Theoretical Basis for Low Energy Quantum Gravity and Quantum Information Phenomena

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Part 1: Quantum Gravity: Quest for the micro/meso structures of spacetime from low energy up. Very different route from Top-down theories: Emergent Gravity

Part 2: **Grav. Decoherence and GravCats** Based on

C. **Anastopoulos** and B. L. Hu, "***Probing a Gravitational Cat State***" *Class. Quant. Grav. 32, 165022 (2015).* [*arXiv:1504.03103*]

Manneli **Derakhshani**, Charis **Anastopoulos** and B. L. Hu, "*Probing a Gravitational Cat State: Experimental Possibilities*" [arXiv:1603.04430]

Part 3: Graviton Noise

H. T. Cho and B. L. Hu, Quantum noise of gravitons and stochastic force on geodesic separation, Phys. Rev. D 105, 086004 (2022).

____Graviton Noise on Tidal Forces and Geodesic Congruence, Phys. Rev. D107, 084005 (2023)

Disclaimers:

1) This talk is not about quantum experiments in gravity or analog gravity. See nice reviews, C. Barceló, S. Liberati, M. Visser, Analogue Gravity, LivRevRel **14**, 3 (2011) Jacquet MJ, Weinfurtner S, König F. 2020 The next generation of analogue gravity experiments. Phil.Trans.R.Soc.A378: 20190239.
2) I'm not an experimentalist, nor am I a versatile theorist capable of doing quality experiments. Listen to the experts: Markus' talk
3) This is not a review. There are many excellent reviews in the literature E.g., de Boer, Dittrich et al: *Frontiers of Quantum Gravity: shared challenges, converging directions* [arxiv:2207.10618]. (Another review with references is given in the next 3 slides.)
4) \`Table-top' can be as big as LIGO/Virgo, extending to deep space at earth-moon distance: e.g., DSQI and further, e.g. LISA.
5) So, *what am I doing here?* Instead, I'd like to discuss some fundamental theoretical issues of low energy (perturbative) quantum gravity when quantum information issues (like decoherence and entanglement) are invoked in any proposed table-top experiment.

Review

Can We Detect the Quantum Nature of Weak Gravitational Fields?

Universe **2021**, 7, 414.

Francesco Coradeschi [1], Antonia Micol Frassino [2], Thiago Guerreiro [3,*] and Jennifer Rittenhouse West [4] and Enrico Junior Schioppa [5,6]

Prior to the 2016 GW discovery papers, proposals for experimental probes of quantum gravity included gamma-ray bursts [5], Michelson interferometers of laboratory scale [6], ultra-high energy cosmic rays and colliders [7], gravitons in hadron collider signatures [8], experimental inconsistencies with an alternative to quantum gravity (the semiclassical Einstein equations) [9], the running of the gravitational coupling G [10,11], quantum corrections to gravitational scattering [12,13], molecular interferometry [14], Lorentz violating signatures and constraints [15], and many others [16], spanning both model-dependent spaces (e.g., string theory or loop quantum gravity-dependent) and model independent parameter spaces.

From 2016 and onward, an increase in the already widespread interest in detecting signatures of quantum gravity was seen in a growing number of new (or renewed) experimental solutions including those—such as interferometers—that could detect possible weak signals in the already weak realm of GW. In fact, although GR correctly explains all current GW observations [17–19] and laboratory tests of gravity [20], it is still possible that GW is a window into signatures of the quantization of gravity [21]. However, GW observatories such as LIGO and Virgo are not the only candidate detectors. Cavity opto-mechanical systems offer complementary windows to astrophysical and spectroscopic measurements [22]. Recent proposals in phenomenological quantum gravity range from quantum gravity in electromagnetic cavities [23], quantum gravity in gravitational wave detectors [24,25], quantum table-top experiments in the lab [26], quantum gravity induced quantum entanglement [27] to interferometers with rotational sensitivity [28,29], and those sensitive to quantum spacetime geometry [30,31] (for conjectured holographic quantum

5. Amelino-Camelia, G.; Ellis, J.R.; Mavromatos, N.E.; Nanopoulos, D.V.; Sarkar, S. Tests of quantum gravity from observations of gamma-ray bursts. *Nature* **1998**, *393*, 763–765. [CrossRef]
6. Hogan, C.J. Interferometers as Probes of Planckian Quantum Geometry. *Phys. Rev. D* **2012**, *85*, 064007, [CrossRef]
7. Hossenfelder, S.; Bleicher, M.; Hofmann, S.; Ruppert, J.; Scherer, S.; Stoecker, H. Collider signatures in the Planck regime. *Phys. Lett. B* **2003**, *575*, 85–99. [CrossRef]
8. Mirabelli, E.A.; Perelstein, M.; Peskin, M.E. Collider signatures of new large space dimensions. *Phys. Rev. Lett.* **1999**, *82*, 2236–2239. [CrossRef]
9. Page, D.N.; Geilker, C.D. Indirect Evidence for Quantum Gravity. *Phys. Rev. Lett.* **1981**, *47*, 979–982. [CrossRef]
10. Hamber, H.W.; Williams, R.M. Non-perturbative gravity and the spin of the lattice graviton. *Phys. Rev. D* **2004**, *70*, 124007, [CrossRef]
11. Hamber, H.W. *Quantum Gravitation: The Feynman Path Integral Approach*; Springer: Berlin, Germany, 2009. [CrossRef]
12. Donoghue, J.F. General relativity as an effective field theory: The leading quantum corrections. *Phys. Rev. D* **1994**, *50*, 3874–3888. [CrossRef] [PubMed]
13. Burgess, C.P. Quantum gravity in everyday life: General relativity as an effective field theory. *Living Rev. Relativ.* **2004**, *7*, 5–56. [CrossRef]
14. Carlip, S. Is Quantum Gravity Necessary? *Class. Quantum Gravity* **2008**, *25*, 154010, [CrossRef]
15. Jacobson, T.; Liberati, S.; Mattingly, D. Threshold effects and Planck scale Lorentz violation: Combined constraints from high-energy astrophysics. *Phys. Rev. D* **2003**, *67*, 124011, [CrossRef]

Planck Scale Effects in Astrophysics and Cosmology, edited by G. Amelino-Camelia and J. Kowalski-Glikman (Springer, Berlin, 2005). Hossenfelder, S.; Smolin, L. *Phenomenological Quantum Gravity*. Phys. Can. 2010, 66, 99-102. Amelino-Camelia, G. *Quantum-Spacetime Phenomenology*. Living Rev. Relativ. 2013, 16, 5

16. Hossenfelder, S. (Ed.) *Experimental Search for Quantum Gravity*; FIAS Interdisciplinary Science Series; Springer: Berlin, Germany, 2018.
17. Abbott, B.P.; Abbott, R.; Abbott, T.D.; Acernese, F.; Ackley, K.; Adams, C.; Adams, T.; Addesso, P.; Adhikari, R.X.; Adya, V.B.; et al. Gravitational Waves and Gamma-Rays from a Binary Neutron Star Merger: GW170817 and GRB 170817A. *Astrophys. J.* **2017**, *848*, L13. [CrossRef]
18. Callister, T.; Biscoveanu, A.S.; Christensen, N.; Isi, M.; Matas, A.; Minazzoli, O.; Regimbau, T.; Sakellariadou, M.; Tasson, J.; Thrane, E. Polarization-Based Tests of Gravity with the Stochastic Gravitational-Wave Background. *Phys. Rev. X* **2017**, *7*. [CrossRef]
19. Abbott, B.; Abbott, R.; Abbott, T.; Acernese, F.; Ackley, K.; Adams, C.; Adams, T.; Addesso, P.; Adhikari, R.; Adya, V.; et al. Search for Tensor, Vector, and Scalar Polarizations in the Stochastic Gravitational-Wave Background. *Phys. Rev. Lett.* **2018**, *120*. [CrossRef]
20. Sushkov, A.O.; Kim, W.J.; Dalvit, D.A.R.; Lamoreaux, S.K. New Experimental Limits on Non-Newtonian Forces in the Micrometer Range. *Phys. Rev. Lett.* **2011**, *107*. [CrossRef]
21. Calcagni, G.; Kuroyanagi, S.; Marsat, S.; Sakellariadou, M.; Tamanini, N.; Tasinato, G. Quantum gravity and gravitational-wave astronomy. *J. Cosmol. Astropart. Phys.* **2019**, *2019*, 012. [CrossRef]
22. Bonaldi, M.; Borrielli, A.; Chowdhury, A.; Di Giuseppe, G.; Li, W.; Malossi, N.; Marino, F.; Morana, B.; Natali, R.; Piergentili, P.; et al. Probing quantum gravity effects with quantum mechanical oscillators. *Eur. Phys. J. D* **2020**, *74*. [CrossRef]
23. Guerreiro, T. Quantum effects in gravity waves. *Class. Quantum Gravity* **2020**, *37*, 155001. [CrossRef]
24. Parikh, M.; Wilczek, F.; Zahariade, G. Quantum Mechanics of Gravitational Waves. *Phys. Rev. Lett.* **2021**, *127*, 081602. [CrossRef]
25. Parikh, M.; Wilczek, F.; Zahariade, G. Signatures of the quantization of gravity at gravitational wave detectors. *Phys. Rev. D* **2021**, *104*, 046021. [CrossRef]
26. Carney, D.; Stamp, P.C.E.; Taylor, J.M. Tabletop experiments for quantum gravity: A user's manual. *Class. Quantum Gravity* **2019**, *36*, 034001. [CrossRef]
27. Krisnanda, T.; Tham, G.Y.; Paternostro, M.; Paterek, T. Observable quantum entanglement due to gravity. *NPJ Quantum Inf.* **2020**, *6*, 12. [CrossRef]

Quantum Gravity

- **In agreement: QG: theories for the microscopic structure of spacetime.** Same noble quest, but the
- **Approaches & Methods toward this lofty goal differ widely,** well represented in this conference.

- **Conventional approach** (from the 50s on) as practiced in the GR community: Quantize the **metric g** or **connection Γ** form.

One crucial question

remains unanswered, or not directly addressed

- We know **General Relativity** works very well in the description of the **large scale structure** and dynamics of spacetime.
- Metrics and connections are **macroscopic variables.**

Why do you believe that quantizing these variables will reveal the microscopic structures?

EM field, yes, Helium 4, yes.

But is this a universal paradigm? No.

Quantization of a collective variable does **not** lead to **micro-structure**

- There is an abundance of examples in condensed matter physics, which deals mainly with the dynamics of **collective variables of atomic interactions**: phonons, rotons, plasmons, excitons, many `on’s:
- One can quantize sound, and study the interaction of phonons with electrons, etc.
- **These `on’s are all quantum entities**, but
- **Quantum =/=> microscopic**.

A crucial question to ask, more important than quantization

Are metric and connection forms

- **fundamental**: depicting the microscopic constituents at the Planck scale? or
- **collective** variables constructed from them?

Is general relativity an effective theory valid only at large scales / low energies? Like hydrodynamics with regard to molecular dynamics

General Relativity is Geometro-**hydro**-dynamics

- **My view:**

GR as Hydro (1996), Can spacetime be a condensate? (2005)
Stochastic gravity as mesoscopic physics (1994)
Cosmology as `condensed matter' physics (1988)

- In reference to Quantum Gravity, General Relativity is of the nature of a **hydrodynamic theory,** valid only in the long wavelength, low energy limits of the microscopic theories at the Planck scale **=>**

- Geometry, manifold structures are derived properties

- Spacetime is an **emergent entity,** so are its symmetries

- **(g, Γ) are collective variables:** Quantizing them gives phonons, not atoms. In fact, sound ceases to exist at the atomic scale, however quantum you make it.

Similar view, **Volovik**: He3 ; **Wen**: tensor network, fermions, light are emergent.
Gravity & Thermodynamics: **Jacobson:** Einstein equation of state (1995)

Alternative: **Posit the micro-variables**

- Strings
- Loops
- Causal sets
- Simplices
- Asymptotic safety (RG)
- Group field theory …

Q: Does your favorite theory depend on some background geometry? If so, where does it come from? (e.g., "string cosmology" – can you show how your FLRW or de Sitter metric emerges from string interactions? String field theory?)

Quantum-Classical vs micro-Macro

Quantum → decoherence, robustness, stability → **Classical**

← **Traditional effort:** quantizing the metric or connection forms

Quantum Gravity (Strings, Loops, Simplices, Causets – micro const.)

micro

fluctuations

coarse-|-graining *MESO* **kinetic theory**

v *Emergent* *spacetime* **hydrodynamics**

MACRO

General Relativity

Issues: Coherence, Correlations, Fluctuations, **Stochasticity**; Collectivity, Variability, Nonlinearity, **Nonlocality**

Besides Quantum-> Classical, an added conceptual dimension need be included in our considerations: micro -> Macro. This points to

the necessity of first identifying the correct micro or meso variables before consideration of quantization.

A. Role of **Fluctuations and Noise**. Need nonequilibrium statistical and stochastic mechanics. Open quantum systems. Even if we think we knew what the microscopic theories are, we still need tools from many-body dynamics and ideas from condensed matter physics to get the macro behavior.

B. Role of **Topology:** more resistant to environmental decoherence

Tasks of
Emergent vs Quantum Gravity

- ***Emergent Gravity*: Top-down** from above Plank energy down
  understand how the macroscopic structure of spacetime emerges/evolves from unknown microscopic structures

- ***Quantum Gravity*: Bottom-up** from today's low energy up
  how to induce/infer micro structures from Macro phenomena

[B. L. Hu, "*Emergent /Quantum Gravity: Macro/Micro Structures of Spacetime*"
DICE 2008. J. Phys. Conf. Ser. 174 (2009) 012015 [arXiv:0903.0878]:

Complementary tasks: need working in Both directions

Emergent Spacetime

- Spacetime emergent from `matter' – geometric structure deductible from interactions of sub-structural constituents.

  -- *Sakharov's `**metric elasticity**',*

  *spurred `induced gravity' in the 80s (*Adler, Zee *et al, not too successful).*

E.g., Build up lattice or condensate from atoms.
Large scale features like crystals or elasticity can have nice geometric depictions, but that fails for the substructures.

Derive hydrodynamics – thermodynamics from molecular dynamics.
Hydrodynamic equations of motion, thermodynamic laws.

- Spacetime is a derived construct, manifold is a representation of the resultant large scale structure of many (likely strongly-interacting) constituent particles *– strings, loops, spin-nets.*

- *HOW are they organized? How do they manifest collectively?*

Issues all Top-Down models need to deal with:

Coarse-graining (CG), RG and Collectivity:

CG measures leading to emergent structure:

a) stability wrt repetition of same CG and

b) robustness wrt variation of CG measures.

Likely new physics appears at successive levels off structure described by nested set of collective variables ffrom micro to meso to macro structures

Beware: Backreaction engenders dissipative dynamics. Quantum open system techniques needed.

Renormalization Group approach:

Reuter and Saueressig et al, Ambjørn, Jurkiewicz, & Loll's long term research program ...
Editorial: Coarse graining in quantum gravity: Bridging the gap between microscopic models and spacetime physics Astrid Eichhorn, Benjamin Bahr, and Antonio D. Pereira, arXiv:2103.14605

- *Beware*: RG running may not be smooth throughout, as new levels of structure may appear with new collective variables describing new interactions.
- Coarse graining measures may need to be adjusted with the appearance of each new level of structure.
- Dissipative dynamics from Backreaction requires real time nonequilibrium RG

Micro-theorists would be working on

Emergent Gravity! **micro =>Macro**

NOTE: Macro-structure is largely insensitive to the details of the underlying micro-structures.

Many micro-theories can share the same macro-structure.

It is the collective properties of these micro structures which show up at the long wavelength, low energy limit.

We should look first at the commonalities of all competing micro-theories rather than their differences, namely, their hydrodynamic limits (at the lowest order approximation) rather than the detailed micro-behavior

This sentiment seems to be shared by Jan de Boer, Bianca Dittrich et al, Frontiers of Quantum Gravity: Shared challenges, converging directions [arxiv:2207.10618]

Need to Deal with Strongly Interacting and Correlated Systems

B. For deductive emergent behavior, path could be tortuous,

- Usually encounters **nonlinear interactions in strongly correlated systems**.
- Need to identify collective variables at successive levels of structure. Cumbersome to deduce M from m-dynamics

(e.g., intermediate between m (molecular) and M (hydro) are kinetic variables. Use maximal entropy laws at stages – but how are they related to each other, becomes maximal when? To identify the collective variables is almost an art.)

- Alert: **nonlocal properties** can emerge. Very involved.
  - requires not just hard work of deduction from one level, but new ideas at every level. Interesting challenge.

Micro locality =/= Macro locality

same for molecular-hydrodynamics and for quantum gravity

No simple correspondence, let alone equivalence, between **locality at the micro and the Macro levels**. NOT even for simple examples of emergence like molecular to hydro- dynamics.

Very different **senses of locality** at the microscopic level of **strings, loops** or causets versus **sense of locality** in our macroscopic spacetime, presumably emergent.

My attitude:

- Take this **inequivalence of micro and macro locality** seriously.

  This is often a rule rather than an exception. We need to depart, even radically, from familiar concepts in our macro world.
- There is **new physics** to be uncovered**!**

Our conception and the construct of the macro world may bear little resemblance to the micro world.

- **(Non) locality at one level may have little to do with (Non) locality at other levels.**
- **The easy way of m→M (weaving) or c→Q (quantizing) may not be the true way.**

Bottom-Up:

Start from low energy theory:

General Relativity as

Geometro-Hydrodynamics

- Gravity/GR is a classical theory which makes sense only at the macro scale. Like phonons, but not atoms,

- Metric and connection forms are collective, not "fundamental", variables. Geometry known in the classical context. (meaning of quantum geometry?)

- GR is an effective theory, the low energy, long wave-length limit of some as yet unknown (unconfirmed) theory (many to one) for the micro structure of spacetime and matter (def of quantum gravity)

- Manifold and metric structure, together with the symmetry of physical laws defined thereby (e.g., Lorentz and gauge invariance), are emergent.

Going **from macro looking for micro** structure is always difficult, if not impossible. BUT not hopeless.. that is how physics has progressed through centuries!

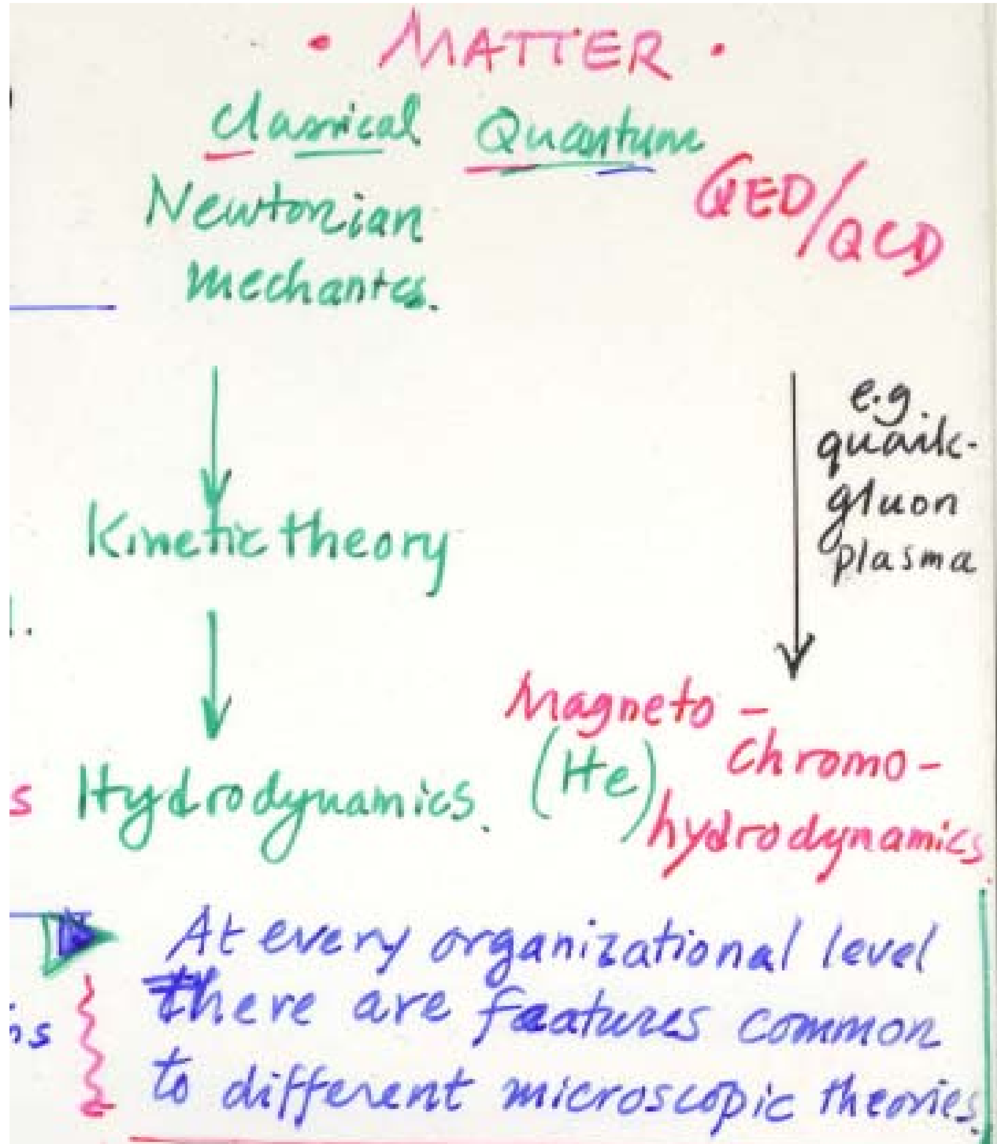
Common Features of Macroscopic Phenomena:

Micro

There are commonalities in the MACroscopic collective behavior of different MICroscopic constituents

Macro

Separate the common features to pinpoint the particulars



Bottom-up: Macro to Micro

We choose to rely on:

A. Topological structures:

More resilient to evolutionary or environmental changes. Many excellent work on topological features in manifolds (wormholes)

Wen, Cirac et al: tensor network, string-nets, emergent spacetime

B. Noise-fluctuations: Fluctuations can reveal some sub-structural contents and behavior (**critical phenomena**).

Information contained in remnants or leftovers.
Reconstruction from corrupted and degraded information

May get some glimpses of the nature of micro-structure.

Deciphering microstructure's generic properties from

Noise and Fluctuation Phenomena

A modest way to move from BOTTOM UP is:

Stochastic Gravity

Main advantage: Minimize speculative assumptions

A natural extension of well known and tested theories:

- Quantum field theory in curved spacetime (e.g., Hawking effect)
- Semiclassical gravity (e.g.,inflationary cosmology)

Part 2 & 3

Quantum Fluctuations in Gravity

Low energy Q G + Quantum Information:

- Part 2:

  Gravitational Decoherence: Energy Basis
  Gravitational Entanglement: Cat states.

- Part 3: Graviton Noise Effects

- Theoretical Basis: Stochastic Gravity, even for low energy GQI phenomena

G (Gravity) Q (Quantum) I (Information) :

3 new developments in the 90s

impacting current research in G-Q-I

- GR+QFT: QFTCST 70s → Semiclassical Gravity 80s → Stochastic Gravity 94 - now.

  For Large fluctuations: Spacetime Foam (Wheeler)

  Carlip's nice review: arXiv:2209.14282v3

  * Nonequilibrium Quantum Processes:

  - Neq. QFT e.g., Calzetta Hu, PRD1988
  - Open Q Systems, esp. nonMarkovian

    e.g.,Hu Paz Zhang PRD1992

- Quantum Information ~ 1995

Quantum (+ or x) Gravity & Information

- **Quantum** ← Quantum Mechanics ← **Quantum Field Theory**

Schroedinger Equation | **GR+QFT= Semiclassical Gravity** | *micro*

- **Gravity** ← Newton Mechanics ← **General Relativity** | *Macro*

Low Energy | *High Energy*

- **Laboratory** conditions: | **Strong Field** Conditions:

Weak field, nonrelativistic limit: | Early Universe, Black Hole

Newton Schroedinger Eq (NSE) | **Semiclassical Einstein Eq**

[*Grav. Q physics*] **Quantum Information**

Gravitational **Decoherence** | Decoherence in Q cosmology (80-90),

Entanglement | structure formation / cosmic radiation inflation

Gravitational Cat State | Stochastic Gravity

(Bottom Up: Quantum Gravity | Emergent Gravity: Top down) [*It from Bit*?]

QI: Two fundamental issues:

Decoherence and Entanglement

1. How does the environment affect the quantum coherence of a system.
2. How an entangled state evolves under the influence of its environment

Rel QI: with S or G Relativity or Gravity

A. Gravitational decoherence by noises of gravitational origin

B. Gravitational cat states from entangling two masses

Part 2: Quantum Info

A. GravDec & GravCat: Key Findings

B. Stochastic Gravity, a schema: Fluctuations

C. Influence Functional method illustrated by Quantum Brownian Motion (QBM)

Part 3: Graviton Noise.

- Direct Detection of Gravitons: *not easy*

> Effects of Graviton Noise on Geodesic Separation

Gravitational Decoherence

C. Anastopoulos and B. L. Hu, A master equation for gravitational decoherence: Probing the textures of spacetime Class. Quant. Grav. 30, 165007 (2013).

M. P. Blencowe, Effective Field Theory Approach to Gravitationally Induced Decoherence Phys. Rev. Lett. 111, 021302 (2013).

V. A. De Lorenci and L. H. Ford, Decoherence induced by long wavelength gravitons Phys. Rev. D 91, 044038 (2015).

F. Suzuki and F. Queisser, Environmental gravitational decoherence and a tensor noise model J. Phys. Conf. Ser. 626, 012039 (2015).

T. Oniga and C. H.-T. Wang, Quantum gravitational decoherence of light and matter Phys. Rev. D 93, 044027 (2016). Quantum coherence, radiance, and resistance of gravitational systems, Phys. Rev. D 96, 084014 (2017).

D. A. Quinones, T. Oniga, B. T. H. Varcoe, and C. H. T. Wang, Quantum principle of sensing gravitational waves: From the zero-point fluctuations to the cosmological stochastic background of spacetime Phys. Rev. D 96, 044018 (2017).

A. Bassi, A. Groardt, and H. Ulbricht, Gravitational decoherence, Classical Quantum Gravity 34, 193002 (2017).

M. Lagouvardos and C. Anastopoulos, Gravitational decoherence of photons, Class. Quant. Grav. 38, 115012 (2021).

Z. Haba, State-dependent graviton noise in the equation of geodesic deviation, Eur. Phys. J. C81 (2021) 40; Graviton noise: the Heisenberg picture [arXiv:2202.06125]

S. Kanno, J. Soda, and J. Tokuda, Noise and decoherence induced by gravitons, Phys. Rev. D 103, 044017 (2021). Indirect detection of gravitons through quantum entanglement, Phys. Rev. D 104, 083516 (2021)

Gravitational Decoherence:

- by noises of gravitational origin.
- Many popular alternative quantum or gravity theories (AQT) invoke this mechanism, e.g., Diosi-Penrose, GRW-Pearle-Bassi (continuous collapse)
- Here: QM (nonrelativistic) + GR (weak field)
- Noise from weak gravitational perturbations of Minkowski space. Only dynamical (propagating) dof : gravitons.

Grav. Decoherence in Energy Basis

1) Many Alertnative Quantum Theories want to see localization in space (collapse of wave function) . If achieved via a decoherence process (gravitational or otherwise) this corresponds to decoherence in position superpositions.

2) Our master equation as well as that of Blencowe (ABH) is Markovian. The Lindblad operators are quadratic in momentum, so gravity can decohere the quantum particles in energy basis superpositions, but not position or momentum superpositions.

3) Some AQTs invoke d*ecoherence due to space-time fluctuations*. We show that they are gauge effects. This whole class of STFI theories is unphysical.

The action for a classical scalar field theory describing the matter degrees of freedom ϕ interacting with the gravitational field is

$$S[g,\phi] = \frac{1}{\kappa}\int \mathrm{d}^4x\sqrt{-g}R + \int \mathrm{d}^4x\sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}m^2\phi^2\right), \qquad (1)$$

where ∇_μ is the covariant derivative defined on a background spacetime with Lorentzian metric $g_{\mu\nu}$, R is the spacetime Ricci scalar and m is the scalar field's mass.

Non-relativistic limit

3.5. The non-relativistic limit

The master equation (48) is still very complex. However, it simplifies significantly in the non-relativistic limit. For $|\mathbf{p}| << m$, the matrix elements of the operators $\mathbf{A}_a$ become

$$\langle \mathbf{p}|\hat{A}_r(\mathbf{k})|\mathbf{p}'\rangle \simeq L^r_{ij}(\mathbf{k})\frac{p^i p^j}{m}(2\pi)^3\delta(\mathbf{p}' - \mathbf{p} - \mathbf{k}), \qquad (49)$$

and thus $\hat{A}_r$ can be expressed as

$$\hat{A}_r(\mathbf{k}) = L^r_{ij}(\mathbf{k})\frac{\hat{p}^i\hat{p}^j}{m}e^{ik_i\hat{X}^i}, \qquad (50)$$

where $\hat{x}^i$ is the position and $\hat{p}_j$ the momentum operators of a non-relativistic particle.

Markovian master equation for a non-relativistic particle (subscript 1 one particle state) interacting with gravity, valid to first order in κ.

$$\frac{\partial\hat{\rho}_1}{\partial t} = -\frac{i}{2m_R}[\hat{\mathbf{p}}^2, \hat{\rho}_1] - \frac{\kappa\Theta}{18m_R^2}(\delta^{ij}\delta^{kl} + \delta^{ik}\delta^{jl})[\hat{p}_i\hat{p}_j, [\hat{p}_k\hat{p}_l, \hat{\rho}_1]]$$

the renormalized mass $m_R = m(1 + \frac{9\kappa\Lambda}{2\pi^2}).$ UV cutoff

Gravitational Decoherence Time

$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{2m_R}[\hat{p}^2, \hat{\rho}] - \frac{4\pi G\Theta}{9m_R^2}[\hat{p}^2, [\hat{p}^2, \hat{\rho}]]. \tag{61}$$

where we reinserted Newton's constant by setting $\kappa = 8\pi G$.

This master equation is exactly solvable in the momentum representation

$$\rho_t(p, p') = \exp\left[-\frac{i}{2m_R}(p^2 - p'^2)t - \frac{4\pi G\Theta}{9m_R^2}(p^2 - p'^2)^2 t\right]\rho_0(p, p') \tag{62}$$

It is evident that the master equation leads to decoherence in the momentum basis. Let us assume that the initial state is a superposition of two states, localized in momentum p_1 and p_2. Define the mean momentum $p = (p_1 + p_2)/2$ and $\Delta p = |p_2 - p_1|$. Then, after time of order of

Energy

$$t_{dec} = \frac{m_R^2}{G\Theta p^2 \Delta p^2} = \frac{1}{G\Theta m_R^2 v^2 \delta v^2}, \tag{63}$$

the momentum superpositions will have been destroyed; v and δv refer to the mean velocity and the velocity difference, respectively. Inserting back c and $\hbar$ the decoherence time is

$$t_{dec} = \frac{\hbar^2 c^5}{G\Theta m_R^2 v^2 \delta v^2}. \tag{64}$$

Two possibilities

1) view the Minkowski spacetime as the lowest energy microstate, or

2) the lowest energy macrostate of the collective variables derived from an underlying theory of quantum gravity: a theory for the basic constituents of spacetime.

Grav. Decoherence: Main Features

- Decoherence time ~ 1/Θ a free parameter (temperature) : Planck scale if gravity is fundamental => negligible effect.
- But if gravity is emergent Θ is much lower. Thus

  if gravitational decoherence is observed, its magnitude can in principle discern whether Gravity is fundamental or emergent ?

- Quantum Brownian Motion: Spectral density of environment (gravity) ➔ Texture of Spacetime

Gravitational decoherence may also r*eveal the textures (meso structure) of spacetime*.

Entanglement:

Gravitational Cat State

It is not just an inalienable feature of QM,
but a deeper reflection of the
innate tension between Q & G (e.g., Penrose)

- Anastopoulos, C. & Hu, B.-L. Probing a gravitational cat state. *Class. Quantum Gravity* 32, 165022 (2015).
- Derakhshani, M., Anastopoulos, C. & Hu, B.-L. Probing a gravitational cat state: experimental possibilities. *J. Phys. Conf. Ser.* 701, 012015 (2016).
- Krisnanda, T., Zuppardo, M., Paternostro, M. & Paterek, T. Revealing nonclassicality of inaccessible objects. *Phys. Rev. Lett.* 119, 120402 (2017).
- Bose, S. et al. Spin entanglement witness for quantum gravity. *Phys. Rev. Lett.* 119, 240401 (2017).
- Marletto, C. & Vedral, V. Gravitationally induced entanglement between two massive particles is sufficient evidence of quantum effects in gravity. *Phys. Rev. Lett.* 119, 240402 (2017). Witness gravity's quantum side in the lab. *Nature* 547, 156–158 (2017).
- Comments on these two papers by Anastopoulos and B. L. Hu [arXiv:1804.11315]
- Belenchia, A. et al. Quantum superposition of massive objects and the quantization of gravity. *Phys. Rev. D* 98, 126009 (2018).
- Christodoulou M and Rovelli C 2019 On the possibility of laboratory evidence for quantum superposition of geometries *Phys. Lett.* B **792** 64
- Carlesso, M., Bassi, A., Paternostro, M. & Ulbricht, H. Testing the gravitational field generated by a quantum superposition. New J. Phys. 21, 093052 (2019).
- T. Krisnanda et al, Observable quantum entanglement due to gravity, NPJ Quantum Information (2020) 6:12
- C. Anastopoulos and B. L. Hu, Quantum Superposition of Two Gravitational Cat States Class Quant Gravity 37, 235012 (2020)

& More…

The most basic element in quantum information: **Quantum Entanglement**

Examine the **expectation value** not *wrt* a vacuum state (vev), but, say, a **cat state:**

| +- > = 1/V2 (|left> +- |right>) |…..0…..|

-x +x

Semi-Classical Gravity

One should know that **SCG is not sufficient for QI**, since it gives the **mean value** of the stress energy tensor T_{mn}, which predicts wrongly that the cat is at x=0. No Superposition, not quantum.

Semiclassical Gravity

Semiclassical Einstein Equation

(this nomenclature is preferred over Moller-Rosenfeld Eq.):

$$\tilde{G}_{\mu\nu}(g_{\alpha\beta}) = \kappa\langle\hat{T}_{\mu\nu}\rangle_q + \kappa\,(T_{\mu\nu})_c$$

$\tilde{G}_{\mu\nu}$ is the Einstein tensor (plus covariant terms associated with the renormalization of the quantum field)

$\kappa = 8\pi G_N$ and G_N is Newton's constant

Free massive scalar field $(\Box - m^2 - \xi R)\hat{\phi} = 0.$

$\hat{T}_{\mu\nu}$ is the stress-energy tensor operator

$\langle\,\rangle_q$ denotes the expectation value

Semiclassical theory can't cope

- **SCG does not admit cat states.**
  It gives only the mean value, not a coherent quantum superposition.

➢ **Q Information in the face of gravity :**

Entanglement: Gravitational Cat State

Need to go beyond the mean field theory (<Tmn> **SCG)**

➢ Must include contributions from the **Fluctuations** in quantum matter field in addition to the mean.

- **Correlations of the stress energy tensor** $\langle T_{mn} T_{rs} \rangle$ are needed to address issues in **quantum information with gravity** (Relativistic QI, or **RQI**)

- There is such a theory, **Stochastic Semiclassical Gravity (SSG)**, **based solely on GR+QFT**.
  No new invention needed (or allowed), as in Alternative Quantum Theories (**AQTs**).

- Noise and fluctuations in quantum field induce metric fluctuations of spacetime (meso-structure) described by the Einstein-Langevin Equation.

Stochastic Gravity

Einstein- Langevin Equation (schematically):

$$\tilde{G}_{\mu\nu}(g_{\alpha\beta}) = \kappa \left(T^{\rm c}_{\mu\nu} + T^{\rm qs}_{\mu\nu} \right)$$

$T^{c}_{\mu\nu}$ is due to classical matter or fields

$$T^{\rm qs}_{\mu\nu} \equiv \langle \hat{T}_{\mu\nu} \rangle_{\rm q} + T^{\rm s}_{\mu\nu}$$

$T^{\rm qs}_{\mu\nu}$ is a new stochastic term
related to the quantum fluctuations of $T_{\mu\nu}$

Einstein-Langevin Equation

- Consider a weak gravitational perturbation h off a background g $g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu},$ The ELE is given by (The ELE is Gauge invariant)

$$G_{ab}[g+h] + \Lambda(g_{ab} + h_{ab}) - 2(\alpha A_{ab} + \beta B_{ab})[g+h] = 8\pi G\left(\langle \hat{T}^{R}_{ab}[g+h]\rangle + \xi_{ab}[g]\right).$$

- Nonlocal dissipation and colored noise

Nonlocality manifests with **stochasticity**

because the gravitational sector is an open system

NOISE KERNEL

- Exp Value of 2-point correlations of stress tensor: bitensor

- Noise kernel measures quantum flucts of stress tensor

It can be represented by (shown via influence functional to be equivalent to) a classical stochastic tensor source $\xi_{ab}[g]$

$$\langle \xi_{ab} \rangle_s = 0 \qquad \langle \xi_{ab}(x)\xi_{cd}(y)\rangle_s = N_{abcd}(x,y)$$

- **Symmetric, traceless** (for conformal field), **divergenceless**

Now, look for the

Gravitational Quantum Cat

from the nonrelativistic, weak field limit of the

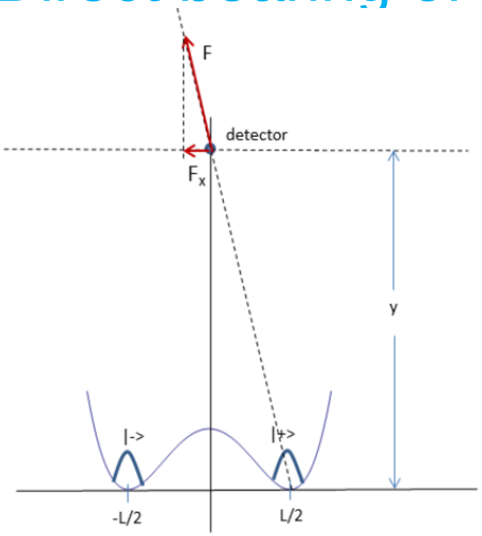
Fluctuations of energy density, or the correlator of the stress energy tensor:

Noise Kernel

(Not the full Schrodinger cat – but definitely contains the quantum attributes of the cat)

Gravitational Cat State:

Direct bearing of Quantum Optomechanics



A particle of **mass m** in a confining potential with two minima. Assume that the **distance L** between the minima is much larger than the width of the localization region.

The system can be approximated by a qubit with defining states |+> and |->.

Hamiltonian $\hat{H} = \nu\ \hat{\sigma}_1$, where v is the tunneling rate between the two minima.

- The famous ***atomic*** cat of Wineland et al (1996) had L = 80nm and m = 8 amu.
- **M. Ardnt** 2012 diffraction experiment with L = 100nm and m =1300 amu.
- **M. Arndt and K. Hornberger**, *Testing the limits of quantum mechanical superpositions*, Nat. Phys. **10**, 271 (2014). Bassi's review has data till 2013.
- Record for weakest force measured from **CalTech** ? (2014), ~ 4 x 10^{-23} N.
- **Aspelmeyer:** Measurement of gravitational force of milligram masses arXiv:1602.07539.
- **Romero-Isart**'s superconducting microsphere gives the most stringent limit on AQTs, should be able to rule out Diosi-Penrose. The latest in arXiv:1603.01553

We have considered both a classical and a quantum probe: Rabi osc.

Mass density operator of a *non-relativistic N-Particle System*

The Hamiltonian then becomes

$$\hat{\mathrm{H}} = m \int d\mathbf{r} \hat{\psi}^{\dagger}(\mathbf{r}) \hat{\psi}(\mathbf{r}) - \frac{1}{2m} \int d\mathbf{r} \hat{\psi}^{\dagger}(\mathbf{r}) \nabla^2 \hat{\psi}(\mathbf{r}). \tag{11}$$

We will denote the second term in Eq.(11) as $\hat{H}_0$ because it corresponds to the Hamiltonian for N non-relativistic particles. The first term in Eq.(11) corresponds to Nm, for an N-particle state. Hence, the number operator $\hat{N}$ is

$$\hat{N} = \int d\mathbf{r} \hat{\psi}^{\dagger}(\mathbf{r}) \hat{\psi}(\mathbf{r}) \tag{12}$$

This suggests that $m\hat{\psi}^{\dagger}(\mathbf{r})\hat{\psi}(\mathbf{r})$ can be identified as the mass-density operator $\hat{\mu}(\mathbf{r})$.

We include the effect of a confining potential $V(\mathbf{r})$, by modifying the field Hamiltonian

$$\hat{\mathrm{H}} = m \int d\mathbf{r} \hat{\psi}^{\dagger}(\mathbf{r}) \hat{\psi}(\mathbf{r}) + \int d\mathbf{r} \hat{\psi}^{\dagger}(\mathbf{r}) \left[-\frac{1}{2m} \nabla^2 + V(\mathbf{r}) \right] \hat{\psi}(\mathbf{r}). \tag{13}$$

Mass Density Correlations

2.2. Mass-density correlations, noise kernel

In the non relativistic limit, the dominant component of the stress tensor $T_{\mu\nu}$ is the energy density, which is dominated by the mass density, namely

$$T_{\mu\nu}(\mathbf{r},t) = \delta^0_\mu \delta^0_\nu \mu(\mathbf{r},t) \tag{14}$$

Thus, it suffices to calculate the correlation functions of the Heisenberg-picture operator

$$\hat{\mu}(\mathbf{r},t) = e^{i\hat{H}t}\hat{\mu}(\mathbf{r})e^{-i\hat{H}t}. \tag{15}$$

We assume an one-particle state

$$|\phi\rangle = \int d\mathbf{r}\phi(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r})|0\rangle, \tag{16}$$

where $\phi(\mathbf{r})$ is the one-particle wave-function.

We find

$$\langle\mu(\mathbf{r},t)\rangle = \langle\phi|\hat{\mu}(\mathbf{r},t)|\phi\rangle = m\phi^*(\mathbf{r},t)\phi(\mathbf{r},t) \tag{17}$$

$$\langle\mu(\mathbf{r},t)\mu(\mathbf{r}',t')\rangle = \phi^*(\mathbf{r},t)\phi(\mathbf{r}',t')G(\mathbf{r},t;\mathbf{r}',t') \tag{18}$$

where $\phi(\mathbf{r},t)$ is the time-evolved single particle wave function and $G(\mathbf{r},t;\mathbf{r}',t')$ is the one-particle propagator,

$$G(\mathbf{r},t;\mathbf{r}',t') = \langle\mathbf{r}'|e^{-i\hat{H}(t'-t)}|\mathbf{r}\rangle. \tag{19}$$

Noise Kernel

For a free particle,

$$G(\mathbf{r}, t; \mathbf{r}', t') = \left(\frac{m}{2\pi i t}\right)^{3/2} \exp\left[\frac{im(\mathbf{r} - \mathbf{r}')^2}{2(t' - t)}\right] \tag{20}$$

We note that the two-point correlation function is complex valued. In general, it does not define a stochastic process. However, the real part,

$$\xi(\mathbf{r}, t; \mathbf{r}', t') = Re\langle \mu(\mathbf{r}, t)\mu(\mathbf{r}', t')\rangle, \tag{21}$$

known as the noise kernel, corresponds in some cases to the two-point correlation function of a stochastic process.

Of importance is also the connected two-point correlation function for the mass densities

$$\eta(\mathbf{r}, t; \mathbf{r}', t') = \langle \mu(\mathbf{r}, t)\mu(\mathbf{r}', t')\rangle - \langle \mu(\mathbf{r}, t)\rangle\langle \mu(\mathbf{r}', t')\rangle. \tag{22}$$

Smeared Mass-Density Function

- In realistic systems the mass density is not defined at a sharp spacetime point but smeared over a finite spacetime region.
- In actual experiments, the particles under consideration (atoms) have a finite size d and it is meaningless to talk about mass densities at scales smaller than d, unless one has a detailed knowledge of the particle's internal state.

- For this reason, rather than the exact mass density function, we consider a smeared mass density function:

$$\hat{\mu}_s(\mathbf{r}, t) = \int d\mathbf{r}' f(\mathbf{r} - \mathbf{r}')\hat{\mu}(\mathbf{r}', t), \qquad (23)$$

for some smearing function $f(\mathbf{r})$ of dimension $[\text{length}]^{-3}$, centered around $\mathbf{r} = 0$. The smearing scale ℓ is defined by the condition $\ell^3 = 1/f(0)$.

We define the positive operator

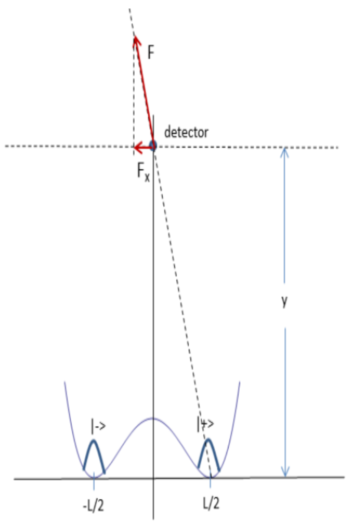
$$\hat{P}_{\mathbf{r}} = \int d\mathbf{r}' g(\mathbf{r} - \mathbf{r}')|\mathbf{r}'\rangle\langle\mathbf{r}'|, \qquad (24)$$

where $g(\mathbf{r}) := f(\mathbf{r})/f(0)$.

Low Energy Quantum Probes (e.g., Optomechanics)

Into the Q Information issues of gravitational systems: GravCats

Treat small values of ν as perturbations of the adiabatic solution.



4.2.2. Rabi oscillations A finite value of ν allows for transitions between the two gravitational quantum states, which induce transitions among the phase space paths of the oscillator. While the model is not exactly solvable, we can estimate the rate of such transitions using perturbation theory with respect to the tunneling rate ν. In Appendix B, we show that to leading order in ν, $e^{-i\hat{H}t} = e^{-i\hat{H}_0 t}\hat{O}_t$, where

$$\hat{O}_t = \begin{pmatrix} \cos\nu t & -i\sin\nu t\hat{D}(2\zeta_0) \\ -i\sin\nu t\hat{D}(-2\zeta_0) & \cos\nu t \end{pmatrix}. \quad (79)$$

As an estimate of the transition between the two gravitational quantum states, we compute the amplitude $\langle -\zeta_0, -|\hat{O}_t|\zeta_0, +\rangle$, between the stationary states $|\zeta_0, +\rangle$ and $|-\zeta_0, -\rangle$. We find

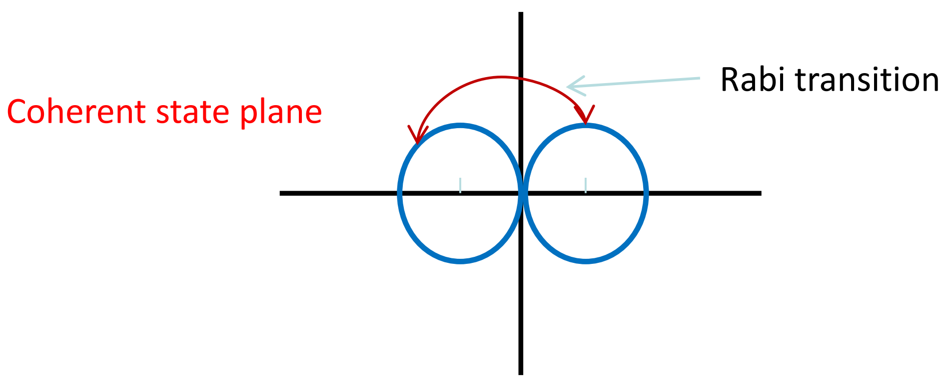
$$\langle -\zeta_0, -|\hat{O}_t|\zeta_0, +\rangle = -i\sin\nu t, \quad (80)$$

and thus the associated probability

$$p(t) = |\langle -\zeta_0, -|\hat{O}_t|\zeta_0, +\rangle|^2 = \sin^2\nu t, \quad (81)$$

exhibits Rabi-type oscillations, with frequency ν.

Rabi oscillations of frequency ν: transition between the two stationary gravitational quantum states $|\zeta_0\rangle$, $|-\zeta_0\rangle$



The Bigger Picture

- *Quantum Gravity (= theories for the microscopic structures of spacetime) "Top-Down" (Planck energy) theories are not needed.* → *Focus on nonrelativistic systems under laboratory conditions: weak gravitational field.*

- *Bottom-Up (low energy) theories: Semiclassical Gravity is inadequate.* → *Focus on fluctuations and correlations of mass density -- incorporated in Stochastic Gravity Theory*

Big Quest: *Can we infer attributes of spacetime fluctuations from quantum experiments even at the level of Newtonian gravity without appealing to new theories of QM or GR (AQTs)?*

Part 3: Graviton Noise

Effect on Geodesic Separation

Direct Graviton Detection is not easy

- F. Dyson, *Is a graviton detectable?* (2012)

More hopeful schemes?:

- M. Parikh, F. Wilczek, and G. Zahariade, **The noise of gravitons,** Int. J. Mod. Phys. D 29, 2042001 (2020). Phys. Rev. Lett. 127, 081602 (2021). Signature of the quantization of gravity at **gravitational wave detectors**, Phys. Rev. D 104, 046021 (2021).

- S. Kanno, J. Soda, and J. Tokuda, Indirect detection of gravitons through **quantum entanglement**, Phys. Rev. D 104, 083516 (2021).

- T. Guerreiro**, Quantum effects in gravity waves**, Class.Quant. Gravity 37, 155001 (2020). F. Coradeschi, A. M. Frassino, T. Guerreiro, J. R. West, and E. J. Schioppa, Can we detect the quantum nature of weak gravitational fields? Universe 7, 414 (2021).

Quantum noise of gravitons and stochastic force on geodesic separation

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In this work we consider the effects of gravitons and their fluctuations on the dynamics of two masses using the Feynman-Vernon influence functional formalism, applied earlier to nonequilibrium quantum field theory [Calzetta and Hu, *Nonequilibrium Quantum Field Theory* (Cambridge University Press, Cambridge, England, 2008)] and semiclassical stochastic gravity [Hu and Verdaguer, *Semiclassical and Stochastic Gravity: Quantum Field Effects on Curved Spacetime* (Cambridge University Press, Cambridge, England, 2020)], and most recently, to this problem by Parikh *et al.*, [Phys. Rev. Lett. **127**, 081602 (2021); Phys. Rev. D **104**, 046021 (2021)]. The Hadamard function of the gravitons yields the noise kernel acting as a stochastic tensorial force in a Langevin equation governing the motion of the separation of the two masses. The fluctuations of the separation due to the graviton noise are then solved for various quantum states including the Minkowski vacuum, thermal, coherent and squeezed states. The previous considerations of Parikh *et al.* are only for some selected modes of the graviton, while in this work we have included all graviton modes and polarizations. We comment on the possibility of detecting these fluctuations in primordial gravitons using interferometers with long baselines in deep space experiments.

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PHYSICAL REVIEW D 105, 086004 (2022)

- **Closed-time-path (in-in) formalism** and **Schwinger-Keldysh** Green Functions applied to

Cosmological problems: E.g.,

Calzetta & Hu (1987): Particle production backreaction,
Weinberg (2005): Correlation functions

Charge and mass motion: Gravitational Radiation Reaction Johnson & Hu (2001), Galley & Hu (2006, 2009)

Monographs related to **QFT, Gravitation and Cosmology**:

Calzetta & Hu, *Nonequilibrium Quantum Fields* (Cambridge U 2008)

Hu & Verdaguer, *Semiclassical and Stochastic Gravity*(Cambridge 2020)

Quantum Open Systems conceptual framework**:**

Easier to understand via **Quantum Brownian Motion**

Quantum Brownian Motion via the Influence Functional / closed-time path integral Methods

Feynman Vernon 1963,
Caldeira & Leggett 1983,
Grabert, Ingold Schramm 1988
Hu, Paz & Zhang 1992 ...

slides courtesy Prof. Hing-Tong Cho

Easier to first understand the stochastic field theory approach via Quantum Brownian Motion

System (1HO) interacting *bilinearly* with an Environment of N Harmonic Oscillators (later we'll switch to a scalar field)

$$
\begin{aligned}
S[x] &= \int_0^t ds \left[\frac{1}{2} M \dot{x}^2 - V(x)\right] \\
S_e[q_n] &= \int_0^t ds \sum_n \left[\frac{1}{2} m_n \dot{q}_n^2 - \frac{1}{2} m_n \omega_n^2 q_n^2\right] \\
S_{int}[x, \{q_n\}] &= \int_0^t ds \sum_n (-C_n x\, q_n)
\end{aligned}
$$

Gaussian Systems permit exact solutions

Closed-Time-Path /Schwinger-Keldysh/ in-in Effective Action

[Schwinger 61, Keldysh 63, Chou, Hao, Su, Yu 1981, Calzetta Hu 1987, Weinberg 2005 …]

$$
\begin{aligned}
e^{i\Gamma[x_+,x_-]} &= e^{iS[x_+]-iS[x_-]} \times \\
&\int_{CTP} \prod_n Dq_{n+} Dq_{n-} \left(e^{iS_e[\{q_{n+}\}]-iS_e[\{q_{n-}\}]} \right. \\
&\qquad\qquad \left. e^{iS_{int}[x_+,\{q_{n+}\}]-iS_{int}[x_-,\{q_{n-}\}]} \right) \\
&= e^{iS[x_+]-iS[x_-]+iS_{IF}[x_+,x_-]}
\end{aligned}
$$

where S_IF is the influence action.

$$
\begin{aligned}
S_{IF}[x_+, x_-] = \; & \sum_n \frac{1}{2} \int ds\, ds' \\
& \left[x_+(s) G_{n++}(s,s') x_+(s') - x_+(s) G_{n+-}(s,s') x_-(s') \right. \\
& \left. -x_-(s) G_{n-+}(s,s') x_+(s') + x_-(s) G_{n--}(s,s') x_-(s') \right]
\end{aligned}
$$

where Gn are the Schwinger-Keldysh or closed time path CTP (+, -) propagators:

$$
\begin{aligned}
G_{n++}(s,s') &= -\eta_n(s-s')\,\mathrm{sgn}(s-s') + i\nu_n(s-s') \\
G_{n+-}(s,s') &= \eta_n(s-s') + i\nu_n(s-s') \\
G_{n-+}(s,s') &= -\eta_n(s-s') + i\nu_n(s-s') \\
G_{n--}(s,s') &= \eta_n(s-s')\,\mathrm{sgn}(s-s') + i\nu_n(s-s')
\end{aligned}
$$

The influence action S_{IF} can be written as

$$e^{iS_{IF}} = e^{-i\int_0^t ds \int_0^s ds' [\Delta x(s)\eta(s-s')\Sigma x(s')]} e^{-\frac{1}{2}\int_0^t ds \int_0^t ds' [\Delta x(s)\nu(s-s')\Delta x(s')]}$$

where $\Delta x(s) = x_+(s) - x_-(s)$ and $\Sigma x(s) = x_+(s) + x_-(s)$, and

$$\eta(s-s') = \sum_n \eta_n(s-s') = -\sum_n \frac{C_n^2}{2m_n\omega_n} \sin\omega_n(s-s')$$

$$\nu(s-s') = \sum_n \nu_n(s-s') = \sum_n \frac{C_n^2}{2m_n\omega_n} \cos\omega_n(s-s')$$

Spectral density $\mathcal{I}(\omega)$ (later)

Feynman-Vernon Gaussian Integral Identity, Noise Kernel

Rewriting the imaginary part of S_{IF} as

$$
\begin{aligned}
& e^{-\frac{1}{2}\int \Delta x \nu \Delta x} \\
= \quad & N \int D\xi e^{-\frac{1}{2}\int \xi \nu^{-1} \xi} e^{-\frac{1}{2}\int \Delta x \nu \Delta x} \\
= \quad & N \int D\xi e^{-\frac{1}{2}\int (\xi - i\nu\Delta x)\nu^{-1}(\xi - i\nu\Delta x)} e^{-\frac{1}{2}\int \Delta x \nu \Delta x} \\
= \quad & N \int D\xi P[\xi] e^{i\int \xi \Delta x}
\end{aligned}
$$

where $P[\xi] = e^{-\frac{1}{2}\int \xi \nu^{-1} \xi}$ is the Gaussian probability density of the stochastic force ξ.

Due to this probability density one has the stochastic average $\langle \xi(s)\xi(s')\rangle_s = \nu(s - s')$ which is called the noise kernel.

Equation of Motion from the influence action

After this procedure the effective action

$$
\begin{aligned}
\Gamma[x_+, x_-] &= S[x_+] - S[x_-] \\
&\quad - \int_0^t ds \int_0^s ds' \Delta x(s) \eta(s - s') \Sigma x(s') \\
&\quad + \int_0^t ds \Delta x(s) \xi(s)
\end{aligned}
$$

The equation of motion for the particle is then given by

$$
\left. \frac{\delta \Gamma[x_+, x_-]}{\delta x_+} \right|_{x_+ = x_- = x} = 0
$$

Langevin Equation
Dissipation kernel

The equation of motion is a Langevin equation with the stochastic force $\xi(t)$,

$$M\ddot{x} + V'(x) + \int_0^t ds\, \eta(t-s)x(s) = \xi(t)$$

The integral term is related to dissipation as one can write

$$\eta(t) = \frac{d}{dt}\gamma(t) \Rightarrow \gamma(t) = \sum_n \frac{C_n^2}{2m_n\omega_n^2}\; cos\omega_n t$$

and we have

$$M\ddot{x} + V'(x) + \int_0^t ds\, \gamma(t-s)\dot{x}(s) = \xi(t)$$

$\eta(s-s')$ is called the dissipation kernel.

Fluctuation-Dissipation Relation

$$\nu(s) = \int_{-\infty}^{\infty} ds' \, K(s - s')\gamma(s')$$

in this simple case

$$K(s) = \int_0^{\infty} \frac{d\omega}{\pi} \omega \, \cos \omega s$$

Note the existence of FDR is a condition of self-consistency between the system dynamics with backaction from the environment.
This relation originates from the unitarity In the original closed system.

The coarse-grained environmental variables are now represented by noise and fluctuations. Their backreaction on the system imparts to it dissipative dynamics in the now opened system.

η for dissipation kernel in HPZ92

$$\mathcal{F}_n^{(1)}[x,x'] = \exp\left\{ -\frac{2i}{\hbar}\int_0^t ds_1 \int_0^{s_1} ds_2 \, [x(s_1)-x'(s_1)]\, \mu_n(s_1-s_2)\, [x(s_2)+x'(s_2)] - \frac{1}{\hbar}\int_0^t ds_1 \int_0^{s_1} ds_2\, [x(s_1)-x'(s_1)]\, \nu_n(s_1-s_2)\, [x(s_2)-x'(s_2)] \right\}$$

Summing up the modes, get:

(Noise kernel)

$$\nu(s) = \sum_n \nu_n(s) = \int \frac{d\omega}{\pi} I(\omega) \coth \tfrac{1}{2}\beta\hbar\omega \cos\omega s \qquad s = s_1 - s_2$$

(Dissipation kernel)

$$\mu(s) = \sum_n \mu_n(s) = \frac{d}{ds}\gamma(s), \quad \gamma(s) = \int \frac{d\omega}{\pi} I(\omega) \frac{1}{\omega} \cos\omega s$$

(damping kernel)

Spectral density function $I(\omega) = \rho_D(\omega)\cdot \frac{\pi c^2(\omega)}{2m\omega}$

density of state $\rho_D(\omega)$

simplest case: $= \sum \delta(\omega-\omega_n)$

- Different environments with the same spectral density function will have the same influence on the Brownian particle

A common form for $I(\omega) \sim \omega^\alpha e^{-\omega/\Gamma}$ Γ: cutoff frequency

$\alpha = 1$ ohmic   **QBM at high temp.** → $\delta(s)$ **white** noise

$\alpha > 1$ superohmic

$0 < \alpha < 1$ subohmic

$$\nu(s) = \frac{2kT}{\hbar}\gamma(s)$$
$$\gamma(s) = 2m\gamma_0\,\delta(s)$$
(Markov)

Ohmic with high T ⇒ local dissipation

Open Q System Approach in 3 Classes of Problems: Case A: Backreaction of Cosmological Particle Creation

$$S_{\rm int}(g;\Phi] = \int d^4x\sqrt{-g}\,h_{\mu\nu}T^{\mu\nu}$$

(Eq. (2.5) of [45] Gravitational Perturbations System | Environment Quantum Field

(weak inhomogeneities or anisotropy) |

- [45] J. B. Hartle, Effective-potential Approach to Graviton Production in the Early Universe, Phys. Rev. Lett. 39, 1373 (1977). "In-Out" effective action
- [46] M. V. Fischetti, J. B. Hartle, and B. L. Hu, Quantum effects in the early universe. I. Influence of trace anomalies on homogeneous, isotropic, classical geometries, Phys. Rev. D 20, 1757 (1979).
- [47] J. B. Hartle and B. L. Hu, Quantum effects in the early universe. II. Dissipation of anisotropy by scalar particle production, Phys. Rev. D 20, 1772 (1979). "In-Out" eff. action
- [16] E. Calzetta and B. L. Hu, Closed time-path functional formalism in curved spacetime: Application to cosmological back-reaction problems, Phys. Rev. D 35, 495 (1987). In-in
- [48] A. Campos and E. Verdaguer, Semiclassical equations for weakly inhomogeneous cosmologies, Phys. Rev. D 49, 1861 (1994). In-in

Noise of Quantum Fields: bkrn on spacetime: Einstein-Langevin Equation: Stochastic Semiclassical Gravity

- [52] E. Calzetta and B. L. Hu, Noise and fluctuations in semiclassical gravity, Phys. Rev. D 49, 6636 (1994).
- [53] B. L. Hu and A. Matacz, Einstein-Langevin equation for backreactions in semiclassical cosmology, Phys. Rev. D 51, 1577 (1995).
- [54] B. L. Hu and S. Sinha, Fluctuation-dissipation relation in cosmology, Phys. Rev. D 51, 1587 (1995).
- [49] A. Campos and E. Verdaguer, Stochastic semiclassical equations for weakly inhomogeneous cosmologies, Phys. Rev. D 53, 1927 (1996).
- [55] F. C. Lombardo and F. D. Mazzitelli, Einstein-Langevin equations from running coupling constants, Phys. Rev. D 55, 3889 (1997).
- [56] R. Martin and E. Verdaguer, On the semiclassical Einstein-Langevin equation, Phys. Lett. 465B, 113 (1999); Stochastic semiclassical gravity, Phys. Rev. D 60, 084008 (1999); Stochastic semiclassical fluctuations in Minkowski spacetime, Phys. Rev. D 61, 124024 (2000).
- [57] Nicholas. G. Phillips and B. L. Hu, Noise kernel in stochastic gravity and stress energy bi-tensor of quantum fields in curved spacetimes, Phys. Rev. D 63, 104001 (2001).

Case B: Moving Charges and Masses in a Quantum Field

Interaction action

$$S_{int}(z; \mathbf{\Phi}(\mathbf{x})) = \int d^4x \sqrt{-g}\, j(x;z)\, \mathbf{\Phi}(x)$$

where z *is* the worldline coordinates *(system)* of the charge or mass, $j(x;z)$ is its current density, $\Phi(\mathbf{x})$ is the scalar quantum field (*Environment*)

Charge: vector current $j^{\mu}(x;z)$, potential A_{μ}, Eq. (3.2) [25]

Small Mass : $h_{\mu\nu}T^{\mu\nu}$, , where $T^{\mu\nu} \sim u^{\mu}u^{\nu}$ Eq. (4.2) of [25]

Electromagnetic & Gravitational Radiation Reaction (Self Force)

[24] Chad R. Galley and B. L. Hu, Self-force with a stochastic component from radiation reaction of a scalar charge moving in curved spacetime, Phys. Rev. D 72, 084023 (2005).

[25] Chad R. Galley, B. L. Hu, and S. Y. Lin, Electromagnetic and gravitational radiation reaction in curved spacetime: Self-force derivation from stochastic field theory, Phys. Rev. D 74, 024017 (2006).

Here, our system is Geodesic Deviation

For this problem, our system is the geodesic deviation vector z^i, our environment is the graviton field, each of its two polarizations is represented by a massless minimally-coupled scalar field. The interaction action is

$$\int dt \frac{m\kappa}{4} \ddot{h}_{ij} z^i z^j = \alpha \int d^4x \sum h^{(s)}(x) X^{(s)}(x)$$

$$\alpha = m\kappa / 2\sqrt{2}(2\pi)^3$$

The second equality here [Eq. (15)] comes after an integration by parts on the \ddot hij, passing one time derivative to each z, thus ending up with an interaction action of the same form hμν Tμν, with $T^{\mu\nu} \sim u^\mu u^\nu$ for the mass as in Case B and

$$X^{(s)}(x) = \int d^3k e^{i\vec{k}\cdot\vec{x}} \frac{d^2}{dt^2} (\epsilon_i^{(s)*}(\vec{k}) z^i(t))^2$$

Quantum noise and stochastic force from gravitons

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Outline

I. Motivation
II. Influence action and Langevin equation
III. Separation fluctuations
IV. Conclusions

I. Motivations

1. Interferometric observations of gravitational waves since 2015.
2. Is it possible to explore the quantum nature of gravity by interferometric observations?
3. Proposal by Parikh, Wilczek, and Zahariade to study the quantum noise due to gravitons.
4. A more general framework in which all graviton modes and polarizations are taken into account.

II.Influence action and Langevin equation

Einstein action

$$S_g = \frac{1}{\kappa^2} \int d^4x \sqrt{-g}\, R$$

where $\kappa^2 = 16\pi G$.

With $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, the graviton action

$$S_{grav} = -\frac{1}{2} \int d^4x \sum_s \partial_\alpha h^{(s)}(x) \partial^\alpha h^{(s)}(x)$$

sum over 2 polarizations: massless
minimally coupled scalar field

Fermi normal coordinate system $(t, \vec{z})$ along the geodesic of the first mass with the metric

$$\begin{aligned} g_{00}(t,\vec{z}) &= -1 - R_{i0j0}(t,0)z^i z^j + \cdots \\ g_{0i}(t,\vec{z}) &= -\frac{2}{3}R_{0ijk}(t,0)z^j z^k + \cdots \\ g_{ij}(t,\vec{z}) &= \delta_{ij} - \frac{1}{3}R_{ikjl}(t,0)z^k z^l + \cdots \end{aligned}$$

Action of the second mass,

$$S_m = -m\int \sqrt{-ds^2}$$

In terms of the gravitational perturbation $\kappa h_{\mu\nu}$, the Riemann tensor component

$$R_{i0j0} = -\frac{\kappa}{2}\ddot{h}_{ij}$$

$$S_m = \int dt \left[\frac{m}{2}\,\delta_{ij}\,\dot{z}^i\dot{z}^j + \frac{m\kappa}{4}\ddot{h}_{ij}\,z^i z^j\right] + \cdots$$

Closed-time-path formalism

$$
\begin{aligned}
e^{iS_{IF}} &= \int_{CTP} Dh_+ \, Dh_- \, e^{i(S_g[h_+]-S_g[h_-]+\int J_+ h_+ - \int J_- h_-)} \\
&= e^{-\frac{i}{2}\int (J_+ G_{++} J_+ - J_+ G_{+-} J_- - J_- G_{-+} J_+ + J_- G_{--} J_-)}
\end{aligned}
$$

Schwinger-Keldysh Green functions

$$
\begin{aligned}
G_{++}(x,x') &= -i\langle T h_+(x) h_+(x')\rangle \\
G_{+-}(x,x') &= -i\langle h_-(x') h_+(x)\rangle \\
G_{-+}(x,x') &= -i\langle h_-(x) h_+(x')\rangle = G_{+-}(x',x) \\
G_{--}(x,x') &= -i\langle \bar{T} h_-(x) h_-(x')\rangle = -G^*_{++}(x,x')
\end{aligned}
$$

Influence action

$$S_{IF} = \int dt\,dt'\,\Delta^{ij}(t)D_{ijkl}(t,t')\Sigma^{kl}(t') + \frac{i}{2}\int dt\,dt'\,\Delta^{ij}(t)N_{ijkl}(t,t')\Delta^{kl}(t')$$

where

$$\Sigma^{ij}(t) = \frac{1}{2}\left[z_+^i(t)z_+^j(t) + z_-^i(t)z_-^j(t)\right]$$

$$\Delta^{ij}(t) = z_+^i(t)z_+^j(t) - z_-^i(t)z_-^j(t)$$

The dissipation and the noise kernels,

$$D_{ijkl}(t,t') = \left(\frac{m^2\kappa^2}{512\pi^6}\right)\frac{d^2}{dt^2}\frac{d^2}{dt'^2}\int d^3k\, d^3k' \int d^3x\, d^3x' \; e^{-i\vec{k}\cdot\vec{x}}e^{-i\vec{k}'\cdot\vec{x}'}\sum_s \epsilon_{ij}^{(s)}(\vec{k})\epsilon_{kl}^{(s)}(\vec{k}')G_{ret}(x,x')$$

$$N_{ijkl}(t,t') = \left(\frac{m^2\kappa^2}{1024\pi^6}\right)\frac{d^2}{dt^2}\frac{d^2}{dt'^2}\int d^3k\, d^3k' \int d^3x\, d^3x' \; e^{-i\vec{k}\cdot\vec{x}}e^{-i\vec{k}'\cdot\vec{x}'}\sum_s \epsilon_{ij}^{(s)}(\vec{k})\epsilon_{kl}^{(s)}(\vec{k}')G^{(1)}(x,x')$$

$G_{ret}(x,x') = i\theta(t-t')\langle[h(x),h(x')]\rangle$ is the retarded Green function, and $G^{(1)}(x,x') = \langle\{h(x),h(x')\}\rangle$ is the Hadamard function.

Feynman-Vernon formalism

$$
\begin{aligned}
& e^{-\frac{1}{2}\int \Delta^{ij} N_{ijkl} \Delta^{kl}} \\
= \; & \mathcal{N} \int D\xi \; e^{-\frac{1}{2}\int (\xi_{ij} + i\Delta^{mn} N_{ijmn})(N^{-1})^{ijkl}(\xi_{kl} + iN_{klpq}\Delta^{pq})} \; e^{-\frac{1}{2}\int \Delta^{ij} N_{ijkl} \Delta^{kl}} \\
= \; & \int D\xi \; P[\xi] \; e^{-i\int \xi_{ij}\Delta^{ij}}
\end{aligned}
$$

with the gaussian probability density

$$
P[\xi] = \mathcal{N} \, e^{-\frac{1}{2}\int \xi_{ij}(N^{-1})^{ijkl}\xi_{kl}}
$$

where ξ_{ij} is a stochastic force and $\mathcal{N}$ is a normalization constant.

Stochastic effective action for the geodesic separation

$$
\begin{aligned}
S_{SEA} &= S_m[z_+] - S_m[z_-] + S_{IF} \\
&= \frac{m}{2}\int dt\,\delta_{ij}\dot{z}^i_+(t)\dot{z}^j_+(t) - \frac{m}{2}\int dt\,\delta_{ij}\dot{z}^i_-(t)\dot{z}^j_-(t) \\
&\quad + \int dt\,dt'\,\Delta^{ij}(t)D_{ijkl}(t,t')\Sigma^{kl}(t') - \int dt\,\xi_{ij}(t)\Delta^{ij}(t)
\end{aligned}
$$

Langevin equation

$$\frac{\delta S_{SEA}}{\delta z_+^i}\bigg|_{z_+=z_-=z} = 0$$

$$\Rightarrow \quad m\ddot{z}^i(t) + 2\delta^{im}\int dt'\, D_{mnkl}(t,t')\, z^n(t) z^k(t') z^l(t') - 2\delta^{ik}\xi_{kl}(t)\, z^l(t) = 0 \tag{35}$$

Note that the second term in Eq. (35) involving the dissipation kernel is history dependent. This is a nonlinear integral differential equation which is very difficult to solve analytically. In the following we shall use a perturbative approach. Without the graviton effects, we have the homogeneous equation,

$$m\ddot{z}_0^i(t) = 0 \tag{36}$$

where $z_0^i(t)$ corresponds to the geodesic motion in the background spacetime. In this perturbative approach, the next order effect comes from the noise term which is linear in $z^i(t)$. Suppose we take $z^i = z_0^i + \delta z^i$. Then, We shall concentrate on the noise effect due to the stochastic force

$$m\ddot{z}^i(t) = 2\delta^{ik}\xi_{kl}(t)\, z^l(t)$$

with the correlation function

$$\begin{aligned} \langle \xi_{ij}(t)\xi_{kl}(t')\rangle_s &= \int D\xi \ P[\xi]\, \xi_{ij}(t)\xi_{kl}(t') \\ &= N_{ijkl}(t,t'). \end{aligned}$$

Imposing the initial conditions,

δz^{i}= \dot z^{i} = 0;

the solution δz can be written as

Geodesic separation

$$\delta z^i(t) = \frac{2}{m}\int_0^t dt' \, (t-t')\delta^{ik}\xi_{kl}(t')z_0^l(t')$$

Geodesic separation correlation

$$\langle \delta z^i(t)\delta z^j(t')\rangle_s = \frac{4}{m^2}\delta^{ik}\delta^{jl}\int_0^t dt'' \int_0^{t'} dt'''(t-t'')(t'-t''') \, z_0^m(t'')z_0^n(t''')N_{kmln}(t'',t''')$$

III. Separation fluctuations

Minkowski vacuum

$$N_{ijkl}^{(0)}(t,t') = -\left(\frac{m^2\kappa^2}{240\pi^2}\right)\Lambda^6\left[2\,\delta_{ij}\delta_{kl} - 3(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\right]F[\Lambda(t-t')]$$

where

$$\begin{aligned} F(x) &= \frac{1}{x^6}[(5x^4 - 60x^2 + 120)\cos x \\ &\quad + x(x^4 - 20x^2 + 120)\sin x - 120] \end{aligned}$$

Set, without loss of generality, $z_0^i = (0, 0, z_0)$,

$$\langle(\delta z^3(t))^2\rangle^{(0)} \overset{\text{Minkowski}}{\underset{}{}}{}^{\text{vac}} = \left(\frac{\kappa^2 z_0^2}{60\pi^2 t^2}\right)[\Lambda^4 t^4 + 4\Lambda^2 t^2(1 + 2\cos(\Lambda t)) - 24\Lambda t \sin(\Lambda t) + 24(1 - \cos(\Lambda t))]$$

Also, $\langle(\delta z^1(t))^2\rangle^{(0)} = \langle(\delta z^2(t))^2\rangle^{(0)} = \frac{3}{4}\langle(\delta z^3(t))^2\rangle^{(0)}$.

Estimation, with $\Lambda \sim 1/z_0$ and $t \sim z_0$,

$$\sqrt{\langle(\delta z^3(t))^2\rangle^{(0)}} \sim \kappa \sim 10^{-35}\mathrm{m}$$

which is the order of Planck length.

Thermal state

For low temperature,

$$\langle(\delta z^3(t))^2\rangle^{(\beta)} = \left(\frac{16\pi^4\kappa^2 z_0^2}{945\beta^6}\right)\left(\frac{t^4}{4} - \frac{7\pi^2 t^6}{120\beta^2} + \frac{7\pi^4 t^8}{660\beta^4} + \cdots\right)$$

and $\langle(\delta z^1(t))^2\rangle^{(\beta)} = \langle(\delta z^2(t))^2\rangle^{(\beta)} = \frac{3}{4}\langle(\delta z^3(t))^2\rangle^{(\beta)}$.

We have

$$\sqrt{\langle(\delta z^i(t))^2\rangle^{(\beta)}} \sim T^3$$

For high temperature,

$$\langle(\delta z^3(t))^2\rangle^{(\beta)} = \left(\frac{64\pi^8\kappa^2 z_0^2}{22275\beta^5}\right)\left(t^3 - \frac{5\beta t^2}{4} + \frac{5\beta^3}{32} + \cdots\right)$$

and $\langle(\delta z^1(t))^2\rangle^{(\beta)} = \langle(\delta z^2(t))^2\rangle^{(\beta)} = \frac{3}{4}\langle(\delta z^3(t))^2\rangle^{(\beta)}$.

We have

$$\sqrt{\langle(\delta z^i(t))^2\rangle^{(\beta)}} \sim T^{5/2}$$

Enhancement only at very high temperature

Squeezed state

$$|\xi\rangle = \prod_{\vec{k}} e^{\frac{1}{2}\xi(\hat{a}_{\vec{k}}^2 - \hat{a}_{\vec{k}}^{\dagger 2})}|0\rangle$$

$$\begin{aligned}
&\langle(\delta z^3(t))^2\rangle^{(\xi)} \\
= \quad & (\cosh 2\xi)\left(\frac{\kappa^2 z_0^2}{60\pi^2 t^2}\right) \\
& \quad \left[\Lambda^4 t^4 + 4\Lambda^2 t^2(1 + 2\cos(\Lambda t)) - 24\Lambda t \sin(\Lambda t) + 24(1 - \cos(\Lambda t))\right] \\
& \quad -(\sinh 2\xi)\left(\frac{\kappa^2 z_0^2}{60\pi^2 t^2}\right) \\
& \qquad [-\Lambda^4 t^4 + 2\Lambda^2 t^2(1 - 4\cos(\Lambda t)) + 4\Lambda t \sin(\Lambda t)(2 + \cos(\Lambda t)) \\
& \qquad\qquad -2(5 - 4\cos(\Lambda t) - \cos^2(\Lambda t))]
\end{aligned}$$

Enhancement by the squeeze parameter,

$$\sqrt{\langle(\delta z^3(t))^2\rangle^{(\xi)}} \sim e^{\xi}\sqrt{\langle(\delta z^3(t))^2\rangle^{(0)}}$$

Primordial gravitons from the inflationary era are in a squeezed quantum state.

IV. Conclusions

1. Quantum effect of gravitons could manifest as noise on the detector.
2. For the vacuum state, the noise effect is too small to be detectable. This is also true for the thermal state at low temperature.
3. For the squeezed state, there is a possibility in detecting the primordial gravitons from the inflationary era.

Summary: **Quantum fluctuations in Gravity**

Noise enters in **Decoherence** processes,

Correlation enters in **Entanglement** processes.

Graviton Noise effects on geodesic deviations.

Stochastic Semiclassical Gravity as a reliable
Theoretical Basis for Low Energy Quantum Gravity
and Quantum Information Phenomena

Thank You!

A. Quantum Gravity: Probing the micro/meso structures of spacetime from low energy up: What are the main challenges?

Very different from Emergent Gravity: theories postulated at Planck scale, with the correct low energy limit (ground state) matching observed spacetime and matter properties (not so easy).

B. What is a suitable theoretical basis for analyzing experiments on quantum systems in a weak gravitational field including quantum information considerations, in attempts such as

1) detecting the quantum nature of perturbative gravity,
2) formulating equivalence principle for quantum systems,
3) gravitational decoherence and entanglement (GravCats)
4) graviton noise effects?

Semiclassical Gravity (classical GR+ QFT) is the natural foundation, but not enough. Fluctuations/Correlations

Quantum vs Emergent Gravity

complementary tasks:

Top Down <=>Bottom Up

Hu, *Emergent /Quantum Gravity: Macro/Micro Structures Spacetime* [arXiv:0903.0878]

cf: Jan de Boer, Bianca Dittrich et al: *Frontiers of Quantum Gravity: shared challenges, converging directions* [arxiv:2207.10618]

Quantum Gravity: Theories of the microscopic structures of spacetime - How to identify/unravel these micro-structures from low energy physics **Bottom(energy)Up**: very challenging: need to **retrieve lost information.**

From assumed basic constituents: Strings,Loops,Sets,Groups,Simplices How to derive/deduce the spacetime manifold structure we see today -- **Emergent Gravity: Top Down.** Not so easy. **Emergence mechanisms**

II. Emergence

(Top-down: Micro to Macro)

Emergence: The Challenge lies in Explaining HOW and WHAT from?

- One needs to identify the mechanisms and processes whereby different levels of structures and the laws governing them, including the symmetry principles, emerge. That depends on deeper interplay of collectivity, complexity, stochasticity and self-organization at all levels.
- This is where non-equilibrium statistical mechanics -- including molecular chaotic dynamics, theories of open systems and fluctuations phenomena enter in a nontrivial way

Emergence vs Reduction

- Not a strange new concept. Prevalent in condensed matter physics, biology
  It is observed to be more commonly instrumental in nature than the simpler (simplistic?) constructionist**- reductionist view.** E.g., **TOE:**

  If we knew all the elementary particles and their interactions,
  we could deduce everything in the physical world. QCD→nuclear physics, QED → atomic, String → QG; physics → chemistry → biology, etc.

- In reality, there are new EMERGENT rules/laws (of organization, forms) and new modes of interactions and dynamics **at every level of structure**.
  "More is different!" (Anderson 1972) in an exclamatory tone!
  "**The Theory of Everything?**" (Laughlin and Pines 2000) in an ironic tone!
  Progress in particle physics needs ideas from CMP (e.g., symmetry breaking)

- In truth these two aspects are always present in our quest to understand nature: Elementary at low energy (resolution) is composite at higher energy (resolution).
  For few body systems it is easy to construct models but the real world is governed by many body interactions. (e.g., make nucleons from quark-gluon)

- **Equally challenging both ways.** (Molecular dynamics from hydrodynamics) More difficult to unravel, but that is what we've been doing in physics for centuries.

Emergence: Deductive versus Non-Deductive

Emergent behavior of macroscopic structure which **can or cannot** be deduced from microscopic structure and dynamics.

Deductive – Predictive

Hydrodynamics from kinetic theory from molecular dynamics.
-- But not always easy e.g., Turbulence.
-- Reverse is always more difficult.

Non-Deductive – **Un** (or not easily) **predictable from sub-structure / subdynamics**: e.g., Fractional Quantum Hall effect. Only after its discovery can one construct theories to `deduce'.

Of course, it depends on whether one can find the rules of emergence to deduce or predict. But how are these rules related to those of their substructure and dynamics, given there are possibly infinite of them?

--- gauge hierarchy problem, effective field theory, constituent vs collective.

Emergent Gravity

Difficulties in top-down:

[tasks of string theory/ loops /spin-nets]

The micro constituents are believed to be known,
need to get the macro limits --
Here we are dealing with deductive-predictive theories. Still, not an easy treat.

A. For nondeductive emergent behavior, even micro → macro / sub → super structure is difficult, bordering on the impossible.