Quantum gravitational decoherence from fluctuating minimal length

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Abstract

We introduce a decoherence process due to quantum gravity effects. We assume a foamy quantum spacetime with a fluctuating minimal length coinciding on average with the Planck scale. Considering deformed canonical commutation relations with a fluctuating deformation parameter, we derive a Lindblad-type master equation that yields localization in energy space and decoherence times consistent with the currently available observational evidence. Finally, we discuss possible experimental tests of our model based on cavity optomechanics setups with ultracold massive molecular oscillators.

Introduction

Following early pioneering studies [1], the investigation of the quantum-to-classical transition via the mechanism of decoherence has become a very active area of research, playing an increasingly central role in the research area on the foundations of quantum mechanics. In broad terms, decoherence appears to be due to the inevitable interaction and the ensuing creation of entanglement between a given quantum system and the environment in which it is embedded. Among the various ideas that revolve around the concept of decoherence induced by gravity, a significant portion deals with the high-energy regime in which quantum and gravitational effects are deemed to be comparably important. Although a complete and consistent theory of quantum gravity is still lacking, different current candidates predict the existence of a minimal length at the Planck scale. An immediate consequence of this common aspect is the breakdown of the Heisenberg uncertainty principle (HUP). Therefore, as firstly derived in the framework of string theory [2], at the Planck scale the HUP must be superseded by a generalized uncertainty principle (GUP), whose minimal one-dimensional expression reads

$$\Delta X \Delta P \geq \frac{\hbar}{2}\left(1+\beta\,\ell_{\mathrm{p}}^2\,\frac{\Delta P^2}{\hbar^2}\right), \tag{1}$$

where β denotes the so-called deformation parameter and ℓ_{p} is the Planck length.

Here, we introduce a model accounting for a universal quantum-gravitational decoherence process [3]. The model assumes deformed canonical commutation relations (DCCRs) leading to the deformed uncertainty relation (1) that accounts for the presence of a minimal length scale. Furthermore, the model assumes the minimal length scale to be a fluctuating quantity, induced by a conjectured foamy character of quantum spacetime, whose average magnitude is fixed at the Planck length scale.

Generalized uncertainty principle and Lindblad-type master equation

In order to define a consistent quantum mechanical theory in Hilbert space from Eq. (1), one can start from the β-deformed canonical commutation relation. For mirror-symmetric states, the uncertainty relation (1) can be straightforwardly derived from the following commutator:

$$[\hat{X},\hat{P}] = i\hbar\left(1+\beta\,\ell_{\mathrm{p}}^2\,\frac{\hat{P}^2}{\hbar^2}\right), \tag{2}$$

where the capital letters label the high-energy position and momentum operators, which are basically different from the usual (low-energy) ones of ordinary quantum mechanics (QM). In the three-dimensional case, the most general counterpart of the previous expression that preserves rotational isotropy is given by

$$[\hat{X}_{\mathrm{j}},\hat{P}_{\mathrm{k}}] = i\hbar\left(\delta_{\mathrm{jk}}+\beta\,\ell_{\mathrm{p}}^2\frac{\hat{P}^2}{\hbar^2}\,\delta_{\mathrm{jk}}+\beta'\ell_{\mathrm{p}}^2\,\frac{\hat{P}_{\mathrm{j}}\hat{P}_{\mathrm{k}}}{\hbar^2}\right), \qquad \mathrm{j,k}=\{1,2,3\}\,, \tag{3}$$

where $\hat{P}^2=\sum_{\mathrm{k}}\hat{P}_{\mathrm{k}}^2$. As frequently done in the literature, in the following we will consider the case $\beta'=2\beta$. In light of Eq. (3), it is not possible to simply recover the position and momentum operators of ordinary QM. However, a suitable choice for the above operators that complies with the deformed canonical commutation relations (DCCRs) yields $\hat{X}_{\mathrm{j}}=\hat{x}_{\mathrm{j}}$, $\hat{P}_{\mathrm{k}}=\left(1+\beta\,\ell_{\mathrm{p}}^2\,\hat{p}^2/\hbar^2\right)\hat{p}_{\mathrm{k}}$, where $\hat{x}_{\mathrm{j}}$ and $\hat{p}_{\mathrm{k}}$ satisfy the original HUP. Collecting the above results, we can now seek the proper generalization of the Schrödinger time evolution equation. Starting from $\hat{X}_{\mathrm{j}}$, $\hat{P}_{\mathrm{j}}$ and introducing $H=H_0+V$ with $H_0=p^2/2m$, it is immediate to achieve the modified quantum mechanical evolution equation for pure quantum states $|\psi\rangle$ in the form

$$i\hbar\,\partial_t|\psi\rangle = \left(H+H_\beta\right)|\psi\rangle\,, \qquad H_\beta = 4\,\frac{\beta\,m\,\ell_{\mathrm{p}}^2}{\hbar^2}\,H_0^2\left(1+\frac{\beta\,m\,\ell_{\mathrm{p}}^2}{\hbar^2}\,H_0\right). \tag{4}$$

Now, the magnitude of the deformation parameter β is commonly estimated to be of order unity [2]. From a more general perspective, β may be regarded as a dynamical variable whose sign in several significant works is taken as either positive or negative. The above wide spectrum of estimates is compatible with the possibility that spacetime fluctuates as the Planck scale is approached, as predicted by major theoretical frameworks. In turn, this suggests that spacetime fluctuations fix the minimal length scale only on average, thereby making the associated deformation parameter itself a fluctuating quantity. Equipped with a random β, Eq. (4) is promoted to a (linear) stochastic Schrödinger equation. Since the fluctuations in β should be related to the fluctuations of the metric tensor near the Planck threshold, within the framework of non-relativistic quantum mechanics it is natural to assume β to be a white noise with a fixed mean and a sharp auto-correlation, so that

$$\beta=\sqrt{t_{\mathrm{p}}}\,\chi(t)\,, \qquad \langle\chi(t)\rangle=\bar{\beta}\,, \qquad \langle\chi(t)\,\chi(t')\rangle=\delta(t-t')\,, \tag{5}$$

where $\langle\ldots\rangle$ denotes an average over fluctuations whose intensity is provided by the Planck time t_{p} and $\bar{\beta}$ is the fixed mean, which is set to $1/\sqrt{t_{\mathrm{p}}}$ to obtain $\langle\beta\rangle=1$. Finally, from the β-deformed stochastic Schrödinger equation (4) for the state vector $|\psi\rangle$, it is straightforward to derive the ensuing β-deformed stochastic Liouville-von Neumann equation for the dynamics of the quantum density matrix $\varrho=|\psi\rangle\langle\psi|$, that is

$$\partial_t\varrho = -\frac{i}{\hbar}\left[H+H_\beta,\varrho\right]. \tag{6}$$

By switching back and forth from the interaction picture to enforce the Born-Markov approximation before averaging over fluctuations to study the evolution of the averaged density matrix ρ, one eventually ends up with the Lindblad-type [4] master equation

$$\partial_t\rho(t) = -\frac{i}{\hbar}\left[H_0,\rho(t)\right]-\sigma\left[H_0^2,\left[H_0^2,\rho(t)\right]\right], \qquad \sigma=\frac{16\,m^2\,\ell_{\mathrm{p}}^4\,t_{\mathrm{p}}}{\hbar^6}. \tag{7}$$

In the previous expression, the so-called dissipator is handily identified as the second term in the r.h.s.. As required, in the limit $\sigma\approx 0$ we recover the standard Liouville-von Neumann equation and the corresponding unitary dynamics.

Quantum gravitational decoherence

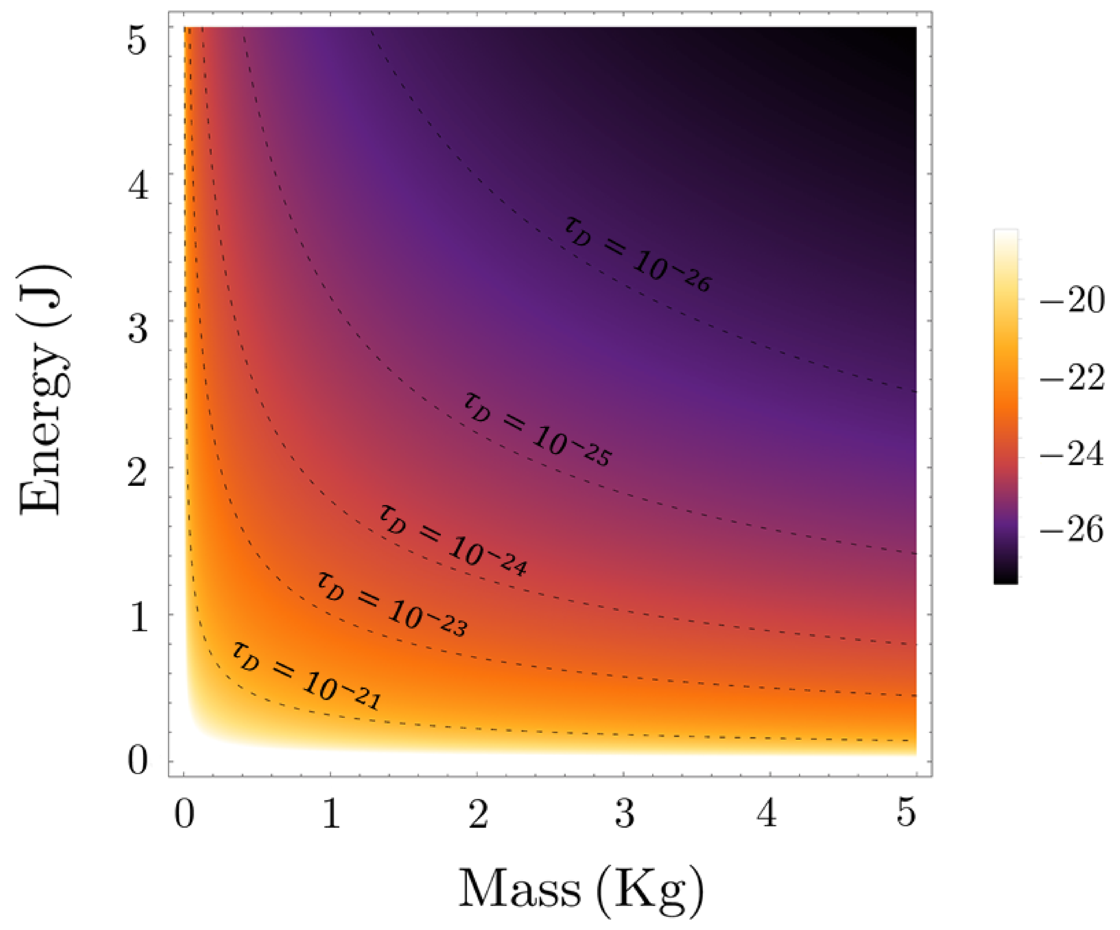
In order to estimate the decoherence time, it is convenient to work in the momentum representation, where the density matrix elements are given by $\rho_{p,p'}(t)=\langle\mathbf{p}|\,\rho(t)\,|\mathbf{p}'\rangle$. By using the notation $E(p)=p^2/2m$ and solving the differential equation (7), one arrives at

$$\rho_{p,p'}(t)=\exp\left[-\frac{i\left(E(p)-E(p')\right)t}{\hbar}-\sigma\left(\Delta E^2\right)^2 t\right]\rho_{p,p'}(0)\,, \tag{8}$$

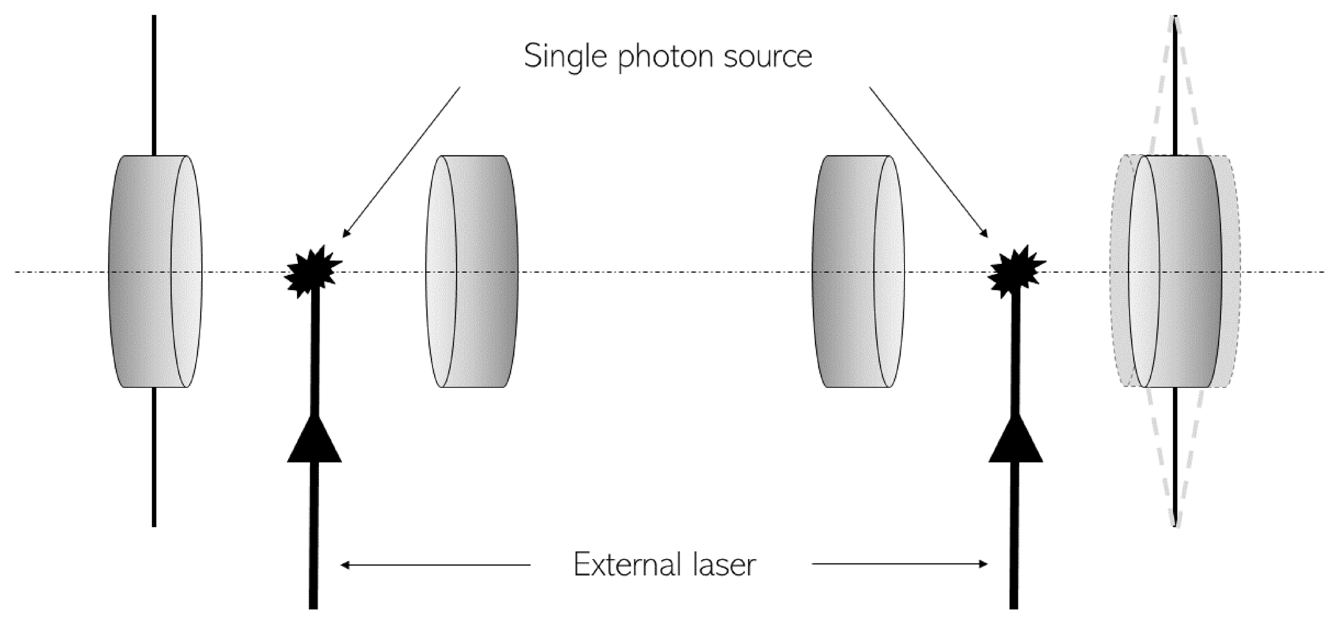
with $\Delta E^2=\left(E^2(p)-E^2(p')\right)$. From Eq. (8), we observe that the time evolution preserves the diagonal elements whereas, as long as $\Delta E^2\neq 0$, the off-diagonal terms of the density matrix decay exponentially, thereby realizing an effective localization in energy. We can then identify the ensuing decoherence time τ_{D}:

$$\tau_{\mathrm{D}}=\frac{1}{\sigma\,(\Delta E^2)^2}=\frac{\hbar^6}{16\,m^2\,\ell_{\mathrm{p}}^4\,t_{\mathrm{p}}\,(\Delta E^2)^2}. \tag{9}$$

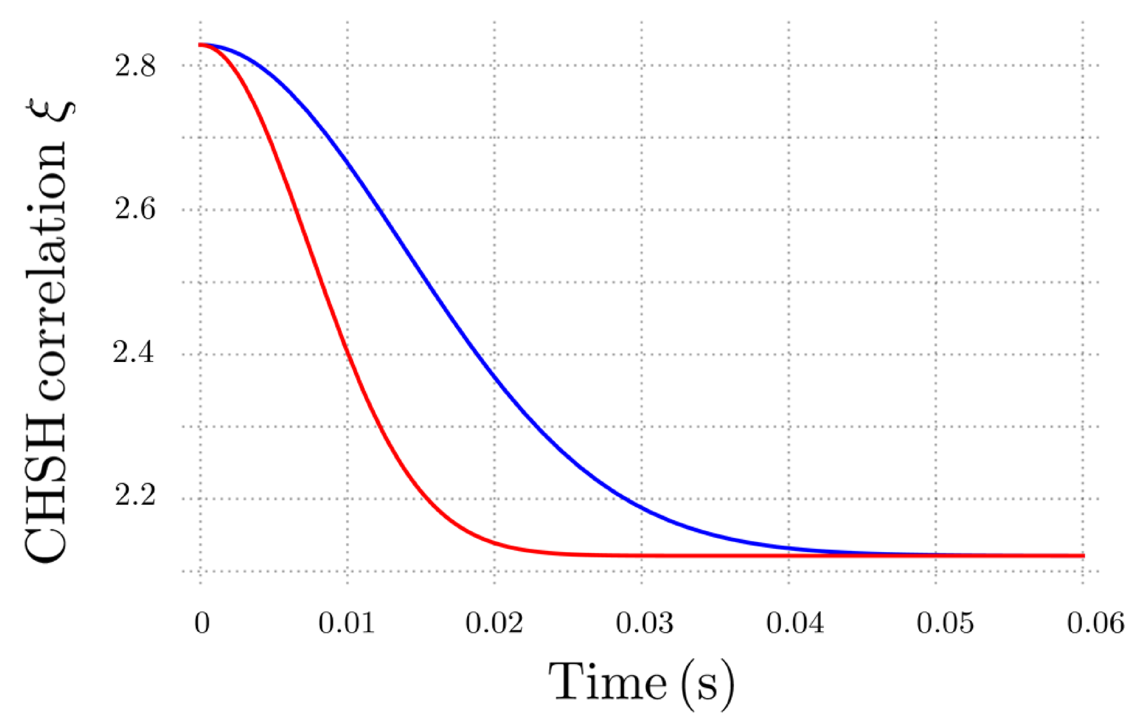
The decoherence time is strongly influenced by the actual energy regime: the larger the deviation from the Planck scale is, the longer τ_{D} becomes, as the picture below conveys.



Concerning the experimental verification, a promising route consists in relying on cavity optomechanics, which has the advantage of allowing for the implementation of universal schemes for decoherence detection. Specifically, in Ref. [5] the authors have introduced a very general method to verify any gravitational decoherence model starting from a readout of the loss of entanglement between two prepared subsystems. This phenomenon can be revealed by means of Clauser-Horne-Shimony-Holt (CHSH) correlation measurements. Building on the above framework for the quantification of decoherence, we now discuss an optomechanical scheme for putting our model of quantum gravitational decoherence to the test. Let us consider two optomechanical systems where entangled photonic qubits are created in different conditions. In particular, one of the cavities possesses two fixed mirrors, while the other one is prepared so as to be subject to gravitational decoherence by allowing one of the mirrors to be movable [5]. In both cavities, the mechanical oscillator is an atom or a molecular structure trapped in a harmonic potential and the two systems are prepared initially in an entangled state. By applying an external laser field, these oscillators jump from the ground state to an excited level; they then decay back to the ground state by emitting the cavity radiation that allows for an accurate study of their vibrational modes.



In a realistic environment, besides gravitational decoherence, we must necessarily take into account the phenomenon of mechanical heating, which further contributes to the loss of quantum coherence between the perfectly correlated photon sources. The two effects, however, can be made comparable with a suitable choice of the experimental parameters [3], although the required values are still demanding: temperatures of the order of 10 nK, cavity quality factor of 10^{10}, photon sources with a mass of 10^{-16} Kg, etc. Nevertheless, under these circumstances, the discrepancy between the presence (red curve in the plot below) or absence (blue curve in the plot below) of a gravitational decoherence mechanism would be sharp and easily accessible, in that it would predict a faster drop for the correlation function on time scales of the order of 10^{-2} s.



On a final note, it is worth stressing that the very same setting described above can in principle be used to test different shapes of gravitationally-induced deformations of quantum mechanics,which in turn are associated with different models of quantum gravity. Along this line, a surprising result is related to the fact that, in some of these scenarios, the required experimental parameters are within the reach of the available technology [6].

References

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