76; Adams *et al.* '01; …
- Caveat: 'Naturalness' assumption $G_{4F}^{qqql}(M_X) \sim 1$.

This work: First-principles estimate of $G_{4F}^{qqql}(M_X)$
$\to$ Assumption: QFT(SM + metric) holds at scales $M_X < k < k_{\mathrm{UV}}$
non-perturbatively renormalizable, implement using functional RG
$\to$ Applicable, e.g., to (effective) Asymptotic Safety

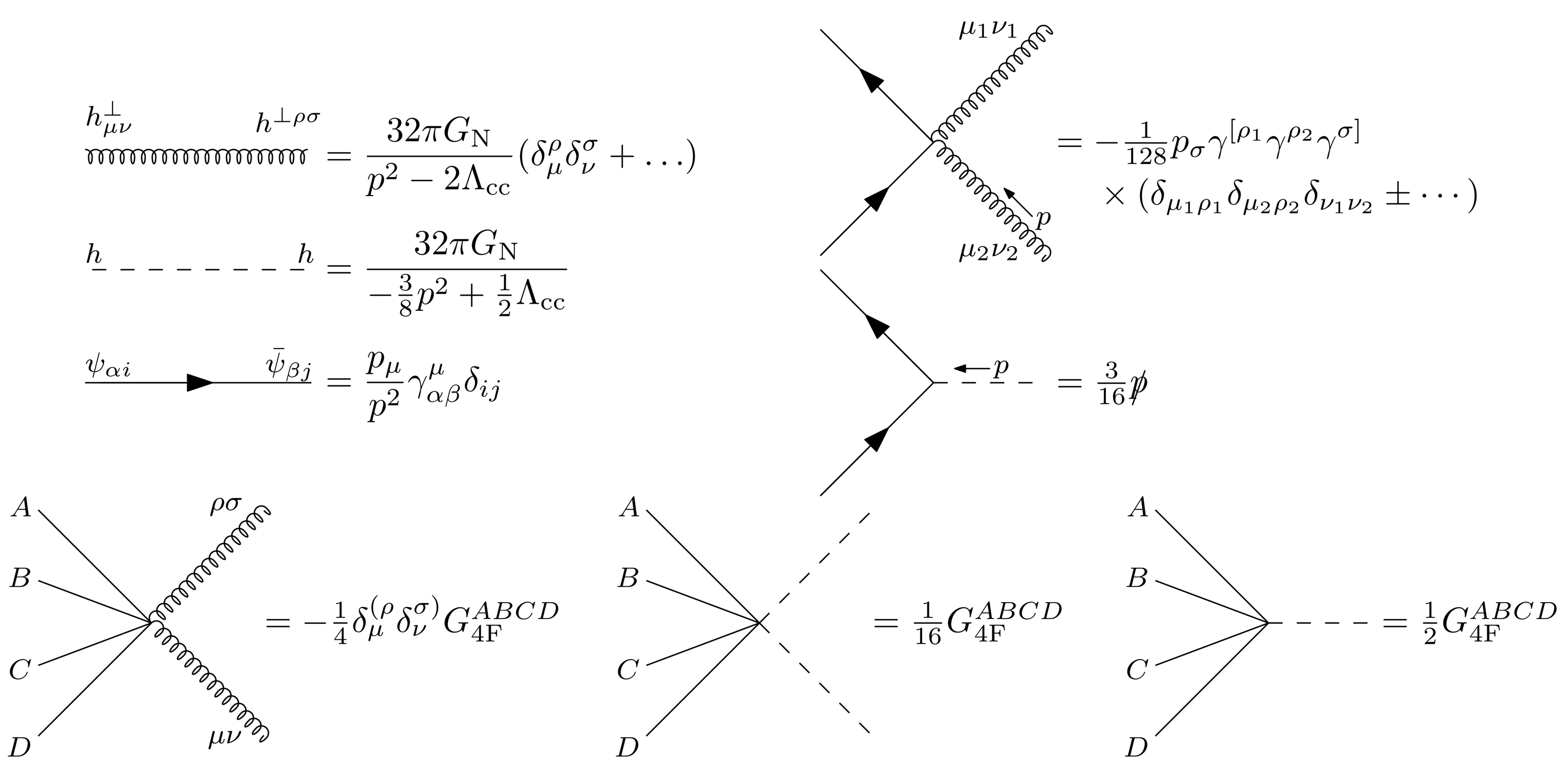
Model, computational set-up, technical background

Toy model QFT(SM + metric):

$$S = S_{\mathrm{EH}} + S_{\mathrm{kin,F}} + S_{4F}$$
$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}}\int_x \sqrt{g}(-R + 2\Lambda_{\mathrm{cc}})$$
$$S_{\mathrm{kin,F}} = \int_x \sqrt{g}\bar\psi i\nabla\!\!\!/\,\psi$$
$$S_{4F} = \bar{G}_{4F}^{ABCD}\int_x \sqrt{g}\,\Psi_A\Psi_B\Psi_C\Psi_D \qquad \Psi = \begin{pmatrix}\psi\\ \psi^c\end{pmatrix}$$

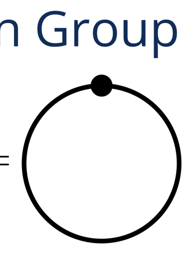
- ψ: contains all SM fermions
  Ψ = Nambu–Gor'kov spinor
  …Dirac fermions, right-handed neutrinos included; SU(2)$_L$ gauge coupling asymptotically free in ASQG
- split $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
  …in general: eigenvalues of $-\Delta(\bar g)$ defines notion of scale
  …often in practice (i.e., here): $\bar{g}_{\mu\nu}\to\delta_{\mu\nu}$ $\Longrightarrow$ momentum is 'good quantum number' after all …
- G_{4F}^{ABCD}: most general 4-Fermi, proton decay: $\sim qqql$ Grzadkowski et al. '10

Propagators and vertices: N.B.: Landau-deWitt gauge: only spin-2 and spin-0 modes of metric fluct.



Technique – Functional Renormalization Group (FRG):

$$\frac{\partial\Gamma_k[\Phi]}{\partial\ln k} = \frac{1}{2}\mathrm{STr}\left[\left(\frac{\delta^2\Gamma_k[\Phi]}{\delta\Phi\delta\Phi^\top} + R_k[\Phi]\right)^{-1}\frac{\partial R_k[\Phi]}{\partial\ln k}\right] =$$

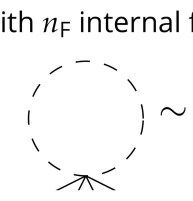


cf., e.g.: Berges *et al.* Phys. Rep. '02; Metzner *et al.* Rev. Mod. Phys. '12; Dupuis *et al.* Phys. Rept. '21; and refs. therein

$\Phi = (h^\perp_{\mu\nu}, h, \psi, \psi^c)^\top$; k — RG scale; R — regulator

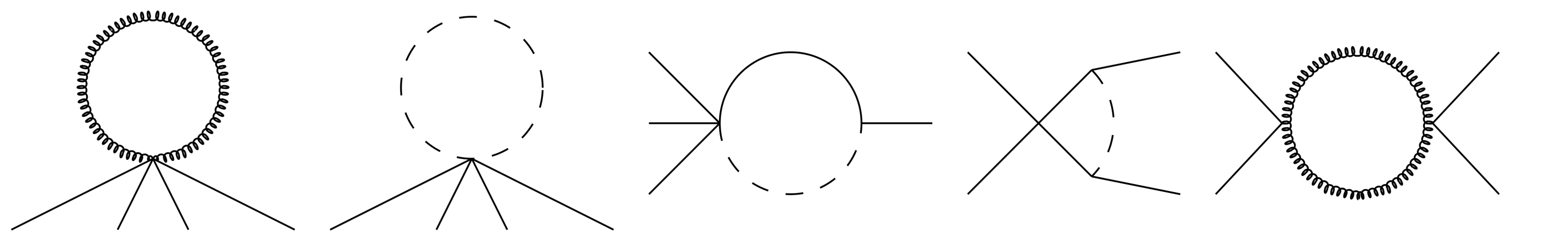
Minimal ('quick and dirty') version:

- Ansatz: $\Gamma_k = S|_{h_{\mu\nu}\to\sqrt{Z_{\mathrm{N}}}h_{\mu\nu}, G_{\mathrm{N}}\to G_{\mathrm{N}}(k), \Lambda_{\mathrm{cc}}\to\Lambda_{\mathrm{cc}}(k), \psi\to\sqrt{Z_\psi}\psi, G_{\mathrm{F}}\to G_{\mathrm{F}}(k)}$
- Draw one-loop diagrams with vertices and propagators from above
- Replace couplings and propagators with 'dressed' versions
- Replace momentum integrals with 'threshold functions'
  diagram with n_{F} internal fermion lines, $n_\perp$ spin-2 lines, n_{conf} conformal mode lines $\Rightarrow I_{n_{\mathrm{F}}, n_\perp, n_{\mathrm{conf}}}$

e.g.:  $\sim \int_p \frac{32\pi G_{\mathrm{N}}}{-\frac{3}{8}p^2 + \frac{1}{2}\Lambda_{\mathrm{cc}}} \mapsto I_{001} \sim \frac{(-6+\eta_{\mathrm{N}})g_{\mathrm{N}}}{(3-4\lambda_{\mathrm{cc}})^2}$

$\eta_{\mathrm{N}} = -k\partial_k \ln Z_{\mathrm{N}}, g_{\mathrm{N}} = G_{\mathrm{N}}k^2; \lambda_{\mathrm{cc}} = \Lambda_{\mathrm{cc}}/k^2$

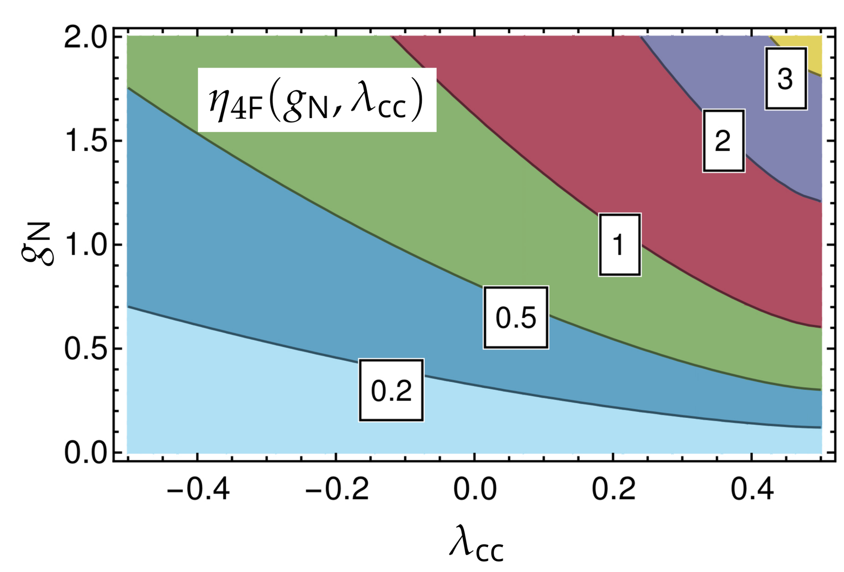
Result



$$k\partial_k G_{4F}^{qqql}(k) = (2 + \eta_{4F})\,G_{4F}^{qqql}(k) + \mathcal{O}((G_{4F}^{qqql})^2)$$
$$\eta_{4F} = 2g_{\mathrm{N}}\left[-\frac{9(2\lambda-3)}{4\pi(3-4\lambda_{\mathrm{cc}})^2} + \frac{6(4\lambda_{\mathrm{cc}}-9)}{5\pi(3-4\lambda_{\mathrm{cc}})^2} + \frac{5}{4\pi(1-\lambda_{\mathrm{cc}})^2} + \frac{3}{2\pi(4\lambda_{\mathrm{cc}}-3)^2}\right]$$
$$= \frac{29g_{\mathrm{N}}}{15\pi} + \frac{32g_{\mathrm{N}}\lambda_{\mathrm{cc}}}{9\pi} + \mathcal{O}(\lambda_{\mathrm{cc}}^2)$$

…η_{4F} independent of index structure $ABCD$ (gravity is 'blind' to internal indices)
…explicit form of η_{4F} calculated in Landau-deWitt gauge and using Litim regulator (agrees with Eichhorn/Gies '11)
…term $\propto g_{\mathrm{N}}\lambda_{\mathrm{cc}}$ in 2nd line reg.-indep.

Discussion



- Generally: $\eta_{4F} > 0$
  'metric fluctuations suppress proton decay'
- hence *a fortiori*: $2 + \eta_{4F} > 0$
  $\Rightarrow$ naïvely 'unnatural' $G_{4F}^{qqql}(M_{\mathrm{QG}}) \ll 1$ is actually 'natural' if QFT(SM + metric) holds at $M_{\mathrm{QG}} < k < k_{\mathrm{UV}}$ for k_{UV} large enough
  N.B.: *Much* milder assumption than (eff.) AS!
  Caveats/assumptions:
  - Einstein–Hilbert truncation should remain good approximation
  - B-violation from UV completion is (at most) 'natural': $G_{4F}^{qqql}(k_{\mathrm{UV}}) \sim 1$

Application to (effective) Asymptotic Safety:

- Neglect running of $g_{\mathrm{N}}(k), \lambda_{\mathrm{cc}}(k)$ for $M_{\mathrm{QG}} < k < k_{\mathrm{UV}}$
  $\leftrightarrow$ quasi-fixed-point regime
- Integrated flow: $G_{4F}^{qqql}(M_X) = (M_X/k_{\mathrm{UV}})^{2+\eta_{4F}}\,G_{4F}^{qqql}(k_{\mathrm{UV}})$

$$\tau_p \sim \left(\frac{k_{\mathrm{UV}}}{M_X}\right)^{4+2\eta_{4F}}\frac{M_X^4}{M_p^5}$$

$\Rightarrow$ increased proton lifetime, e.g.: $k_{\mathrm{UV}} \sim 10^2 M_X \to \tau_p \sim 10^7\,\tau_{p,\mathrm{class}}$

- Corollorary: for $k_{\mathrm{UV}} \to \infty$ (= strict AS limit), $G_{4F}^{qqql}(M_X) = 0$
- Interpretation: ASQG is B-conserving UV completion of GR
  $\Rightarrow$ counter-example to folklore?!
- More generally: ASQG compatible with global symmetry if symmetry-preserving regularization and regulator can be found
  N.B.: sufficient, but not necessary
  $\to$ Truncation-indep. 'proof': Use Quantum Action Principle for coarse-grained effective action Γ_k

$$e^{-\Gamma_k[\Phi]} = \int \mathcal{D}\tilde\Phi\, e^{-S[\tilde\Phi] + (\tilde\Phi_X - \Phi_X)\Gamma_{k,}{}^X[\Phi] - \frac{1}{2}\mathcal{R}_k^{XY}(\tilde\Phi_X - \Phi_X)(\tilde\Phi_Y - \Phi_Y)}$$
$$\delta_\epsilon\Gamma_k[\Phi] = \left\langle \delta_\epsilon\left(S[\tilde\Phi] - (\tilde\Phi_X - \Phi_X)\Gamma_{k,}{}^X[\Phi] + \frac{1}{2}\mathcal{R}_k^{XY}(\tilde\Phi_X - \Phi_X)(\tilde\Phi_Y - \Phi_Y)\right)\right\rangle_{k;\Phi}$$

with

$$\langle\mathcal{F}[\tilde\Phi]\rangle_{k;\Phi} := e^{\Gamma_k[\Phi]}\int \mathcal{D}\tilde\Phi e^{-S[\tilde\Phi] + (\tilde\Phi_X - \Phi_X)\Gamma_{k,}{}^X[\Phi] - \frac{1}{2}\mathcal{R}_k^{XY}(\tilde\Phi_X - \Phi_X)(\tilde\Phi_Y - \Phi_Y)}\mathcal{F}[\tilde\Phi]$$

Ward–Takahashi identity for B-symmetry
N.B.: Assuming reg. preserves B-symmetry, manifest for Dirac fermions (*not* possible in general for Weyl!)
Similar derivations for other symmetries cf. Gies '12; Laporte *et al.* '21

$$\delta_\epsilon S_{\mathrm{kin,F}} = 0 \implies \delta_\epsilon\Gamma_k = 0$$

Conclusion and outlook

- ASQG can accomodate B symmetry $\Rightarrow$ no proton-decay-mediating 4-Fermi operators
  What happens if we throw a quark into a black hole in ASQG?
- B violation is irrelevant perturbation in ASQG
  Further consequences of interplay with GUTs in QG regime?
  Interplay with other 4-Fermi operators ($\to\chi$SB, 'competing order' problem)?
- Technical ToDo list: gauge dependence, reg. dependence, …

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