Causal Structure in Quantum Gravity

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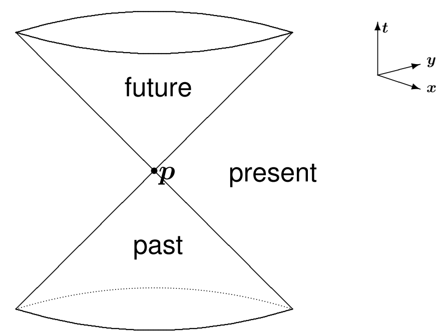
Quantum Gravity 2023
Nijmegen, the Netherlands
July 2023

Causal structure in classical gravity

Lorentzian metric $\Rightarrow$ local light cone structure $\Rightarrow$ causal structure



Classical results

- singularity theorems
- structure of horizons
- cosmic censorship
- asymptotic structures
- propagation of gravitational waves
- ...

Spacetime is a partially ordered set:

$x \prec y \Leftrightarrow x$ is to the causal past of y

This ordering determines most of spacetime structure –

Hawking, King, McCarthy; Malament:

For a causal spacetime,
causal structure + volume element $\Leftrightarrow$ topology + geometry

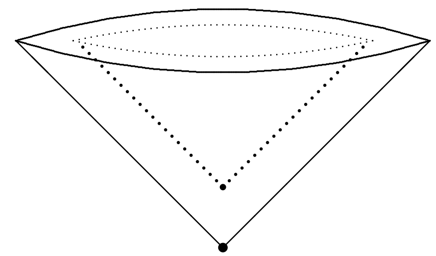
Roughly: order determines metric up to local scale factor
volume element determines scale

But quantum gravity blurs causal structure. . .

Pauli, Klein, Landau (1955): quantum fluctuations “smear” light cones

Wheeler: “spacetime foam” – superpositions in the path integral

Deser, DeWitt: effect on QFT



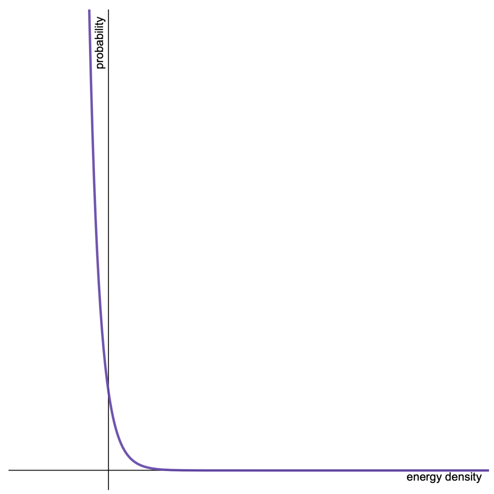
Positive energy fluctuations shrink light cones ⇒ “preserve causality”

But quantum energy fluctuations can be negative…

Effective field theory:

Low energy causality ⇒ conditions on higher curvature terms

But lack of causality still appears at high energies



Fluctuations are highly non-Gaussian:

Fat tail; “gambler’s ruin”

Can construct superpositions of causal order

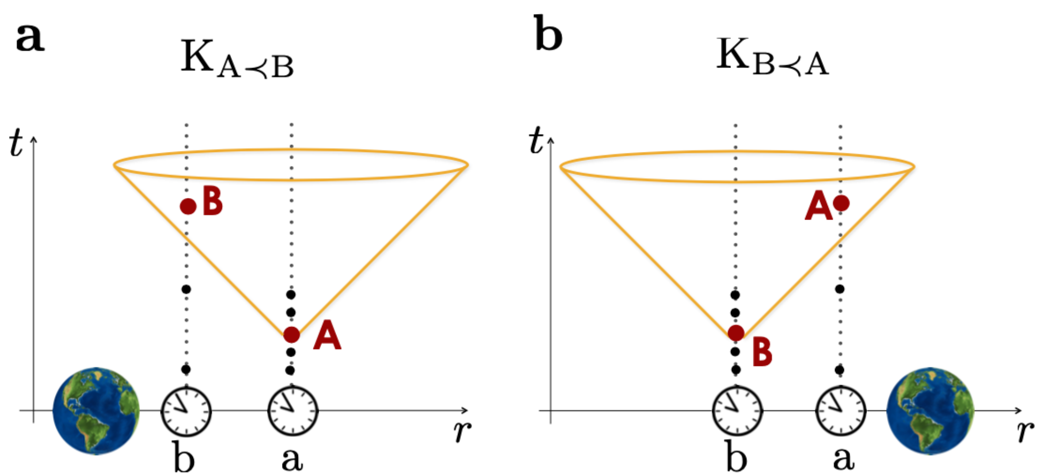


Figure 1: General relativistic engineering of causal relations between space-time events using a massive body. Initially synchronised clocks a and b are positioned at fixed distances from a far-away agent whose time coordinate is t. Event A(B) is defined by the clock of a(b) showing proper time τ^*. **a** In configuration $K_{A\prec B}$ the mass is placed closer to b than to a. Due to gravitational time dilation, event A can end up in a causal past of event B: for a sufficiently large τ^* the time difference between the clocks becomes greater than it takes light to travel between them. Light emitted at event A reaches clock b before the event B occurs. **b** Configuration $K_{B\prec A}$ is fully analogous to $K_{A\prec B}$: the mass is placed closer to clock a and the event B can end up in the causal past of the event A.

Zych et al., *Nature Commun.* 10 (2019) 1

Why this is bad

Microcausality in quantum field theory:

If x and x' are spacelike separated, $[\mathcal{O}_1(x), \mathcal{O}_2(x')] = 0$

$\Rightarrow$ analyticity of S-matrix

Problems of time (Kiefer's talk):

– need acausal surface for inner products/normalized probabilities

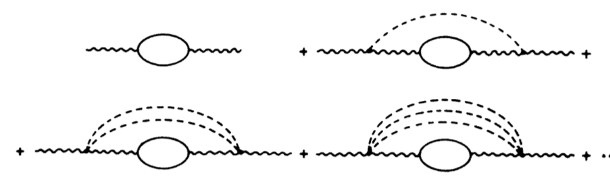
– need propagation between acausal surfaces for unitarity

Why this might be good

Light cone fluctuations may eliminate QFT divergences

First principles: DeWitt, Khriplovich, Isham, Salam, Strathdee

Resum classes of Feynman diagrams



Incomplete; may require reorganizing perturbation theory (Woodard)

Phenomenological: smear light cone by hand

$$\bar{G}_F(x^2) = \frac{1}{16\pi^2 i}\int_0^\infty \frac{ds}{s^2}\exp\left\{-i\left(\frac{x^2-\ell^2}{4s}+m^2 s\right)\right\}$$

or

$$\bar{G}_F(x^2) = \int d\lambda f(\lambda) G_F(x^2-\lambda)$$

(many authors, somewhat different approaches)

Also. . .

– allows topology change: see talks by Dittrich, Asante
  (spatial topology change $\Rightarrow$ closed timelike curves or singularities

– could help with black hole information loss problem
  (recent work on islands and wormholes)

– might have observational consequences: see talk by Amelino-Camelia
  (e.g., loss of coherence in starlight)

Quantum mechanics with indefinite causal structure

Standard QM:

If A and B are spacelike separated: joint state on $\mathcal{H}_1 \times \mathcal{H}_2$
If A and B are causally related: initial state on $\mathcal{H}_1$ and map

Hardy (2005): proposed unified description
Various approaches to "causally neutral QM" since then...
(see also Oeckl's "general boundary formalism")

Analog of Bell: causal superpositions allow violation of classical inequality
(one way signaling on each trial, but observable choice can determine direction)

Laboratory tests claim superpositions of causal order
– but information theoretical causal order $\neq$ spacetime causal order
– no real implementation in fixed spacetime? (e.g., Vilasini and Renner)
– still room for surprises (e.g., Belenchia, Wald et al.)

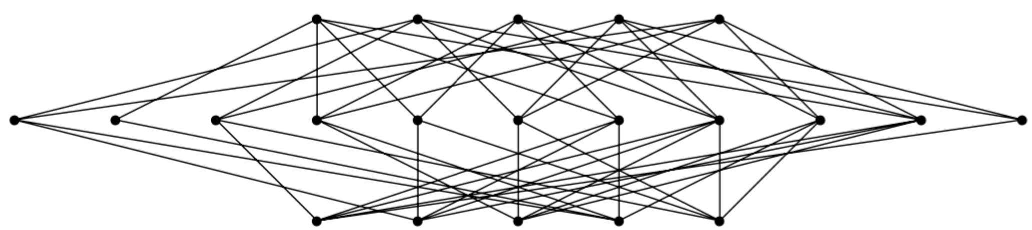
Particular case: causal sets

Discrete version: causal set (as discussed by Surya):

– keep causal order $x \prec y$ as in Hawking, King, McCarthy, Malament
– volume element $\Leftrightarrow$ number of points in a region
– add causality ($x \nprec x$) and finiteness ($|\{z : x \prec z \prec y\}| < \infty$)

Can approximate a spacetime by a causal set (“Poisson sprinkling”)
$\Rightarrow$ dimension, coarse-grained topology, geometry, Greens functions, . . .

But most causal sets are nothing like spacetimes



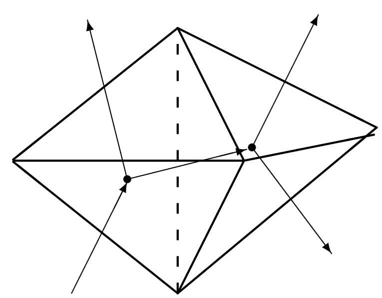
Recent result (P. Carlip, S. Carlip, S. Surya):

Standard (discrete) Einstein-Hilbert path integral
very strongly suppresses most “bad” causal sets
(but there may be other “bad” sets we don’t know about)

Causal sets come from *classical* causality
But path integral certainly includes
superpositions of different classical causal structures

Causal sets occur in many places

Simplicial complexes and spin foams (Bianchi, Martin-Dussaud)



$\Rightarrow$ signs of dihedral angles in Regge calculus, causal EPRL model

Particular case: Bulk metrics and boundary causality

AdS/CFT and similar holographic approaches:
maybe only boundary causality matters

But bulk path integral includes metrics that violate boundary causality
Need extraordinary cancellation: $\langle [\mathcal{O}_1(x), \mathcal{O}_2(x')] \mathcal{O}_3 \mathcal{O}_4 \mathcal{O}_5 \ldots \rangle = 0$

Hernández-Cuenca, Horowitz, Treviño, and Wang:
need to carefully account for out-of-time-order correlators

Causal structure elsewhere

- Lorentzian path integral
  - Teitelboim: causality vs. gauge invariance in Lorentzian path integral

    $$\int [dN][dN^i][dq] \exp\left\{i \int dt \int d^3x \left(\pi^{ij}\dot{q}_{ij} - N^i\mathcal{H}_i - N\mathcal{H}\right)\right\}$$

    What range of integration of N?
  - Dittrich's talk: implementation of Lorentzian path integral
- Asymptotic safety
  - efforts to move to Lorentzian signature
  - Banerjee: IR renormalization group flow is state-dependent

    Reichert: Lorentzian spectral function
- Causal Dynamical Triangulations
  - Ambjorn: next talk

But...

Individual causal spacetimes in path integral $\not\Leftrightarrow$ causal quantum theory

Question remains

How does quantum gravity
give rise to observed classical causality
to such enormous precision?