Lorentzian Quantum Gravity and the Graviton Spectral Function

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Quantum Gravity 2023

Radboud University, Nijmegen, 10. July 2023

Fehre, Litim, Pawlowski, MR: PRL, 2111.13232

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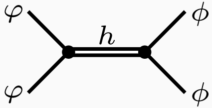
Towards testing Unitarity

A unitary theory requires

- Well-behaved propagators without ghost or tachyonic instabilities
- Bounds on scattering amplitudes, e.g., violated by GR



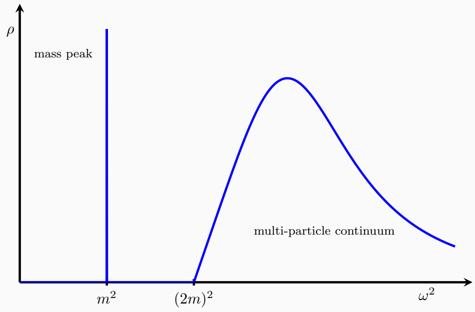
$$\overset{\text{GR}}{\propto} (p_1 + p_2)^2 \not\lesssim 1$$

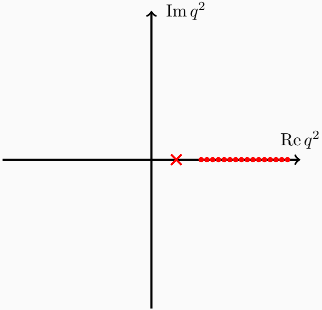
Need access to correlation functions at time-like momenta

Källén-Lehmann spectral representation

[Källén '52; Lehmann '54]

$$\mathcal{G}(q^2) = \int_0^\infty \frac{\mathrm{d}\lambda^2}{\pi} \frac{\rho(\lambda^2)}{q^2 - \lambda^2} \qquad \text{with} \qquad \rho(\omega^2) = -\lim_{\varepsilon \to 0} \operatorname{Im} \mathcal{G}(\omega^2 + i\varepsilon)$$

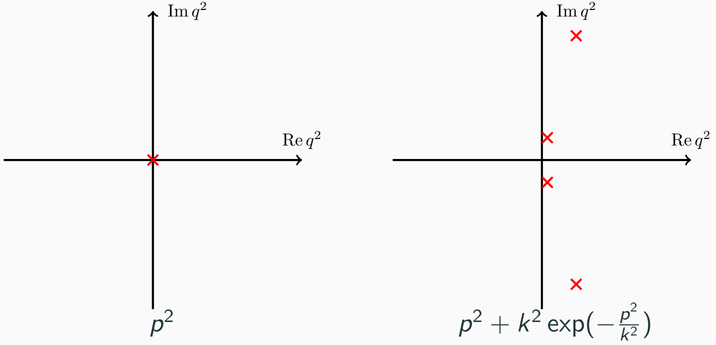




Most non-perturbative methods only provide numerical data for $q^2 < 0$

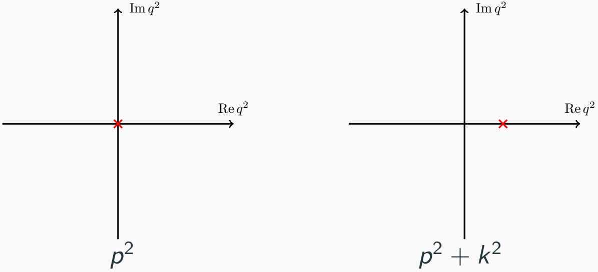
Standard Euclidean Functional Renormalisation Group

- Regulator $R_k(p^2)$ implements integrating out of fluctuations
- Modified dispersion $p^2 \to p^2 + R_k(p^2)$ introduces poles and cuts
- Can not use spectral representation at finite k
- Analytic continuation possible at $k = 0$ [Bonanno, Denz, Pawlowski, MR '21]



Spectral Renormalisation Group

- Callan-Symanzik cutoff $R_k \sim k^2$ allows use of spectral representation



Spectral Renormalisation Group

[Fehre, Litim, Pawlowski, MR '21; Braun, ..., MR, et al '22]

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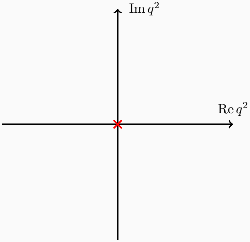
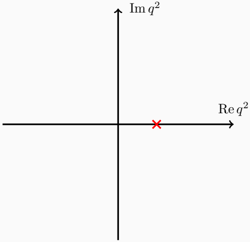


p^2



$p^2 + k^2$

- Finite flow equation with counterterms

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr}\, \mathcal{G}_k\, \partial_t R_k - \partial_t S_{\mathrm{ct},k}$$

[Fehre, Litim, Pawlowski, MR '21; Braun, ..., MR, et al '22]

- Dimensional regularisation of UV divergences in $d = 4 - \varepsilon$ possible

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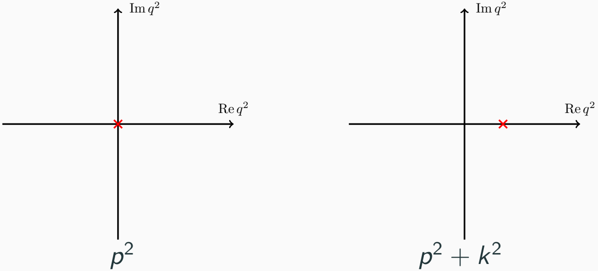

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- Related approach: talk by Rejzner [D'Angelo, Drago, Pinamonti, Rejzner 22; D'Angelo, Rejzner 23]

Lorentzian setup

[Fehre, Litim, Pawlowski, MR '21]

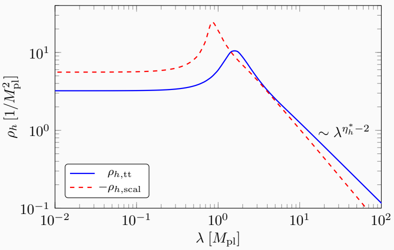
- Einstein-Hilbert action expanded about flat Minkowski background
- Direct flow of ρ_h with $m_h^2 = k^2(1+\mu)$ and $Z_h = Z_h(p^2 = -m_h^2)$

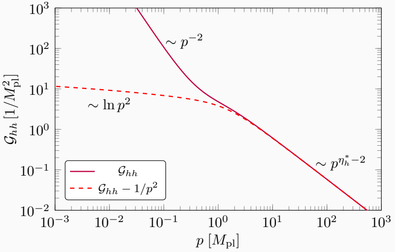
$$\rho_h = \frac{1}{Z_h}\Big[2\pi\delta(\lambda^2 - m_h^2) + \theta(\lambda^2 - 4m_h^2) f_h(\lambda)\Big]$$

- Use ρ_h in flow diagrams

$\partial_t \rho_h \propto$  $+ \dots$ with $\mathcal{G}_h(q^2) = \int_0^\infty \frac{\mathrm{d}\lambda^2}{\pi} \frac{\rho_h(\lambda^2)}{q^2 - \lambda^2}$

Graviton spectral function





[Fehre, Litim, Pawlowski, MR '21; Assant, Litim, MR (in prep)]

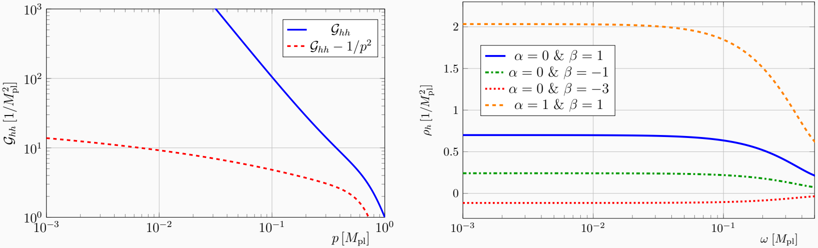
- Massless graviton delta-peak with multi-graviton continuum
- No ghosts and no tachyons
- Good agreement with reconstruction result [Bonanno, Denz, Pawlowski, MR '21]
- Direct relation to form factors $Cf_C(\Box)C$ and $Rf_R(\Box)R$

Comparison to effective field theory

One-loop effective action:

$$\Gamma_{\text{1-loop}} = S_{\text{EH}} + \int_x \sqrt{g}\left(c_1 R \ln(\Box) R + c_2 C_{\mu\nu\rho\sigma} \ln(\Box) C^{\mu\nu\rho\sigma}\right) + \dots$$

Gauge-fixing $\quad S_{\text{gf}} = \frac{1}{\alpha}\int_x F_\mu^2 \quad$ with $\quad F_\mu = \bar{\nabla}^\nu h_{\mu\nu} - \frac{1+\beta}{4}\bar{\nabla}_\mu h^\nu{}_\nu$



[Pawlowski, MR '23]

Computation matches EFT the IR

Propagator is gauge-dependent but pole structure is typically not

[Kluth, Litim, MR '22]

Summary

- First direct Lorentzian fRG computation in QG
- Full 1PI graviton propagator
- EFT in the IR, Safety in the UV
- No ghosts, no tachyons: no indications for unitarity violation

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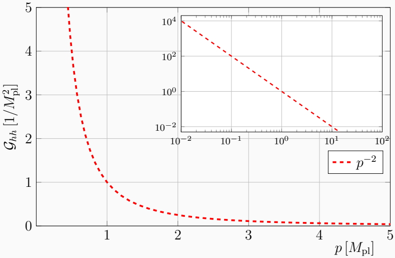
Thank you for your attention!

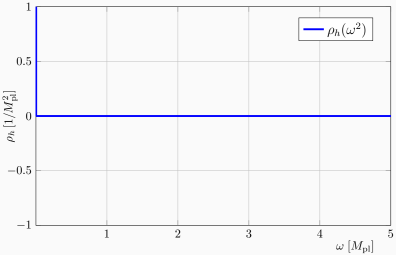
Back-up slides

Classical graviton spectral function

Einstein-Hilbert action: $S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int_x \sqrt{g}\,(2\Lambda - R)$

Flat Minkowski background: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$



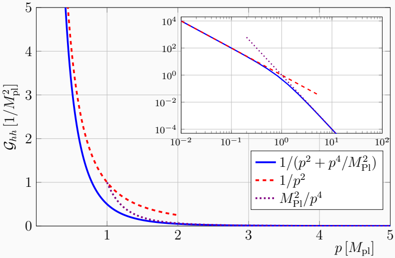


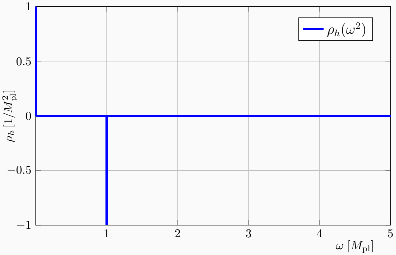
$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2}$

$\rho_h(\omega^2) \sim \delta(\omega^2)$

Classical graviton spectral function

Higher-derivative action: $S_{\text{HD}} = S_{\text{EH}} + \int_x \sqrt{g}\,(aR^2 + bC^2_{\mu\nu\rho\sigma})$

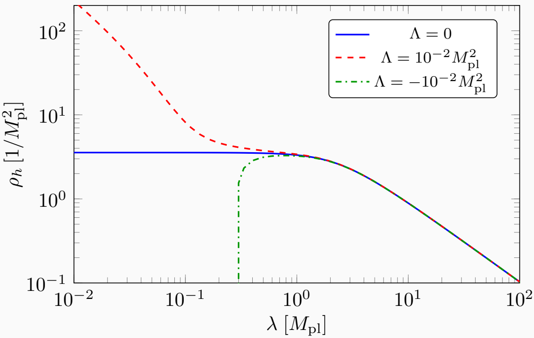




$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2} - \frac{1}{M_{\text{Pl}}^2 + p^2}$$

$$\rho_h(\omega^2) \sim \delta(\omega^2) - \delta(\omega^2 - M_{\text{Pl}}^2)$$

Finite cosmological constant



The cosmological constant acts as off-shell (negative) mass-squared term

Asymptotic behaviour of propagator and spectral function

IR – effective field theory behaviour

- $\mathcal{G}_{hh}(p^2) \sim p^{-2} - c_1 \log p^2 + \dots$
- $\rho_h(\omega^2) \sim \delta(\omega^2) + 2\pi c_1 + \dots$

UV – asymptotically safe scaling

- $\mathcal{G}_{hh}(p^2) \sim p^{-2+\eta_h^*} \left(\log p^2\right)^{-\gamma}$
- $\rho_h(\omega^2) \sim 2\,\omega^{-2+\eta_h^*} \left(\log \omega^2\right)^{-\gamma} \left(\sin\left[\frac{\pi}{2}\eta_h^*\right] - \cos\left[\frac{\pi}{2}\eta_h^*\right] \frac{\pi\gamma}{\log \omega^2}\right)$

Normalisation

$$\int_0^\infty \frac{\mathrm{d}\lambda}{\pi}\, \lambda\, \rho(\lambda) = \begin{cases} 0 & \text{if} \quad \eta < 0 \text{ or } (\eta = 0 \wedge \gamma > 0) \\ 1 & \phantom{\text{if}} \quad \eta = 0 \wedge \gamma = 0 \\ \infty & \phantom{\text{if}} \quad \eta > 0 \text{ or } (\eta = 0 \wedge \gamma < 0) \end{cases}$$