## Common challenges in quantum gravity

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## Diversity of ideas



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Important to learn from each other, and to look for shared challenges and principles.

I will present a biased selection of challenges.

## Our universe is Lorentzian

... most of the times.

## Euclidean Quantum Gravity <br> Why?!?

## Euclidean Quantum Gravity

Why did/do we do Euclidean quantum gravity? $\quad Z_{E}=\int \mathscr{D} \operatorname{geom}_{\mathrm{E}} \exp \left(-\mathrm{S}\left(\mathrm{geom}_{\mathrm{E}}\right)\right) \quad \mathrm{v}_{\mathrm{s}} \quad Z_{L}=\int \mathscr{D} \operatorname{geom}_{\mathrm{L}} \exp \left(\imath \mathrm{S}\left(\operatorname{geom}_{\mathrm{L}}\right)\right)$

- Wick rotation: successful strategy in QFT
- Computational techniques: Monte Carlo vs. few techniques for complex amplitudes
- Thermodynamic interpretation


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No well-defined Wick rotation.
[Ambjorn, Loll, Jurkiewicz et al. 98+: Causal Dynamical Triangulations]
[Talks by Ambjorn, Goerlich]


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- No destructive interference which might cancel out unwanted configurations


## Computational techniques for complex amplitudes

[Talks by Asante, Feldbrugge]

- deformation of integration contour, possible choice - Lefschetz thimbles (Semi-classical and numerical)
- Holomorphic gradient flow / Monte Carlo on Lefschetz thimbles (Numerical,"experimental")
- Acceleration operators for series convergence (for sums and integrals) (Numerical)
- Tensor network renormalization
(Numerical, so far up to three dimensional systems)
- Asymptotic Safety
(Lorentzian configuration space, Lorentzian renormalization flow)
- Machine learning, quantum simulations, ....
(To be explored: Scaling?)
[Witten; Turok, Feldbrugge, Lehners et al ;Asante, BD, Padua-Arguelles, ... ]
[ QCD:Alexandru et al, Spin foams: Han et al, Regge:Jia]
[Effective Spin foam cosmology: BD, Padua-Arguelles 23]
[Cunningham, Delcamp, BD, Martin-Benito, Mizera, Steinhaus; Ito, Kadoh, Sato]
[Biemans, Platania, Saueressig; Knorr, Platania, Schiffer;
Fehre, Litim, Pawlowski, Reichert; Rejzner et al ]
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Challenge: Methods for large scale systems.

## Acceleration techniques for series convergence <br> [Schmidt 4I, Shanks 55, Wynn 56, ...] [BD, Padua-Arguelles 23]

A quite simple technique that allows to treat oscillating sums and integrals.
Contour deformation not necessary. But reproduces results for integrals treated via contour deformation.

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Rel. Error $\sim 10^{-8}$

For computation of expectation value: (Remember this plot)

## Action quadratic in discrete variable

$$
\operatorname{Re} @ \sum_{k}^{N} \exp \left(l\left(\frac{x}{10}-1\right)^{2}\right)
$$

Partial sums for an action quadratic in the summation variable

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$\operatorname{Re} @ \sum_{k}^{N} \exp \left(l\left(\frac{x}{10}-1\right)^{2}\right)$

50
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Partial sums for an action quadratic in the summation variableTrue saddle pointPseudo saddle point

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Lorentzian path integral: deformation of contour/ Picard-Lefschetz

- Convergence criterium automatically selects 'right' contour
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Example: One-loop evaluation of 3D Regge quantum gravity can be reduced to Pachner moves:
Eucl:

$$
\begin{aligned}
& \sim \int \exp \left(-p_{3-2} \lambda^{2}\right) \mathrm{d} \lambda \\
& \sim \int \exp \left(+i p_{3-2} \lambda^{2}\right) \mathrm{d} \lambda
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$\sim \int \exp \left(+p_{4-1} \lambda^{2}\right) \mathrm{d} \lambda$
$+\int \exp \left(-l p_{4-1} \lambda^{2}\right) \mathrm{d} \lambda$
Conformal and gauge modes

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Challenge: Which kind of Euclidean simulations/ calculations to trust?

## Lorentzian vs Euclidean configuration space

What kind of Lorentzian configurations to sum over?

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## Causal Dynamical Triangulations

does forbid such configurations.
Leads to a new universality class of random triangulations and an interesting continuum limit in 4 dimensions.
[Ambjorn, Loll I998+] [without preferred slicing: Jordan, Loll 20I3]

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Lorentzian geometry fully encoded in causal relations and volume=number of points.
[from Surya 2019]

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$b_{1}>b_{2}+a$


Configurations with irregular light cone structure appear to be generic.
Appear even in configurations describing mini-superspace cosmology.
Results indicated that such configurations should be included in the path integral. [Asante, BD, Padua-Arguelles 2022]

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[Talk by Asante]


Which choice of sign for the path integral?

Suppressing sign:
Mechanism for suppressing
light cone irregular configurations.

Possibility to reconcile CDT and Causal Sets positions?

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Complex (boundary) terms in continuum action.
[Neiman I3]

Gravitational thermodynamics from Lorentzian path integrals: Need to include co-dimension 2 singularities. Also for replica copies. These are already there in simplicial approaches/ Regge gravity!

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[Louko-Sorkin 95]
[Witten 21, Lehners 21, ...]

Challenges: Understand better role of complex structures for Lorentzian gravitational path integral. Canonical Formalism?
Should we include configurations describing topology change?
Complex actions for sum over topologies?

## Lorentzian quantum gravity

- More techniques for Lorentzian path integrals
- Lorentzian configurations have additional light cone structure: can be (often) irregular
- Irregular light cone structure leads to branch cuts for the gravitational action and imaginary terms
-Which Lorentzian configurations to include in the path integral? Which side of the branch cut?
-Thermodynamical interpretation?

What is the structure of quantum space time?

## Observables, (non-) locality and quantum space-time

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Quantum Gravity:
Relational observables [Einstein, ...]

Fields $\phi(x, t)$ are not observables. No n-point functions.
$\phi\left(\psi_{1}, \psi_{2}, \psi_{3}, \psi_{4}\right)$
[Talks by Kiefer, Ferrero, Giacomini, Hoehn, Menendez-Pidal, Rastgoo, Ruf, ...]

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But there seem to be no perfect clocks in our (relativistic) universe.

[Giddings, Marolf, Hartle 05; BD, Tambornino 06, Giddings, Donnely I5; Hoehn et al 20+]
Chaos and other aspects: [ Bojowald, Hoehn, et al. I2+, BD, Hoehn, Kosolowski, Nelson I7 ] End of time: [Talks by Gielen, Menendez-Pidal]

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How much can we resolve space-time?

$$
\begin{aligned}
& \begin{array}{l}
\text { Bound on resolution } \\
\text { with relativistic clocks: }
\end{array} \quad[\phi(\psi), \phi(\psi+\epsilon)]=G(\psi, \psi+\epsilon)\left(1+\frac{\operatorname{Energy}(\phi)}{\operatorname{Energy}(\psi))}\right)
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Increasing energy (and size) of clock field leads to black holes. $\Rightarrow$ Locality Bounds


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[Giddings, Marolf, Hartle 05, BD, Tambornino 06] A new uncertainty principle?

New uncertainty for Time of arrival operator $\Delta t>1 / \bar{E}$.
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High energy scattering leading to black holes: "The end of short distance physics". UV-IR mixing.

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[BD,Tambornino 06]

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[Banks,Fischler 99;
Giddings-Thomas 0I, Dvali et al...]

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What is the resulting structure of quantum space time?

- fractal dimensions/ space-times
- non-commutative space-time
- geometric operators (with discrete spectra)
- holography
- local holography
- matrix and tensor models
- group field theories

Space time emergent

- strings
- area metrics
- twistor space
- relative locality and Born duality

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Challenge: Observable algebra and symmetries $\leftrightarrow$ Structure of quantum space-time


## Reconstructing our universe



Challenge: Bridge enormous number of scales. Understand renormalization and develop (common) language for 'emergence' of space-time.

## Understanding our universe

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"So far we have not seen any signatures of quantum gravity."
"Are we now really at the dawn of quantum gravity phenomenology?" [Talk by Amelino-Camelia]

But there are many features of our universe asking for explanation:

- Quantum theory?
- (Macroscopic) four-dimensional smooth space-time
- Lorentzian signature
- Gravitational dynamics
- Small and positive cosmological constant
[Talk:Yasaman]
- Inflation?
- (Simple) initial conditions
- Matter
- Dark Matter
- Types of matter and values of matter couplings



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    [Ambjorn, Nielsen, Rolf, Savvidy 97 .
    [Borissova, BD, Qu, Schiffer,TA]

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