Edge modes as dynamical frames: Charges from post-selection in generally covariant theories

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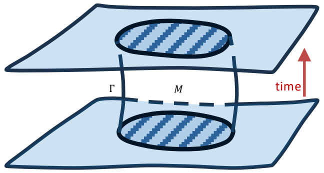
Background

- Gauge symmetry can be broken by the presence of boundaries
- Diffeo invariance in tension with locality of subregions/observables (e.g. [6])
- Adding boundary fields (edge modes) to a theory can restore some gauge invariance, but the physical significance is debated

Goals

- Elucidate the origin and physical relevance of boundary edge modes
- Identify gauge-invariant observables associated with bounded subregions
- Derive subregion theories (actions, symplectic structure, and charge algebras)
- Emphasize consistent embedding within a global theory



Focus is on subregions M of spacetime $\mathcal{M}$ with **timelike boundary** Γ with cylindrical topology $S^{d-1}\times\mathbb{R}$

Dressing with Dynamical Frames

A **Dynamical Reference Frame** is a dynamical subsystem that is acted upon freely by the gauge group G. This subsystem can be used to parametrize G-orbits and construct G-invariant observables.

In the case of diffeomorphisms $\varphi\colon \mathcal{M} \to \mathcal{M}$, fundamental fields like metric g transform via the pullback φ_*

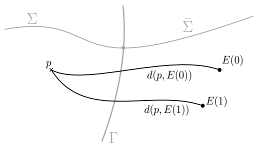
$$\varphi \triangleright g = \varphi_* g \qquad \varphi \triangleright \Upsilon(g,\Psi) = \Upsilon(\varphi_* g, \varphi_* \Psi)$$

A group-valued reference frame $U[g,\Psi]$ can be constructed as a nonlocal functional of g and other fields Ψ, transforming as

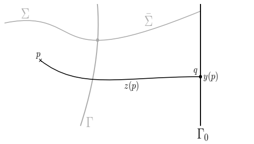
$$\varphi \triangleright U[g,\Psi] = U[\varphi \triangleright g, \varphi \triangleright \Psi] = U \circ \varphi$$

Such U can be constructed locally in field space numerous ways (see [1-3]), e.g. using:

a) distances from fixed spacetime events



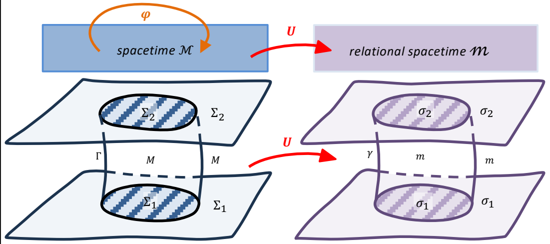
b) geodesics anchored to fixed asymptotics



Spacetime forms α can be *dressed* with U obeying $\varphi \triangleright U = U \circ \varphi$ to create *invariant* forms:

$$\alpha_{inv} \coloneqq U^{-1} \triangleright \alpha. \qquad \varphi \triangleright \alpha_{inv} = \alpha_{inv}$$

This motivates the idea that U maps ***spacetime*** $\mathcal{M}$ to a new manifold, ***relational spacetime*** m:



Decomposing forms relative to frame

U serves function of "dynamical embedding maps" of, e.g. [3,4], here with added interpretation and formalism.

Forms can be decomposed into:

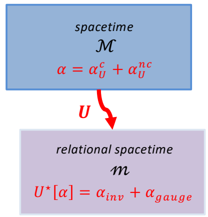
- ***U − covariant // noncovariant*** components on $\mathcal{M}$
- ***invariant // gauge*** components on m.

Decomposition of α on $\mathcal{M}$

$$\alpha = \underbrace{U_\star(U^{-1} \triangleright \alpha)}_{\alpha_U^c} + \underbrace{(\alpha - U_\star(U^{-1} \triangleright \alpha))}_{\alpha_U^{nc}}$$

Decomposition of $U^\star[\alpha]$ on m

$$U^\star[\alpha] = \underbrace{U^{-1} \triangleright \alpha}_{\alpha_{inv}} + \underbrace{(U^\star[\alpha] - U^{-1} \triangleright \alpha)}_{\alpha_{gauge}}$$



If α is a field space 0-form, the invariance of the dressed form is trivial:

$$\alpha_{inv} \coloneqq U^{-1} \triangleright \alpha = U^*\alpha. \qquad \varphi \triangleright \alpha_{inv} = (U \circ \varphi)^*(\varphi_*\alpha) = \alpha_{inv}$$

More generally $U^{-1} \triangleright \alpha \neq U^*\alpha$. For example, dressing $\delta\alpha$ leads to

$$(\delta\alpha)_{inv} \coloneqq U^{-1} \triangleright \delta\alpha = \delta(U^*[\alpha]) = U^*[\delta_\chi \alpha] \qquad \varphi \triangleright (\delta\alpha)_{inv} = (\delta\alpha)_{inv}$$

where $\delta_\chi := \delta + \mathcal{L}_\chi$ and χ is the **Maurer-Cartan form** associated to U, $\chi \coloneqq \delta U^{-1} \circ U$

Variational Principle on Dressed Lagrangian

The boundaries $\Sigma_1, \Sigma_2, \Gamma$ defining the subregion M are not invariant under diffeomorphisms but the images of these surfaces $\sigma_1, \sigma_2, \gamma$ define an invariant subregion m on relational spacetime.

This has implications for the variational principle on the spacetime action

$$\int_M L = \int_m U^\star[L]$$

U is constructed from dynamical fields, so δ passes freely over $\int_{m}$, but not $\int_M$

$$\delta U^*[L] = U^*[\delta_\chi L] = U^\star[E + d(\theta + \chi \lrcorner L)]$$

- This suggests an extended symplectic potential and current density

$$\theta_\chi := \theta + \chi \lrcorner L \qquad \omega_\chi := \delta_\chi \theta_\chi$$

used by [Ciambelli, Leigh, Pai, '21] and [Freidel, '21]

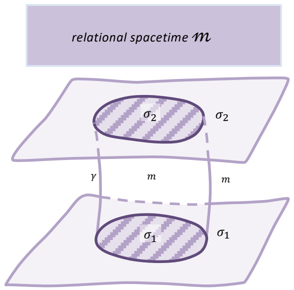
- Decompose θ_χ into its U-covariant and non U-covariant parts:

used by [Speranza, '17]

$$\theta_\chi = \underbrace{(\theta + \hat{\chi} \cdot \theta)}_{(\theta_\chi)_U^c} + \underbrace{(-\hat{\chi} \cdot \theta + \chi \lrcorner L)}_{(\theta_\chi)_U^{nc} = -J_\chi}$$

Where $J_v = C_v + dq_v$ is the Noether current associated to a vector field v, which decomposes in terms of a *constraint* $C_v \approx 0$ and boundary *charge aspect* q_v.

- Implement *post-selection*! (global theory ⟶ subregion theory) [Carrozza, Höhn, '21]



Post-Selection of Subregion Symplectic Structure

Assume a well-defined global symplectic form $\Omega_{m \cup m}$

- Select a gauge-invariant boundary condition on γ:

$$x = x_0$$

- Invariant symplectic flux must vanish on $\mathcal{S}_{x_0}$ up to corner contribution:

$$(\omega_\chi)_{inv}\big|_\gamma \mathrel{\hat{\approx}} d\delta\beta_{inv}$$

- Subregion and complement symplectic forms are locally equivalent to global symplectic form:

$$\Omega_{m\cup m} \mathrel{\hat{\approx}} \Omega_m + \Omega_m \quad \text{with} \begin{cases} \Omega_m \mathrel{\hat{\approx}} \int_\sigma U^\star[\omega_\chi] + \int_{\partial\sigma} \omega_\partial \\ \Omega_m \mathrel{\hat{\approx}} \int_\sigma U^\star[\omega_\chi] - \int_{\partial\sigma} \omega_\partial + \ \ldots \end{cases}$$

- Demand conservation of Ω_m on $\mathcal{S}_{x_0}$:

$$\omega_\partial := -\delta\beta_{inv} + \delta U^\star[q_\chi]\big|_\gamma$$

related to [Harlow, Wu, '19]

- $\Omega(\sigma)$ is defined off-shell, but independent of the choice of cauchy slice σ when fully on shell (including boundary conditions)

$$\Omega(\sigma) \coloneqq \int_\sigma U^\star[\omega_\chi] + \int_{\partial\sigma} (\delta U^\star[q_\chi] - \delta\beta_{inv}) \mathrel{\hat{\approx}} \Omega_m$$

Post-selection:
Foliate solution space $\mathcal{S}$ by a choice of boundary conditions on gauge invariant combinations of fields x on γ.
Restrict to one leaf of this foliation.

$$\mathcal{S} = \bigsqcup_{x_0} \mathcal{S}_{x_0}, \qquad \mathcal{S}_{x_0} = \{\Phi \in \mathcal{S} \mid x = x_0\}$$

Let $\hat{\approx}$ indicate pullback to $\mathcal{S}_{x_0}$

Gauge and Symmetry Charges

Spacetime diffeomorphisms, $\xi \in \mathcal{X}(\mathcal{M})$

Transformation:

$$\hat{\xi} \cdot \delta\Phi = \mathcal{L}_\xi \Phi, \qquad \hat{\xi} \cdot \chi = -\xi$$

$\hat{\xi}$ denotes "field space lift" of vector field ξ

Charges/Integrability:

$$-\hat{\xi} \cdot \Omega(\sigma) = \delta H_\xi(\sigma)$$

$$H_\xi(\sigma) \coloneqq \int_\sigma U^\star[C_\xi] \approx 0$$

Algebra/Bracket:

$$\{H_{\xi_1}(\sigma), H_{\xi_2}(\sigma)\} := -\hat{\xi}_1 \cdot \hat{\xi}_2 \cdot \Omega(\sigma) = -H_{[\xi_1,\xi_2]}(\sigma)$$

(⋆) Complete 1st class constraint algebra closes off-shell.
Anti-homomorphic to the algebra of spacetime vector fields.

Relational diffeomorphisms, $\rho \in \mathcal{X}(m)$

Transformation:

$$\check{\rho} \cdot \delta\Phi = 0, \qquad \check{\rho} \cdot \chi = \rho \qquad (\rho := U_\star[\rho])$$

Charges/Integrability:

$$-\check{\rho} \cdot \Omega(\sigma) = \delta Q_\rho(\partial\sigma) + \mathcal{F}_\rho(\sigma)$$

$$\mathcal{F}_\rho(\sigma) \approx \int_{\partial\sigma} \rho \lrcorner (\theta_{inv} - d\beta_{inv})$$

$$Q_\rho(\partial\sigma) \coloneqq \int_{\partial\sigma} (U^\star[q_\rho] - \check{\rho} \cdot \beta_{inv})$$

Algebra/Bracket (subcases):

<u>If ρ preserves $\partial\sigma$, $\mathcal{F}_\rho$ vanishes:</u>
$\{Q_{\rho_1}, Q_{\rho_1}\}_{\partial\sigma} = -Q_{[\rho_1,\rho_2]}$
on-shell closure of corner algebra
(corner algebra Diff(s)⋉…)

<u>If ρ preserves γ and b.c.s:</u>
$\mathcal{F}_\rho(\sigma) \mathrel{\hat{\approx}} \int_{\partial\sigma}(\rho \lrcorner \ell)$ becomes exact

$$Q_\rho^H \coloneqq \int_{\partial\sigma} (U^\star[q_\rho] - \check{\rho} \cdot \beta_{inv} + \rho \lrcorner \ell)$$

$$\mathcal{K}_{\rho_1,\rho_2} \coloneqq \int_{\partial\sigma} (\rho_1 \lrcorner \Delta_{\rho_2}\ell - \rho_2 \lrcorner \Delta_{\rho_1}\ell)$$

$$\{Q_{\rho_1}^H, Q_{\rho_2}^H\}_\gamma \mathrel{\hat{\approx}} - Q_{[\rho_1,\rho_2]}^H + \mathcal{K}_{\rho_1,\rho_2}$$

(⋆) Charges Q_ρ^H can represent

1. boundary symmetries (preserving b.c.s and $\delta Q_\rho^H \mathrel{\hat{\not\approx}} 0$);
2. boundary gauge transformations (preserving b.c.s and $\delta Q_\rho^H \mathrel{\hat{\approx}} 0$);
3. Metasymmetry (not preserving b.c.s);

Results

- Edge modes are seen to arise from dynamical reference frames, relating a subregion to its complement
- Symplectic structure for subregion theories is identified through post-selection
- Gauge symmetries of the global theory maintained, with closed 1st class constraint algebra
- Physical symmetry algebras identified depending on post-selected theory
- See also [1] for discusion of post-selection of subregion actions, GR example, and comparison to related proposals in the literature

References

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