Generalized spin foam models with matter based on 3-groups

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Abstract

Higher category theory can be employed to generalize the BF action to the so-called nBF action, by passing from the notion of a gauge group to the notion of a gauge n-group. The novel algebraic structures called n-groups are designed to generalize notions of connection, parallel transport and holonomy from curves to manifolds of dimension higher than one. Thus they generalize the concept of gauge symmetry, giving rise to a class of topological actions called nBF actions. Similarly as for the Plebanski action, one can add appropriate simplicity constraints to topological nBF actions, in order to describe the correct dynamics of Yang-Mills, Klein-Gordon, Dirac, Weyl and Majorana fields coupled to Einstein-Cartan gravity. Specifically, one can rewrite the whole Standard Model coupled to gravity as a constrained 3BF or 4BF action. The split of the full action into a topological sector and simplicity constraints sector is adapted to the spinfoam quantization technique, with the aim to construct a full model of quantum gravity with matter.

In addition, the properties of the gauge n-group structures open up a possibility of a nontrivial unification of all fields. An n-group naturally contains additional novel gauge groups which specify the spectrum of matter fields present in the theory, just like the ordinary gauge group specifies the spectrum of gauge bosons in the Yang-Mills theory. The presence and the properties of these new gauge groups has the potential to explain fermion families, and other structure in the matter spectrum of the theory.

3-group and $3BF$ action

Gauge symmetry is represented by a 3-group:

$$(L \xrightarrow{\delta} H \xrightarrow{\partial} G,\ \triangleright,\ \{\_,\_\}_{\rm pf})\,,$$

where G, H, L are Lie groups, $\triangleright$ is an action of G on all three groups, and the map

$$\{\_,\_\}_{\rm pf} : H \times H \to L$$

is the Peiffer lifting map. The maps $\triangleright$, δ, ∂, and $\{\_,\_\}_{\rm pf}$ satisfy a number of axioms. The 3-group gives rise to a 3-connection (α, β, γ) where:

$$\begin{aligned} \alpha &= \alpha^\alpha{}_\mu(x)\, \mathrm{d}x^\mu \otimes \tau_\alpha && \in \Lambda^1(\mathcal{M}) \otimes \mathfrak{g}\,, \\ \beta &= \beta^a{}_{\mu\nu}(x)\, \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \otimes t_a && \in \Lambda^2(\mathcal{M}) \otimes \mathfrak{h}\,, \\ \gamma &= \gamma^A{}_{\mu\nu\rho}(x)\, \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \wedge \mathrm{d}x^\rho \otimes T_A && \in \Lambda^3(\mathcal{M}) \otimes \mathfrak{l}\,, \end{aligned}$$

Corresponding fake 3-curvature $(\mathcal{F}, \mathcal{G}, \mathcal{H})$ is defined as:

$$\mathcal{F} = \mathrm{d}\alpha + \alpha \wedge \alpha - \partial\beta\,, \qquad \mathcal{G} = \mathrm{d}\beta + \alpha \wedge^{\triangleright} \beta - \delta\gamma\,,$$

$$\mathcal{H} = \mathrm{d}\gamma + \alpha \wedge^{\triangleright} \gamma + \{\beta \wedge \beta\}_{\rm pf}\,.$$

Finally, one can use all of the above to introduce the topological $3BF$ action:

$$S_{3BF} = \int_{\mathcal{M}_4} \langle B \wedge \mathcal{F} \rangle_{\mathfrak{g}} + \langle C \wedge \mathcal{G} \rangle_{\mathfrak{h}} + \langle D \wedge \mathcal{H} \rangle_{\mathfrak{l}}\,.$$

Standard Model 3-group

Choice of groups:

$G = SO(3,1) \times SU(3) \times SU(2) \times U(1)$	$\Leftarrow$	Lorentz and internal sym.
$H = \mathbb{R}^4$	$\Leftarrow$	4-translations
$L = \mathbb{R}^4(\mathbb{C}) \times \mathbb{R}^{64}(\mathbb{G}) \times \mathbb{R}^{64}(\mathbb{G}) \times \mathbb{R}^{64}(\mathbb{G})$	$\Leftarrow$	scalars and fermion families

Most 3-group maps are chosen as trivial:

$$\partial h = \mathbb{1}_G\,, \qquad \delta l = \mathbb{1}_H\,, \qquad \{h_1, h_2\}_{\rm pf} = \mathbb{1}_L\,.$$

Action $\triangleright : G \times G \to G$, chosen as conjugation:

$$\triangleright_{[ab][cd]}{}^{[ef]} \equiv f_{[ab][cd]}{}^{[ef]} = \frac{1}{2}\left(\eta_{[a|c}\delta^{[f|}_{|b]}\delta^{|e]}_d - \eta_{[a|d}\delta^{[f|}_{|b]}\delta^{|e]}_c\right)\,,$$

$$\triangleright_{\alpha\beta}{}^\gamma = f_{\alpha\beta}{}^\gamma\,, \qquad \triangleright_{\alpha[ab]}{}^{[cd]} = 0\,, \qquad \triangleright_{[ab]\beta}{}^\gamma = 0\,.$$

Action $\triangleright : G \times H \to H$, chosen as:

$$\triangleright_{[cd]a}{}^b = \frac{1}{2}\eta_{a[d|}\delta^b_{|c]}\,, \qquad \triangleright_{\alpha a}{}^b = 0\,.$$

Action $\triangleright : G \times L \to L$, chosen to define matter field types:

$$\triangleright_{[cd]A}{}^B = \frac{1}{2}\left(\sigma_{[cd]}\right)_A{}^B\,, \qquad \triangleright_{\alpha A}{}^B = \frac{1}{2}\left(\sigma_\alpha\right)_A{}^B\,.$$

Constrained $3BF$ Standard Model action

MAIN INSIGHT: $C = e,\ D = \phi, \psi$ ***!!!***

$$\begin{aligned} S = \int & \overbrace{B_\alpha \wedge F^\alpha + B^{[ab]} \wedge R_{[ab]}}^{\langle B\wedge\mathcal{F}\rangle} + \overbrace{e_a \wedge \nabla\beta^a}^{\langle C\wedge\mathcal{G}\rangle} + \overbrace{\phi^A(\nabla\gamma)_A + \bar\psi_A(\overset{\rightarrow}{\nabla}\gamma)^A - (\bar\gamma\overset{\leftarrow}{\nabla})_A\psi^A}^{\langle D\wedge\mathcal{H}\rangle} && \Leftarrow \text{ topological } 3BF\text{action} \\ & - \int \lambda_{[ab]} \wedge \left(B^{[ab]} - \frac{1}{16\pi l_p^2}\varepsilon^{[ab]cd}\, e_c \wedge e_d\right) + \frac{1}{96\pi l_p^2}\Lambda\, \varepsilon_{abcd}\, e^a \wedge e^b \wedge e^c \wedge e^d && \Leftarrow \text{ gravitational constraint and CC} \\ & + \int \lambda^\alpha \wedge \left(B_\alpha - 12\, C_\alpha{}^\beta M_{\beta ab}\, e^a \wedge e^b\right) + \zeta^{\alpha ab}\left(M_{\alpha ab}\, \varepsilon_{cdef}\, e^c \wedge e^d \wedge e^e \wedge e^f - F_\alpha \wedge e_a \wedge e_b\right) && \Leftarrow \text{ Yang-Mills constraint} \\ & + \int \lambda^A \wedge \left(\gamma_A - H_{abcA}\, e^a \wedge e^b \wedge e^c\right) + \Lambda^{abA} \wedge \left(H_{abcA}\, \varepsilon^{cdef}\, e_d \wedge e_e \wedge e_f - (\nabla\phi)_A \wedge e_a \wedge e_b\right) && \Leftarrow \text{ Higgs constraint} \\ & - \int \frac{1}{12}\chi\left(\phi^A\phi_A - v^2\right)^2 \varepsilon_{abcd}\, e^a \wedge e^b \wedge e^c \wedge e^d && \Leftarrow \text{ Higgs potential} \\ & + \int \bar\lambda_A \wedge \left(\gamma^A + \frac{i}{6}\varepsilon_{abcd}\, e^a \wedge e^b \wedge e^c \left(\gamma^d\psi\right)^A\right) - \lambda^A \wedge \left(\bar\gamma_A - \frac{i}{6}\varepsilon_{abcd}\, e^a \wedge e^b \wedge e^c \left(\bar\psi\gamma^d\right)_A\right) && \Leftarrow \text{ Dirac constraint} \\ & - \int \frac{1}{12} Y_{ABC}\, \bar\psi^A\psi^B\phi^C\, \varepsilon_{abcd}\, e^a \wedge e^b \wedge e^c \wedge e^d && \Leftarrow \text{ Yukawa coupling} \\ & + \int 2\pi i\, l_p^2\, \bar\psi_A\gamma_5\gamma^a\psi^A\, \varepsilon_{abcd}\, e^b \wedge e^c \wedge \beta^d\,. && \Leftarrow \text{ spin-torsion coupling} \end{aligned}$$

3-group topological invariant

$$\begin{aligned} Z = {} & |G|^{-|\Lambda_0|+|\Lambda_1|-|\Lambda_2|}|H|^{|\Lambda_0|-|\Lambda_1|+|\Lambda_2|-|\Lambda_3|}\,|L|^{-|\Lambda_0|+|\Lambda_1|-|\Lambda_2|+|\Lambda_3|-|\Lambda_4|} \\ & \times\left(\prod_{(jk)\in\Lambda_1}\int_G dg_{jk}\right)\left(\prod_{(jk\ell)\in\Lambda_2}\int_H dh_{jk\ell}\right)\left(\prod_{(jk\ell m)\in\Lambda_3}\int_L dl_{jk\ell m}\right) \\ & \times\left(\prod_{(jk\ell)\in\Lambda_2}\delta_G\big(\partial(h_{jk\ell})\, g_{k\ell}\, g_{jk}\, g_{j\ell}^{-1}\big)\right)\left(\prod_{(jk\ell m)\in\Lambda_3}\delta_H\big(\delta(l_{jk\ell m})h_{j\ell m}\,(g_{\ell m} \triangleright h_{jk\ell})\, h_{k\ell m}^{-1}\, h_{jkm}^{-1}\big)\right) \\ & \times\left(\prod_{(jk\ell mn)\in\Lambda_4}\delta_L\big(l_{j\ell mn}^{-1}\, h_{j\ell n} \triangleright' \{h_{\ell mn}, (g_{mn}g_{\ell m}) \triangleright h_{jk\ell}\}_{\rm p}\, l_{jk\ell n}^{-1}(h_{jkn} \triangleright' l_{k\ell mn})l_{jkmn}h_{jmn} \triangleright' (g_{mn} \triangleright l_{jk\ell m})\big)\right). \end{aligned}$$

References

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