Relating Spinfoams to Cosmological Perturbations

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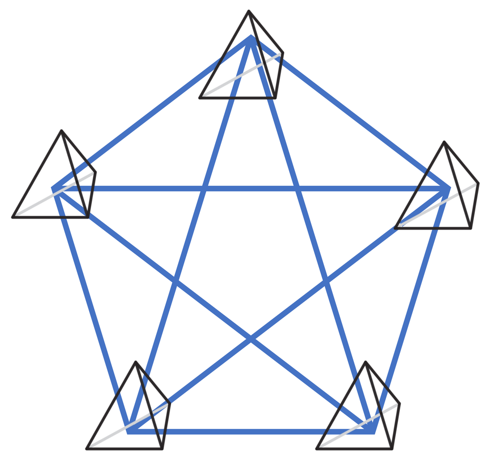
Abstract

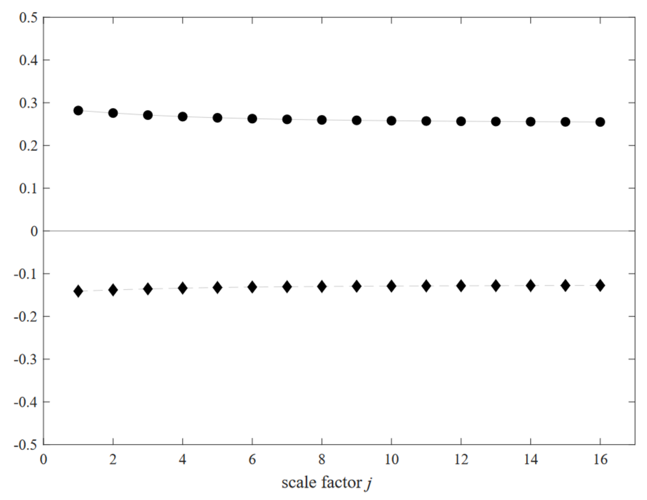
In this master thesis project the spinfoam calculations in [1] will be compared to cosmological calculations of primordial perturbations. In [1] the authors used the EPRL-spinfoam amplitude of Loop Quantum Gravity to calculate the quantum geometric state of the universe shortly after the bounce, using standard cosmological assumptions of homogeneity and isotropy.

Comparing their results to correlations of primordial perturbations several complications arise stemming from fundamental differences of the theories. By performing the calculations of the cosmological perturbations using a hydrodynamic scalar field on a FRW spacetime with positive spatial curvature and calculating the correlations using methods introduced in [2] the two theories should become more comparable.

1. Spinfoam Cosmology

At first order in the vertex expansion the spinfoam amplitude factorizes $W_{\mathcal{C}_1}(\psi_i,\psi_f) = W(\psi_i)W(\psi_f)$ and the probability of the "final" geometry is independent of the "initial" geometrical state. The individual amplitude $W(\psi_f)$ can be interpreted as the spinfoam version of a Hartle-Hawking wave function of the universe. By maximizing this amplitude of going from nothing to a quantum geometrical state the state of the universe shortly after the bounce can be calculated. This was done in [1]. For the calculation they used the graph dual to the regular triangulation of the 3-sphere consisting of five tetrahedra. The spins corresponding to the areas are assumed to be the same for all tetrahedra (homogeneity and isotropy). This spin j can be taken as a proxy for the scale factor.





The results can be summarized as follows:

- The expectation value of the geometry is that of a regular triangulation.
- The variance of the dihedral angles grows with j.
- The correlation between dihedral angles seems to go to a finite asymptotic value for large j.

2. Hydrodynamic scalar field model

The potential for a homogeneous and isotropic hydrodynamic scalar field with parameter of state $w = \frac{1}{3}$ in FRW spacetime with positive spatial curvature is:

$$V(\phi) = \frac{3}{\rho_0}\sinh^4\left(\frac{\phi}{2}\right) \tag{1}$$

We will assume there to be small perturbations $\phi(x,t) = \bar{\phi}(t) + \frac{f(x,t)}{a(t)}$ and calculate the linearized e.o.m. for f [3]:

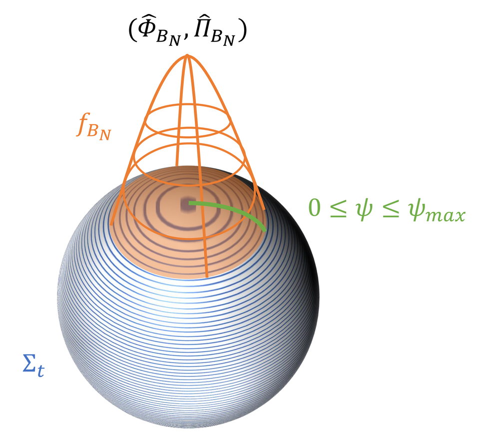
$$\left(\frac{a''}{a} - a^2\frac{d^2V}{d\phi^2}(\bar{\phi})\right) f - f'' + \Delta_{S^3} f = 0 \tag{2}$$

Taking the limit $\bar{\phi} \to \infty$ the equation can be solved. Choosing positive frequency solutions at $\tau = 0$, the perturbations can be quantized:

$$\hat{f}(\tau,\varphi,\theta,\psi) = \sum_{l=0}^{\infty}\sum_{n=0}^{l}\sum_{m=-n}^{n}\left(Y_{lnm}(\varphi,\theta,\psi)g(\tau)\hat{a}_{lmn} + Y^*_{lnm}(\varphi,\theta,\psi)g^*(\tau)\hat{a}^\dagger_{lnm}\right), \tag{3}$$

$$g(\tau) = N\sqrt{\tau}\left(iI_{\frac{4}{7}}\left(\frac{4}{7}\sqrt{5}\tau^{\frac{7}{8}}\right) - I_{-\frac{4}{7}}\left(\frac{4}{7}\sqrt{5}\tau^{\frac{7}{8}}\right)\right). \tag{4}$$

3. Correlations in QFT



In QFT the correlation between two spatial regions is always infinite, as stated in the Reeh-Schlieder theorem. This is to do with the fact that every spatial regions is made out of infinitely many modes. In order to compare QFT correlations to spinfoam correlations it is advantageous to only consider finitely many d.o.f., as spacetime in LQG is discretized. A method to calculate correlations of finitely many d.o.f. was introduced in [2].

In our setting the method is as follows:

- Define the regions of the northern hemisphere B_N and southern hemisphere B_S of the 3-sphere
- Define test functions $f_{N/S}$ that have support in these regions and respect the spherical symmetry
- Smear the field operators $\hat{\phi}$ and $\hat{\Pi}$ with $f_{N/S}$ and calculate the 2-point correlation function and the mutual information

Future Work

Further steps in the project:

- Translate the matter perturbations to metric perturbations
- Calculate the correlations of the metric perturbations using the described method from [2]
- Compare the correlations to the spinfoam correlations calculated in [1]

Beyond the thesis:

- Perform the spinfoam calculation with more nodes (next regular triangulation has 16 nodes) [4]
- Further investigate the relation between the LQG state and a QFT model of the matter perturbations

References

[1] F. Gozzini, F. Vidotto, *Front. Astron. Space Sci.* **2021**, *7*, 629466.

[2] I. Agullo, B. Bonga, P. Ribes-Metidieri, D. Kranas, S. Nadal-Gisbert, **2023**.

[3] V. F. Mukhanov, H. A. Feldman, R. H. Brandenberger, *Physics Reports* **1992**, *215*, 203–333.

[4] P. Frisoni, F. Gozzini, F. Vidotto, *Class. Quantum Grav.* **2023**, *40*, 105001.

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