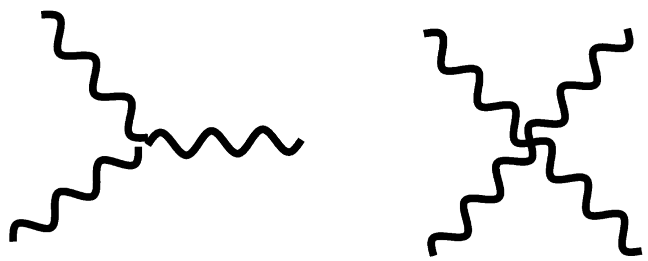
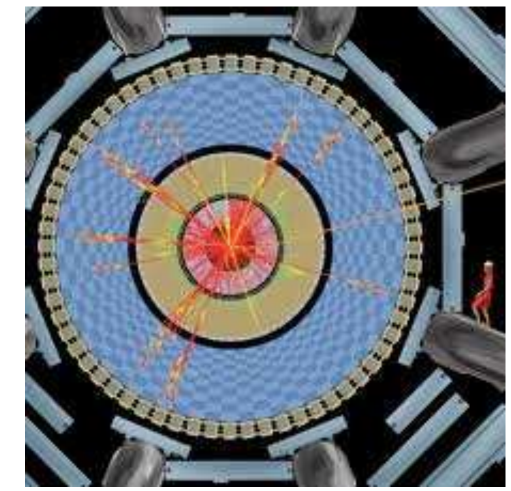
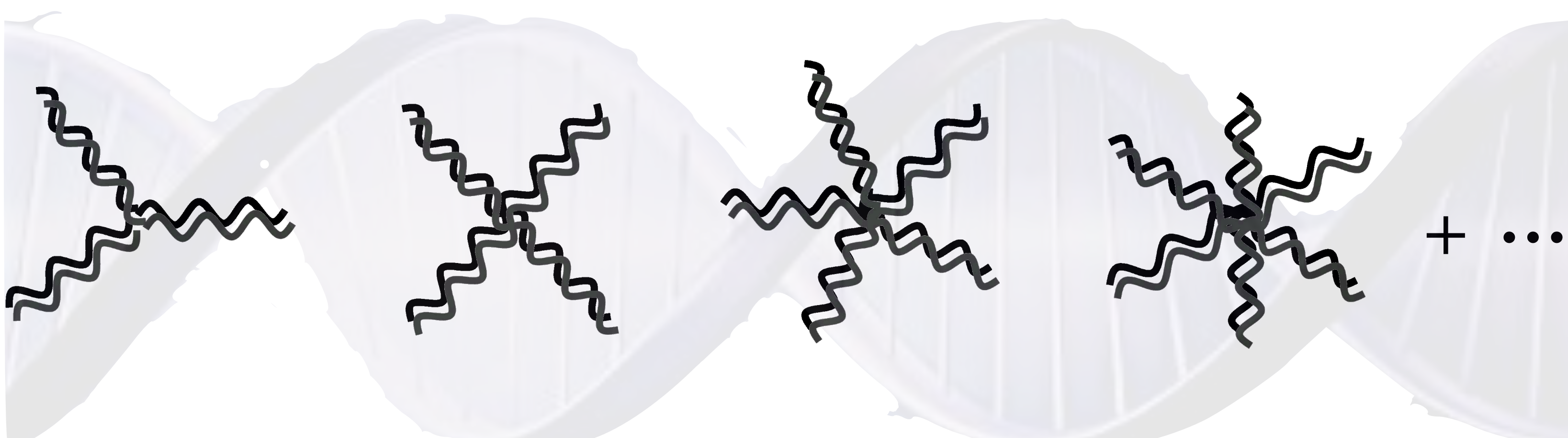
Gravity as a QFT: Decoding the DNA of Prediction

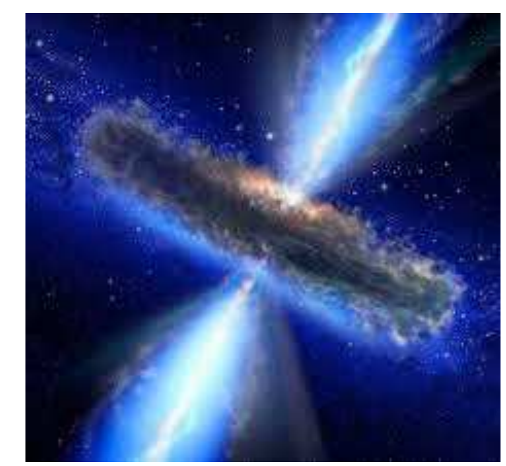


=





=



Northwestern University

John Joseph M Carrasco

Important Questions?

Data drives important physical questions.

Gravitational Wave Astrophysics is an increasingly important source of data re: the classical limit of quantum gravity.

Cosmology is an increasingly data-driven source of our most important physical questions involving the quantum evolution of space and time.

I’m personally excited that people are taking seriously “tabletop” experiment towards novel insight

Consider this talk an invitation, there are many reviews:

Snowmass White Paper: the Double Copy and its Applications, 2204.06547

-gentle overview for broad audience, nice discussion including non-flat backgrounds, classical solns, GW atrophysics, cosmological challenges

The SAGEX review on scattering amplitudes (chapter 2), 2203.13013

-gentle introduction targeting amplitudes expertise

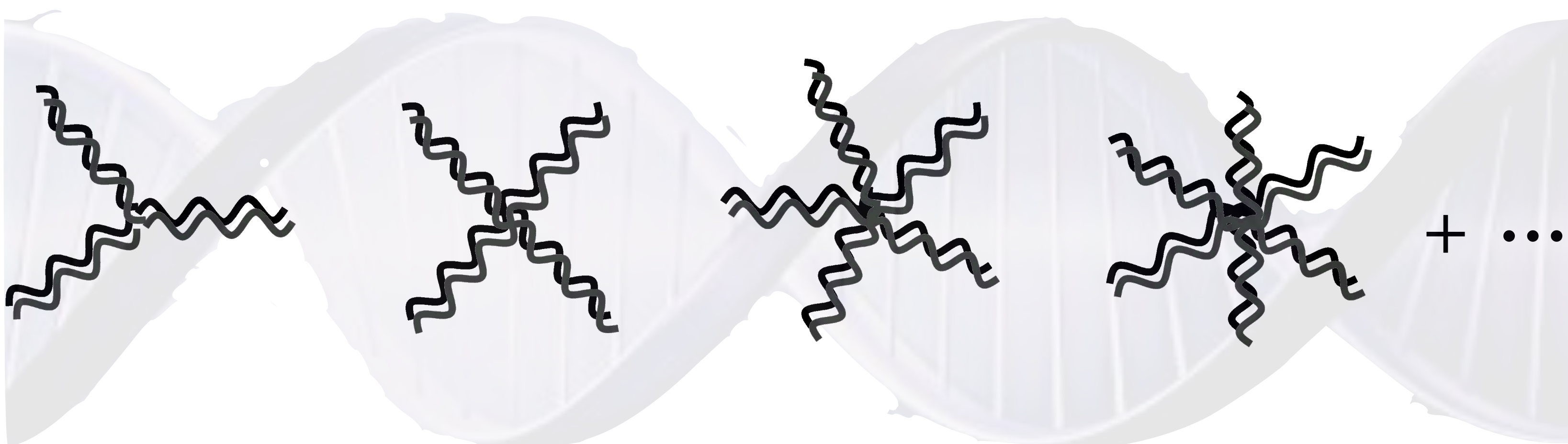
The Duality Between Color and Kinematics and its Applications, 1909.01358

-technical, overview of literature ~ 2019

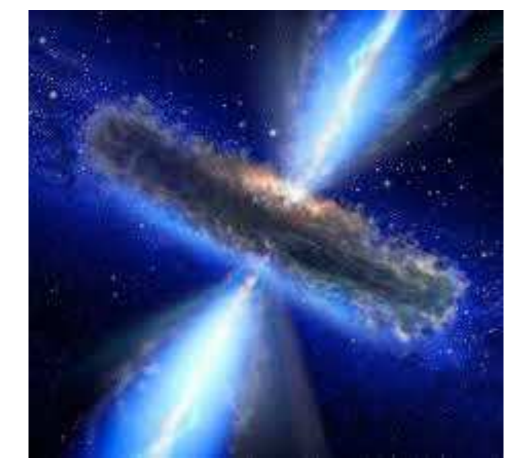
Supergravity amplitudes, the double copy and ultraviolet behavior, 2304.07392

-focus on UV behavior of SG

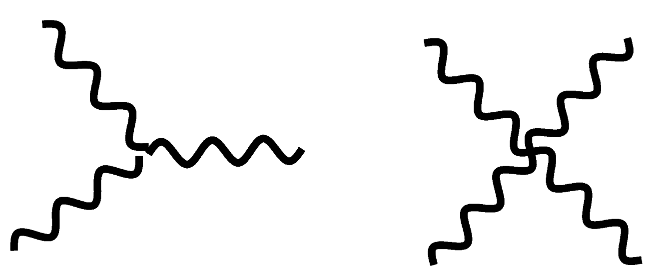
What data-driven questions in:



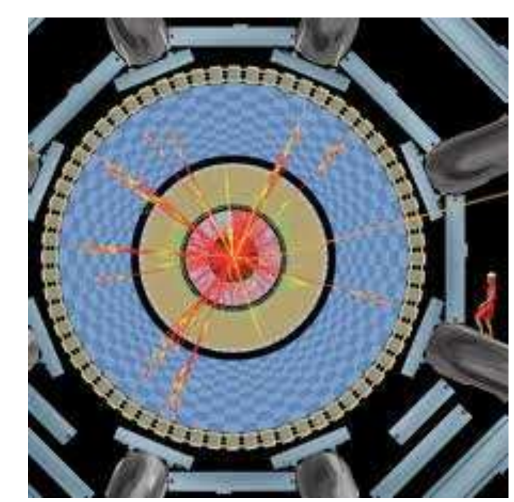
=



cannot be answered by asking questions of:



=



Scattering Amplitudes

IN
free states
(no interactions)

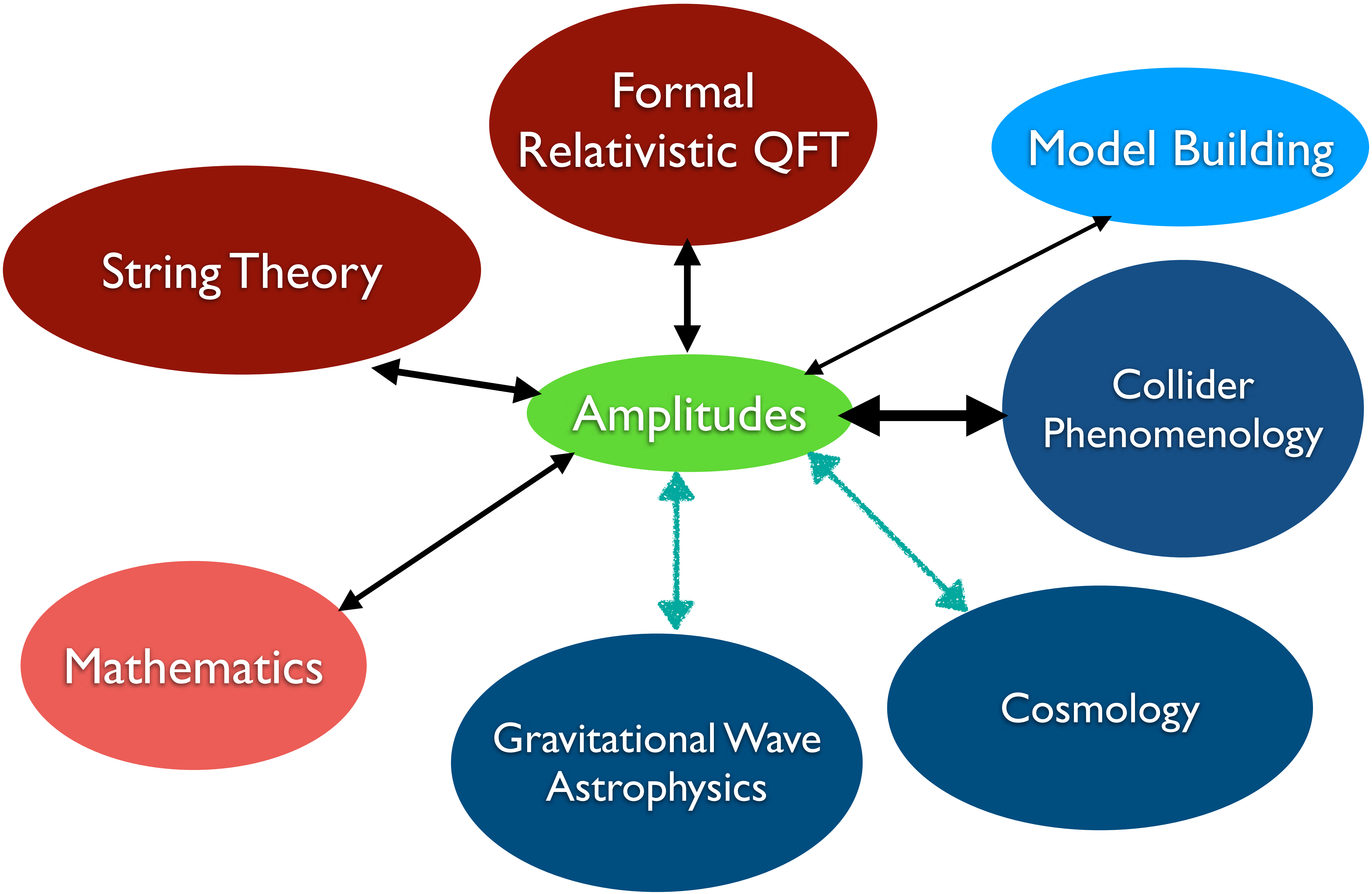
$$\left[\mathrm{S-matrix}\right]$$

OUT
free states
(no interactions)

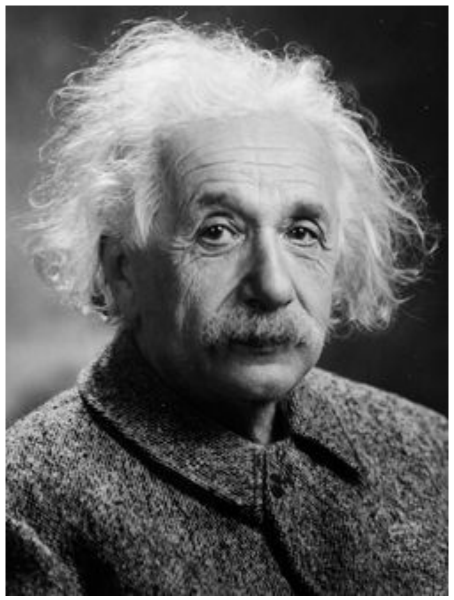
(all the interactions!)

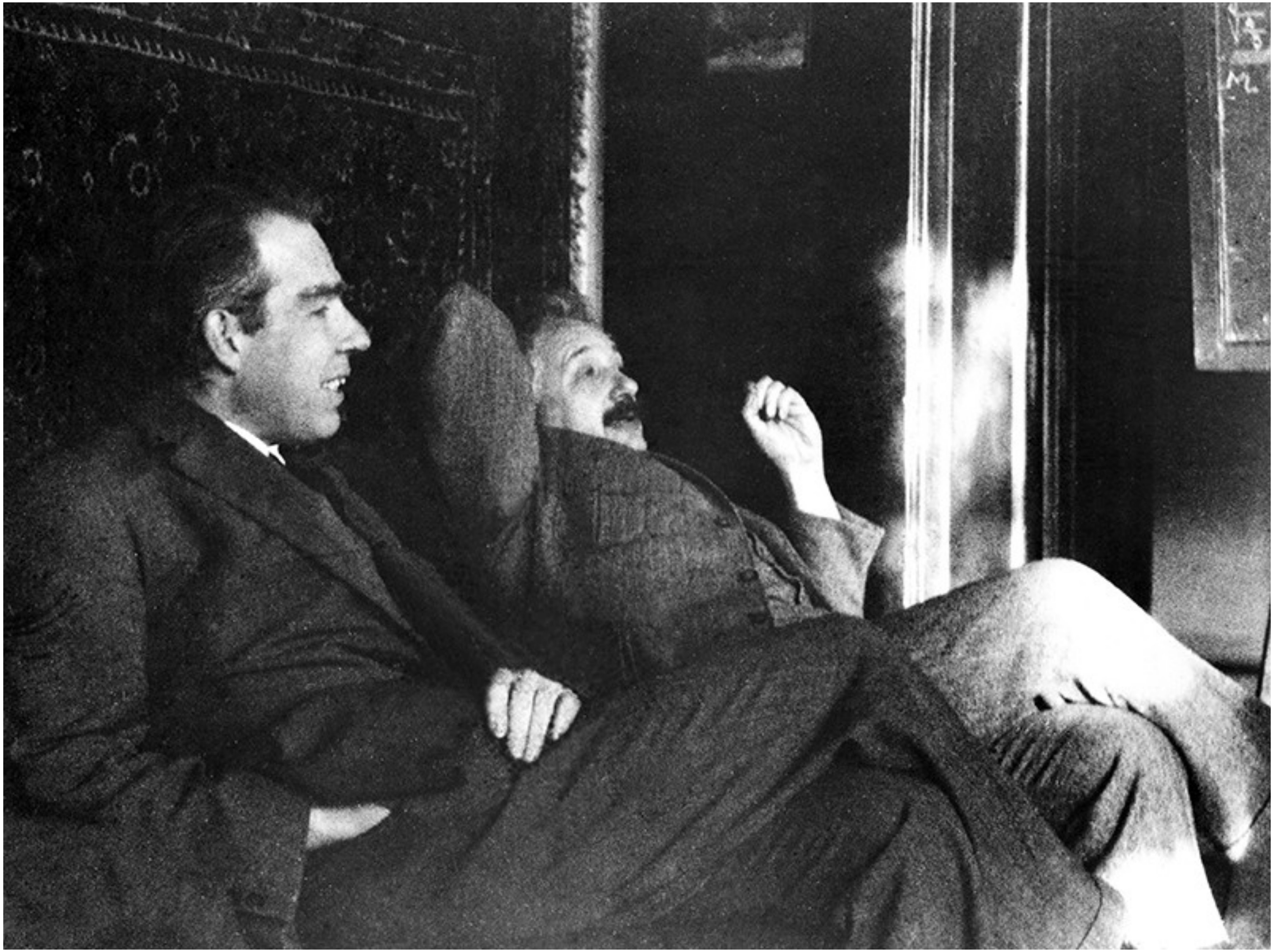
Key Property: GAUGE INVARIANT

Who do we currently connect to?



Can any (3+1)D point-like quantum field theory of gravity be ***predictive*** to all quantum corrections?

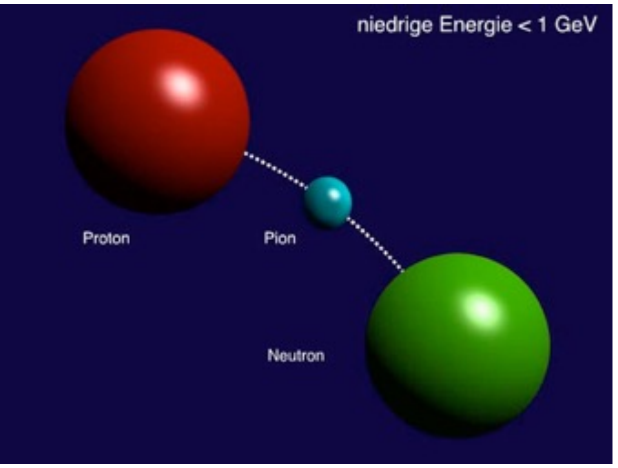




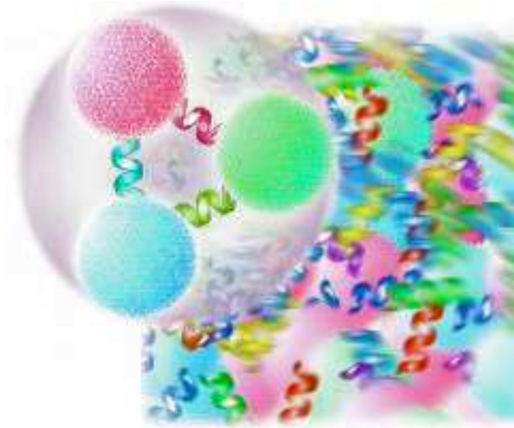
low energy (IR) **high energy (UV)**

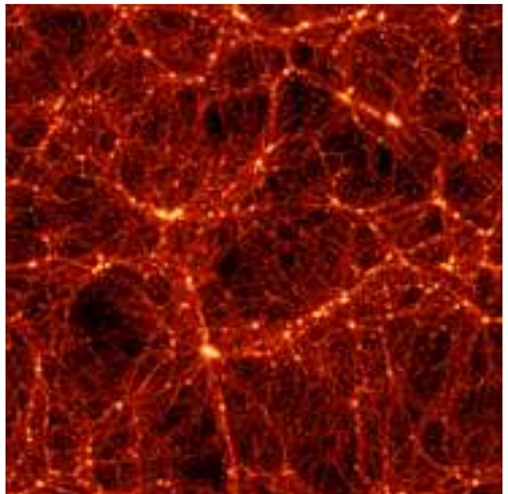
Scales

low energy (IR) **high energy (UV)**



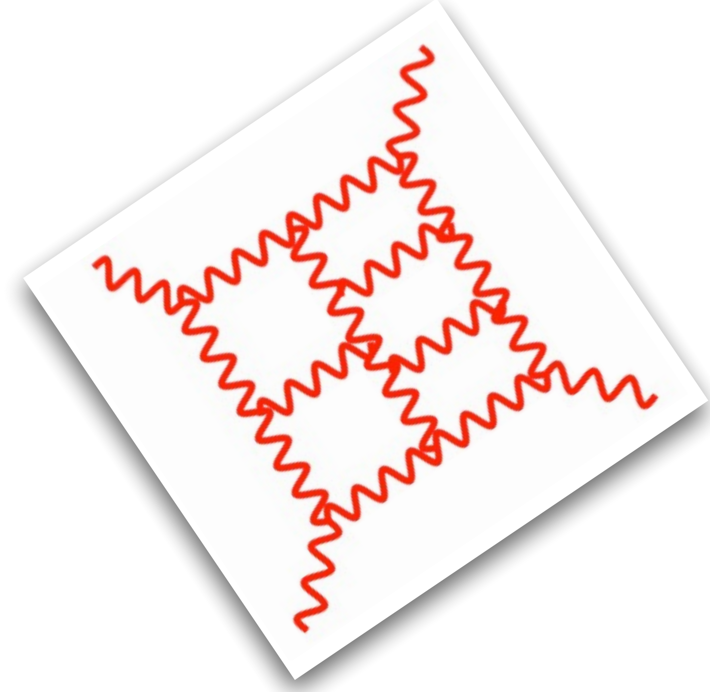
Scales





Evidence of spectacular cancellations in **Maximal (N=8) Supergravity**!

Cremmer, Julia, Scherk



Through the 4th quantum correction (4-loops) it's had the same high energy behavior as Maximal (N=4) Gauge Theory Brink, Schwarz, Scherk

$$D_c = 4 + 6/L$$

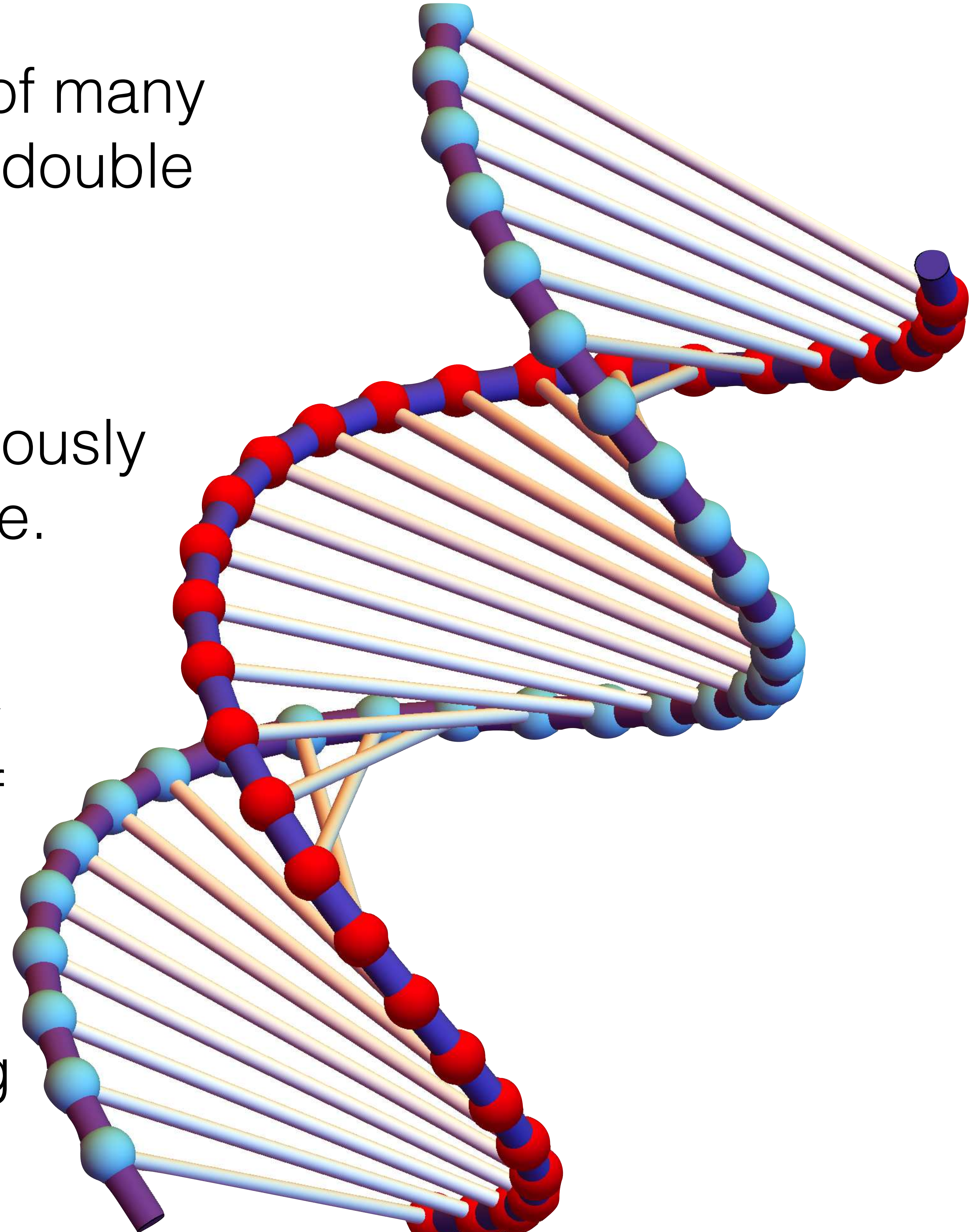
Indeed, pure gravity — while divergent at 2-loops — has surprising cancellations, perhaps only a finite bit of help sufficient for good UV behavior … (N>=5 SG??)

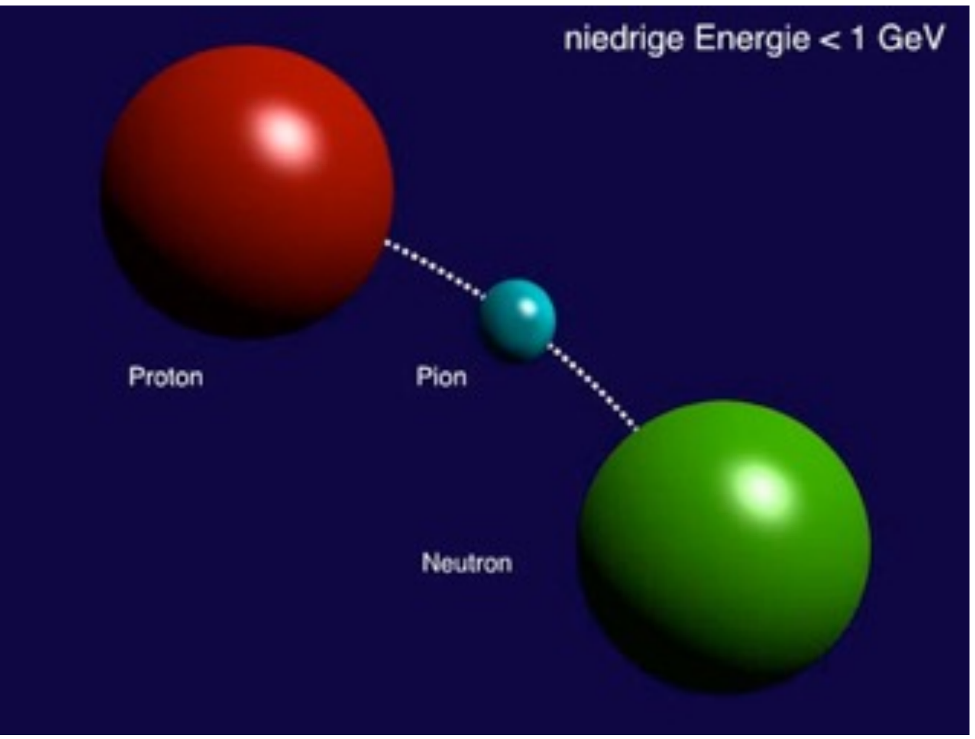
The scattering amplitudes of many relativistic theories admit a double copy description.

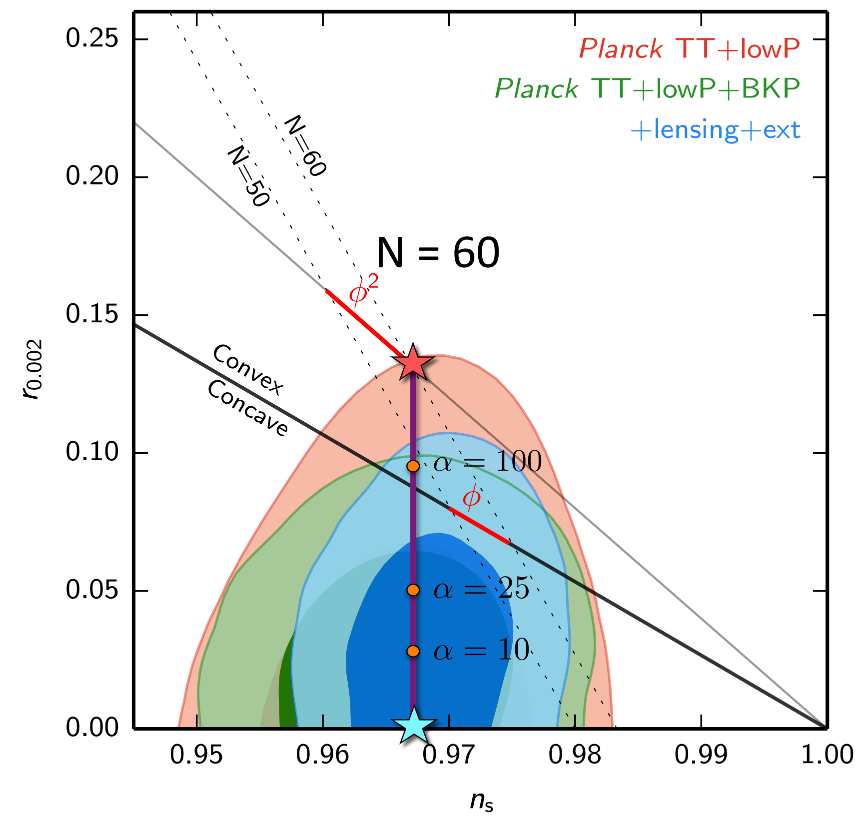
This points to previously hidden structure.

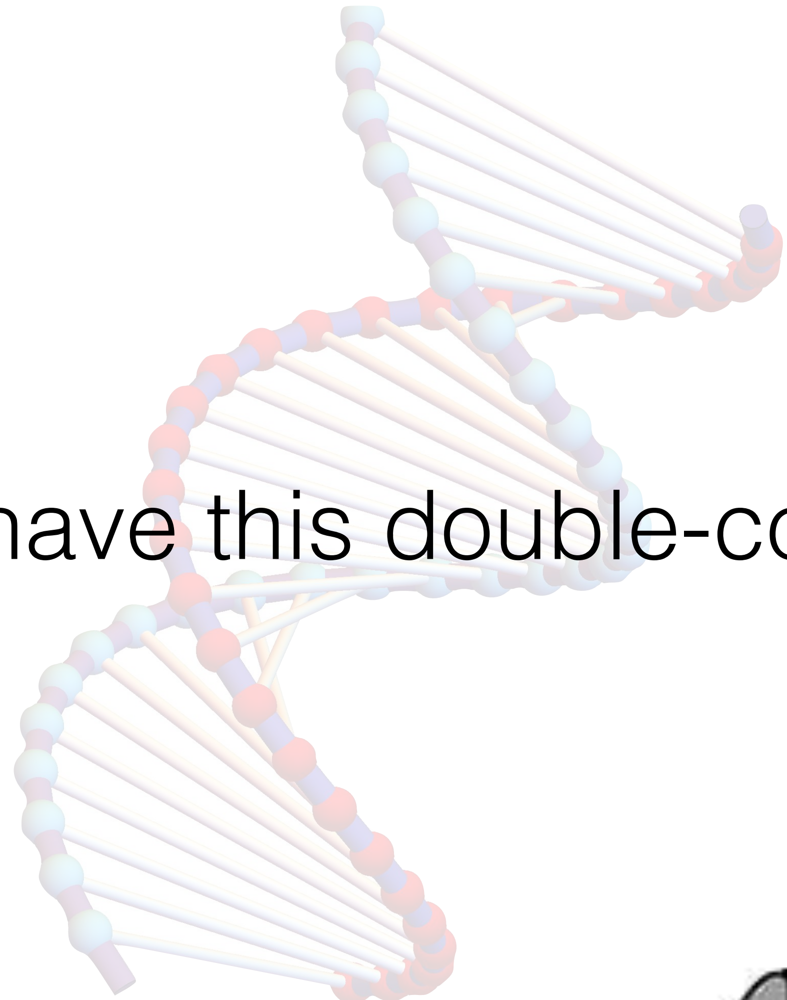
The is yet to be generally understood at the level of the action.

But this is already trivializing previously intractable problems.

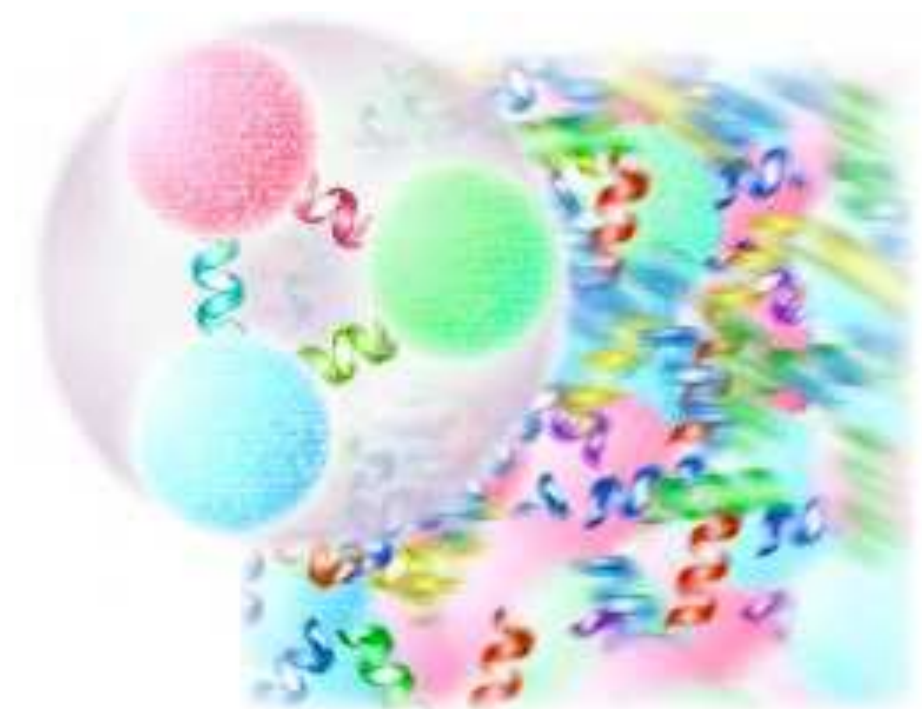


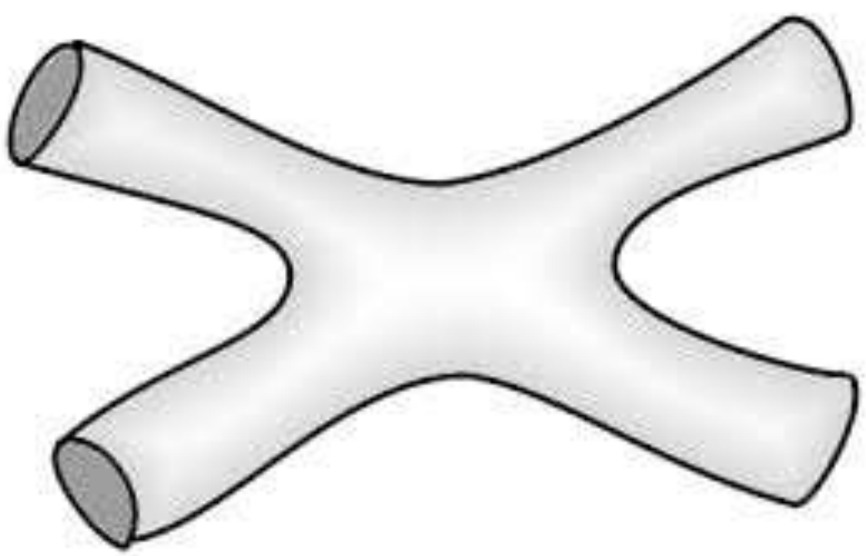






Many theories have this double-copy structure!





Key Point: **MANY Theories are Double Copies**

Advantages:

+ Can exploit for technical simplicity in prediction

+ Exposes a beautiful geometry in S-matrix

+ Structural foundations of predictions in theories with ultra-soft UV behavior.

+ Exposes deep web of relationships between theories

Key Point: **MANY Theories are Double Copies**

Why do such (generalized) gauge choices exist?

How can we exploit the benefits in all circumstances?

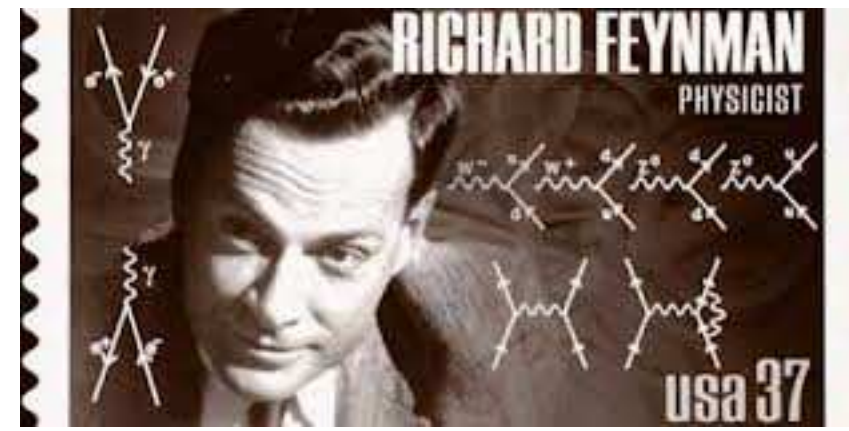
MOTIVATION

Need for Technical Simplicity

(the perils of infinite employment prospects)

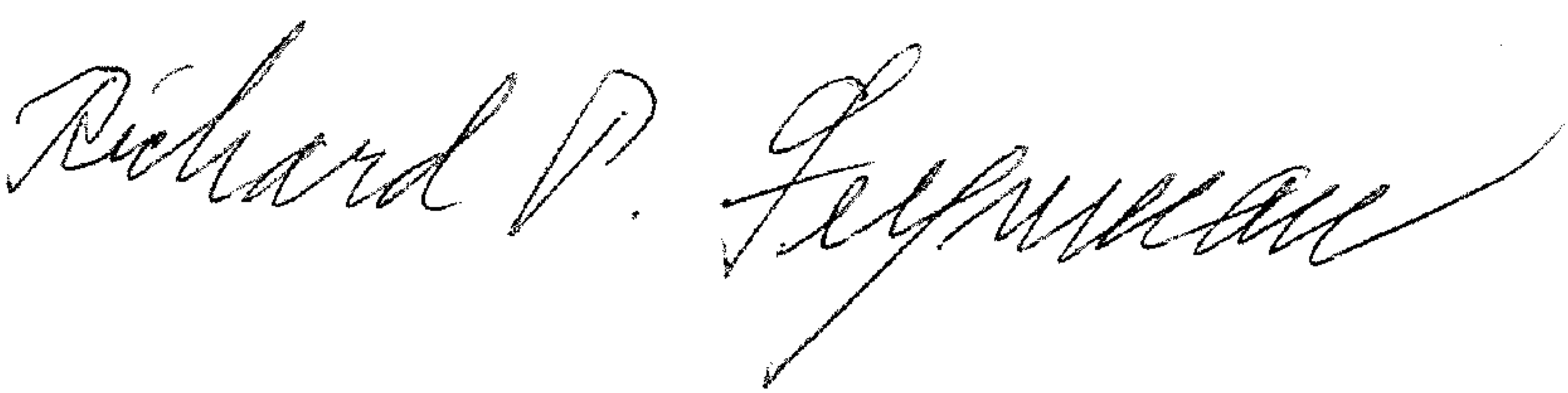
Complexity of Carrying Unphysical Information

"Do Feynman rules represent a useful solution??"

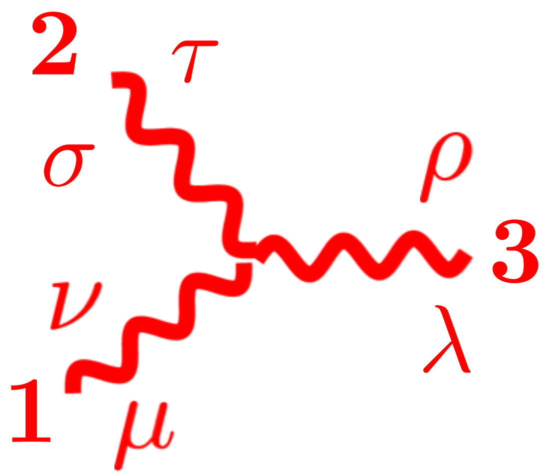


trees: semi-classical

loops: increasing quantum corrections



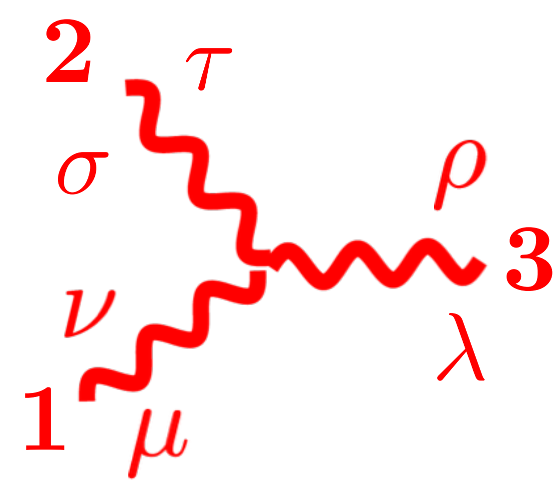
Off-shell three-graviton vertex:



Off-shell three-graviton vertex:

$$
\begin{aligned}
&\frac{\delta S^3}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma\tau}\delta\varphi_{\rho\lambda}} \to 2\eta^{\mu\tau}\eta^{\nu\sigma}k_1{}^{\lambda}k_1{}^{\rho} + 2\eta^{\mu\sigma}\eta^{\nu\tau}k_1{}^{\lambda}k_1{}^{\rho} - 2\eta^{\mu\nu}\eta^{\sigma\tau}k_1{}^{\lambda}k_1{}^{\rho} + \\
&2\eta^{\lambda\tau}\eta^{\mu\nu}k_1{}^{\sigma}k_1{}^{\rho} + 2\eta^{\lambda\sigma}\eta^{\mu\nu}k_1{}^{\tau}k_1{}^{\rho} + \eta^{\mu\tau}\eta^{\nu\sigma}k_2{}^{\lambda}k_1{}^{\rho} + \eta^{\mu\sigma}\eta^{\nu\tau}k_2{}^{\lambda}k_1{}^{\rho} + \eta^{\lambda\tau}\eta^{\nu\sigma}k_2{}^{\mu}k_1{}^{\rho} + \\
&\eta^{\lambda\sigma}\eta^{\nu\tau}k_2{}^{\mu}k_1{}^{\rho} + \eta^{\lambda\tau}\eta^{\mu\sigma}k_2{}^{\nu}k_1{}^{\rho} + \eta^{\lambda\sigma}\eta^{\mu\tau}k_2{}^{\nu}k_1{}^{\rho} + \eta^{\lambda\tau}\eta^{\nu\sigma}k_3{}^{\mu}k_1{}^{\rho} + \eta^{\lambda\sigma}\eta^{\nu\tau}k_3{}^{\mu}k_1{}^{\rho} - \\
&\eta^{\lambda\nu}\eta^{\sigma\tau}k_3{}^{\mu}k_1{}^{\rho} + \eta^{\lambda\tau}\eta^{\mu\sigma}k_3{}^{\nu}k_1{}^{\rho} + \eta^{\lambda\sigma}\eta^{\mu\tau}k_3{}^{\nu}k_1{}^{\rho} - \eta^{\lambda\mu}\eta^{\sigma\tau}k_3{}^{\nu}k_1{}^{\rho} + \eta^{\lambda\nu}\eta^{\mu\tau}k_3{}^{\sigma}k_1{}^{\rho} + \\
&\eta^{\lambda\mu}\eta^{\nu\tau}k_3{}^{\sigma}k_1{}^{\rho} + \eta^{\lambda\nu}\eta^{\mu\sigma}k_3{}^{\tau}k_1{}^{\rho} + \eta^{\lambda\mu}\eta^{\nu\sigma}k_3{}^{\tau}k_1{}^{\rho} + 2\eta^{\mu\nu}\eta^{\rho\tau}k_1{}^{\lambda}k_1{}^{\sigma} + 2\eta^{\mu\nu}\eta^{\rho\sigma}k_1{}^{\lambda}k_1{}^{\tau} - \\
&2\eta^{\lambda\rho}\eta^{\mu\nu}k_1{}^{\sigma}k_1{}^{\tau} + 2\eta^{\lambda\nu}\eta^{\mu\rho}k_1{}^{\sigma}k_1{}^{\tau} + 2\eta^{\lambda\mu}\eta^{\nu\rho}k_1{}^{\sigma}k_1{}^{\tau} + \eta^{\mu\tau}\eta^{\nu\rho}k_1{}^{\sigma}k_2{}^{\lambda} + \eta^{\mu\rho}\eta^{\nu\tau}k_1{}^{\sigma}k_2{}^{\lambda} + \\
&\eta^{\mu\sigma}\eta^{\nu\rho}k_1{}^{\tau}k_2{}^{\lambda} + \eta^{\mu\rho}\eta^{\nu\sigma}k_1{}^{\tau}k_2{}^{\lambda} + \eta^{\nu\tau}\eta^{\rho\sigma}k_1{}^{\lambda}k_2{}^{\mu} + \eta^{\nu\sigma}\eta^{\rho\tau}k_1{}^{\lambda}k_2{}^{\mu} + \eta^{\lambda\tau}\eta^{\nu\rho}k_1{}^{\sigma}k_2{}^{\mu} - \\
&\eta^{\lambda\rho}\eta^{\nu\tau}k_1{}^{\sigma}k_2{}^{\mu} + \eta^{\lambda\nu}\eta^{\rho\tau}k_1{}^{\sigma}k_2{}^{\mu} + \eta^{\lambda\sigma}\eta^{\nu\rho}k_1{}^{\tau}k_2{}^{\mu} - \eta^{\lambda\rho}\eta^{\nu\sigma}k_1{}^{\tau}k_2{}^{\mu} + \eta^{\lambda\nu}\eta^{\rho\sigma}k_1{}^{\tau}k_2{}^{\mu} + \\
&2\eta^{\nu\rho}\eta^{\sigma\tau}k_2{}^{\lambda}k_2{}^{\mu} + \eta^{\mu\tau}\eta^{\rho\sigma}k_1{}^{\lambda}k_2{}^{\nu} + \eta^{\mu\sigma}\eta^{\rho\tau}k_1{}^{\lambda}k_2{}^{\nu} + \eta^{\lambda\tau}\eta^{\mu\rho}k_1{}^{\sigma}k_2{}^{\nu} - \eta^{\lambda\rho}\eta^{\mu\tau}k_1{}^{\sigma}k_2{}^{\nu} + \\
&\eta^{\lambda\mu}\eta^{\rho\tau}k_1{}^{\sigma}k_2{}^{\nu} + \eta^{\lambda\sigma}\eta^{\mu\rho}k_1{}^{\tau}k_2{}^{\nu} - \eta^{\lambda\rho}\eta^{\mu\sigma}k_1{}^{\tau}k_2{}^{\nu} + \eta^{\lambda\mu}\eta^{\rho\sigma}k_1{}^{\tau}k_2{}^{\nu} + 2\eta^{\mu\rho}\eta^{\sigma\tau}k_2{}^{\lambda}k_2{}^{\nu} + \\
&2\eta^{\lambda\tau}\eta^{\rho\sigma}k_2{}^{\mu}k_2{}^{\nu} + 2\eta^{\lambda\sigma}\eta^{\rho\tau}k_2{}^{\mu}k_2{}^{\nu} - 2\eta^{\lambda\rho}\eta^{\sigma\tau}k_2{}^{\mu}k_2{}^{\nu} + \eta^{\mu\tau}\eta^{\nu\sigma}k_1{}^{\lambda}k_2{}^{\rho} + \eta^{\mu\sigma}\eta^{\nu\tau}k_1{}^{\lambda}k_2{}^{\rho} + \\
&\eta^{\lambda\nu}\eta^{\mu\tau}k_1{}^{\sigma}k_2{}^{\rho} + \eta^{\lambda\mu}\eta^{\nu\tau}k_1{}^{\sigma}k_2{}^{\rho} + \eta^{\lambda\nu}\eta^{\mu\sigma}k_1{}^{\tau}k_2{}^{\rho} + \eta^{\lambda\mu}\eta^{\nu\sigma}k_1{}^{\tau}k_2{}^{\rho} + 2\eta^{\mu\tau}\eta^{\nu\sigma}k_2{}^{\lambda}k_2{}^{\rho} + \\
&2\eta^{\mu\sigma}\eta^{\nu\tau}k_2{}^{\lambda}k_2{}^{\rho} - 2\eta^{\mu\nu}\eta^{\sigma\tau}k_2{}^{\lambda}k_2{}^{\rho} + 2\eta^{\lambda\nu}\eta^{\sigma\tau}k_2{}^{\mu}k_2{}^{\rho} + 2\eta^{\lambda\mu}\eta^{\sigma\tau}k_2{}^{\nu}k_2{}^{\rho} + \eta^{\nu\tau}\eta^{\rho\sigma}k_1{}^{\lambda}k_3{}^{\mu} + \\
&\eta^{\nu\sigma}\eta^{\rho\tau}k_1{}^{\lambda}k_3{}^{\mu} - \eta^{\nu\rho}\eta^{\sigma\tau}k_1{}^{\lambda}k_3{}^{\mu} + \eta^{\lambda\tau}\eta^{\nu\rho}k_1{}^{\sigma}k_3{}^{\mu} + \eta^{\lambda\nu}\eta^{\rho\tau}k_1{}^{\sigma}k_3{}^{\mu} + \eta^{\lambda\sigma}\eta^{\nu\rho}k_1{}^{\tau}k_3{}^{\mu} + \\
&\eta^{\lambda\nu}\eta^{\rho\sigma}k_1{}^{\tau}k_3{}^{\mu} + \eta^{\nu\tau}\eta^{\rho\sigma}k_2{}^{\lambda}k_3{}^{\mu} + \eta^{\nu\sigma}\eta^{\rho\tau}k_2{}^{\lambda}k_3{}^{\mu} + \eta^{\lambda\tau}\eta^{\rho\sigma}k_2{}^{\nu}k_3{}^{\mu} + \eta^{\lambda\sigma}\eta^{\rho\tau}k_2{}^{\nu}k_3{}^{\mu} + \\
&\eta^{\lambda\tau}\eta^{\nu\sigma}k_2{}^{\rho}k_3{}^{\mu} + \eta^{\lambda\sigma}\eta^{\nu\tau}k_2{}^{\rho}k_3{}^{\mu} + \eta^{\mu\tau}\eta^{\rho\sigma}k_1{}^{\lambda}k_3{}^{\nu} + \eta^{\mu\sigma}\eta^{\rho\tau}k_1{}^{\lambda}k_3{}^{\nu} - \eta^{\mu\rho}\eta^{\sigma\tau}k_1{}^{\lambda}k_3{}^{\nu} + \\
&\eta^{\lambda\tau}\eta^{\mu\rho}k_1{}^{\sigma}k_3{}^{\nu} + \eta^{\lambda\mu}\eta^{\rho\tau}k_1{}^{\sigma}k_3{}^{\nu} + \eta^{\lambda\sigma}\eta^{\mu\rho}k_1{}^{\tau}k_3{}^{\nu} + \eta^{\lambda\mu}\eta^{\rho\sigma}k_1{}^{\tau}k_3{}^{\nu} + \eta^{\mu\tau}\eta^{\rho\sigma}k_2{}^{\lambda}k_3{}^{\nu} + \\
&\eta^{\mu\sigma}\eta^{\rho\tau}k_2{}^{\lambda}k_3{}^{\nu} + \eta^{\lambda\tau}\eta^{\rho\sigma}k_2{}^{\mu}k_3{}^{\nu} + \eta^{\lambda\sigma}\eta^{\rho\tau}k_2{}^{\mu}k_3{}^{\nu} + \eta^{\lambda\tau}\eta^{\mu\sigma}k_2{}^{\rho}k_3{}^{\nu} + \eta^{\lambda\sigma}\eta^{\mu\tau}k_2{}^{\rho}k_3{}^{\nu} + \\
&2\eta^{\lambda\tau}\eta^{\rho\sigma}k_3{}^{\mu}k_3{}^{\nu} + 2\eta^{\lambda\sigma}\eta^{\rho\tau}k_3{}^{\mu}k_3{}^{\nu} - 2\eta^{\lambda\rho}\eta^{\sigma\tau}k_3{}^{\mu}k_3{}^{\nu} + \eta^{\mu\tau}\eta^{\nu\rho}k_1{}^{\lambda}k_3{}^{\sigma} + \eta^{\mu\rho}\eta^{\nu\tau}k_1{}^{\lambda}k_3{}^{\sigma} + \\
&\eta^{\lambda\nu}\eta^{\mu\rho}k_1{}^{\tau}k_3{}^{\sigma} + \eta^{\lambda\mu}\eta^{\nu\rho}k_1{}^{\tau}k_3{}^{\sigma} + \eta^{\mu\tau}\eta^{\nu\rho}k_2{}^{\lambda}k_3{}^{\sigma} + \eta^{\mu\rho}\eta^{\nu\tau}k_2{}^{\lambda}k_3{}^{\sigma} - \eta^{\mu\nu}\eta^{\rho\tau}k_2{}^{\lambda}k_3{}^{\sigma} + \\
&\eta^{\lambda\tau}\eta^{\nu\rho}k_2{}^{\mu}k_3{}^{\sigma} + \eta^{\lambda\nu}\eta^{\rho\tau}k_2{}^{\mu}k_3{}^{\sigma} + \eta^{\lambda\tau}\eta^{\mu\rho}k_2{}^{\nu}k_3{}^{\sigma} + \eta^{\lambda\mu}\eta^{\rho\tau}k_2{}^{\nu}k_3{}^{\sigma} - \eta^{\lambda\tau}\eta^{\mu\nu}k_2{}^{\rho}k_3{}^{\sigma} + \\
&\eta^{\lambda\nu}\eta^{\mu\tau}k_2{}^{\rho}k_3{}^{\sigma} + \eta^{\lambda\mu}\eta^{\nu\tau}k_2{}^{\rho}k_3{}^{\sigma} + 2\eta^{\lambda\rho}\eta^{\nu\tau}k_3{}^{\mu}k_3{}^{\sigma} + 2\eta^{\lambda\rho}\eta^{\mu\tau}k_3{}^{\nu}k_3{}^{\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}k_1{}^{\lambda}k_3{}^{\tau} + \\
&\eta^{\mu\rho}\eta^{\nu\sigma}k_1{}^{\lambda}k_3{}^{\tau} + \eta^{\lambda\nu}\eta^{\mu\rho}k_1{}^{\sigma}k_3{}^{\tau} + \eta^{\lambda\mu}\eta^{\nu\rho}k_1{}^{\sigma}k_3{}^{\tau} + \eta^{\mu\sigma}\eta^{\nu\rho}k_2{}^{\lambda}k_3{}^{\tau} + \eta^{\mu\rho}\eta^{\nu\sigma}k_2{}^{\lambda}k_3{}^{\tau} - \\
&\eta^{\mu\nu}\eta^{\rho\sigma}k_2{}^{\lambda}k_3{}^{\tau} + \eta^{\lambda\sigma}\eta^{\nu\rho}k_2{}^{\mu}k_3{}^{\tau} + \eta^{\lambda\nu}\eta^{\rho\sigma}k_2{}^{\mu}k_3{}^{\tau} + \eta^{\lambda\sigma}\eta^{\mu\rho}k_2{}^{\nu}k_3{}^{\tau} + \eta^{\lambda\mu}\eta^{\rho\sigma}k_2{}^{\nu}k_3{}^{\tau} - \\
&\eta^{\lambda\sigma}\eta^{\mu\nu}k_2{}^{\rho}k_3{}^{\tau} + \eta^{\lambda\nu}\eta^{\mu\sigma}k_2{}^{\rho}k_3{}^{\tau} + \eta^{\lambda\mu}\eta^{\nu\sigma}k_2{}^{\rho}k_3{}^{\tau} + 2\eta^{\lambda\rho}\eta^{\nu\sigma}k_3{}^{\mu}k_3{}^{\tau} + 2\eta^{\lambda\rho}\eta^{\mu\sigma}k_3{}^{\nu}k_3{}^{\tau} - \\
&2\eta^{\lambda\rho}\eta^{\mu\nu}k_3{}^{\sigma}k_3{}^{\tau} + 2\eta^{\lambda\nu}\eta^{\mu\rho}k_3{}^{\sigma}k_3{}^{\tau} + 2\eta^{\lambda\mu}\eta^{\nu\rho}k_3{}^{\sigma}k_3{}^{\tau} - \eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_1\cdot k_2 - \eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_1\cdot \\
&k_2 - \eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_1\cdot k_2 + \eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_1\cdot k_2 - \eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_1\cdot k_2 + \eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_1\cdot k_2 + \\
&2\eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_1\cdot k_2 - \eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_1\cdot k_2 - \eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_1\cdot k_2 + 2\eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_1\cdot k_2 - \\
&\eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_1\cdot k_2 - \eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_1\cdot k_2 - 2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_1\cdot k_2 + 2\eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_1\cdot k_2 + \\
&2\eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_1\cdot k_2 - \eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_1\cdot k_3 - \eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_1\cdot k_3 - \eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_1\cdot k_3 + \\
&2\eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_1\cdot k_3 - \eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_1\cdot k_3 + 2\eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_1\cdot k_3 + 2\eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_1\cdot k_3 - \\
&\eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_1\cdot k_3 - \eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_1\cdot k_3 + 2\eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_1\cdot k_3 - \eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_1\cdot k_3 - \\
&\eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_1\cdot k_3 - 2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_1\cdot k_3 + \eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_1\cdot k_3 + \eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_1\cdot k_3 - \\
&\eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_2\cdot k_3 - \eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_2\cdot k_3 - \eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_2\cdot k_3 + 2\eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_2\cdot k_3 - \\
&\eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_2\cdot k_3 + 2\eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_2\cdot k_3 + \eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_2\cdot k_3 - \eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_2\cdot k_3 - \\
&\eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_2\cdot k_3 + \eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_2\cdot k_3 - \eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_2\cdot k_3 - \eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_2\cdot k_3 - \\
&2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_2\cdot k_3 + 2\eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_2\cdot k_3 + 2\eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_2\cdot k_3
\end{aligned}
$$

171 terms

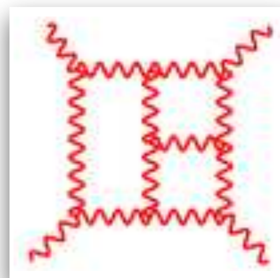


[DeWitt, 1967]

Textbook approach crumbles:

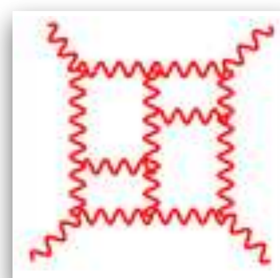
Feynman rules for a graviton: 171 terms per vertex
3 terms per edge

A single 3 loop diagram:



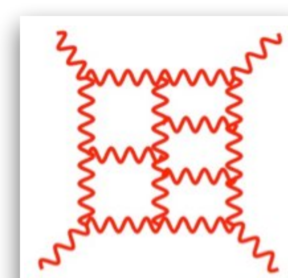
$\sim 10^{20}$ TERMS

4 loop diagram:



$\sim 10^{26}$ TERMS

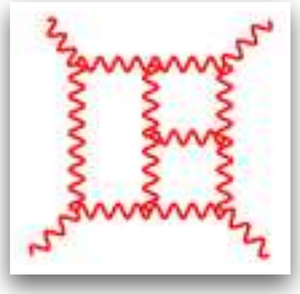
5 loop diagram:



$\sim 10^{31}$ TERMS

BUT FINAL EXPRESSIONS ARE TRACTABLE

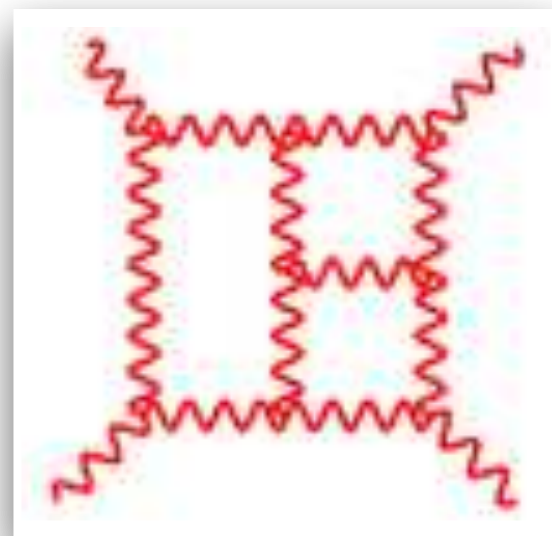
Vast majority of terms: unphysical freedom that must cancel



~10^{20}
TERMS

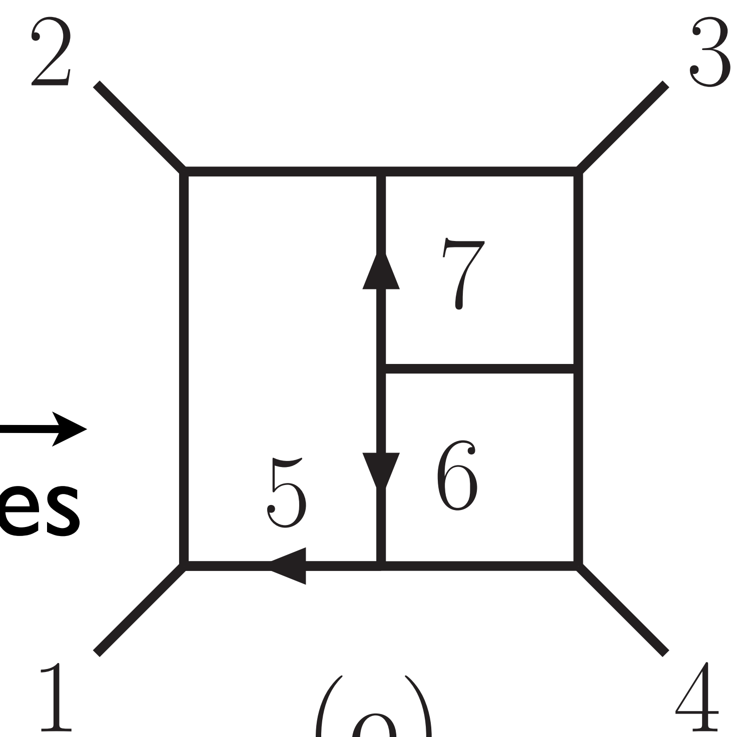
Bern, JJMC, Dixon, Kosower, Roiban '07

MOST SYMMETRIC 4D THEORY, N=8 SUGRA



~10^{20}
TERMS

add all other particles →



(e)

$$\propto \int stu\mathcal{M}_4^{(0)} \frac{\left(s\,(k_4+l_5)^2\right)^2}{d\circ(e) \equiv (l_5^2 l_6^2 l_7^2 (k_1-l_5)^2 \cdots)}$$

A strategy obscured in the standard formalism:

Calculate with physical (on-shell) quantities: $k_i^2 = 0$

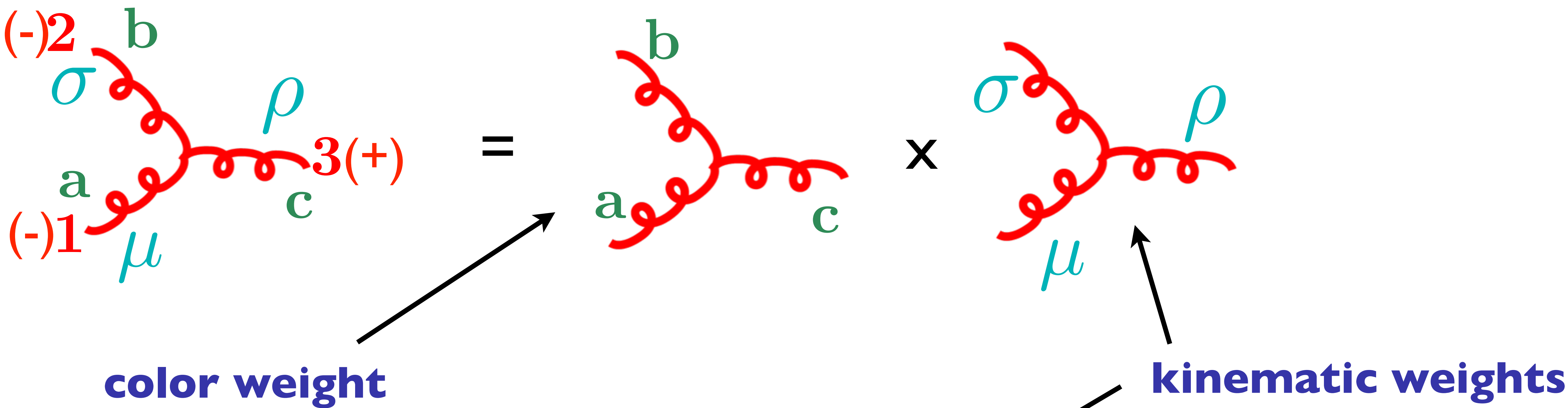
Physical (on-shell) **tree-level amplitudes contain all the information necessary to build *all* loop-level amplitudes**

Bern, Dixon, Dunbar, and Kosower (‘94,’95)

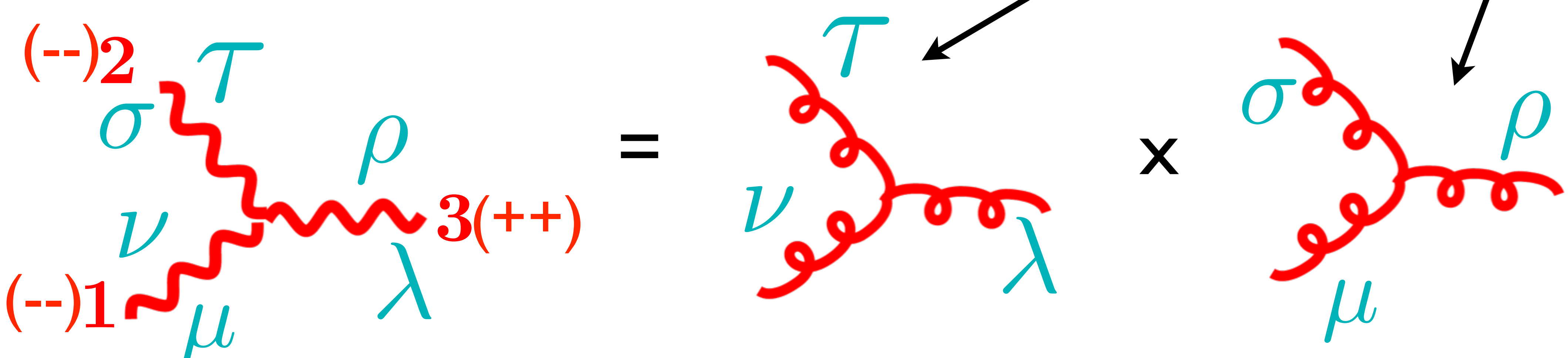
Bern, Dixon, and Kosower (‘96)

$k_i^2 = 0$

Physical gluon 3-vertex: $\mathbf{f^{abc}} \left(k_1{}^{\sigma} \eta^{\mu\rho} - k_2{}^{\mu} \eta^{\rho\sigma} \right)$



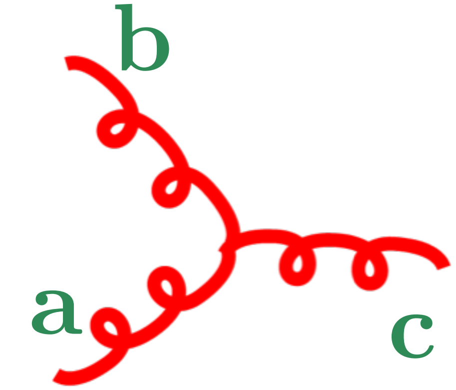
Physical graviton 3-vertex:



$$\left(\mathbf{k_1}{}^{\sigma} \eta^{\mu\rho} - \mathbf{k_2}{}^{\mu} \eta^{\rho\sigma} \right) \left(\mathbf{k_1}{}^{\tau} \eta^{\nu\lambda} - \mathbf{k_2}{}^{\nu} \eta^{\lambda\tau} \right)$$

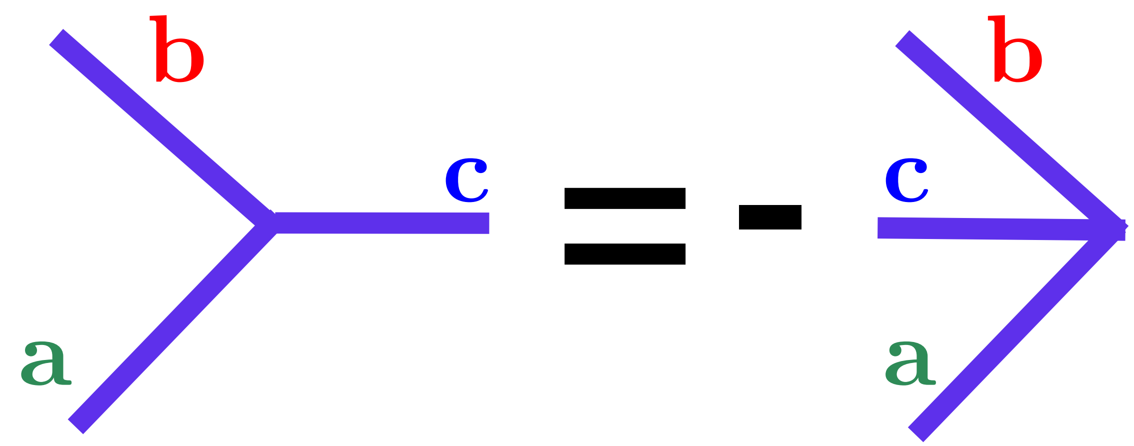
Lie Algebra structure constants:

$$f^{abc}$$



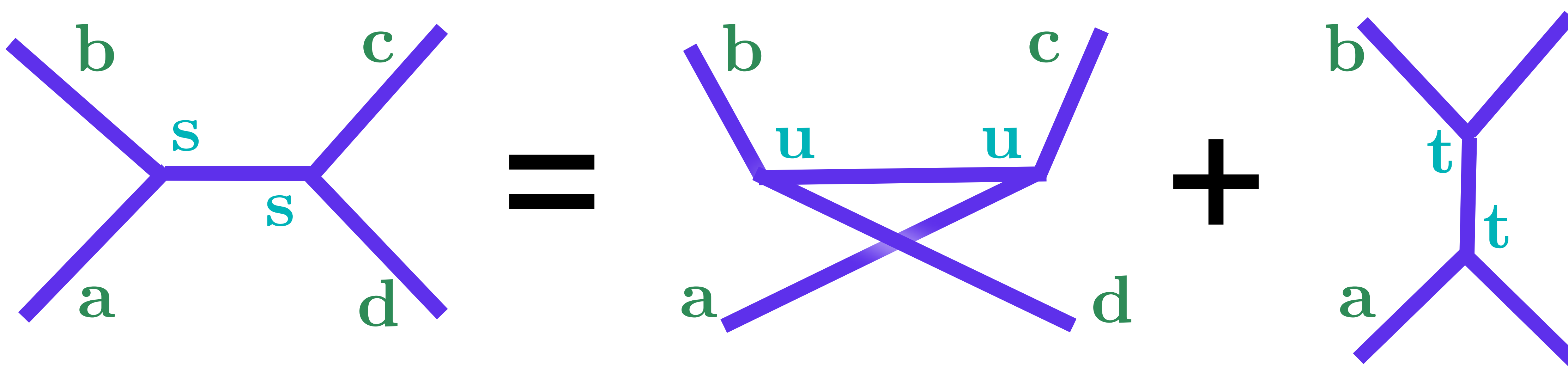
ANTISYMMETRY:

$$f^{abc} = -f^{acb}$$



JACOBI:

$$f^{abs}\, f^{scd} = f^{cau}\, f^{udb} + f^{dat}\, f^{tbc}$$

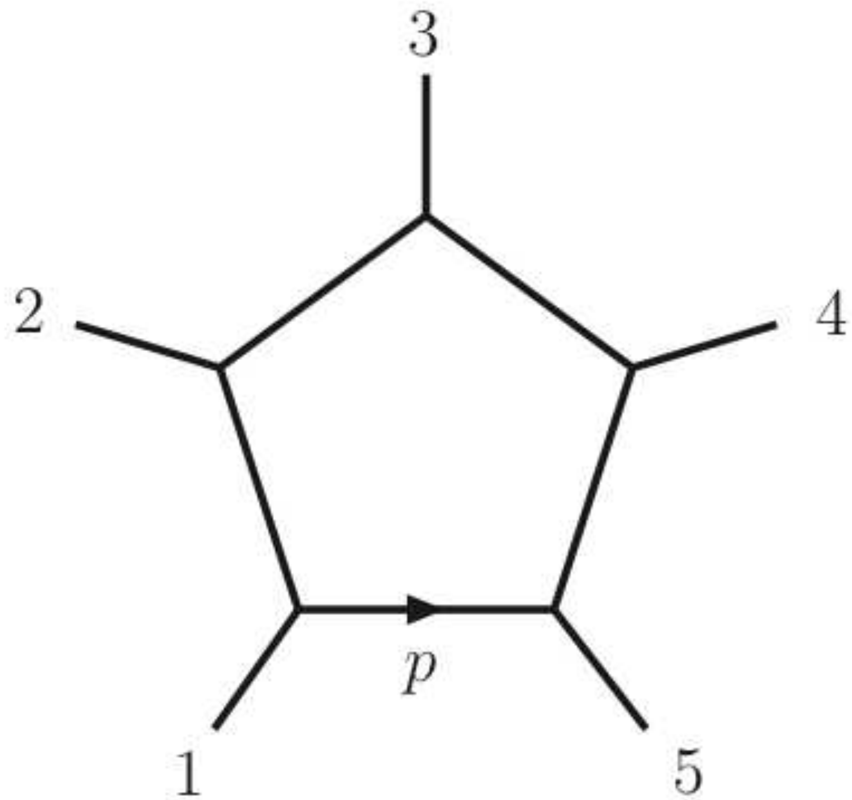


Ready to solve all of life's problems?

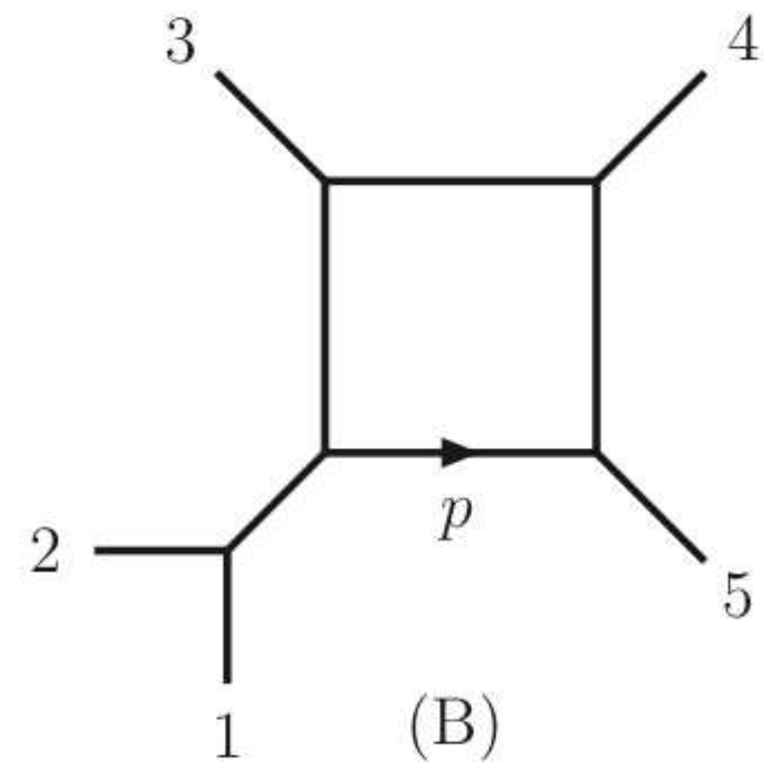
Complexity of Insisting on Local Representations

JJMC, Johansson (2011)

Five point 1-loop (no triangles, no bubbles)



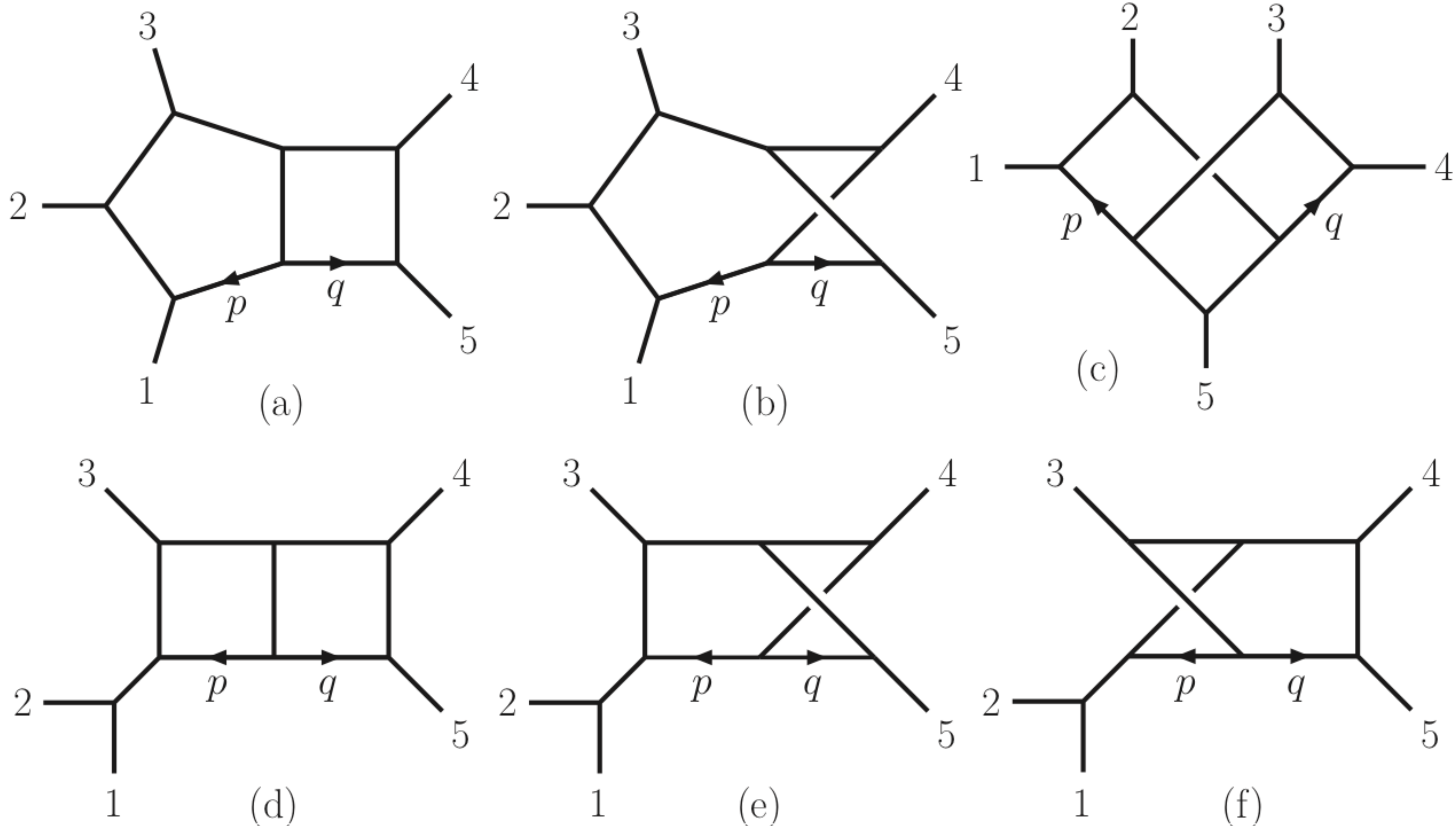
(P)



(B)

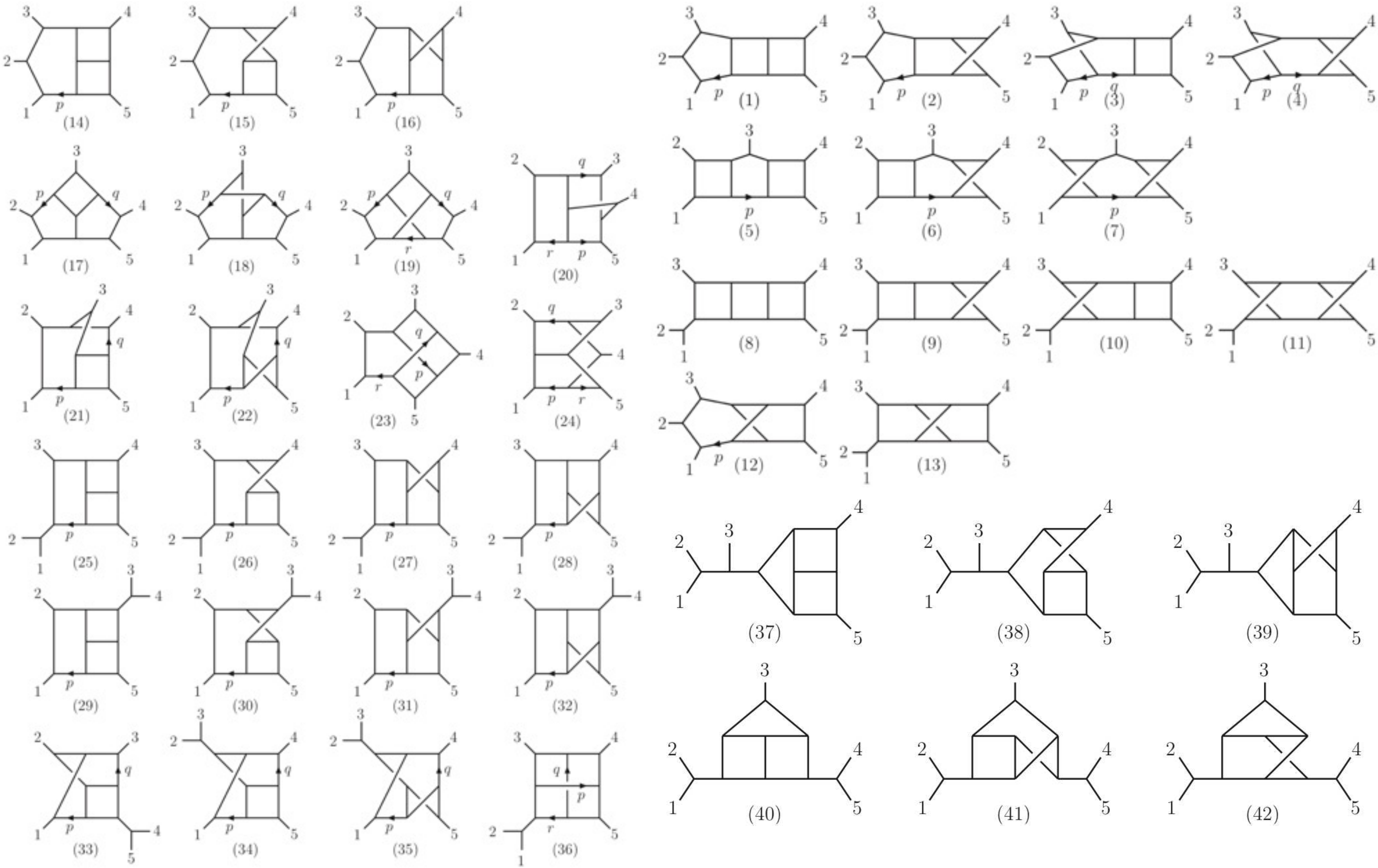
JJMC, Johansson (2011)

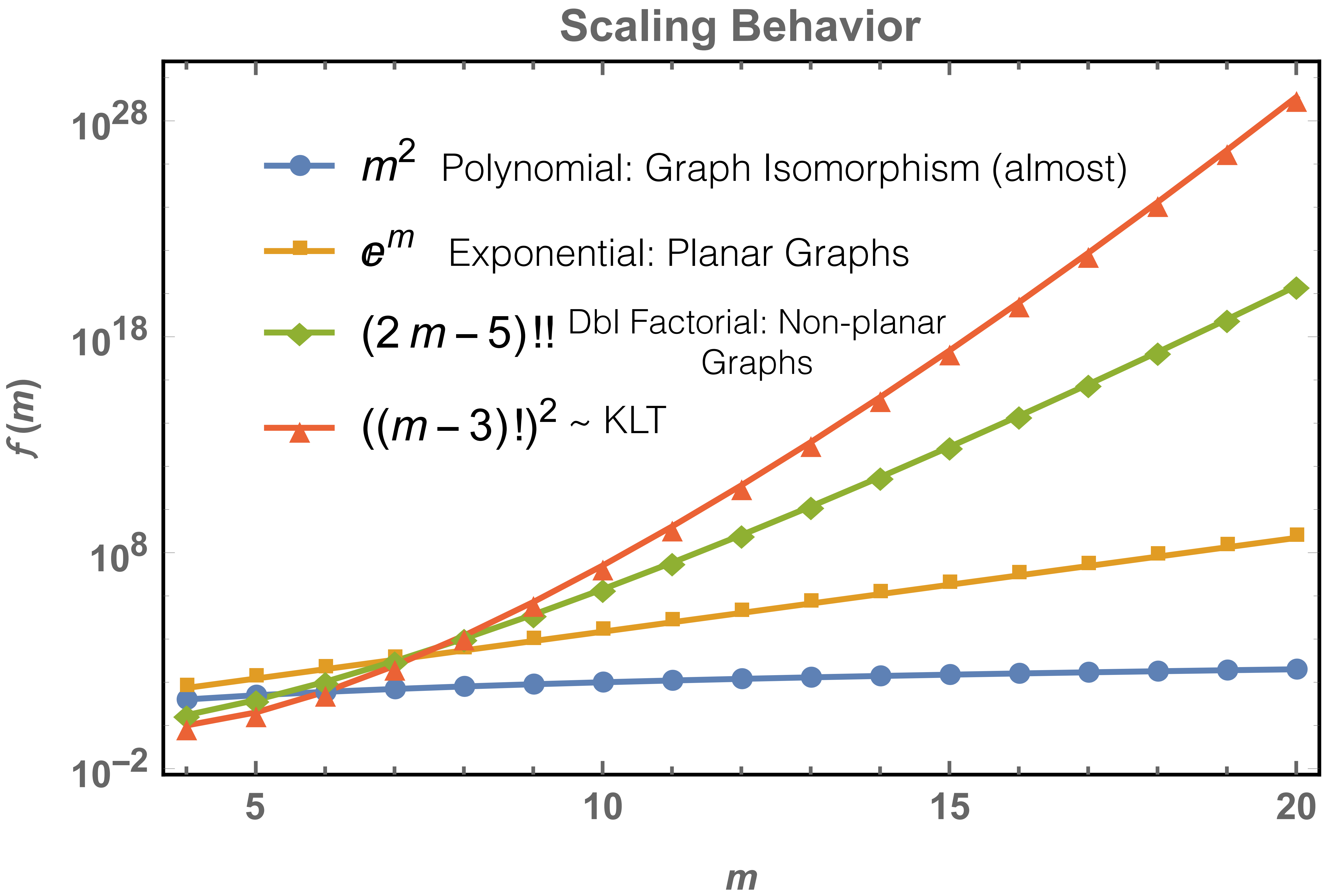
Five point 2-loop (no triangles, no bubbles)



Five point 3-loop (no bubbles, no triangles)

JJMC, Johansson (to appear)





Scaling Behavior
10^{28}
10^{18}
10^{8}
10^{-2}
$f(m)$
m^2 Polynomial: Graph Isomorphism (almost)
e^m Exponential: Planar Graphs
$(2m-5)!!$ Dbl Factorial: Non-planar Graphs
$((m-3)!)^2$ ~ KLT
5
10
15
20
m

Consider a Villanelle

Do Not Go Gentle Into That Good Night

Do not go gentle into that good night,
Old age should burn and rave at close of day;
Rage, rage against the dying of the light.

Though wise men at their end know dark is
right,
Because their words had forked no lightning
they
Do not go gentle into that good night.

Good men, the last wave by, crying how bright
Their frail deeds might have danced in a green
bay,
Rage, rage against the dying of the light.

Wild men who caught and sang the sun in
flight,
And learn, too late, they grieved it on its way,
Do not go gentle into that good night.

Grave men, near death, who see with blinding
sight
Blind eyes could blaze like meteors and be gay,
Rage, rage against the dying of the light.

And you, my father, there on that sad height,
Curse, bless, me now with your fierce tears, I
pray.
Do not go gentle into that good night.
Rage, rage against the dying of the light.

-Dylan Thomas

Do Not Go Gentle Into That Good Night

Do not go gentle into that good night,
Old age should burn and rave at close of day;
Rage, rage against the dying of the light.

Though wise men at their end know dark is right,
Because their words had forked no lightning they
Do not go gentle into that good night.

Good men, the last wave by, crying how bright
Their frail deeds might have danced in a green bay,
Rage, rage against the dying of the light.

Wild men who caught and sang the sun in flight,
And learn, too late, they grieved it on its way,
Do not go gentle into that good night.

Grave men, near death, who see with blinding sight
Blind eyes could blaze like meteors and be gay,
Rage, rage against the dying of the light.

And you, my father, there on that sad height,
Curse, bless, me now with your fierce tears, I pray.
Do not go gentle into that good night.
Rage, rage against the dying of the light.

-Dylan Thomas

Do Not Go Gentle Into That Good Night

Do not go gentle into that good night,
Old age should burn and rave at close of day;
Rage, rage against the dying of the light.

Though wise men at their end know dark is right,
Because their words had forked no lightning they
Do not go gentle into that good night.

Good men, the last wave by, crying how bright
Their frail deeds might have danced in a green bay,
Rage, rage against the dying of the light.

Wild men who caught and sang the sun in flight,
And learn, too late, they grieved it on its way,
Do not go gentle into that good night.

Grave men, near death, who see with blinding sight
Blind eyes could blaze like meteors and be gay,
Rage, rage against the dying of the light.

And you, my father, there on that sad height,
Curse, bless, me now with your fierce tears, I pray.
Do not go gentle into that good night.
Rage, rage against the dying of the light.

-Dylan Thomas

Do Not Go Gentle Into That Good Night

Do not go gentle into that good night,
Old age should burn and rave at close of day;
Rage, rage against the dying of the light.

Though wise men at their end know dark is right,
Because their words had forked no lightning they
Do not go gentle into that good night.

Good men, the last wave by, crying how bright
Their frail deeds might have danced in a green bay,
Rage, rage against the dying of the light.

Wild men who caught and sang the sun in flight,
And learn, too late, they grieved it on its way,
Do not go gentle into that good night.

Grave men, near death, who see with blinding sight
Blind eyes could blaze like meteors and be gay,
Rage, rage against the dying of the light.

And you, my father, there on that sad height,
Curse, bless, me now with your fierce tears, I pray.
Do not go gentle into that good night.
Rage, rage against the dying of the light.

-Dylan Thomas

Do Not Go Gentle Into That Good Night

Do not go gentle into that good night,
Old age should burn and rave at close of day;
Rage, rage against the dying of the light.

Though wise men at their end know dark is
right,
Because their words had forked no lightning
they
Do not go gentle into that good night.

Good men, the last wave by, crying how bright
Their frail deeds might have danced in a green
bay,
Rage, rage against the dying of the light.

Wild men who caught and sang the sun in
flight,
And learn, too late, they grieved it on its way,
Do not go gentle into that good night.

Grave men, near death, who see with blinding
sight
Blind eyes could blaze like meteors and be gay,
Rage, rage against the dying of the light.

And you, my father, there on that sad height,
Curse, bless, me now with your fierce tears, I
pray.
Do not go gentle into that good night.
Rage, rage against the dying of the light.

-Dylan Thomas

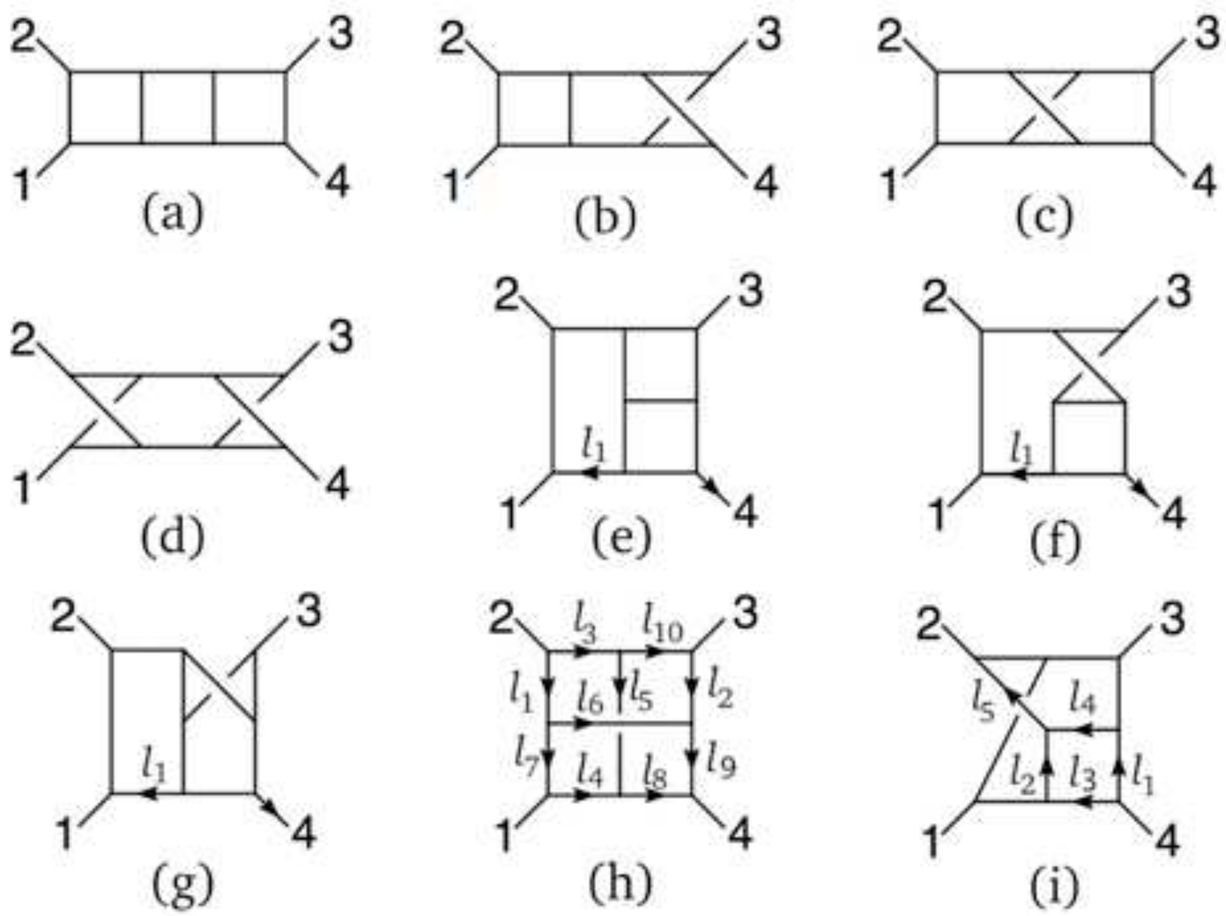
What's going on?

- Minimal information in.
- Relations propagate this information to a full solution.

Consider an Amplitude

Bern, JJMC, Dixon, Kosower, Johansson, Roiban '07

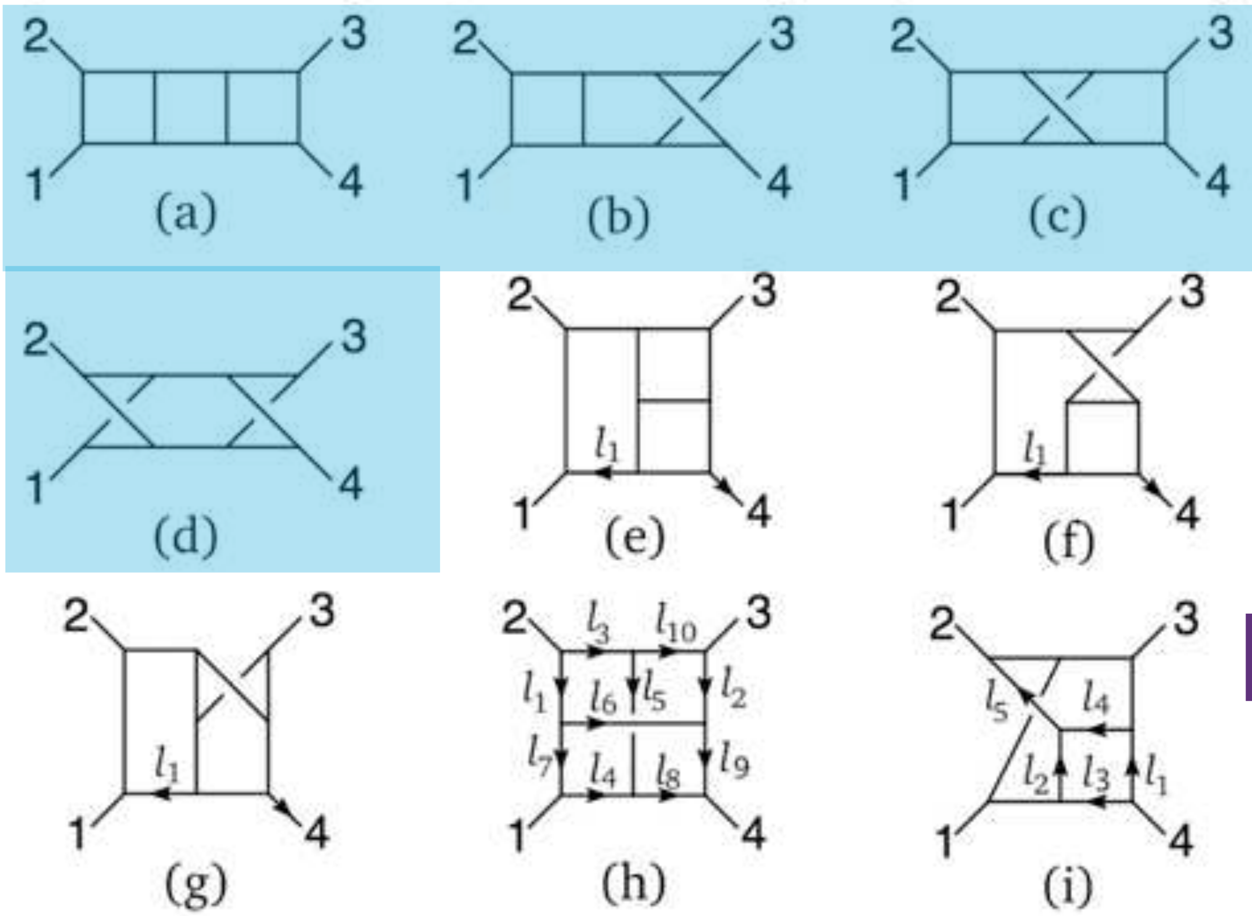
Original solution of three-loop four-point N=4 sYM and N=8 sugra



Integral	$\mathcal{N}=4$ Yang-Mills	$\mathcal{N}=8$ Supergravity
(a)–(d)	s^2	$[s^2]^2$
(e)–(g)	$s(l_1+k_4)^2$	$[s\,(l_1+k_4)^2]^2$
(h)	$s(l_1+l_2)^2+t(l_3+l_4)^2 - sl_5^2 - tl_6^2 - st$	$(s(l_1+l_2)^2+t(l_3+l_4)^2-st)^2 - s^2(2((l_1+l_2)^2-t)+l_5^2)l_5^2 - t^2(2((l_3+l_4)^2-s)+l_6^2)l_6^2 - s^2(2l_7^2l_2^2+2l_1^2l_9^2+l_2^2l_9^2+l_1^2l_7^2) - t^2(2l_3^2l_8^2+2l_{10}^2l_4^2+l_8^2l_4^2+l_3^2l_{10}^2)+2stl_5^2l_6^2$
(i)	$s(l_1+l_2)^2-t(l_3+l_4)^2 - \frac{1}{3}(s-t)l_5^2$	$(s(l_1+l_2)^2-t(l_3+l_4)^2)^2 - (s^2(l_1+l_2)^2+t^2(l_3+l_4)^2+\frac{1}{3}stu)l_5^2$

Bern, JJMC, Dixon, Kosower, Johansson, Roiban ’07

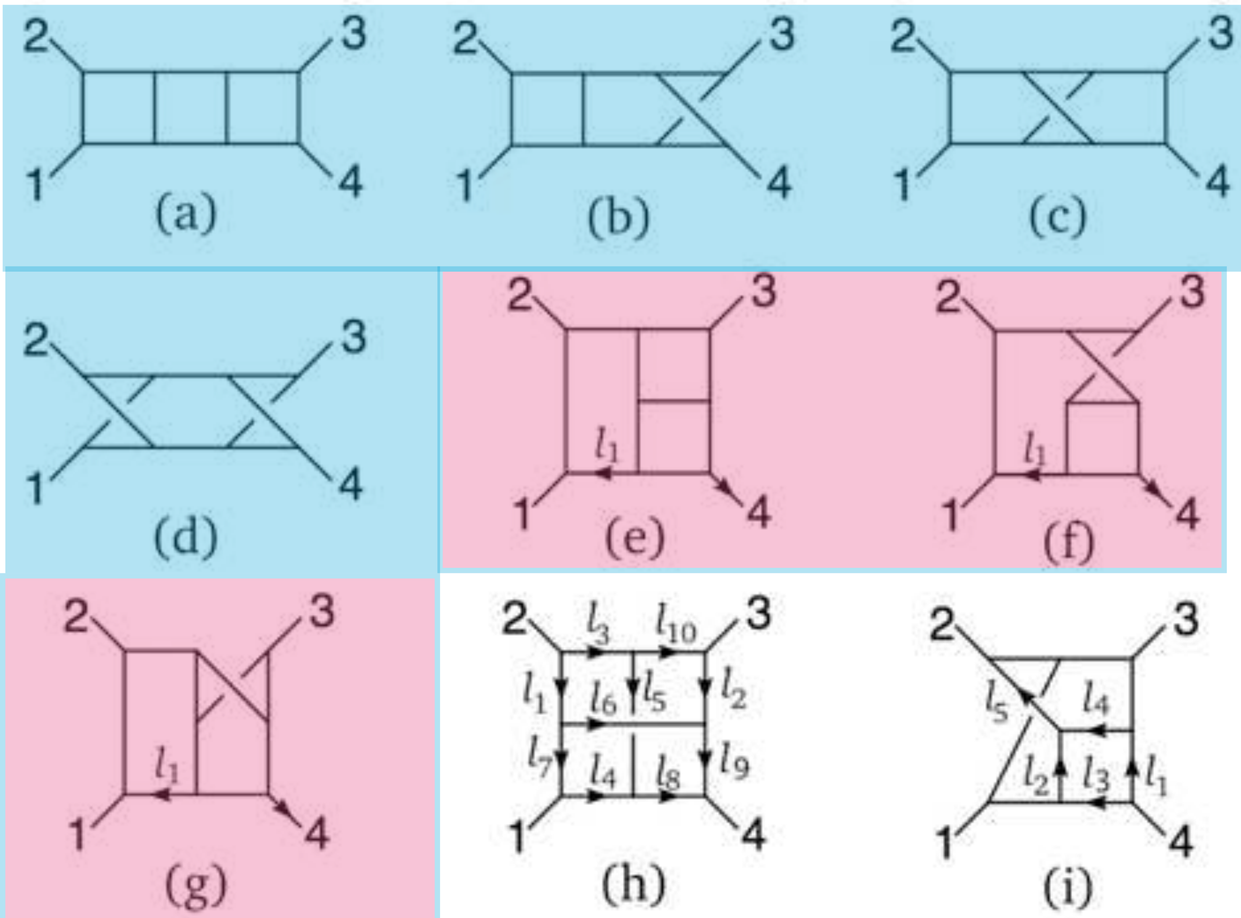
Original solution of three-loop four-point N=4 sYM and N=8 sugra



Integral	$\mathcal{N}=4$ Yang-Mills	$\mathcal{N}=8$ Supergravity
(a)–(d)	s^2	$[s^2]^2$
(e)–(g)	$s(l_1+k_4)^2$	$[s\,(l_1+k_4)^2]^2$
(h)	$s(l_1+l_2)^2+t(l_3+l_4)^2$ $-\,sl_5^2-tl_6^2-st$	$(s(l_1+l_2)^2+t(l_3+l_4)^2-st)^2-s^2(2((l_1+l_2)^2-t)+l_5^2)l_5^2$ $-t^2(2((l_3+l_4)^2-s)+l_6^2)l_6^2-s^2(2l_7^2l_2^2+2l_1^2l_9^2+l_2^2l_9^2+l_1^2l_7^2)$ $-\,t^2(2l_3^2l_8^2+2l_{10}^2l_4^2+l_8^2l_4^2+l_3^2l_{10}^2)+2stl_5^2l_6^2$
(i)	$s(l_1+l_2)^2-t(l_3+l_4)^2$ $-\frac{1}{3}(s-t)l_5^2$	$(s(l_1+l_2)^2-t(l_3+l_4)^2)^2$ $-\,(s^2(l_1+l_2)^2+t^2(l_3+l_4)^2+\frac{1}{3}stu)l_5^2$

Bern, JJMC, Dixon, Kosower, Johansson, Roiban ’07

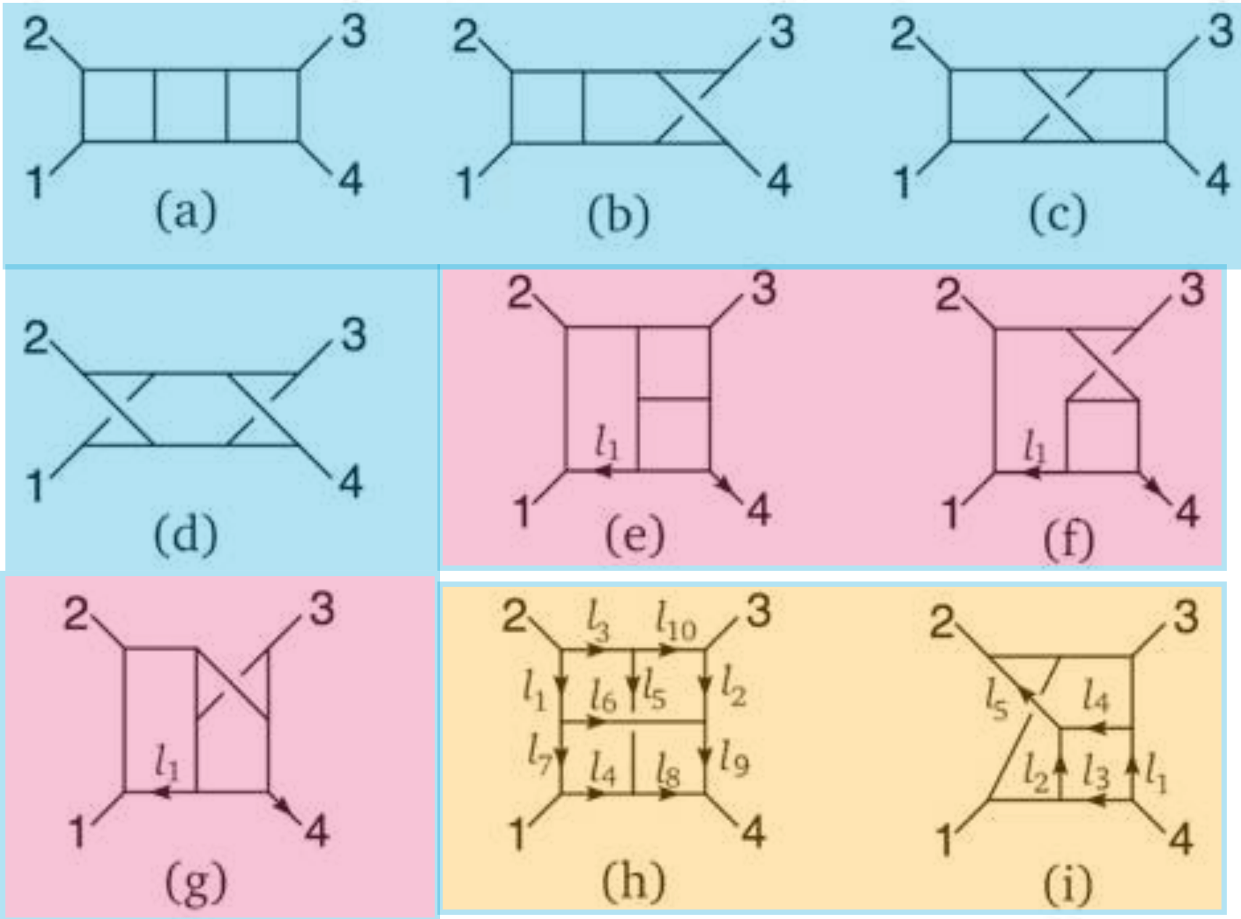
Original solution of three-loop four-point N=4 sYM and N=8 sugra



Integral	$\mathcal{N}=4$ Yang-Mills	$\mathcal{N}=8$ Supergravity
(a)–(d)	s^2	$[s^2]^2$
(e)–(g)	$s(l_1+k_4)^2$	$[s\,(l_1+k_4)^2]^2$
(h)	$s(l_1+l_2)^2+t(l_3+l_4)^2 - sl_5^2 - tl_6^2 - st$	$(s(l_1+l_2)^2+t(l_3+l_4)^2-st)^2 - s^2(2((l_1+l_2)^2-t)+l_5^2)l_5^2 - t^2(2((l_3+l_4)^2-s)+l_6^2)l_6^2 - s^2(2l_7^2l_2^2+2l_1^2l_9^2+l_2^2l_9^2+l_1^2l_7^2) - t^2(2l_3^2l_8^2+2l_{10}^2l_4^2+l_8^2l_4^2+l_3^2l_{10}^2)+2stl_5^2l_6^2$
(i)	$s(l_1+l_2)^2-t(l_3+l_4)^2 - \frac{1}{3}(s-t)l_5^2$	$(s(l_1+l_2)^2-t(l_3+l_4)^2)^2 - (s^2(l_1+l_2)^2+t^2(l_3+l_4)^2+\frac{1}{3}stu)l_5^2$

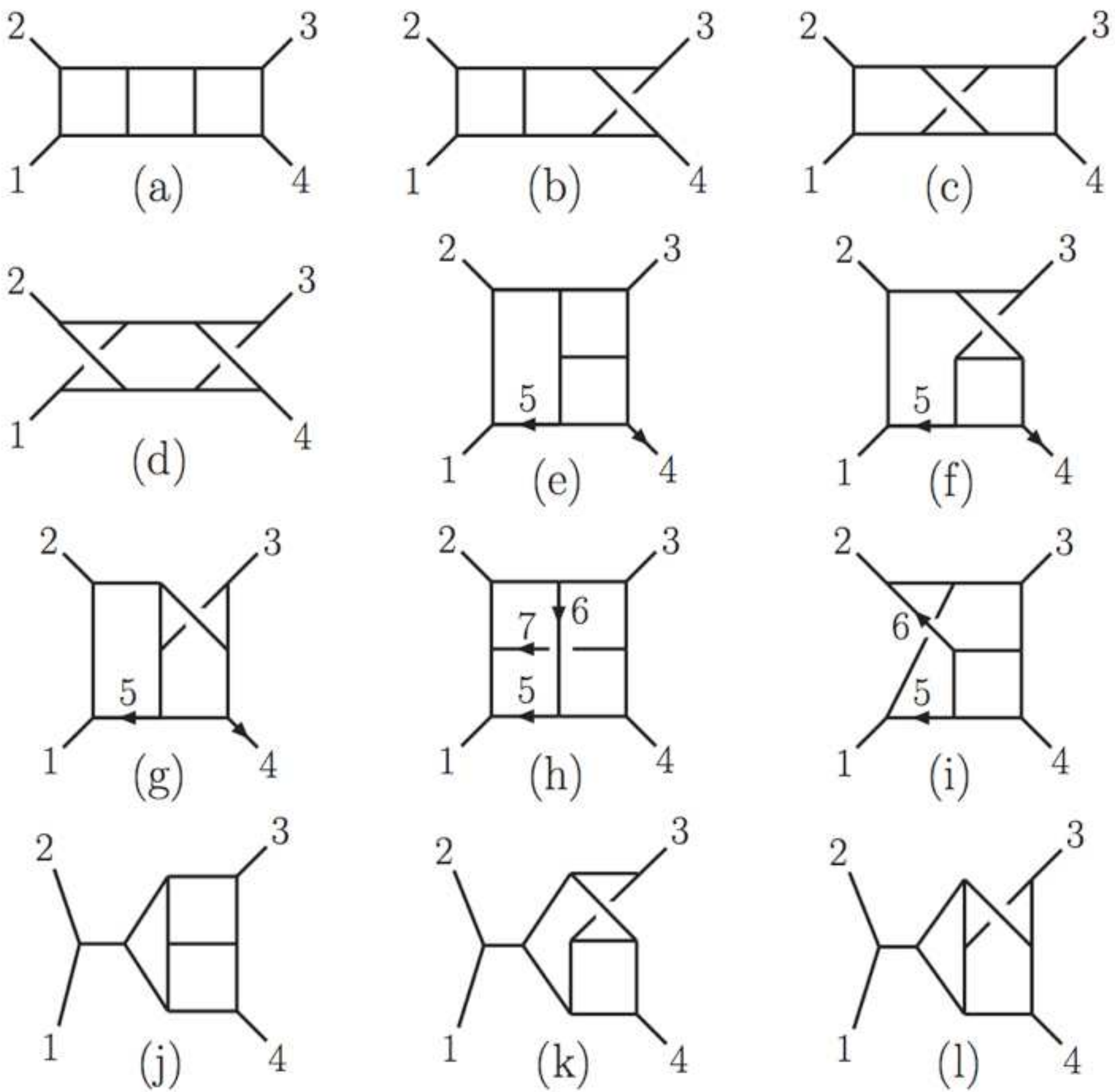
Bern, JJMC, Dixon, Kosower, Johansson, Roiban ’07

Original solution of three-loop four-point N=4 sYM and N=8 sugra

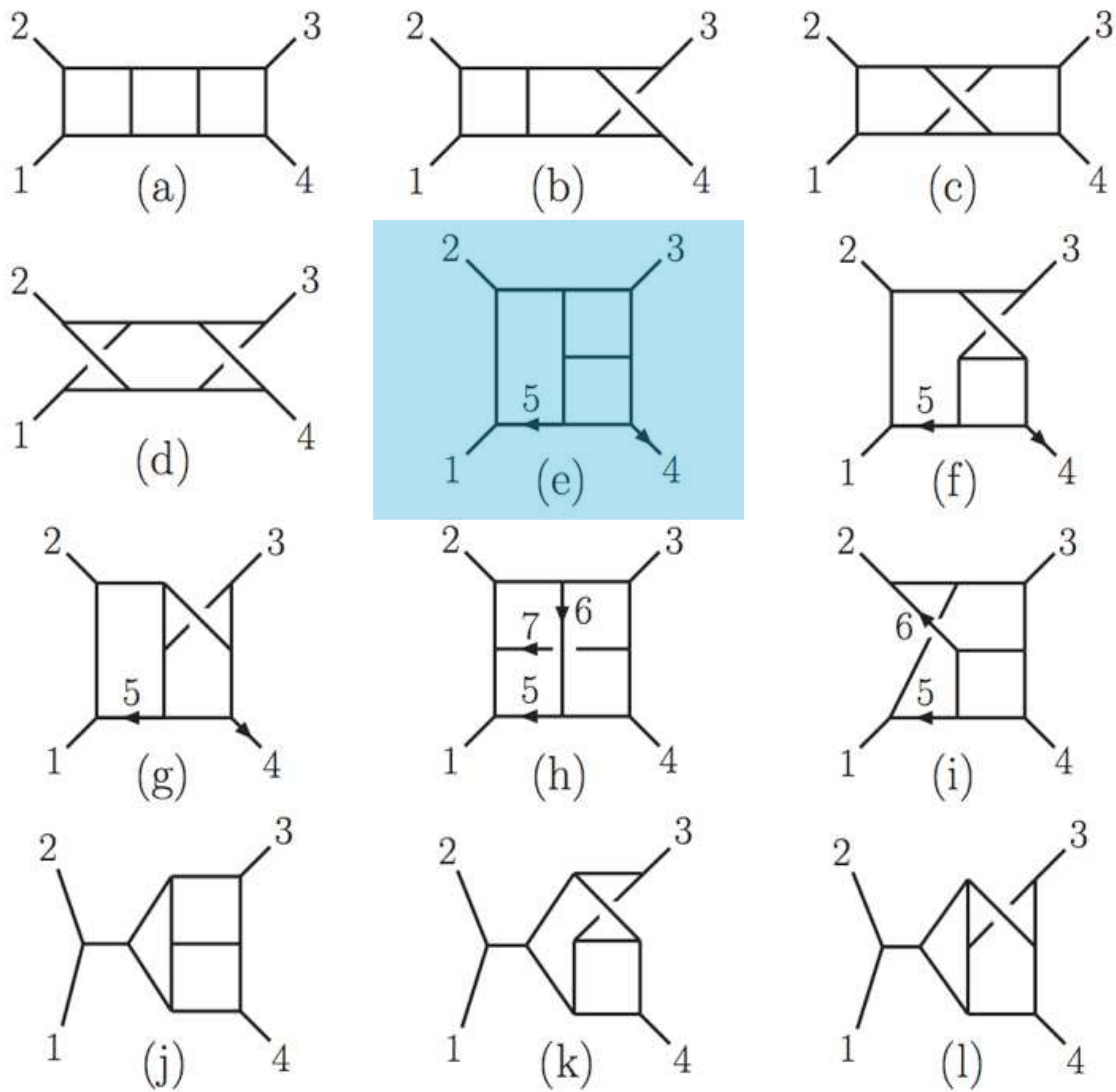


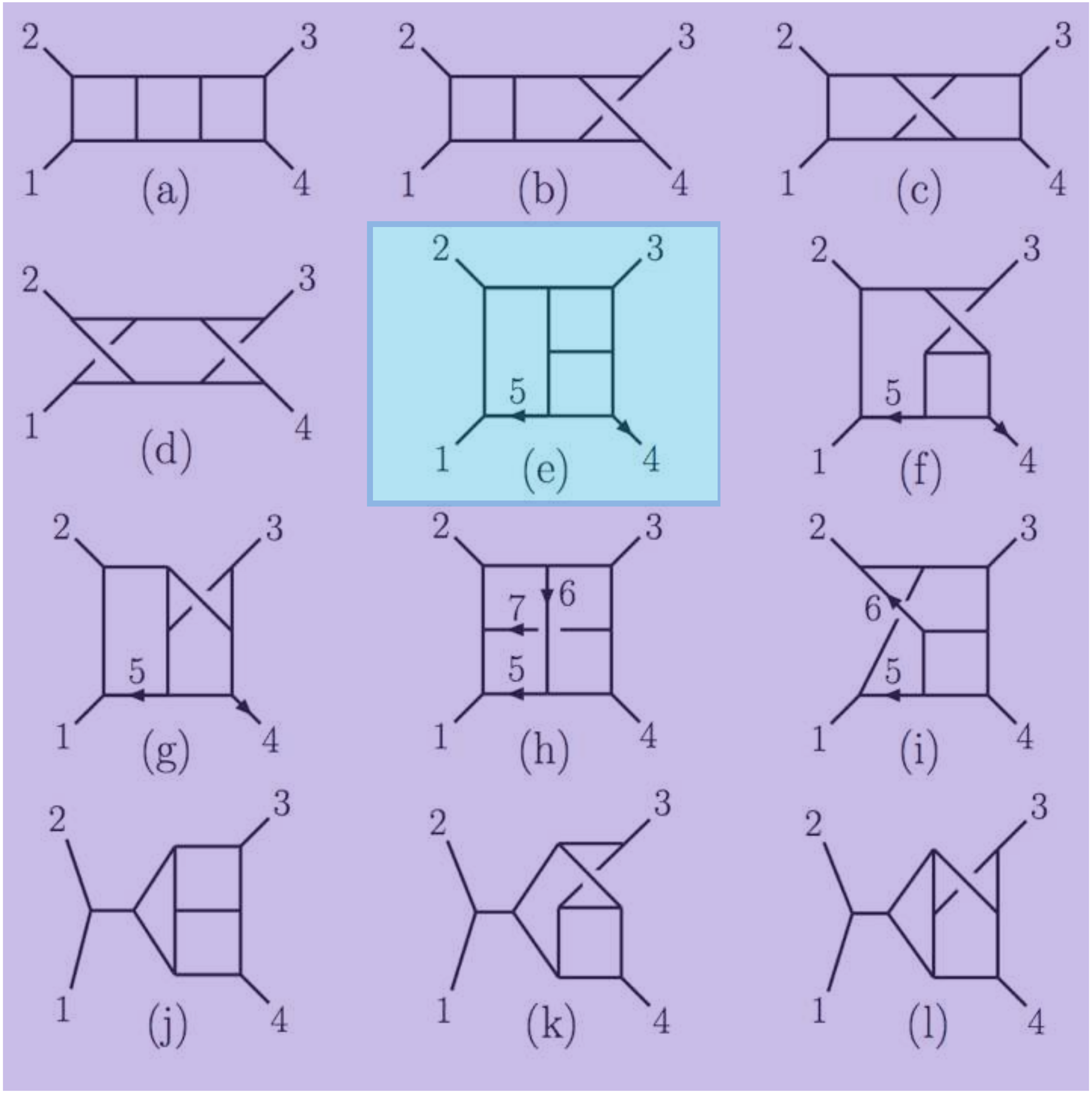
Integral	$\mathcal{N}=4$ Yang-Mills	$\mathcal{N}=8$ Supergravity
(a)–(d)	s^2	$[s^2]^2$
(e)–(g)	$s(l_1+k_4)^2$	$[s\,(l_1+k_4)^2]^2$
(h)	$s(l_1+l_2)^2+t(l_3+l_4)^2 - sl_5^2 - tl_6^2 - st$	$(s(l_1+l_2)^2+t(l_3+l_4)^2 - st)^2 - s^2(2((l_1+l_2)^2-t)+l_5^2)l_5^2 - t^2(2((l_3+l_4)^2-s)+l_6^2)l_6^2 - s^2(2l_7^2l_2^2+2l_1^2l_9^2+l_2^2l_9^2+l_1^2l_7^2) - t^2(2l_3^2l_8^2+2l_{10}^2l_4^2+l_8^2l_4^2+l_3^2l_{10}^2)+2stl_5^2l_6^2$
(i)	$s(l_1+l_2)^2 - t(l_3+l_4)^2 - \frac{1}{3}(s-t)l_5^2$	$(s(l_1+l_2)^2 - t(l_3+l_4)^2)^2 - (s^2(l_1+l_2)^2+t^2(l_3+l_4)^2+\frac{1}{3}stu)l_5^2$

Bern, JJMC, Johansson, '10



Bern, JJMC, Johansson, '10

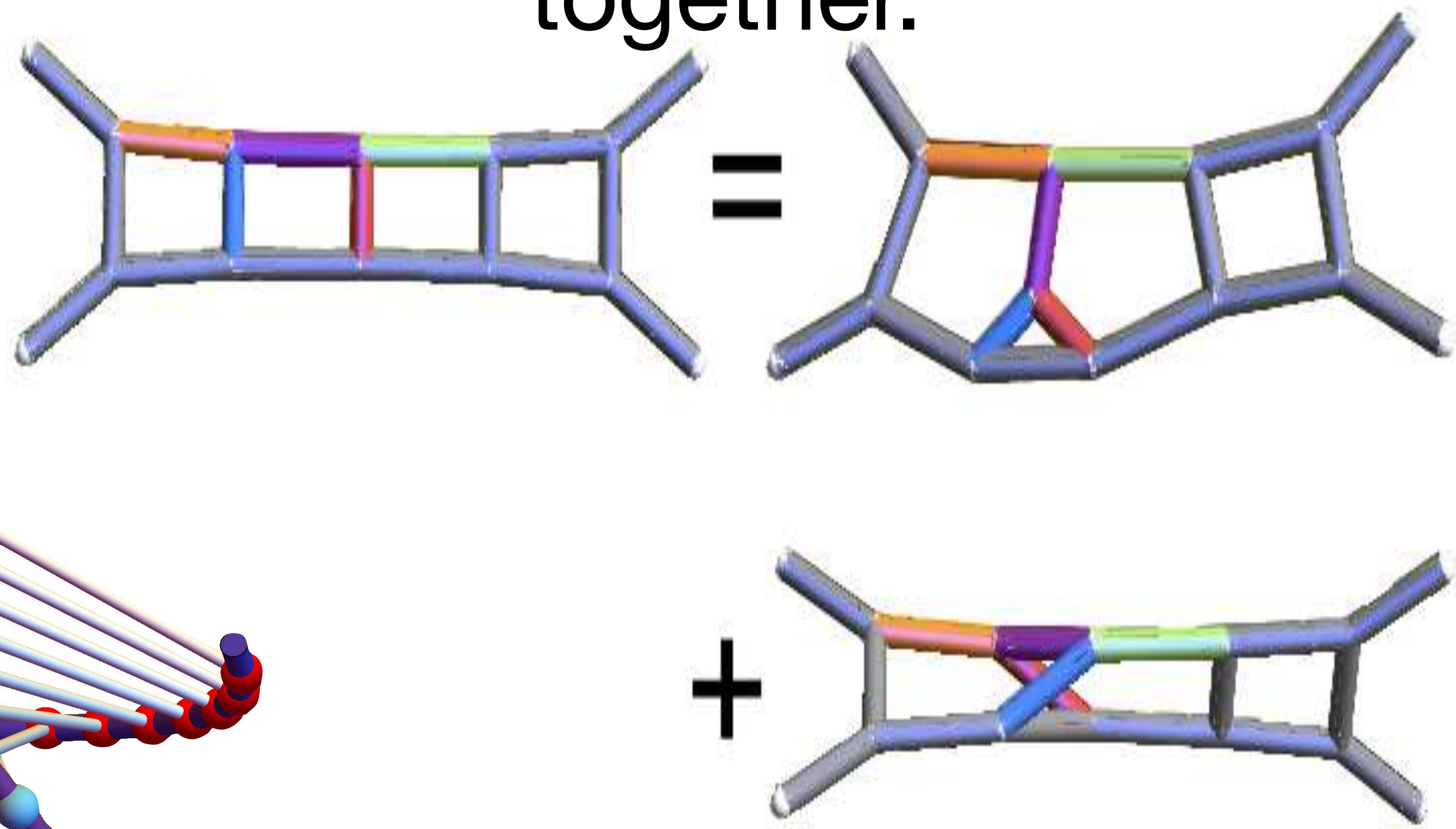


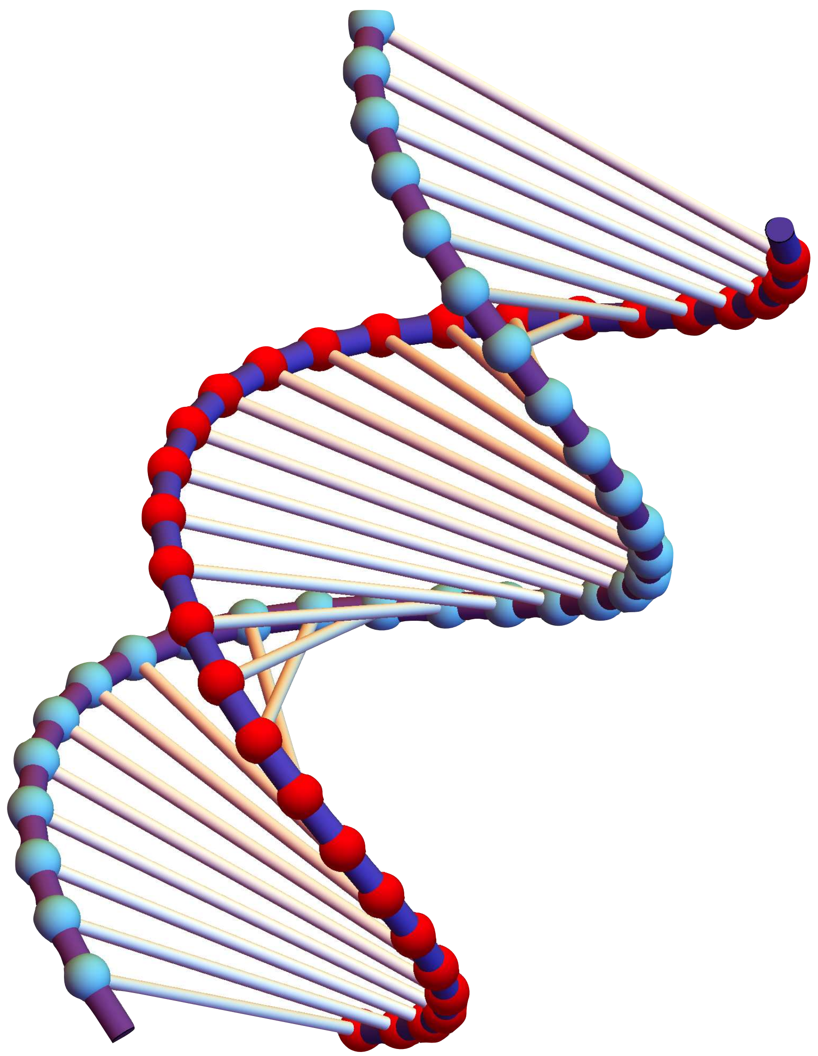


Bern, JJMC, Johansson, '10

Bern, JJMC, Johansson, '10

Color and Kinematics dance together.



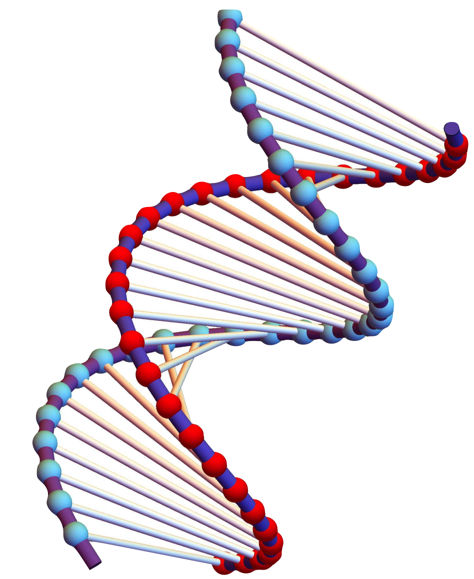


Solving Yang-Mills theories means solving Gravity theories.

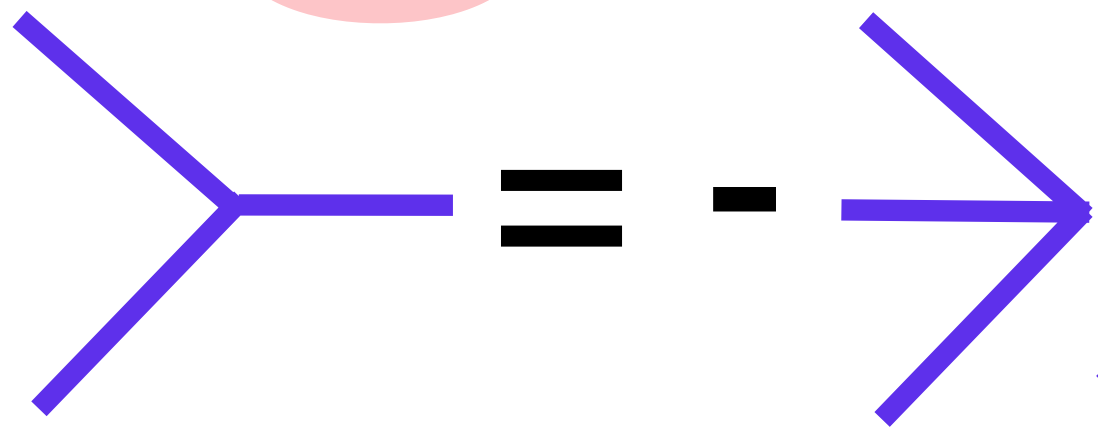
Bern, JJMC, Johansson ('08,'10)

Generic D-dimensional YM theories at tree-level

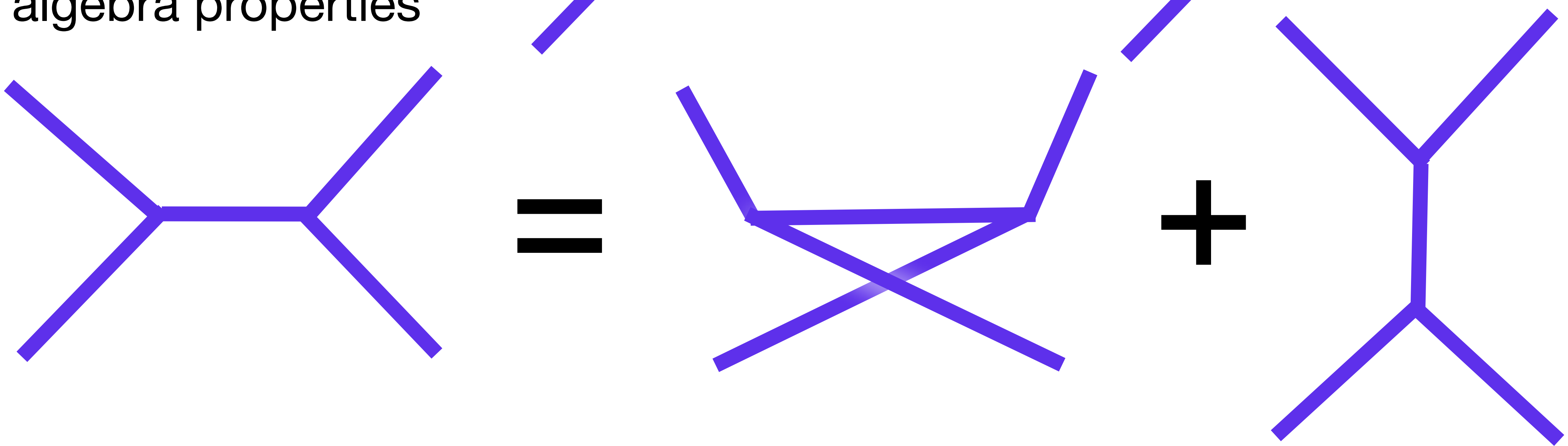
$$\mathcal{A}_m^{\text{tree}} = \sum_{\mathcal{G}\in\text{cubic}} \frac{c(\mathcal{G})\,n(\mathcal{G})}{D(\mathcal{G})}$$



Color factors and numerator factors satisfy similar lie algebra properties



Vertex Antisymmetry



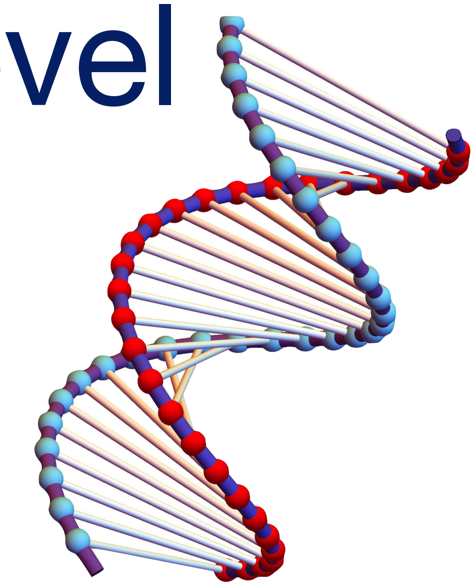
Jacobi

Color-Kinematic Duality!

Bern, JJMC, Johansson ('08,'10)

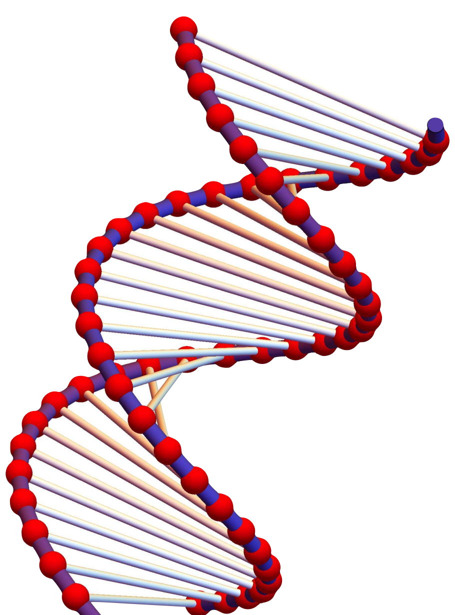
Generic D-dimensional YM theories have a fascinating structure at tree-level

$$\mathcal{A}_m^{\text{tree}} = \sum_{\mathcal{G}\in\text{cubic}} \frac{c(\mathcal{G})n(\mathcal{G})}{D(\mathcal{G})}$$



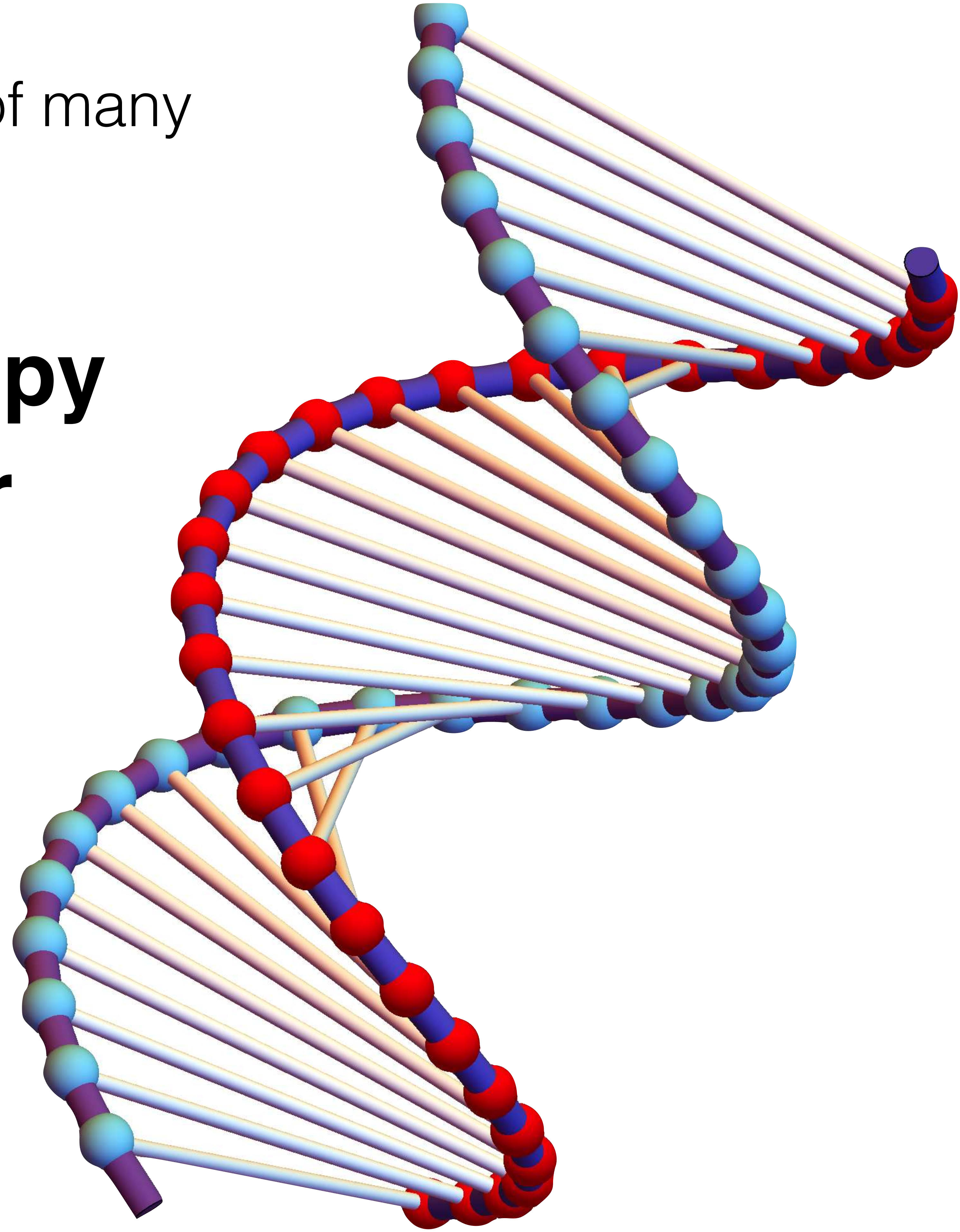
YM: Color-Kinematic Duality, makes manifest gravitational double copy structure:

$$-iM_n^{\text{tree}} = \sum_{\mathcal{G}\in\text{cubic}} \frac{n(\mathcal{G})\tilde{n}(\mathcal{G})}{D(\mathcal{G})}$$



The scattering amplitudes of many relativistic theories admit a:

Double-copy
Numerator
Algebra



Key Point: **MANY Theories are Double Copies**

Bi-Adjoint Scalar: color $\otimes$ color

Bern, de Freitas, Wong ('99); Bern, Dennen, Huang; Du, Feng, Fu; Bjerrum-Bohr, Damgaard, Monteiro, O'Connell

(S)YM (…(S)QCD…): color $\otimes$ spin-1

BCJ ('08) Bjerrum-Bohr, Damgaard, Vanhove; Steiberger; Feng et al; Mafra, Schlotterer, ('08-'11); Johansson, Ochirov

(S)Gr (…(S)Einstein-YM…): spin-1 $\otimes$ spin-1

KLT('86); BCJ ('08); Chiodaroli, **Gunaydin**, Johansson, **Roiban**; Johansson, Ochirov; Johansson, Kälin, Mogull

NLSM / Chiral Lagrangian: "color" $\otimes$ even-spin-0

Chen, Du '13 Cachazo, He, Yuan '14 Cheung, Shen '16

(S)Born-Infeld: spin-1 $\otimes$ even-spin-0

Cachazo, He, Yuan '14

Special Galileon: even-spin-0 $\otimes$ even-spin-0

Cachazo, He, Yuan '14 Cheung, Shen '16

Open String: α' $\otimes$ spin-1

Broedel, Schlotterer, Stieberger

Closed String: spin-1 $\otimes$ α' corrected spin-1

Broedel, Schlotterer, Stieberger;

Z-theory: α' $\otimes$ "color"

Broedel, Schlotterer, Stieberger; JJMC, Mafra, Schlotterer

Key Point: **MANY Theories are Double Copies**

Ingredients:

α'

color

spin 0,1/2,1

For all these theories:

(S) YM
(...(S) QCD...)

Bi-Adjoint Scalar

(S) Gr
(...(S) Einstein-YM...)

NLSM

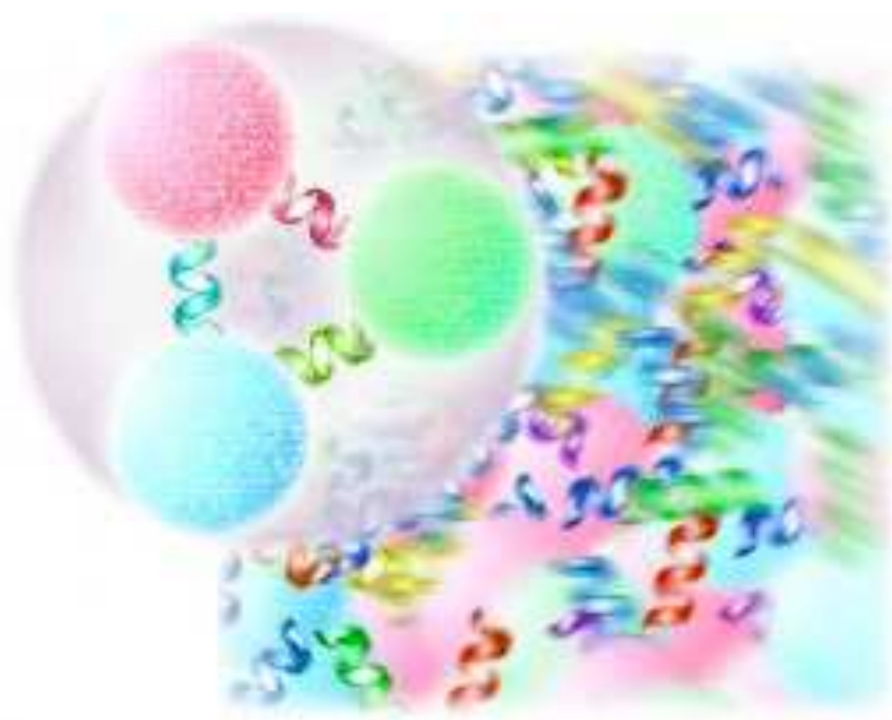
(S) Born-Infeld

Special Galileon

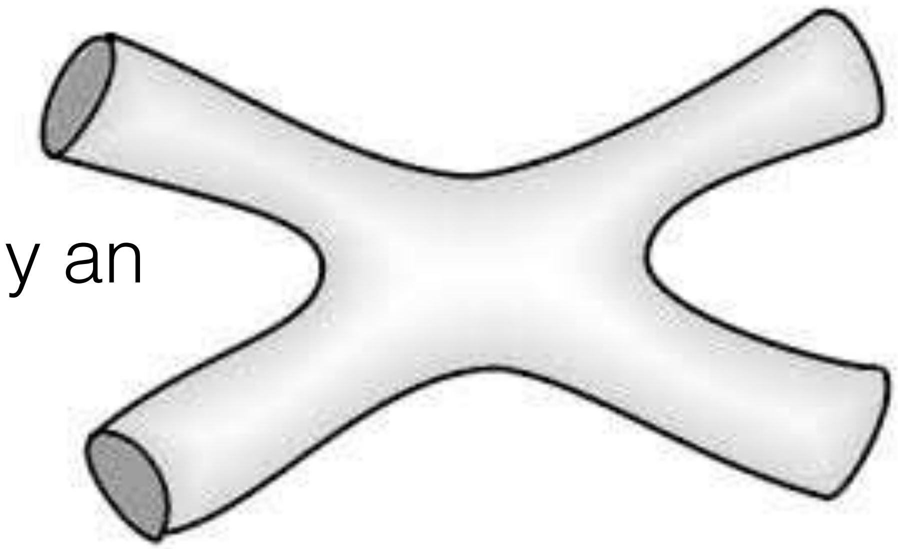
Z-theory

Open String

Closed String



Linked via Double Copy by an
Effective Field Theory



JJMC, Mafra, Schlotterer

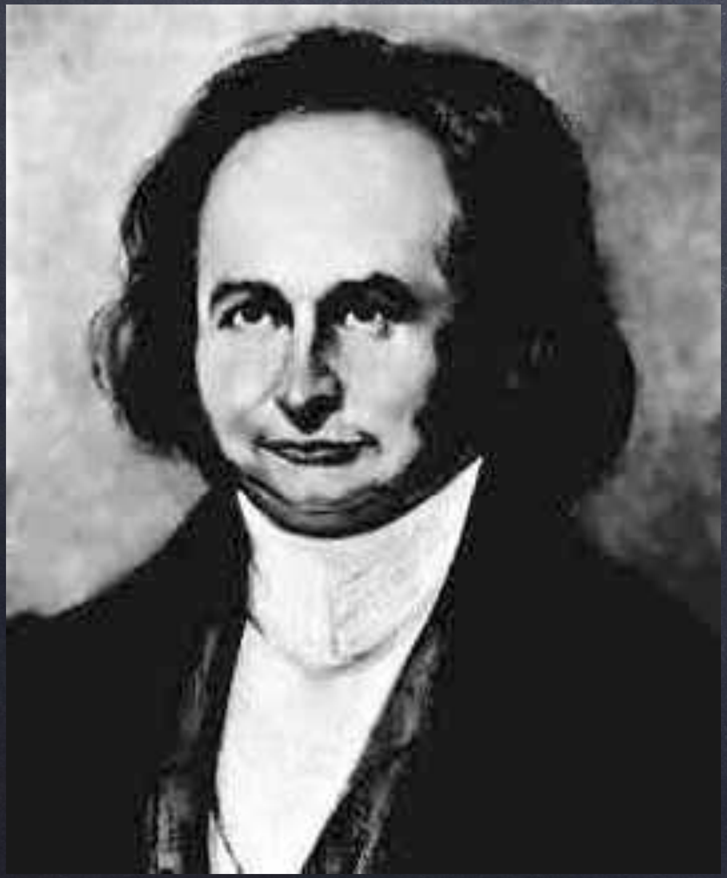
You can build the entirety of this EFT out of very simple building blocks!

At 4pts, only need one simple building block and composition for superstring

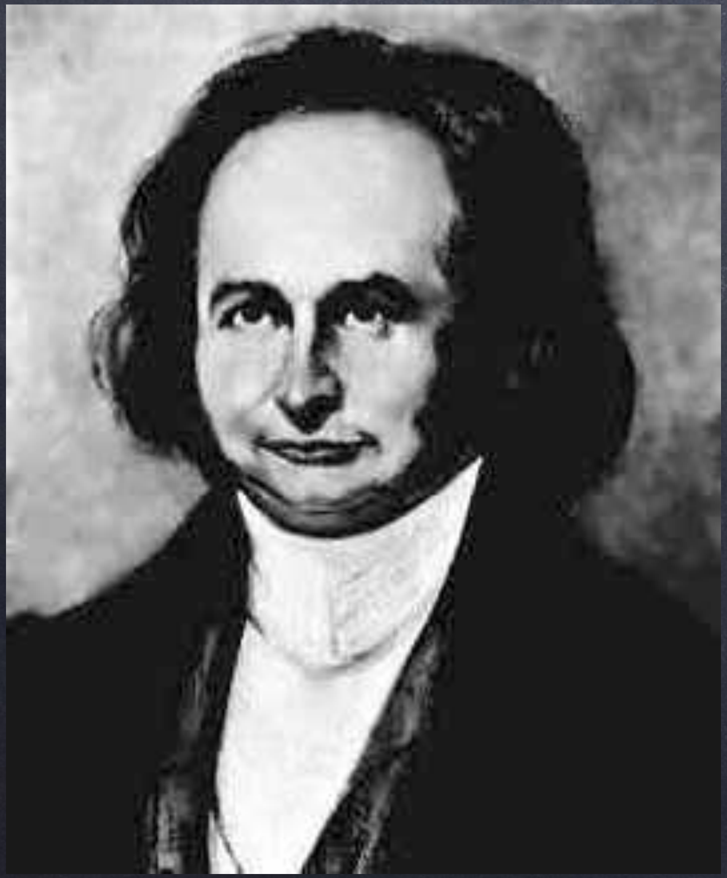
JJMC, Rodina, Yin, Zekioğlu

Only 6 more blocks required for all DC compatible HD operators!

The world is QUANTUM - wouldn’t it be great to generalize to loop-order corrections?



The world is QUANTUM -
wouldn’t it be great to generalize to loop-order corrections?

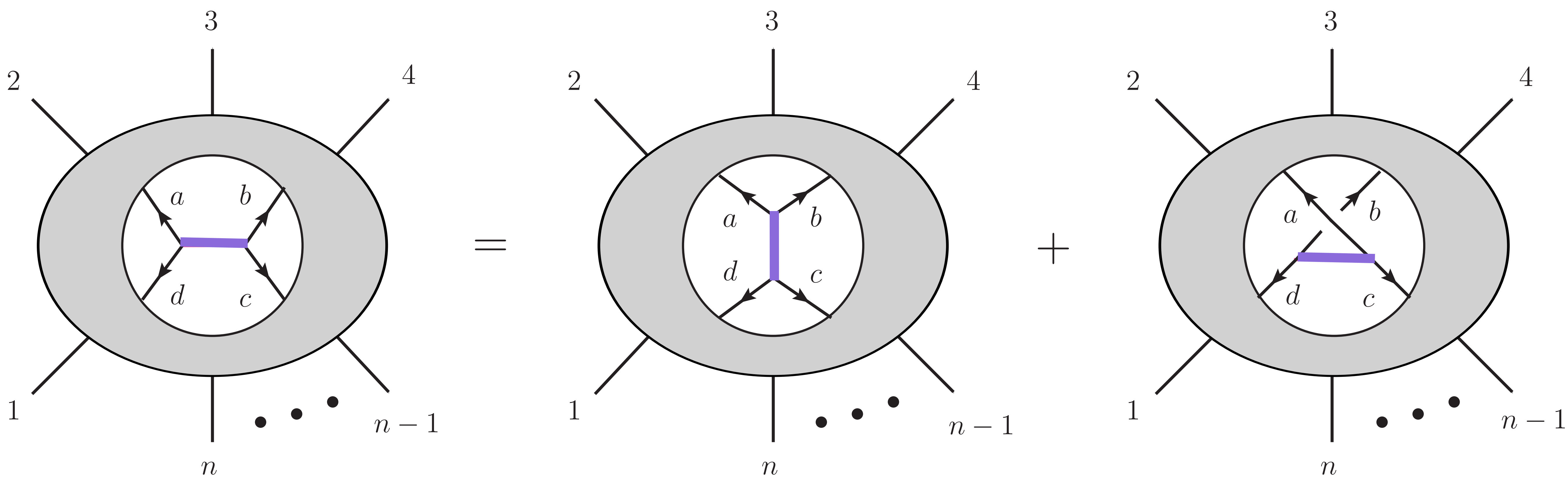


“One should always generalize.” -
C. Jacobi

Valid multi-loop generalization?

$$\frac{(-i)^L}{g^{n-2+2L}}\mathcal{A}^{\text{loop}} = \sum_{\mathcal{G}\in\text{cubic}} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S(\mathcal{G})} \frac{n(\mathcal{G})c(\mathcal{G})}{D(\mathcal{G})}$$

CONJECTURE: for all graphs, can impose CK on every edge:

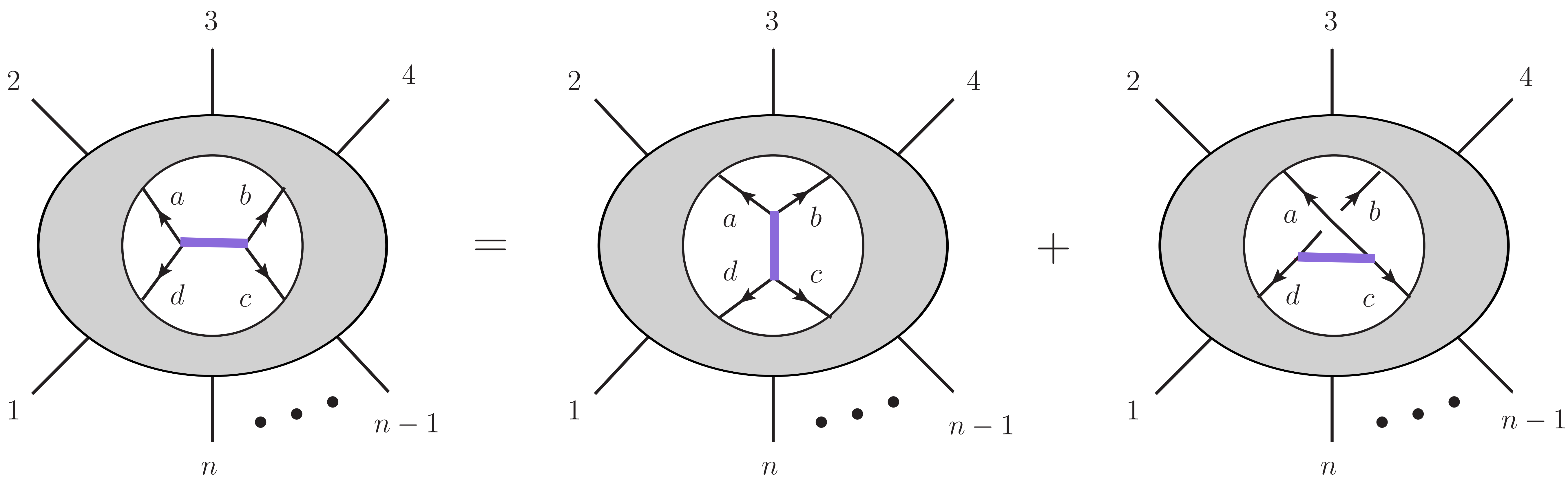


Consequence of unitarity: double copy structure holds.

Valid multi-loop generalization?

$$\frac{(-i)^L}{g^{n-2+2L}}\mathcal{A}^{\text{loop}} = \sum_{\mathcal{G}\in\text{cubic}} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S(\mathcal{G})} \frac{n(\mathcal{G})c(\mathcal{G})}{D(\mathcal{G})}$$

CONJECTURE: for all graphs, can impose CK on every edge:

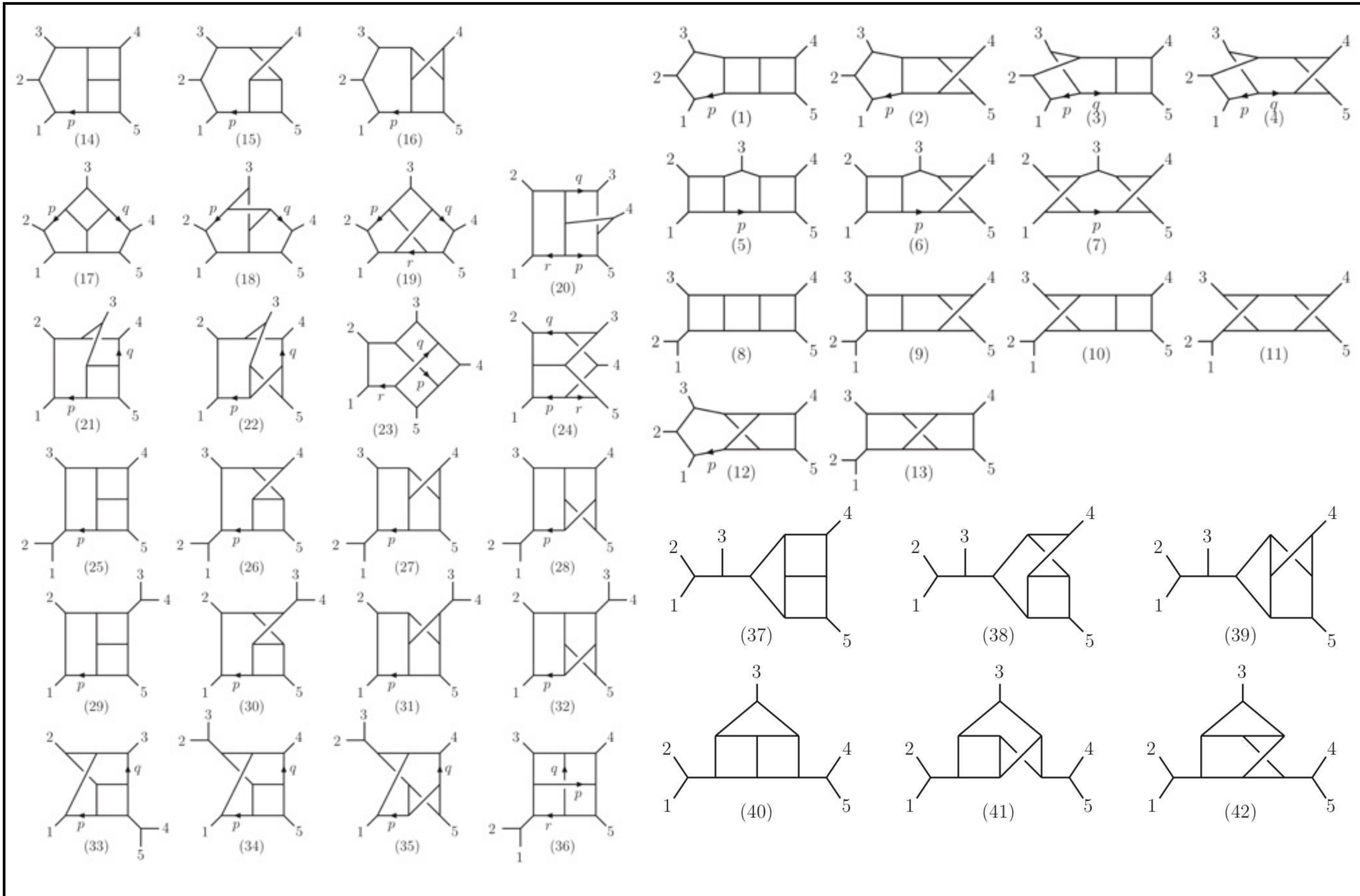


Consequence of unitarity: double copy structure holds.

$$\frac{(-i)^{L+1}}{(\kappa/2)^{n-2+2L}}\mathcal{M}^{\text{loop}} = \sum_{\mathcal{G}\in\text{cubic}} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S(\mathcal{G})} \frac{n(\mathcal{G})\tilde{n}(\mathcal{G})}{D(\mathcal{G})}$$

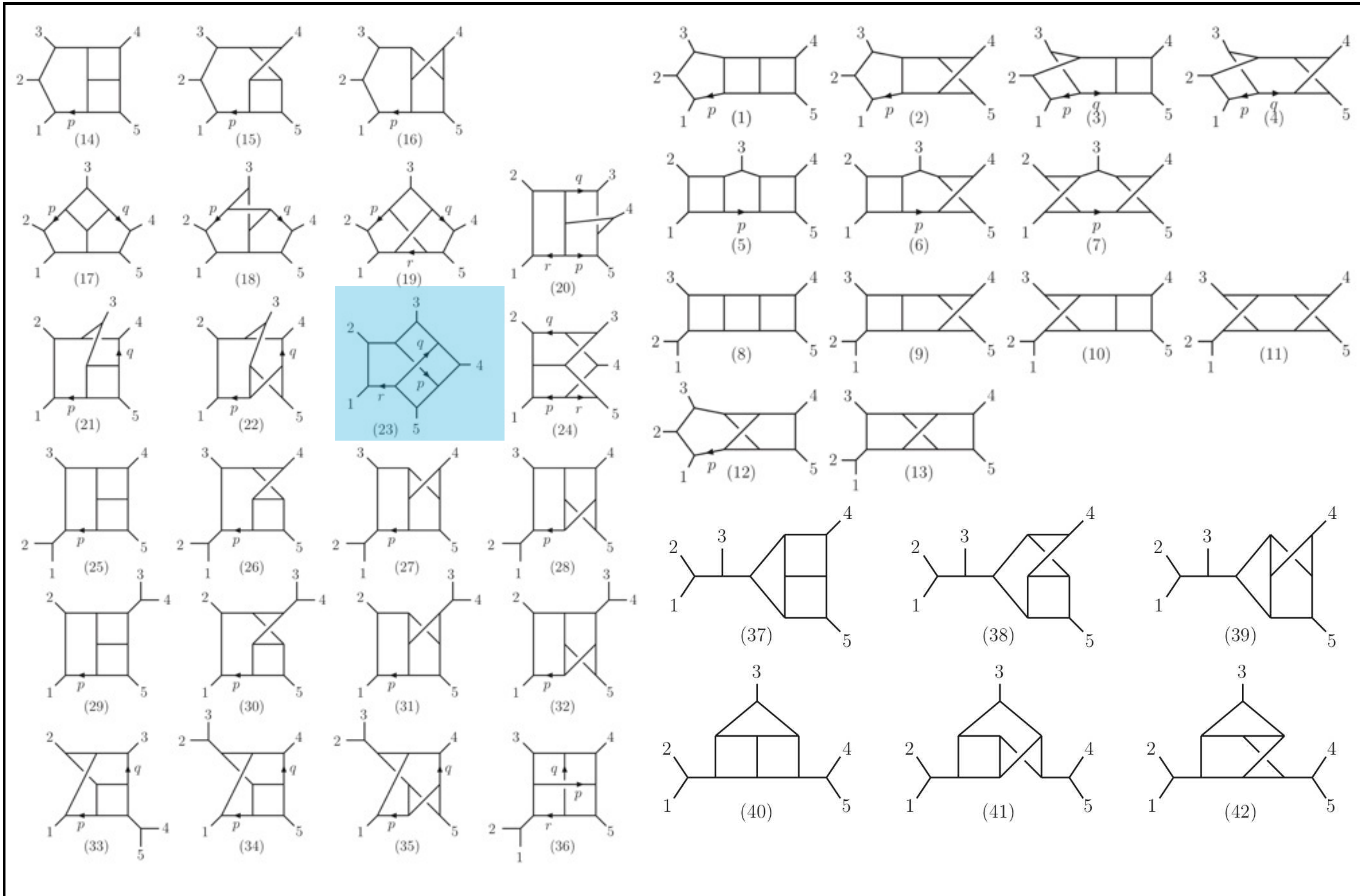
Five point 3-loop N=4 SYM & N=8 SUGRA

JJMC, Johansson (to appear)



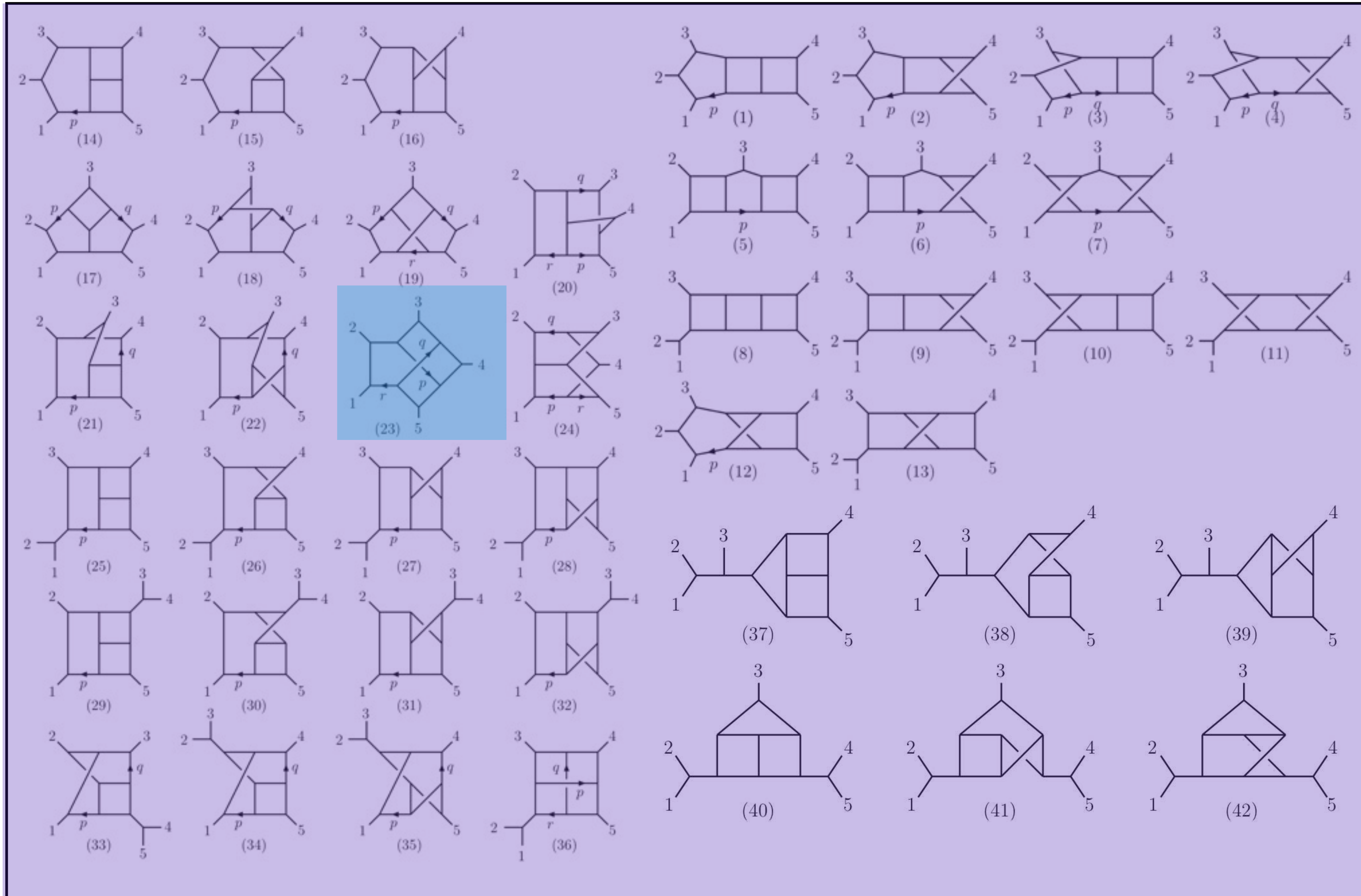
Five point 3-loop N=4 SYM & N=8 SUGRA

JJMC, Johansson (to appear)



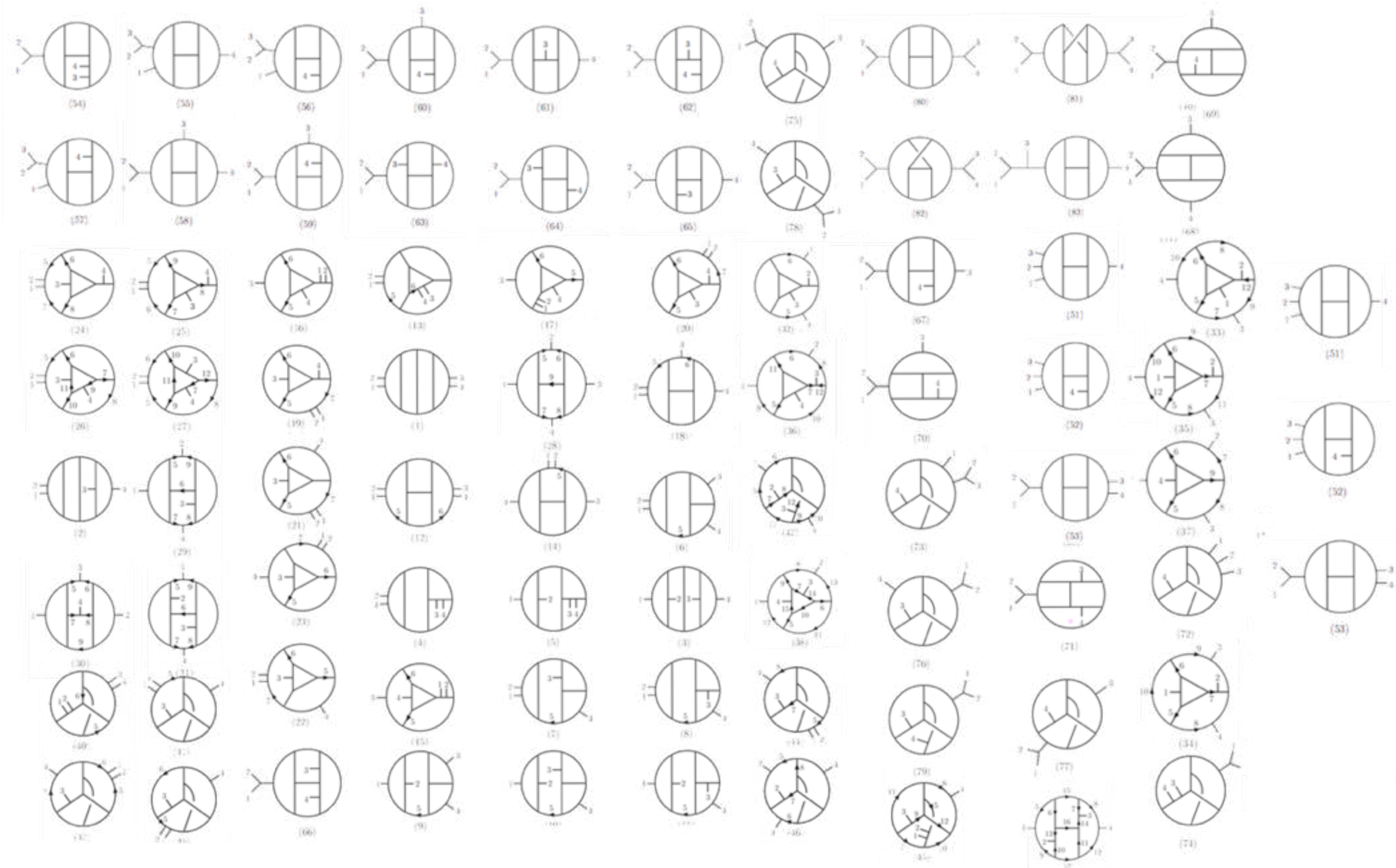
Five point 3-loop N=4 SYM & N=8 SUGRA

JJMC, Johansson (to appear)



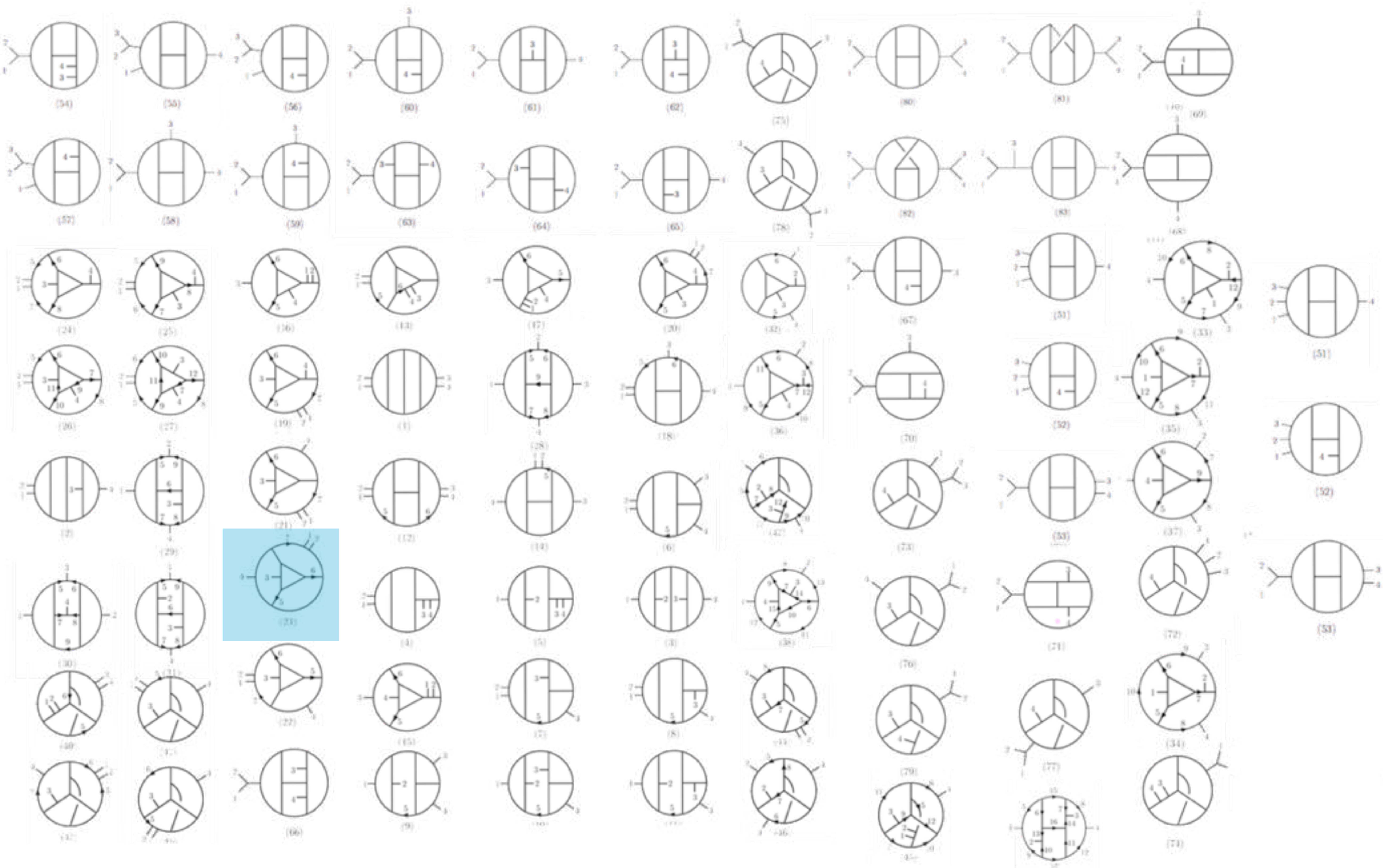
Full four loop N=4 SYM & N=8 SUGRA

Bern, JJMC, Dixon, Johansson, Roiban (2012)



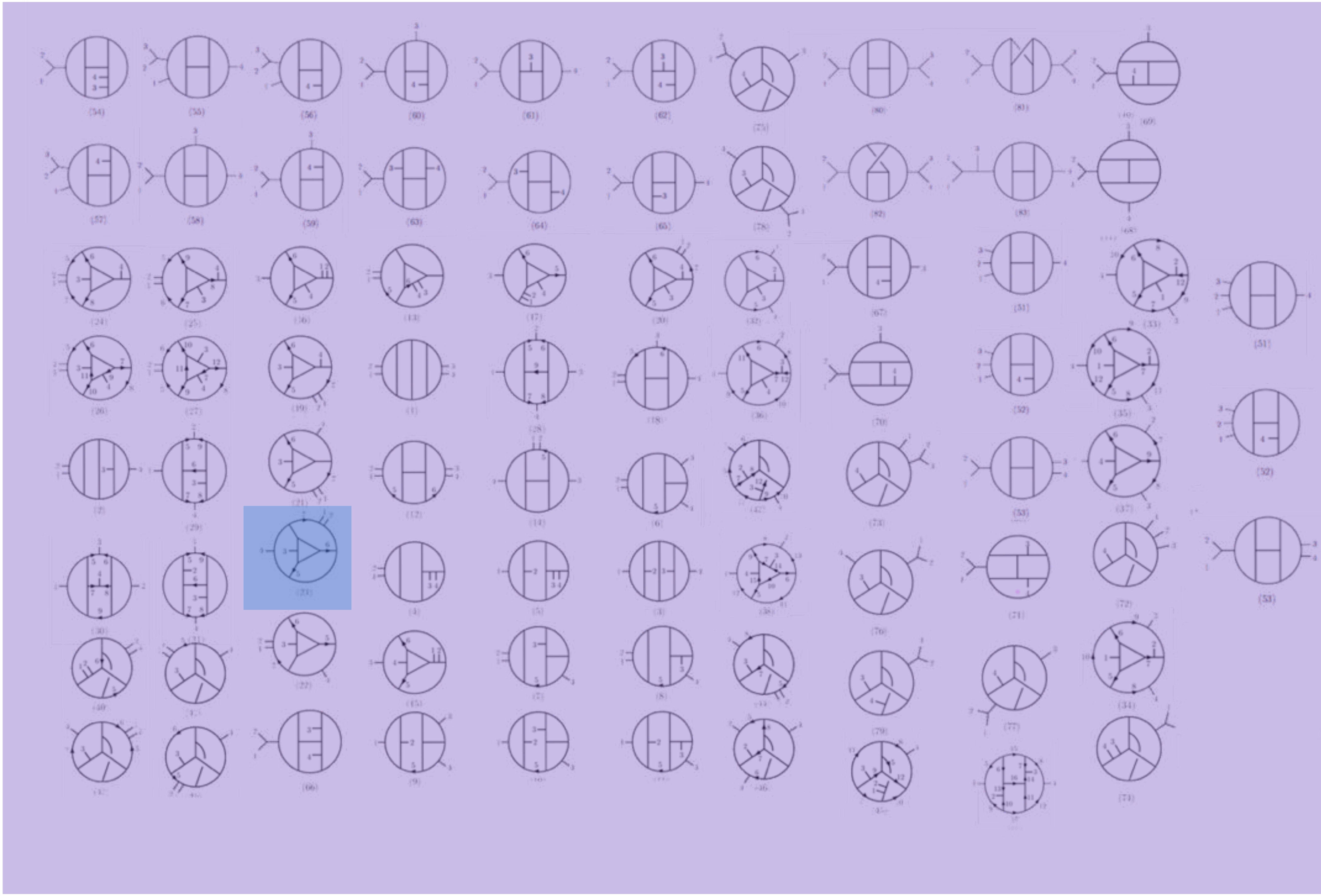
Full four loop N=4 SYM & N=8 SUGRA

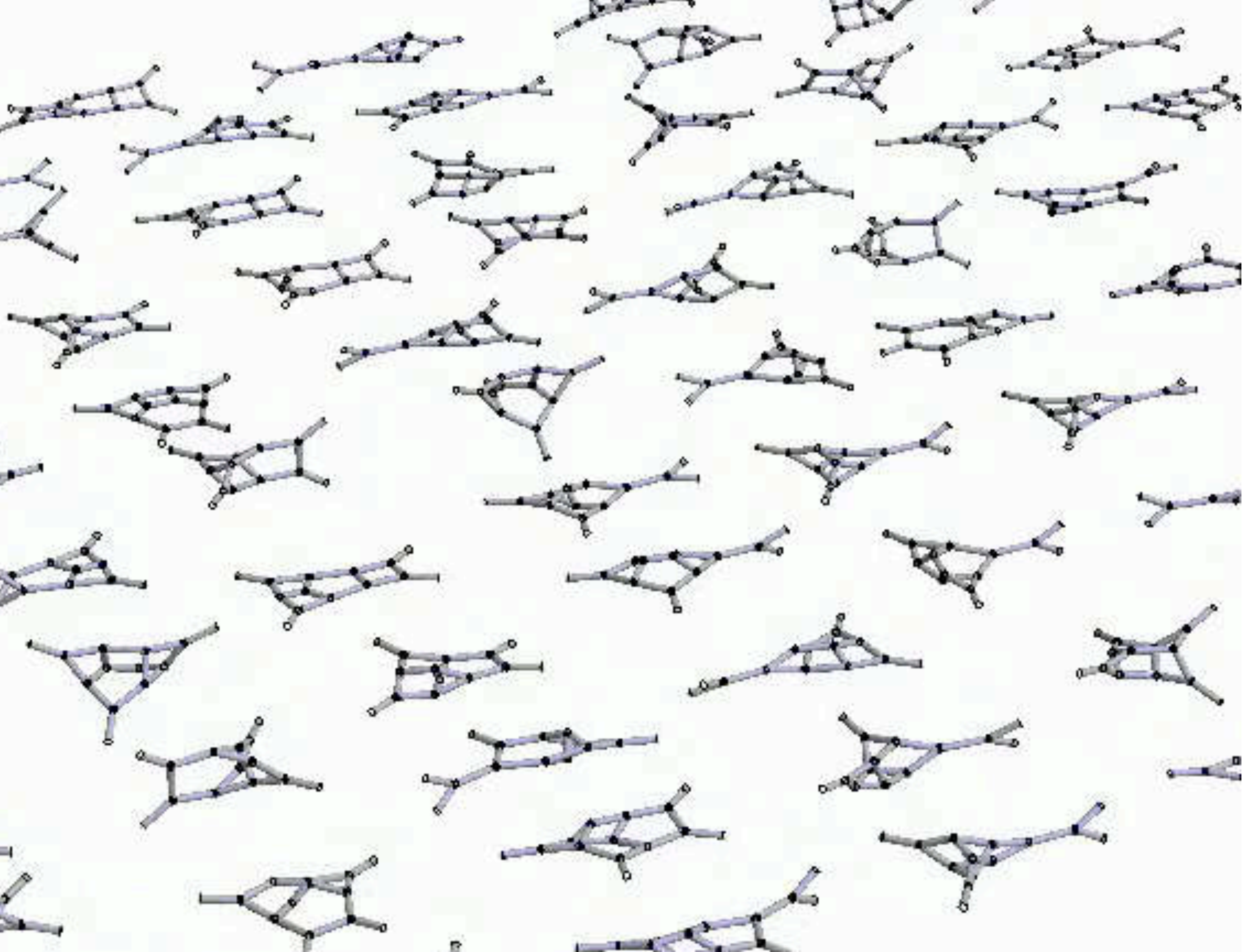
Bern, JJMC, Dixon, Johansson, Roiban (2012)



Full four loop N=4 SYM & N=8 SUGRA

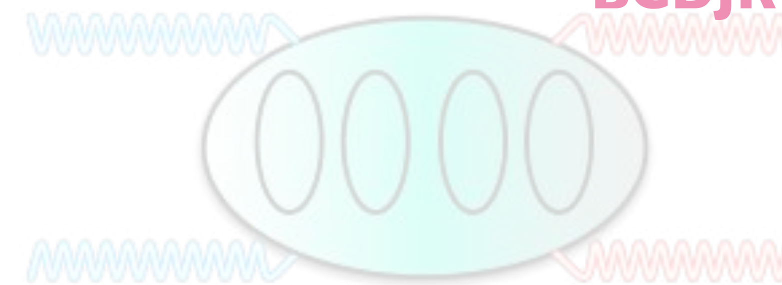
Bern, JJMC, Dixon, Johansson, Roiban (2012)



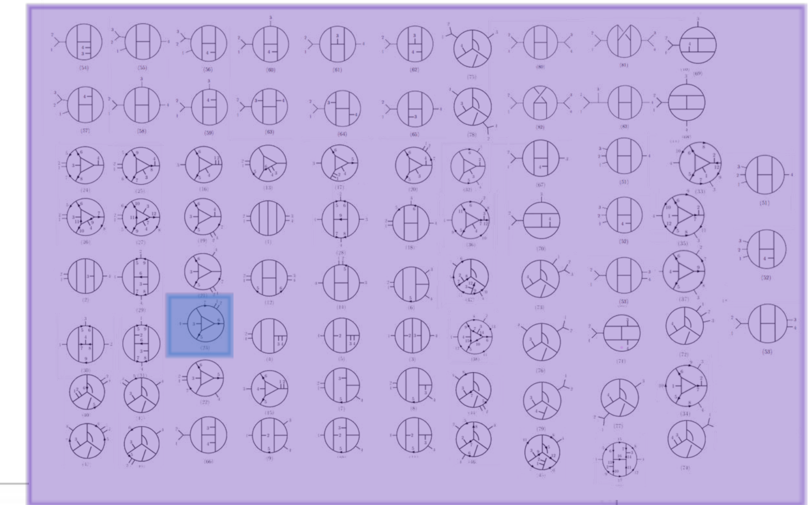


BCDJR

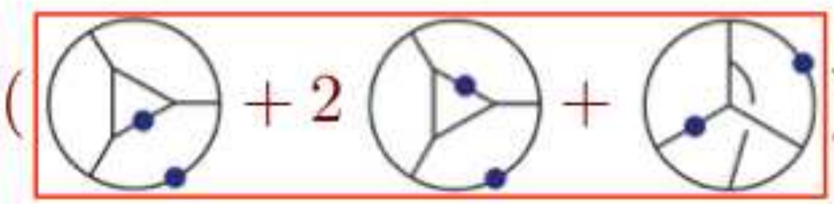
An interesting development at 4-loops!



In the new manifest representation, we have the power to identify remarkable structure between YM and Gravity



$$\mathcal{M}_4^{(4)}\Big|_{\text{pole}} = -\frac{23}{8}\left(\frac{\kappa}{2}\right)^{10} stu(s^2+t^2+u^2)^2\, M_4^{\text{tree}}\,\Big($$



$$\Big)$$

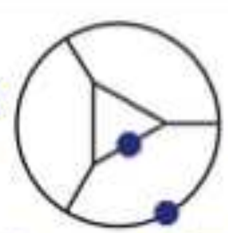
$$-256 + \frac{2025}{8}$$

$\frac{2025}{8}$ ← 12- and 13-propagator integrals

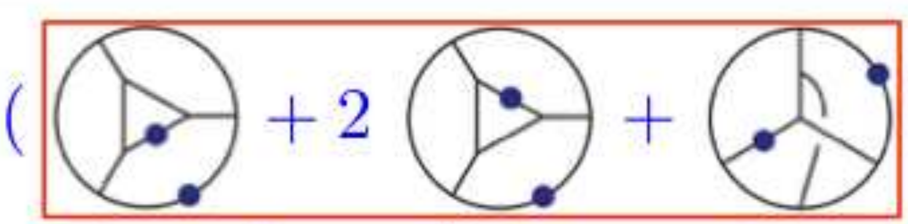
-256 ← 11-propagator integrals; same as in sYM

D=11/2

$$\mathcal{A}_4^{(4)}\Big|_{\text{pole}}^{SU(N_c)} = -6\, g^{10}\, \mathcal{K}\, N_c^2\Big(N_c^2$$



$$+\,12\,\Big($$



$$\Big)\Big)$$

$$\times\Big(s\,(\mathrm{Tr}_{1324}+\mathrm{Tr}_{1423}) + t\,(\mathrm{Tr}_{1243}+\mathrm{Tr}_{1342}) + u\,(\mathrm{Tr}_{1234}+\mathrm{Tr}_{1432})\Big)$$

D=11/2

Color-dual Model Building

The traditional story

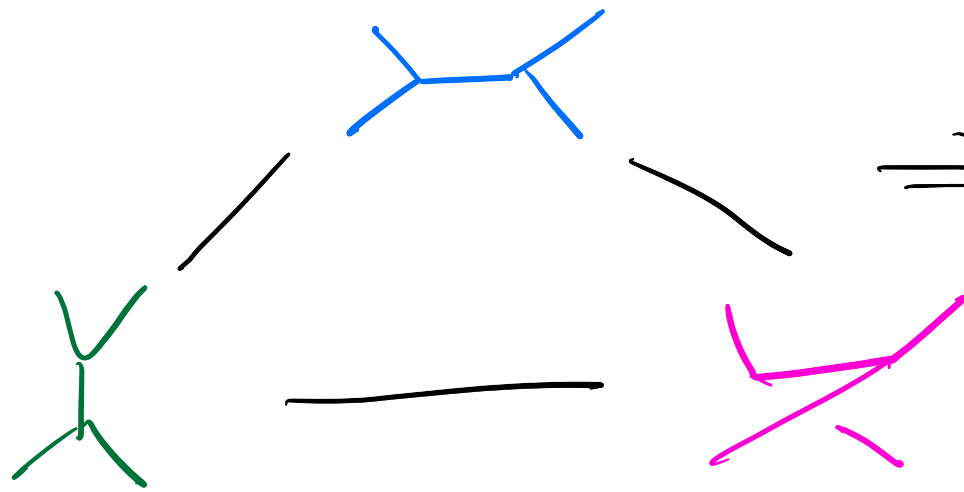
color-dual kinematics = linearized gauge-invariance

$$\mathcal{L} = (\partial A)^2 + g_1 A^2 (\partial A) + g_2 A^4$$

The traditional story

color-dual kinematics = linearized gauge-invariance

$$\mathcal{L} = (\partial A)^2 + g_1 A^2 (\partial A) + g_2 A^4$$

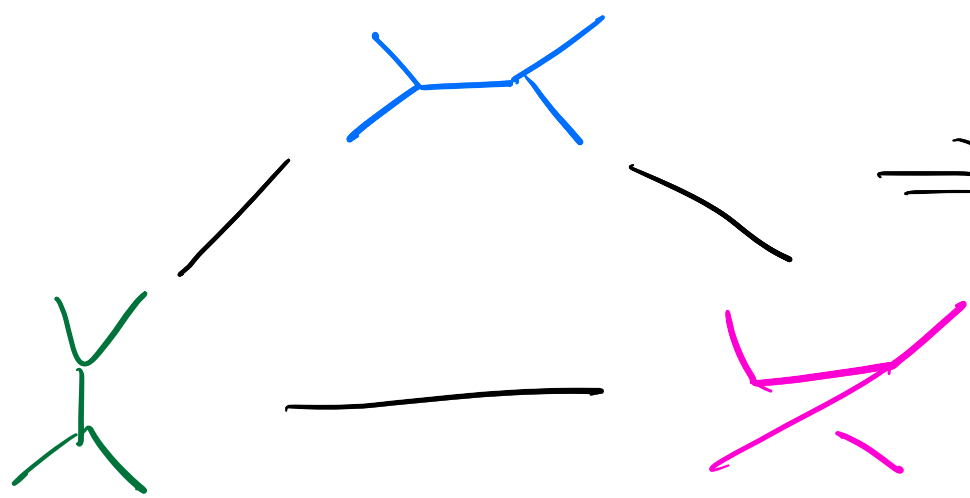


$$\Longrightarrow \; g_2 \sim g_1^{\,2} \Longrightarrow \mathcal{L} = -\frac{1}{4} \operatorname{tr} F^2$$

The traditional story

color-dual kinematics = linearized gauge-invariance

$$\mathcal{L} = (\partial A)^2 + g_1 A^2 (\partial A) + g_2 A^4$$



$$\Rightarrow g_2 \sim g_1^2 \Rightarrow \mathcal{L} = -\frac{1}{4} \mathrm{tr} F^2$$

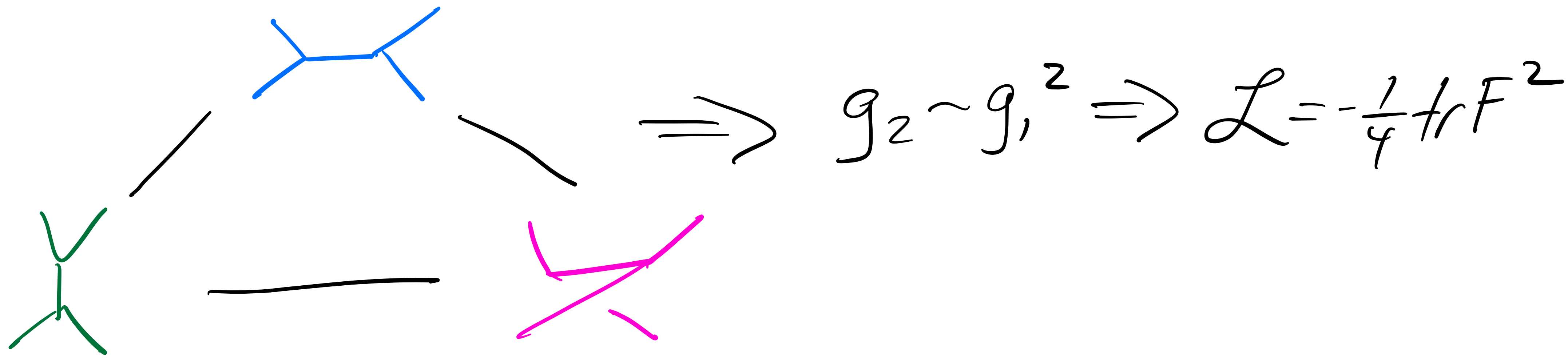
color-dual kinematics + DC = linear diffeo inv.

$$M_5^{GR} = \sum \frac{n(\vdash\!\!\dashv)\, n(\vdash\!\!\dashv)}{d_g}$$

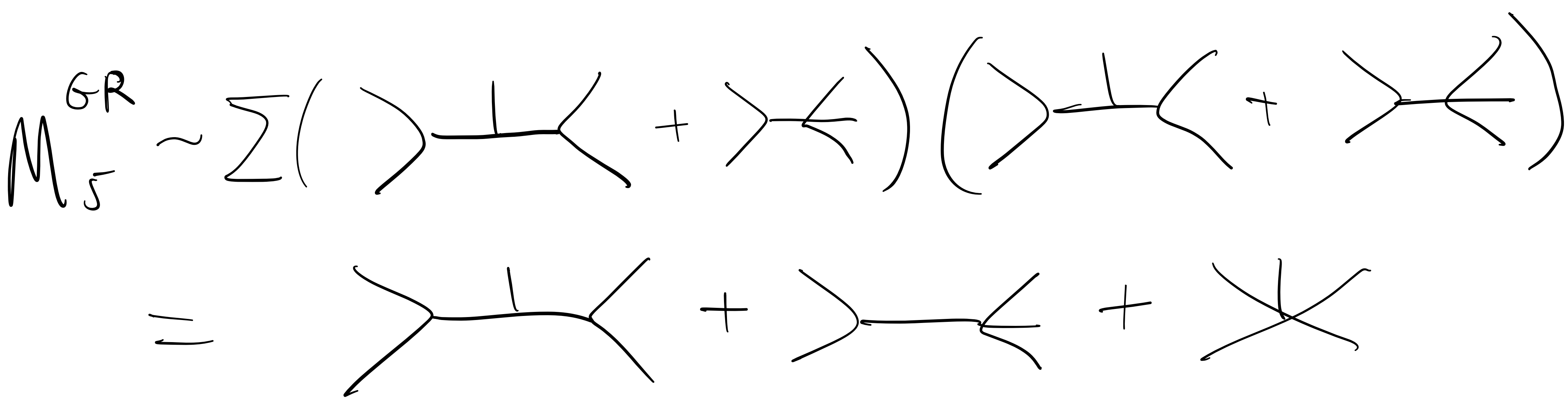
The traditional story

color-dual kinematics = linearized gauge-invariance

$$\mathcal{L} = (\partial A)^2 + g_1 A^2 (\partial A) + g_2 A^4$$



color-dual kinematics + DC = linear diffeo inv.



The traditional story

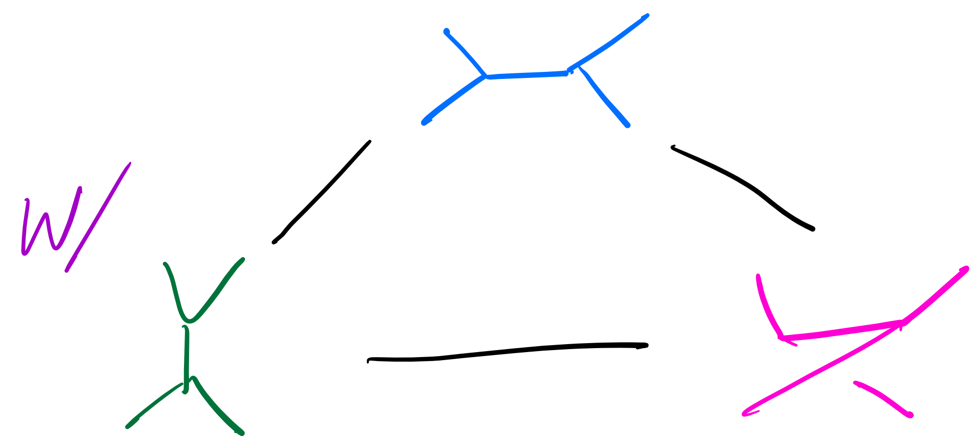
color-dual kinematics = soft bootstrap

$$\mathcal{L} = (\partial\pi)^2 \sum_{k=0} c_k \pi^{2k}$$

The traditional story

color-dual kinematics = soft bootstrap

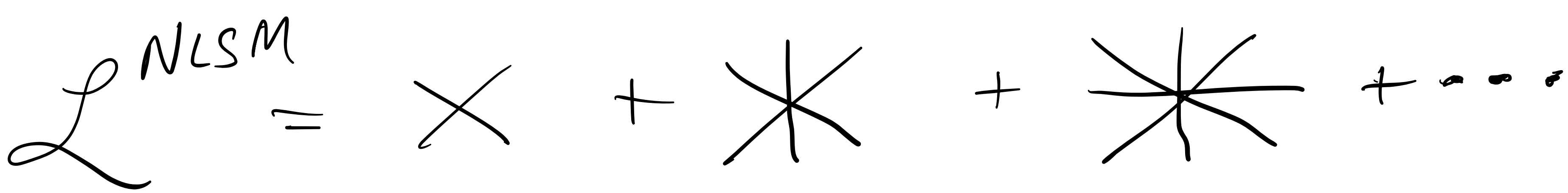
$$\mathcal{L} = (\partial\pi)^2 \sum_{k=0} c_k \pi^{2k}$$



JJMC, Rodina

Cheung, Shen, Wen,...

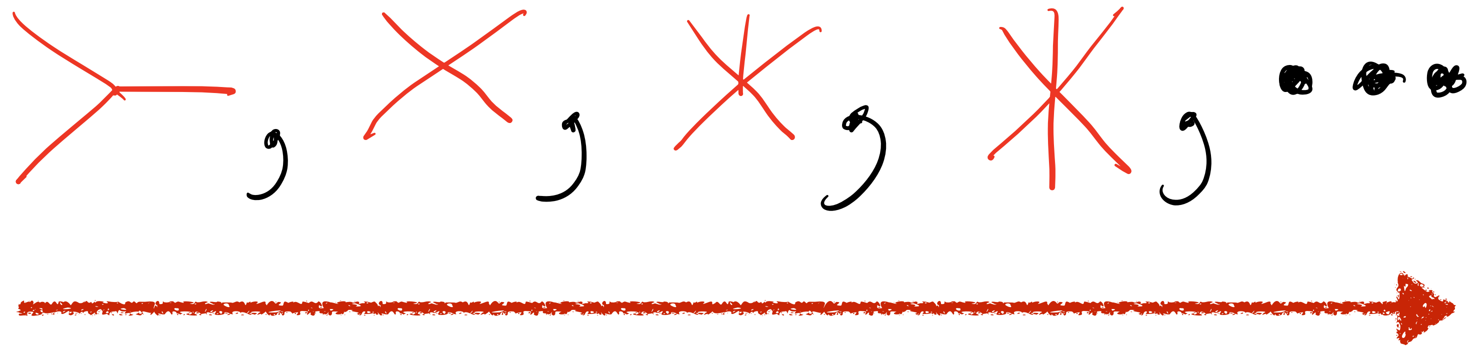
Resums to NLSM:



$$\mathcal{L}^{\text{NLSM}} = (\partial U)^\dagger (\partial U) \qquad U = e^{i\pi}$$

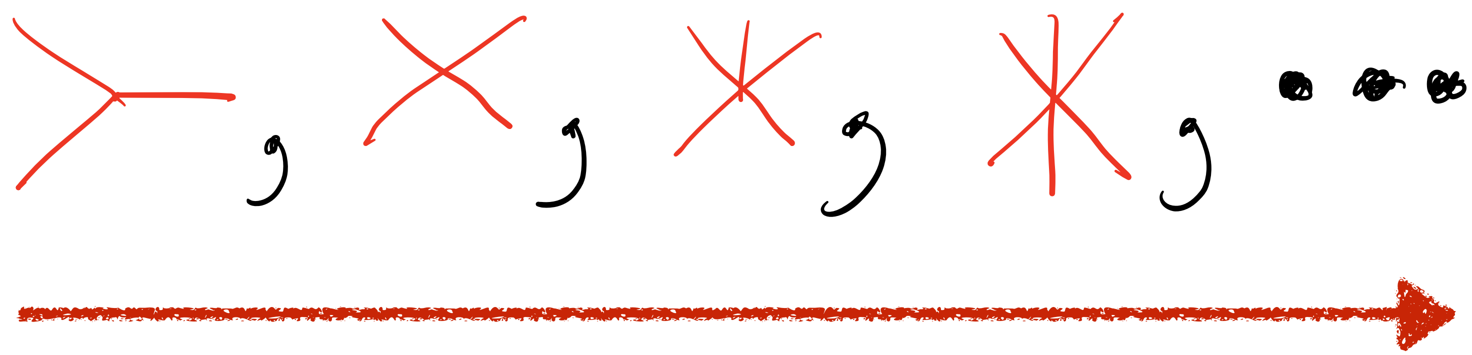
The traditional story

Color/kinematics fixes out…



The traditional story

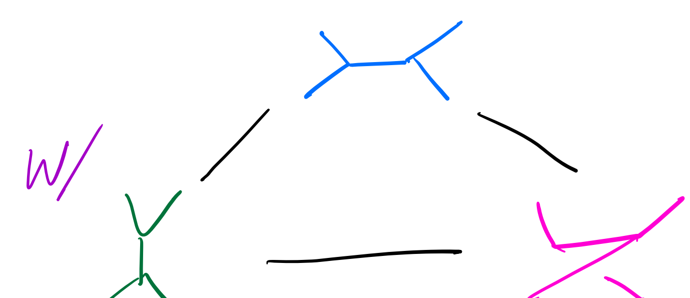
Color/kinematics fixes out…



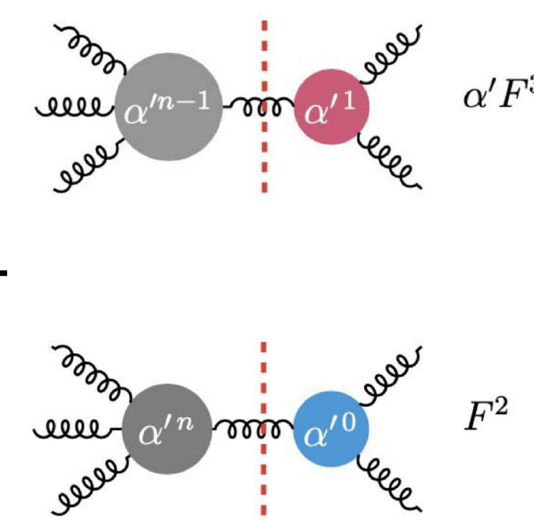
New story in the UV!

Color/Kinematics + EFT (mixed HD contacts) constraints **UP**!

$$\mathcal{L}^{YM+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \alpha'^2 F^4 + \alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4$$

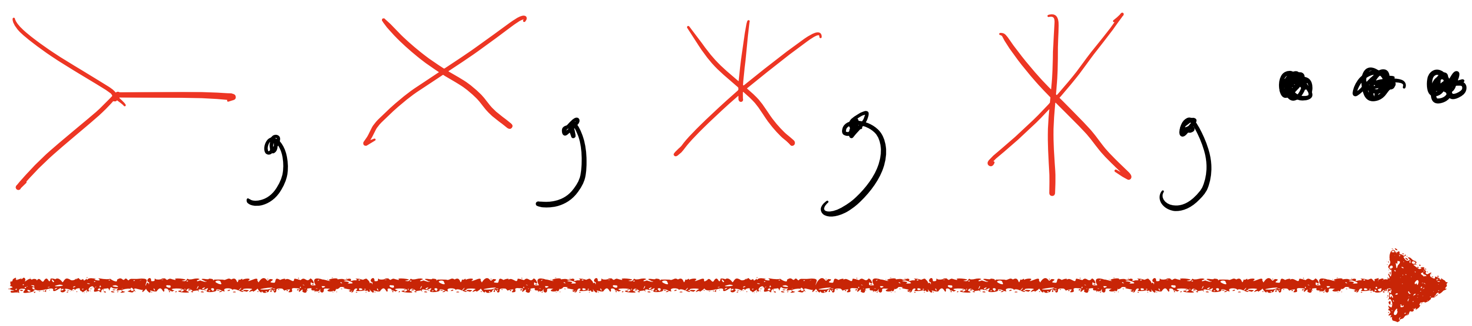


+

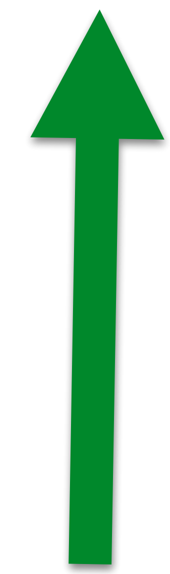


The traditional story

Color/kinematics fixes out…



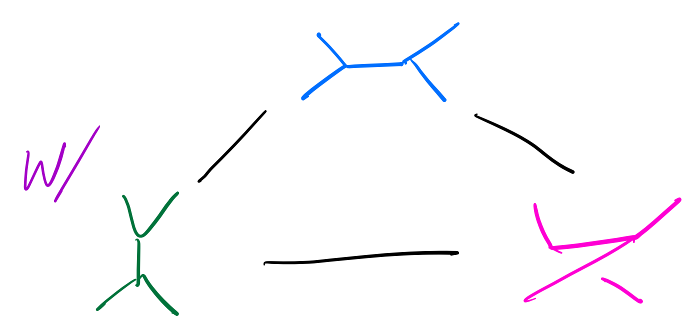
$$\mathcal{L}^{DF^2\text{+YM+HD}}$$



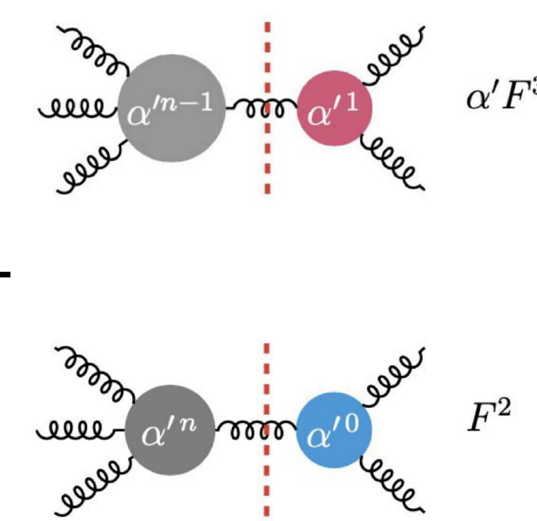
New story in the UV!

Color/Kinematics + EFT (mixed HD contacts) constraints **UP**!

$$\mathcal{L}^{YM+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \alpha'^2 F^4 + \alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4$$



+



$$A_4^{DF^2\text{+YM+HD}} = A_4^{DF^2\text{+YM}}\left(1 + \sum c_{(x,y)} \sigma_3^x \sigma_2^y\right)$$

JJMC, Pavao, Lewandowski
2203.03592, 2211.04441

Takeaway:

Mixed HD color-kinematics + factorization consistency induces a TOWER of EFT operators to the UV

In case of F^3 + YM, double copy lands on so called "twisted" string theory amplitudes — with HD freedom that lands on e.g. open, closed, heterotic

JJMC, Pavao, Lewandowski
2203.03592, 2211.04441

Additional examples explored in scalar EFT

BAS + HD — **Chen, Elvang Herderschee 2302.04895**

NLSM + HD — **Brown, Kampf, Oktem, Paranjape, Trnka 2305.05688**

Exploiting Double-Copy to extract predictions.

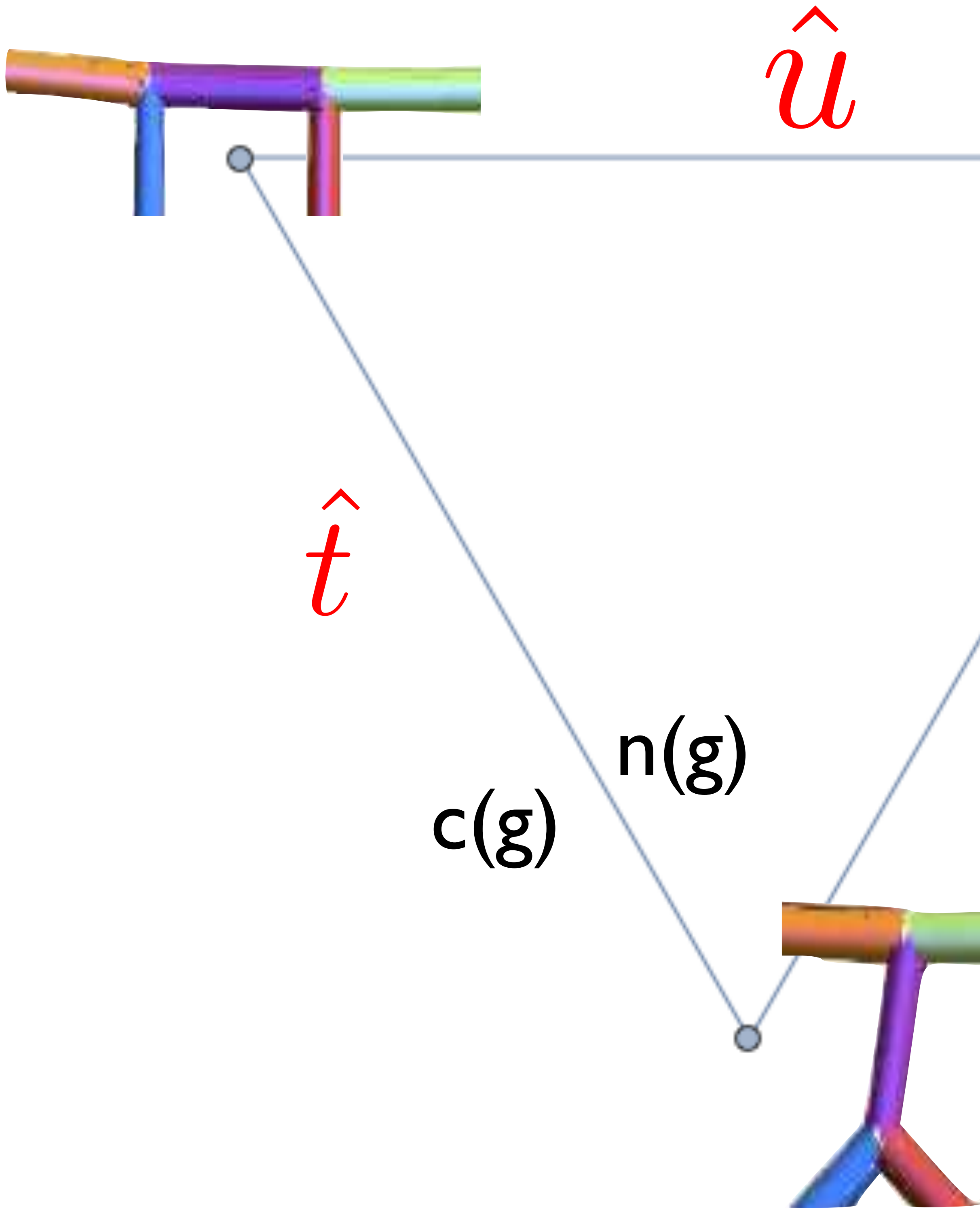
I. Tree level

Physics = Geometry

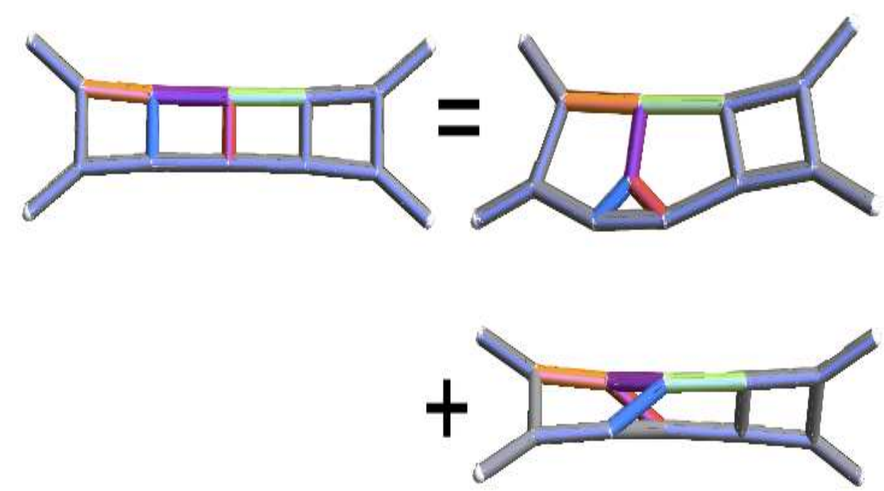
(the best polytopes are built on graphs of graphs!)

JJMC

Convenient language: graphs of graphs



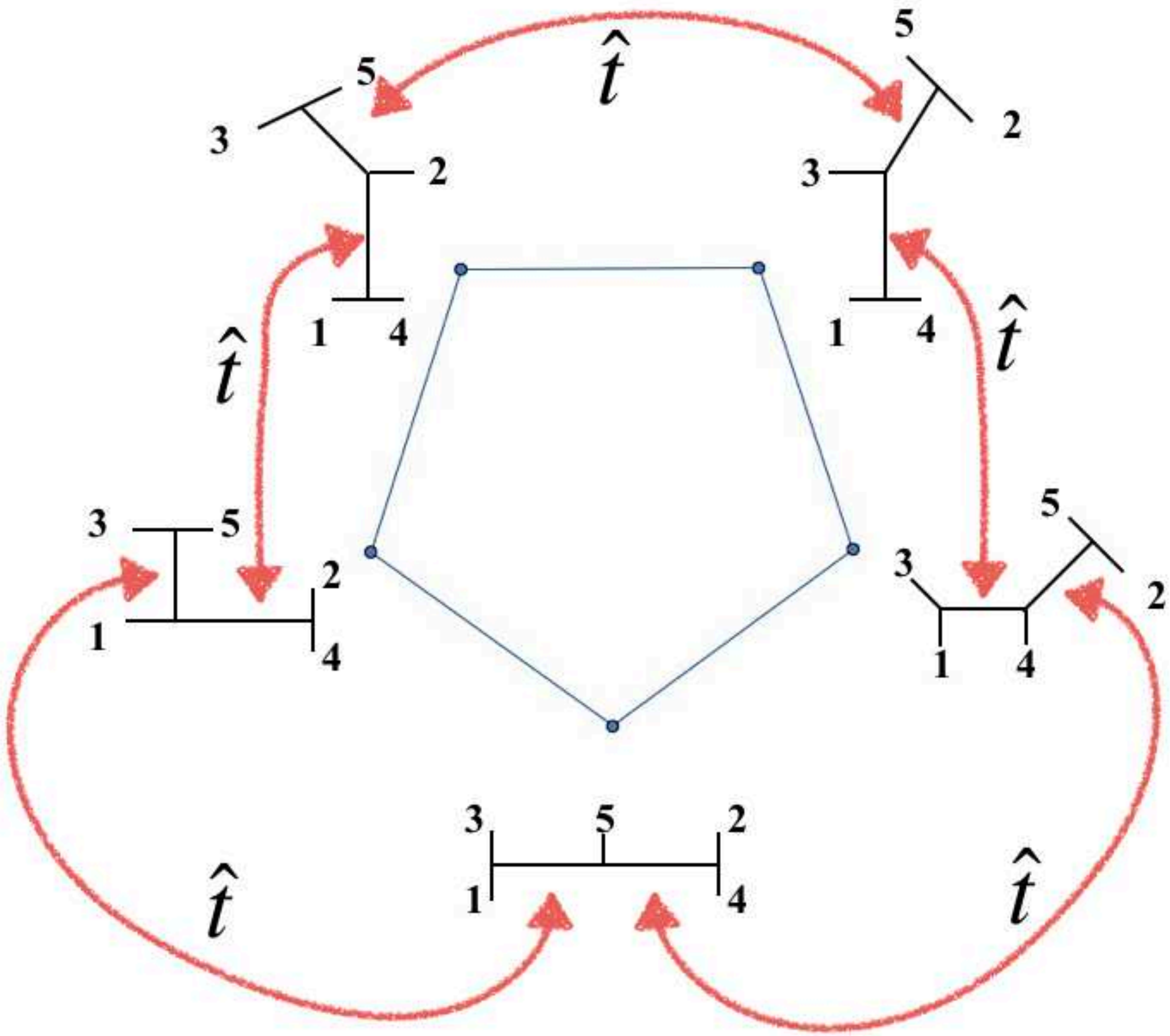
Color-Kinematics



Theory specific input — Partial amplitudes

REQUIREMENT: **graph contributions must reproduce partial amplitudes.**

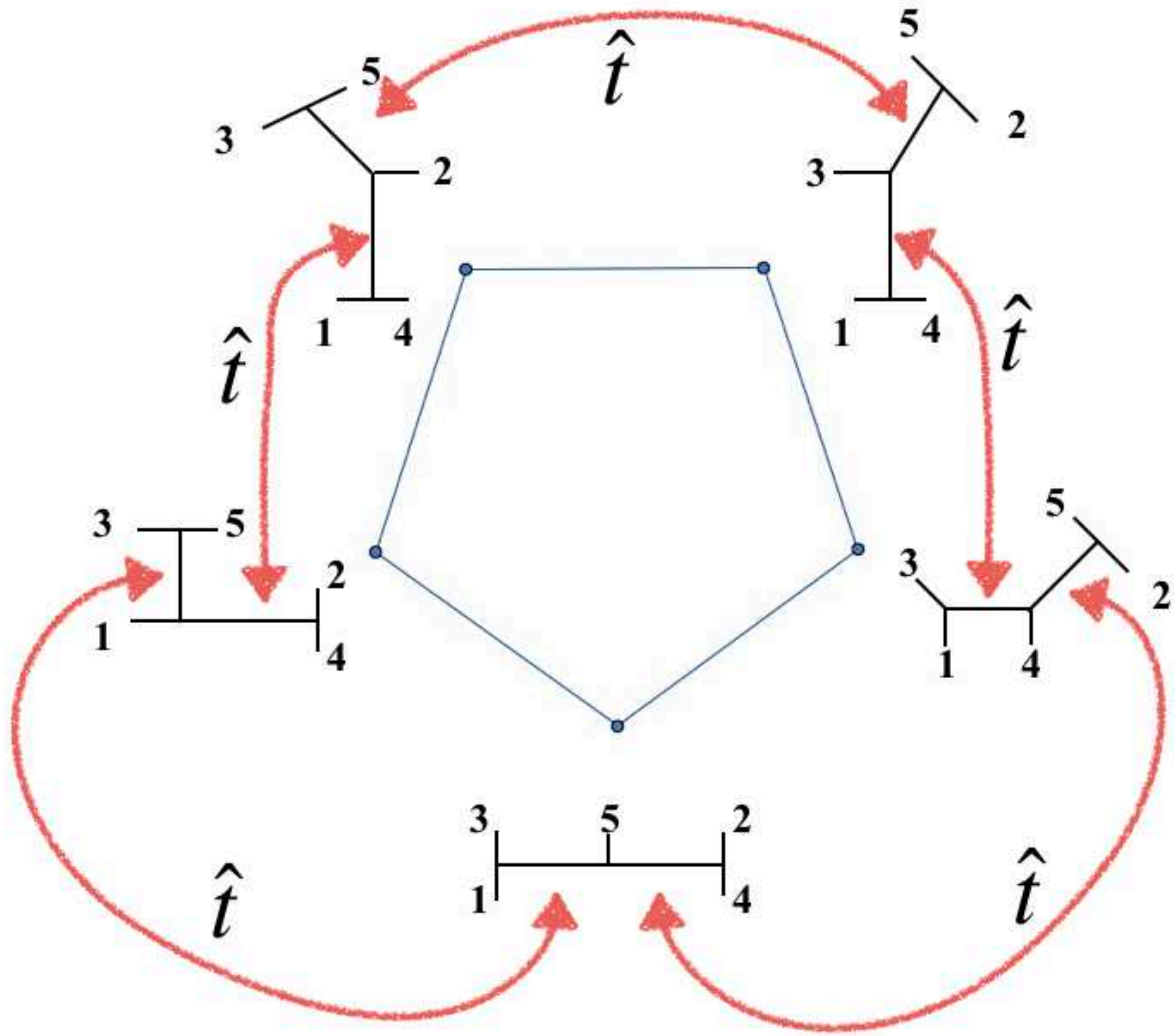
5pt example:



Note: same color-order!

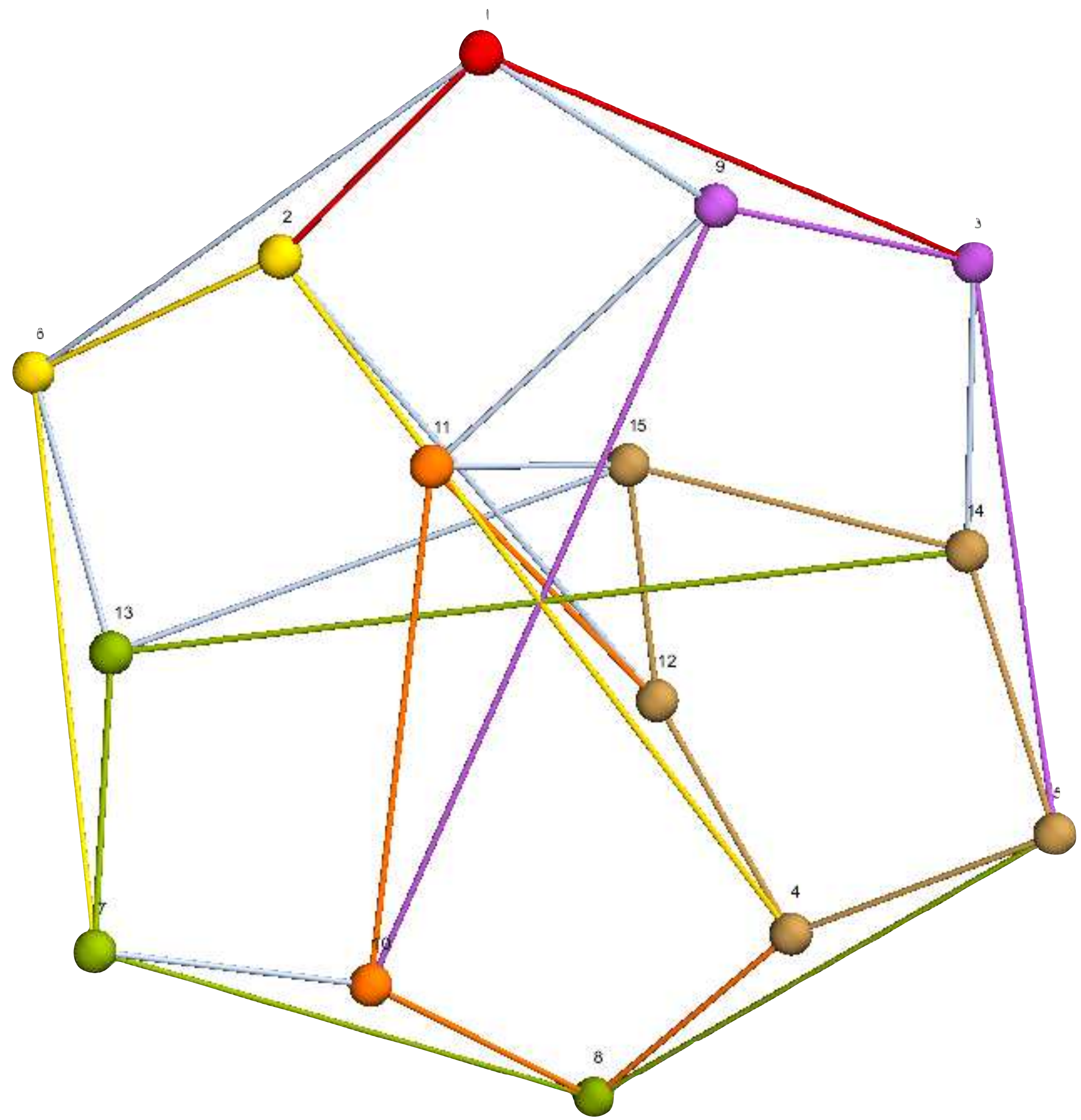
Theory specific input — Partial amplitudes

Graphs contributing to a tree-level **partial amplitudes** frame **Stasheff polytopes** .

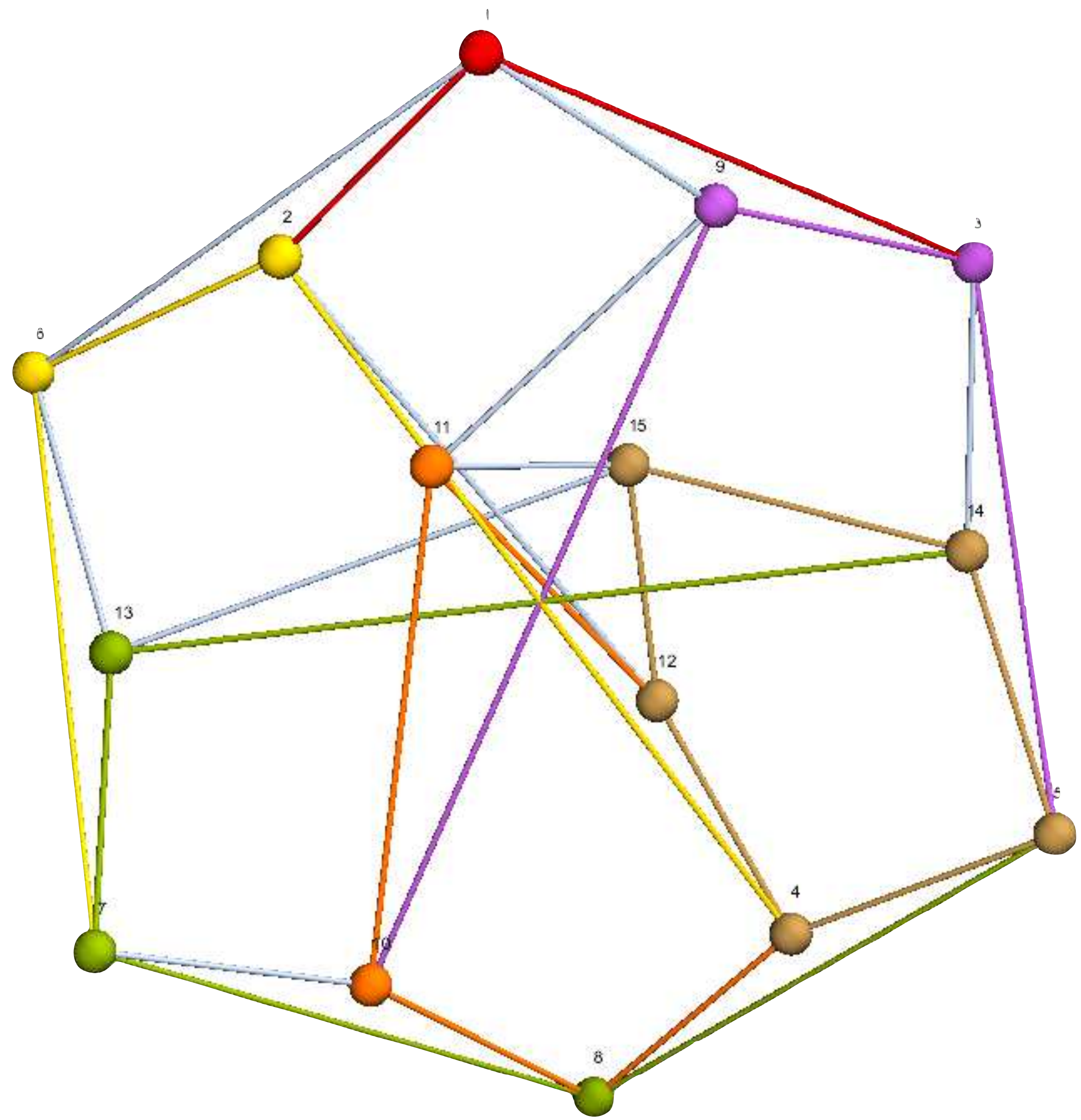


(these polytopes are also called **associahedra**)

JJMC



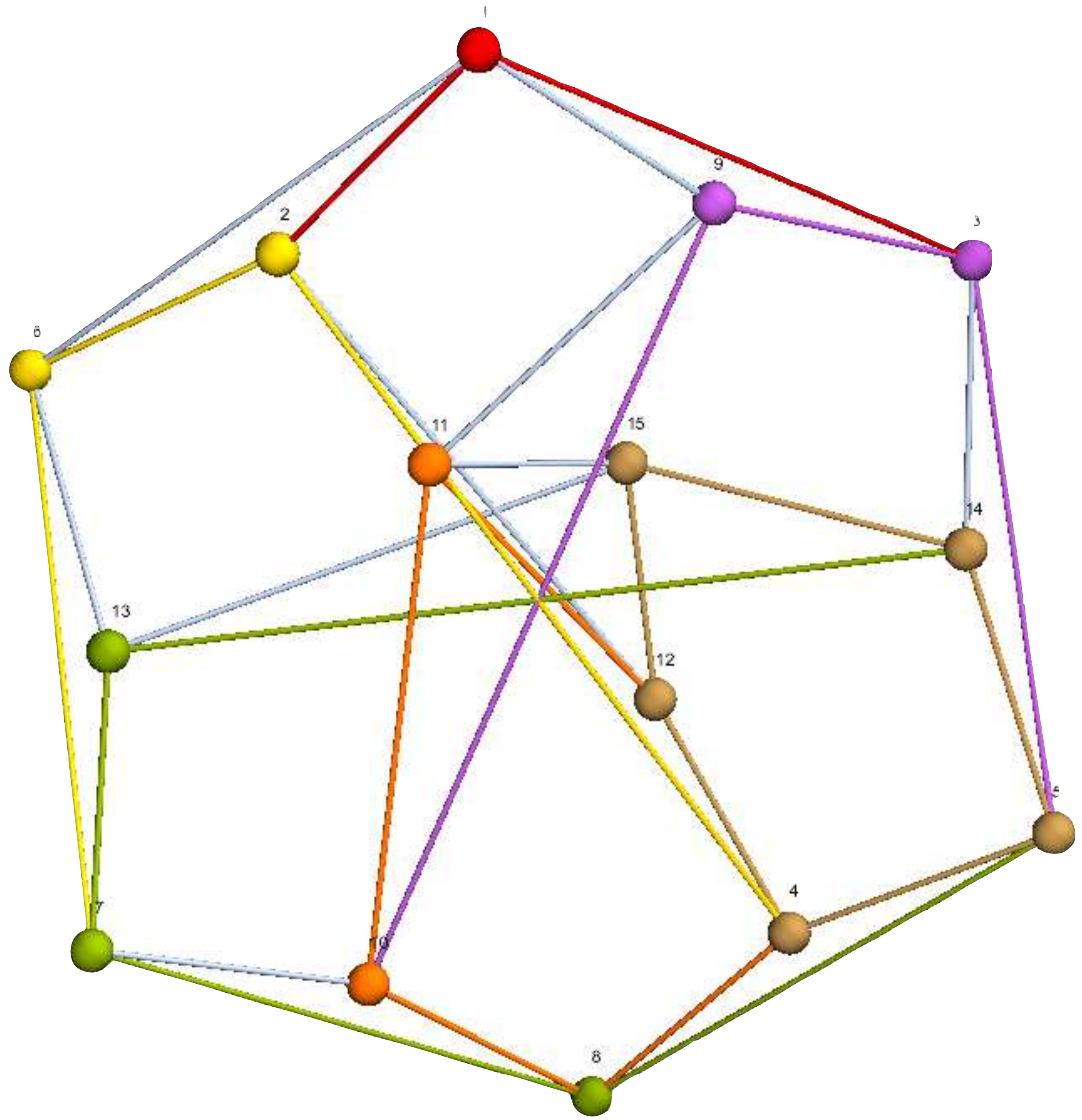
You might think you need (m-2)! of these color-stripped amplitudes to capture everything because this is what is required to include every vertex at least once:



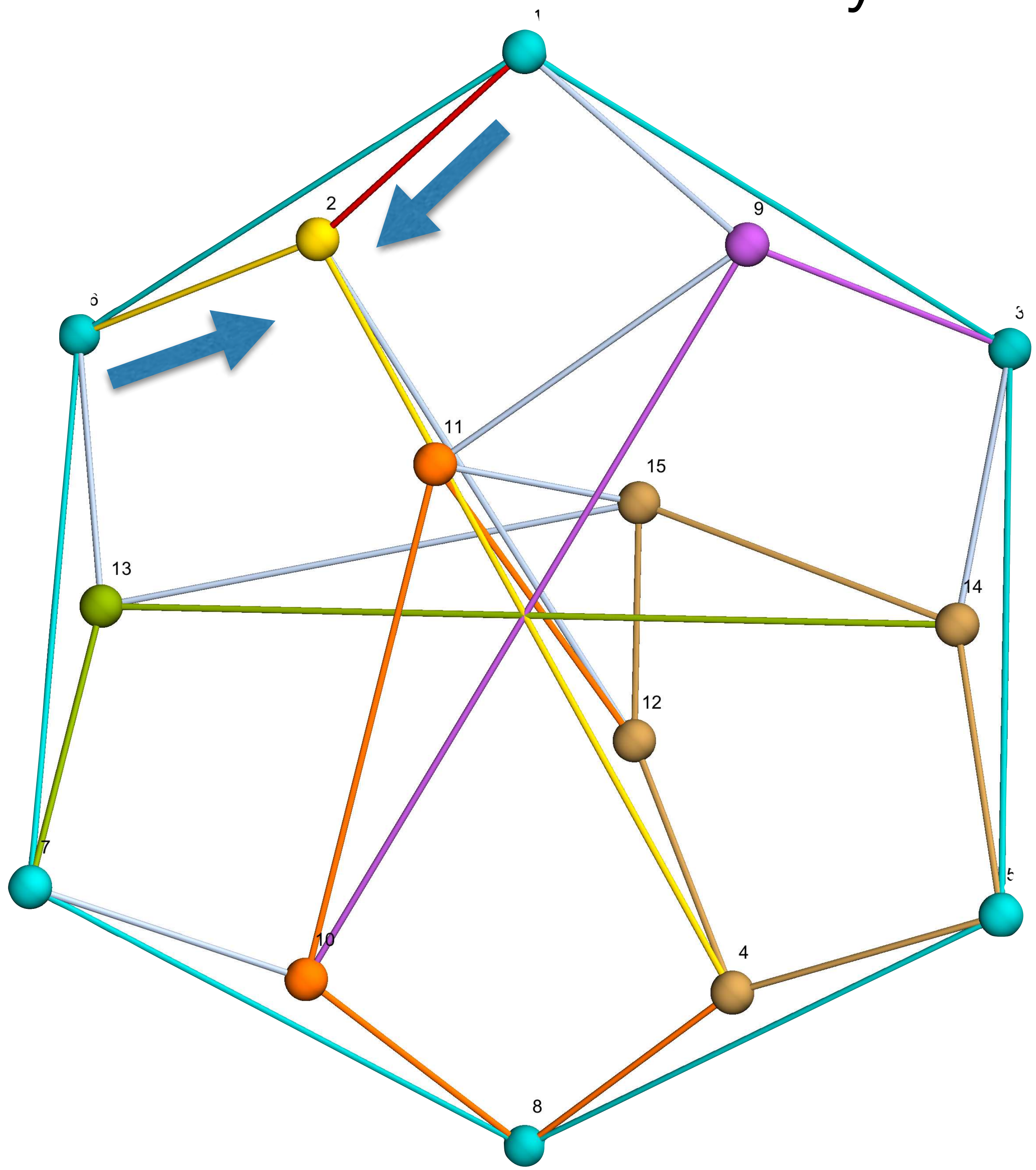
In fact, such a choice is the KK-basis, proven sufficient by Del Duca, Dixon, and Maltoni

JJMC

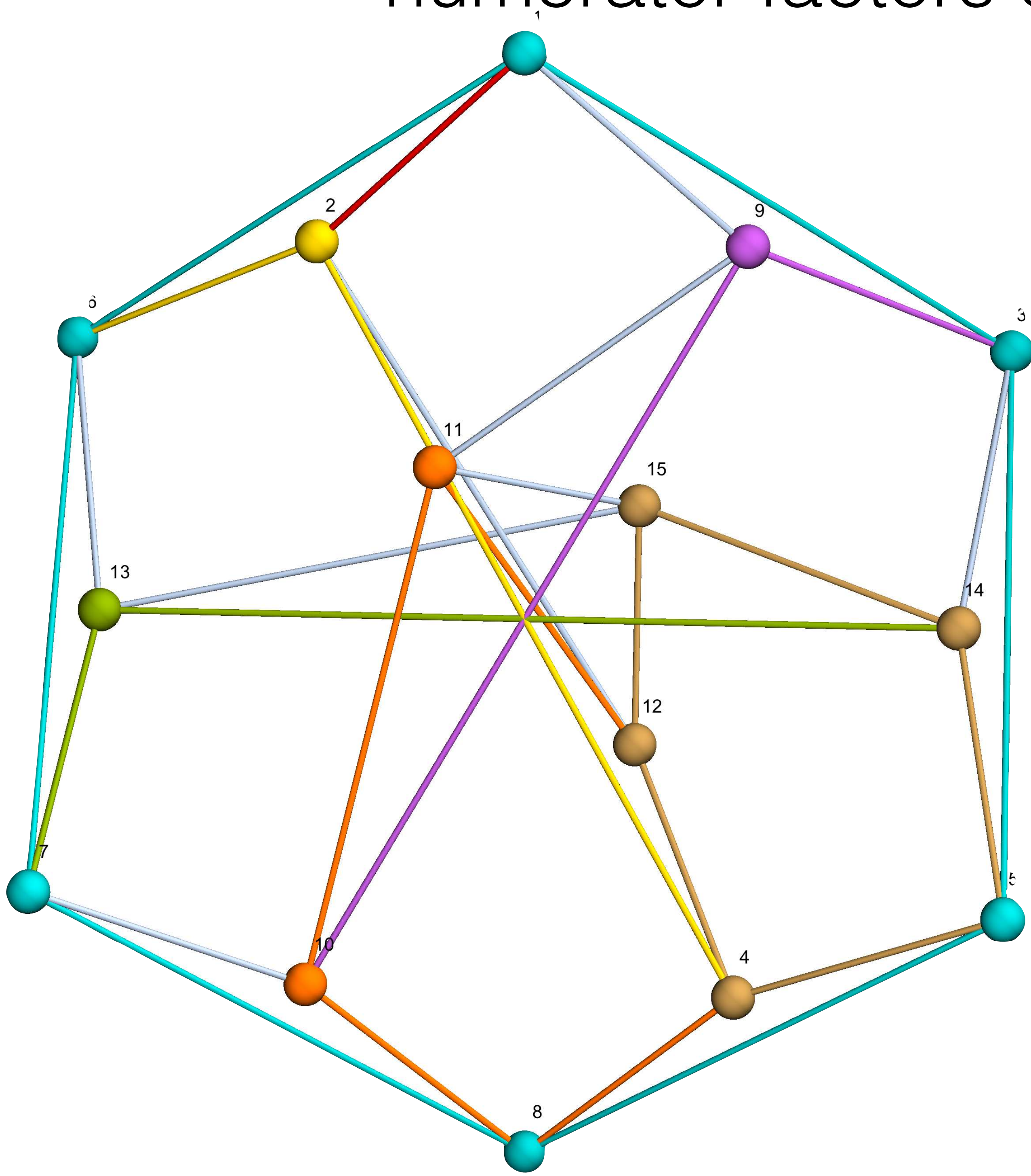
But notice, because of color-kinematics, only (m-2)! nodes are needed to specify both the color factors and numerator factors of everyone

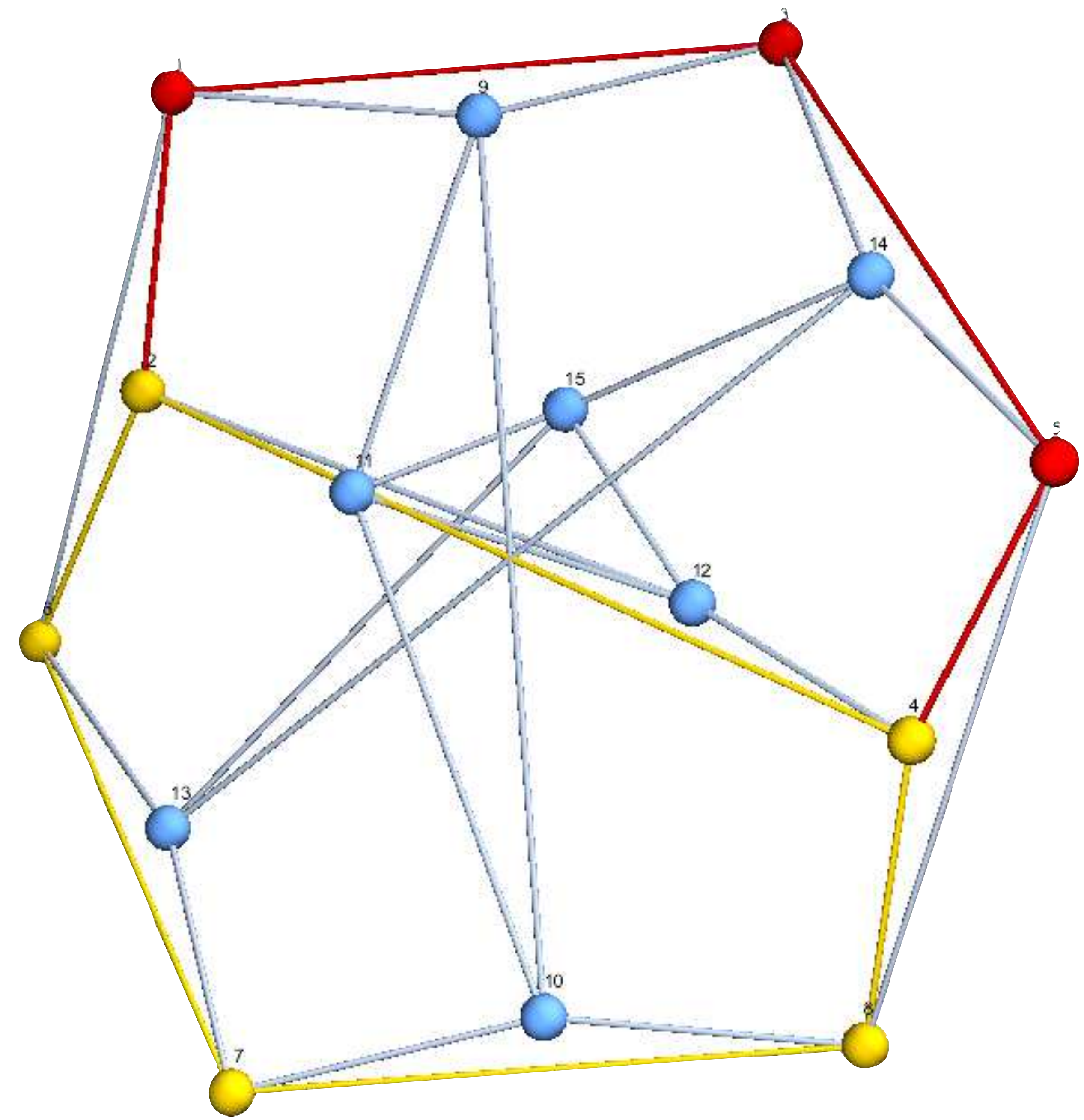


But notice, because of color-kinematics, only (m-2)! nodes are needed to specify both the color factors and numerator factors of everyone



But notice, because of color-kinematics, only (m-2)! nodes are needed to specify both the color factors and numerator factors of everyone

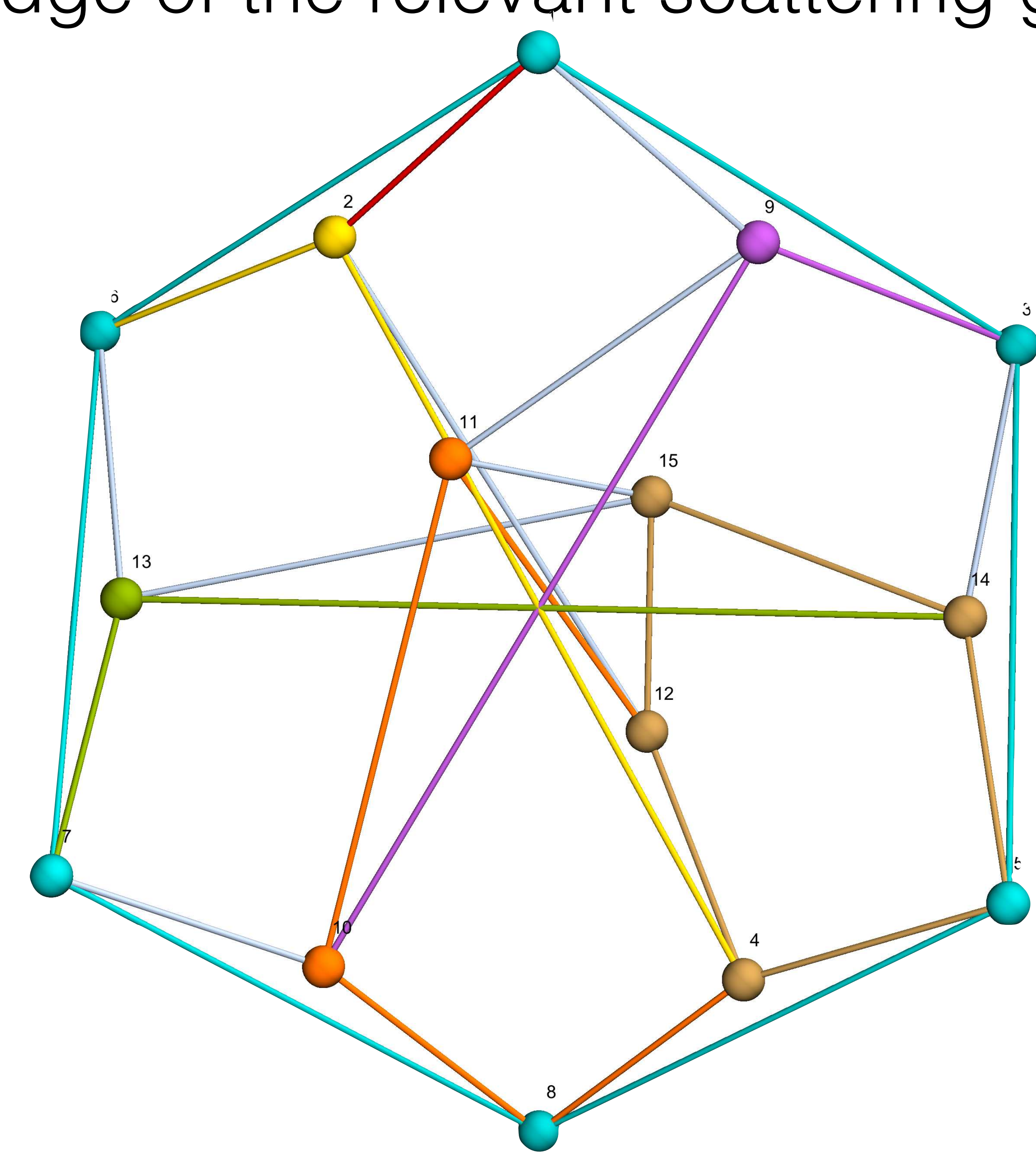


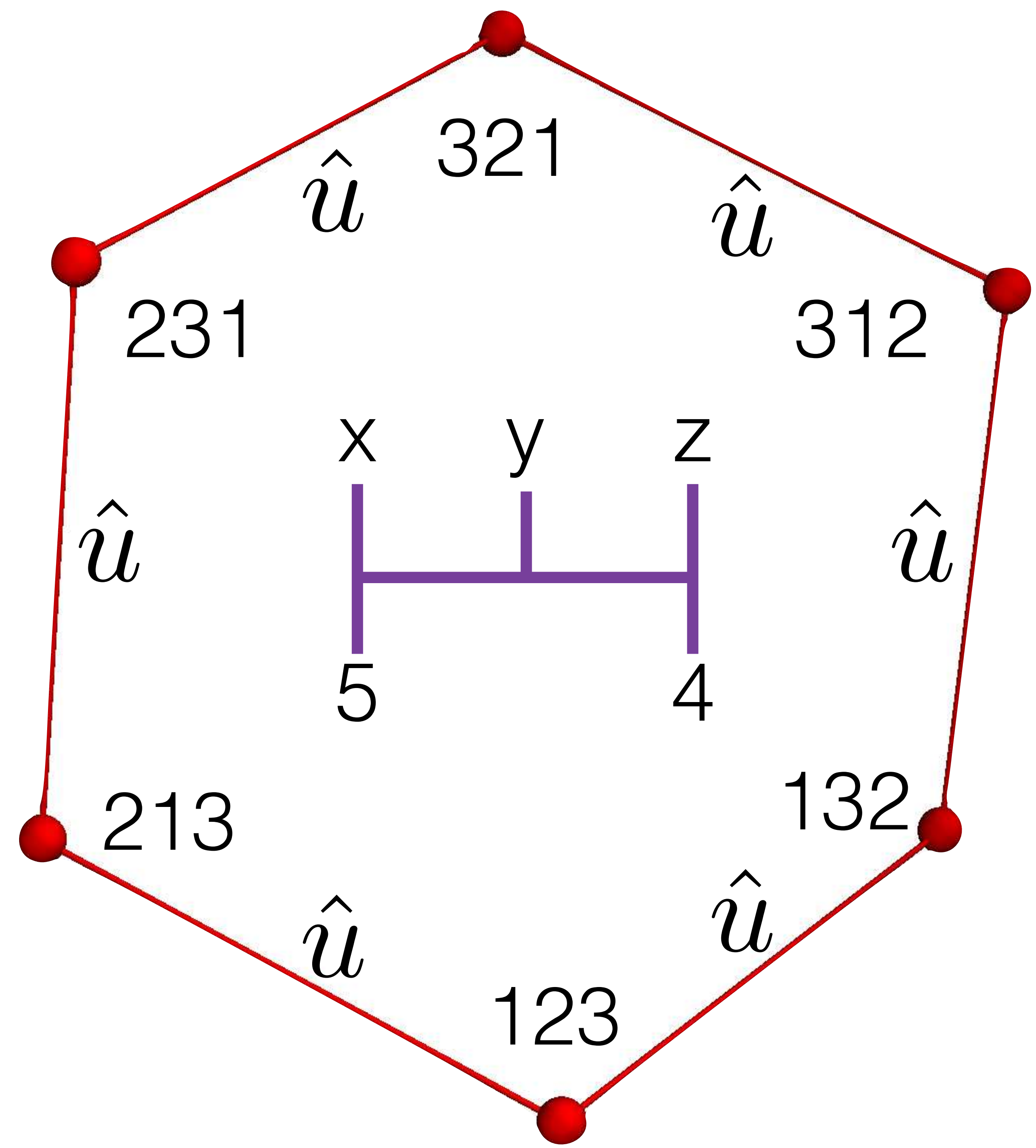


This reduces the set of necessary ordered partial amplitudes (associahedra) to (m-3)! : “BCJ” relations

JJMC

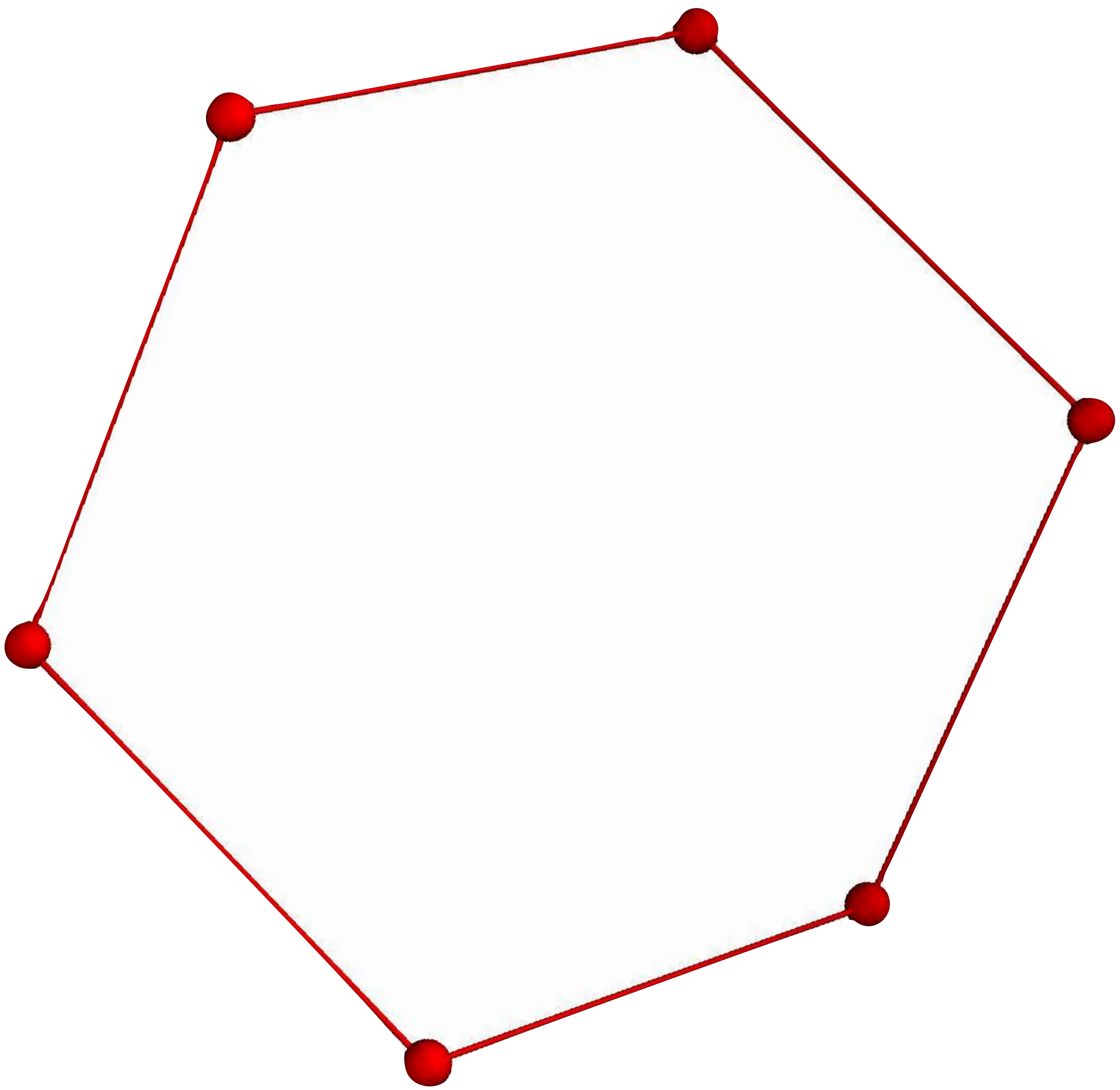
At every multiplicity the masters can be chosen to form the 1-skeleton of a polytope related by $\hat{u}$ on every internal edge of the relevant scattering graphs

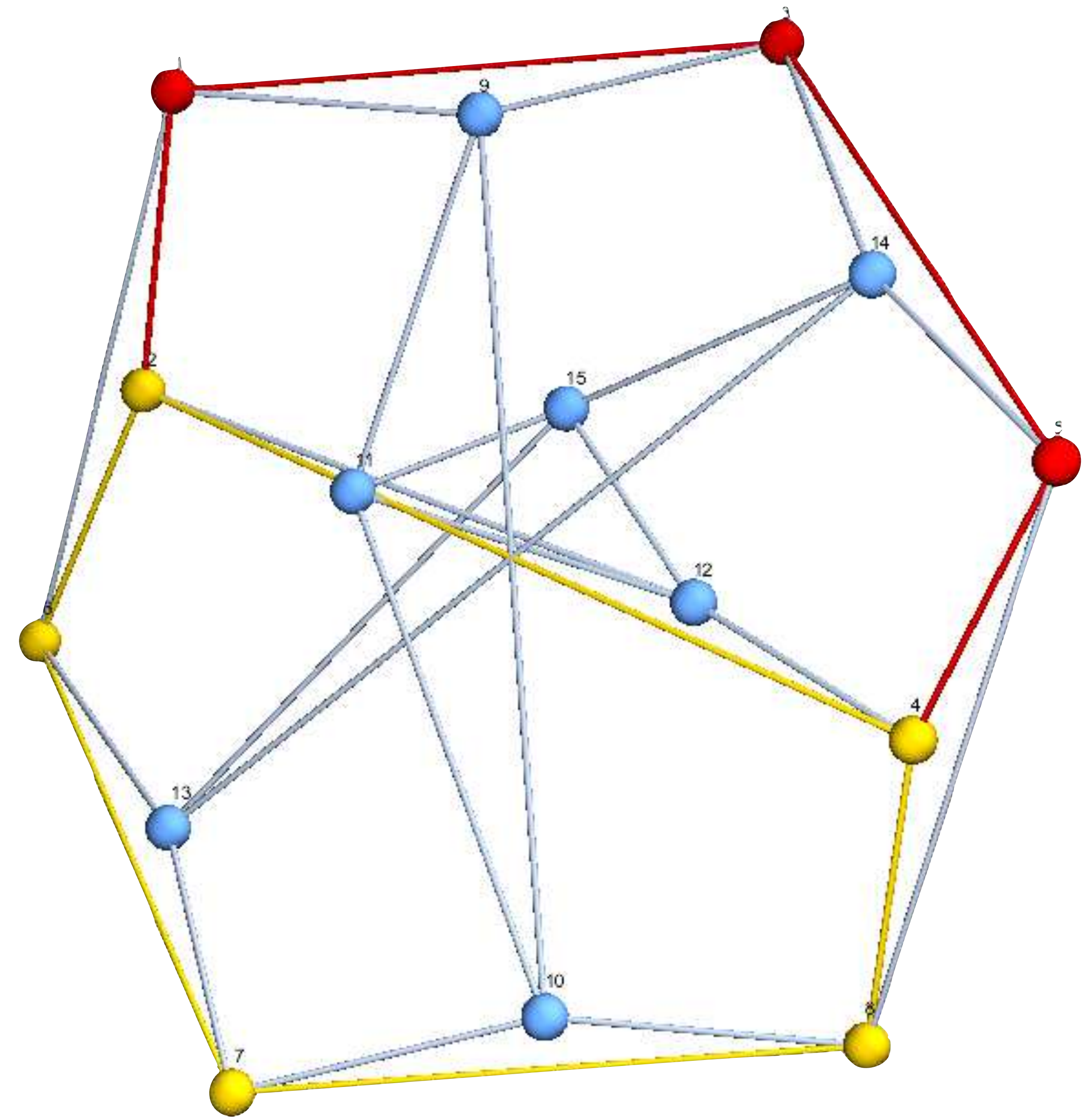




(these polytopes are called **permutahedra**)

Can linearly solve for the (m-2)! numerators of the masters in terms of the (m-3)! “BCJ” independent color-ordered amplitudes. In fact you get (m-3)! numerators in terms of the ordered partial amplitudes and (m-3)(m-3)! free functions. **JJMC**

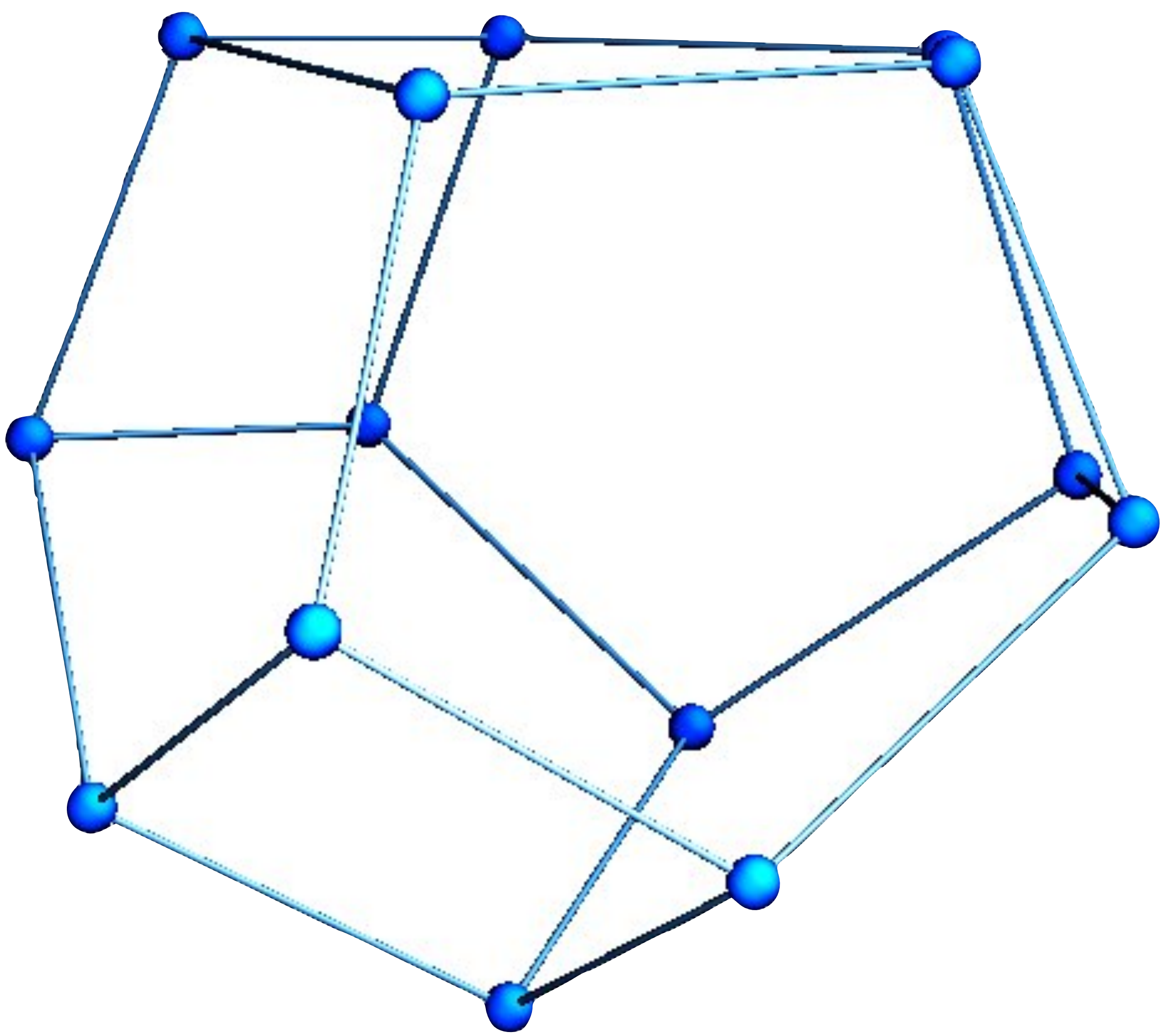




(generalized gauge freedom)

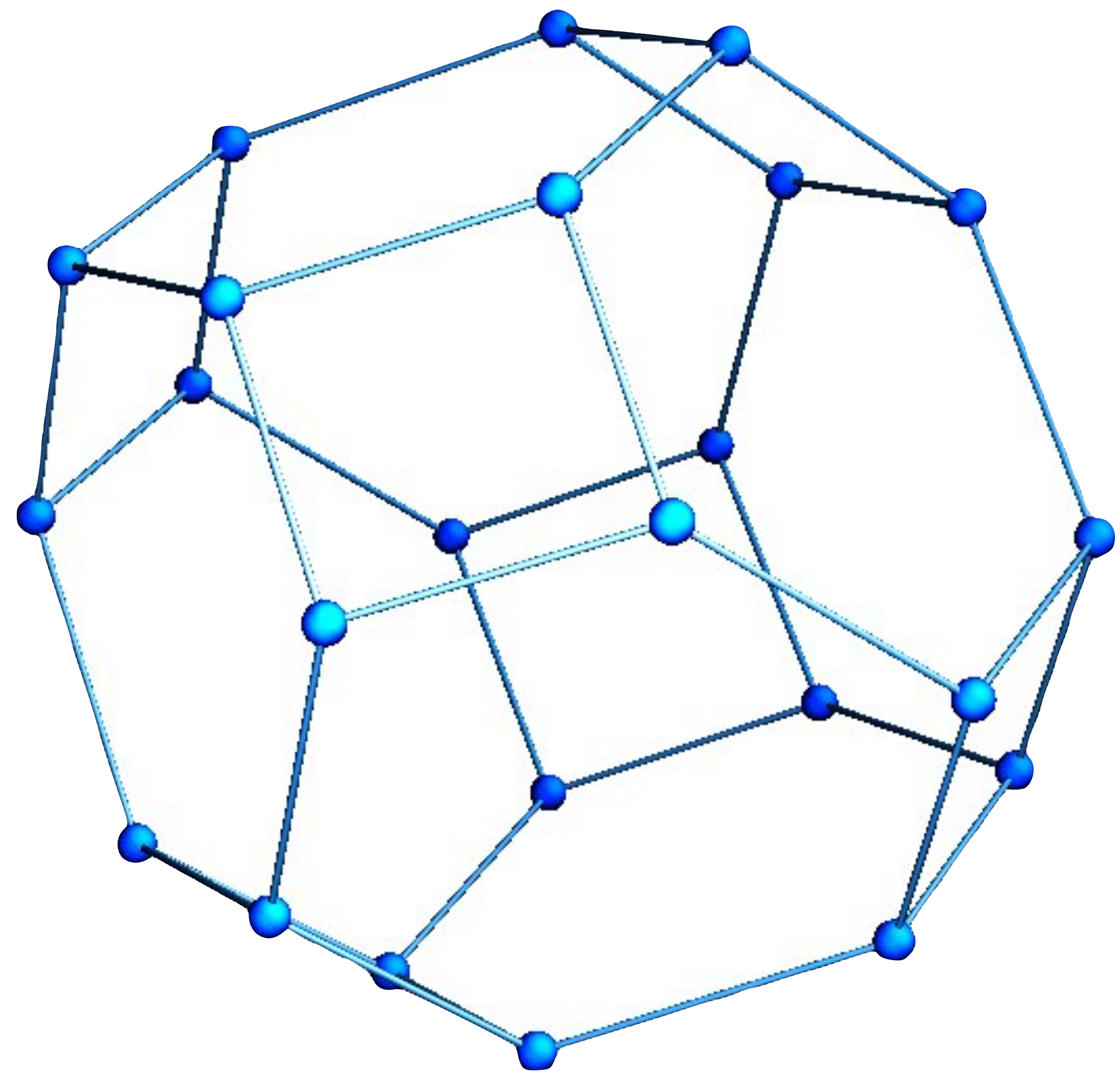
Building blocks at 6-points:

color-ordered amplitude



associahedron

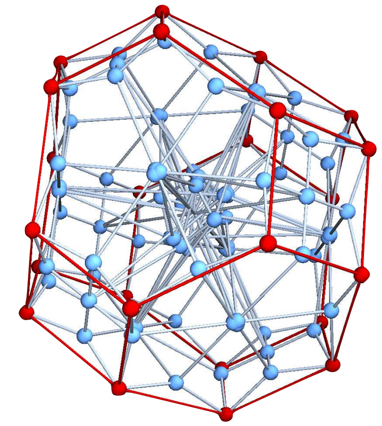
set of masters

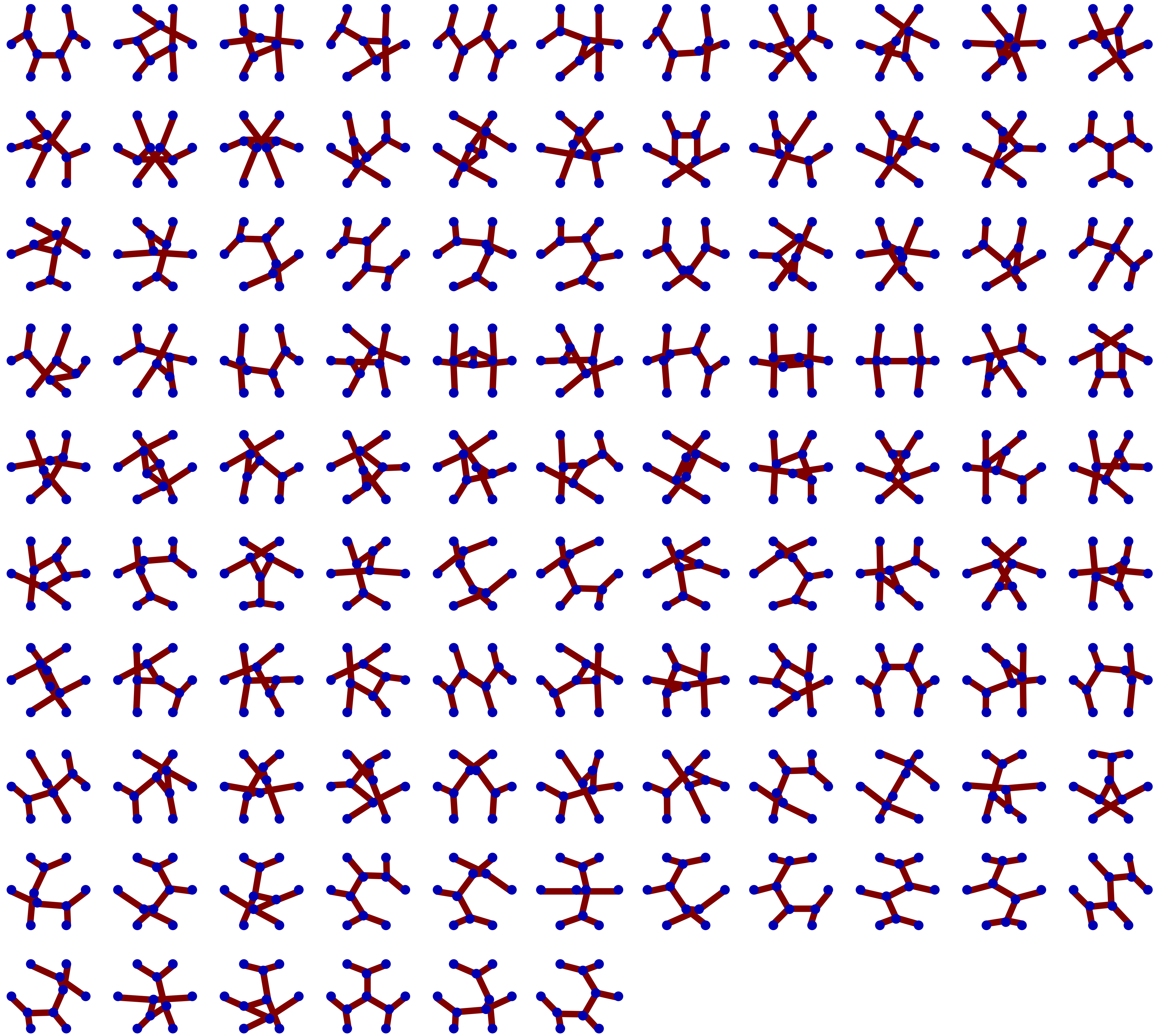


permutahedron

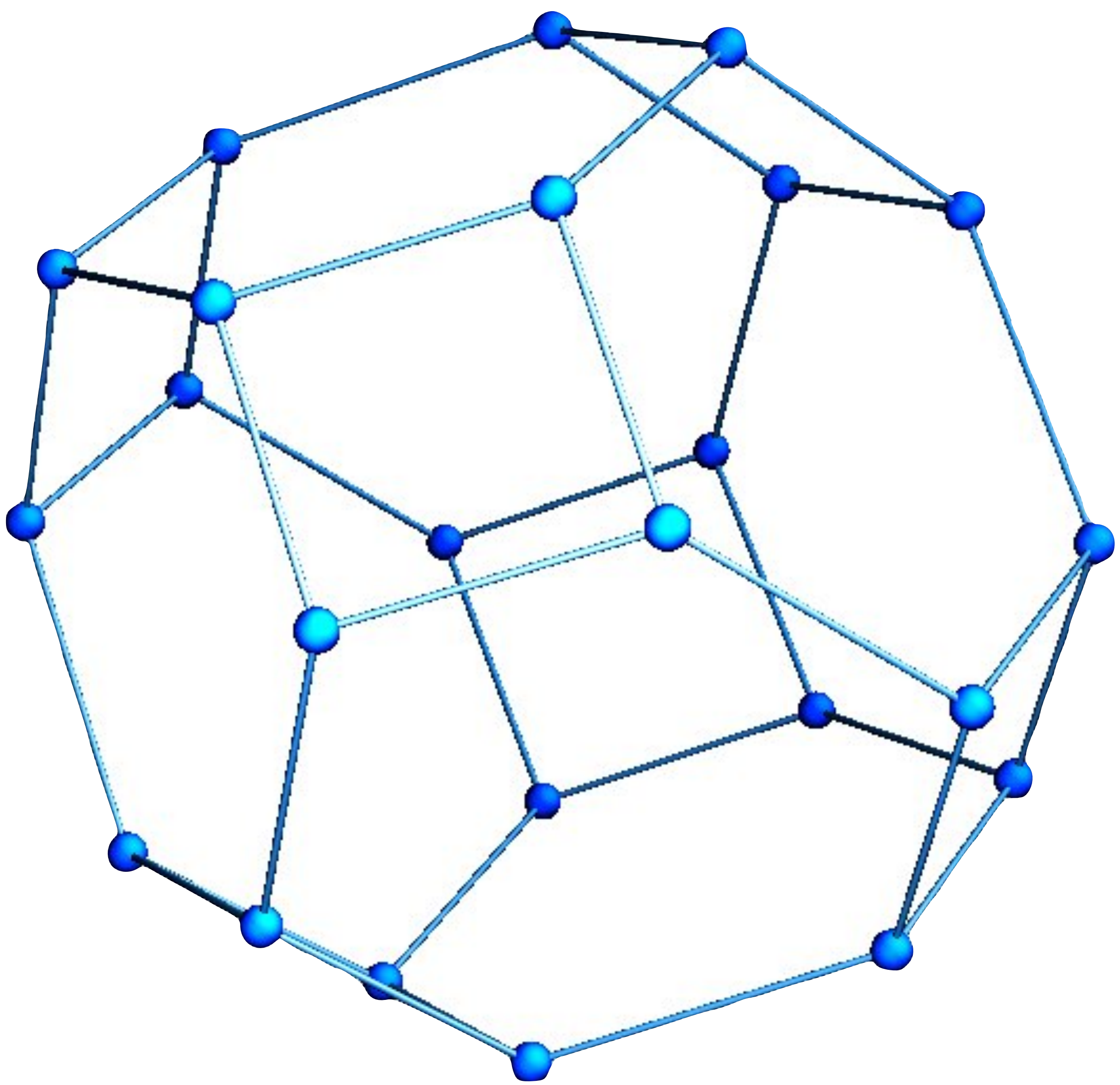
105
cubic graphs at 6 pt

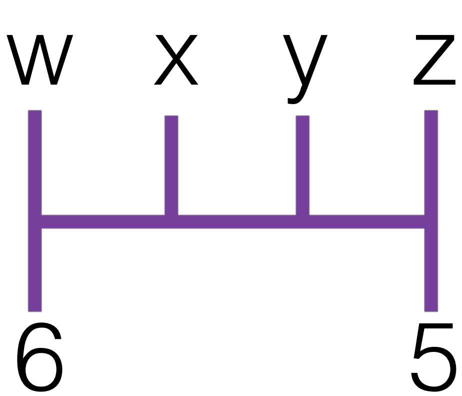
JJMC



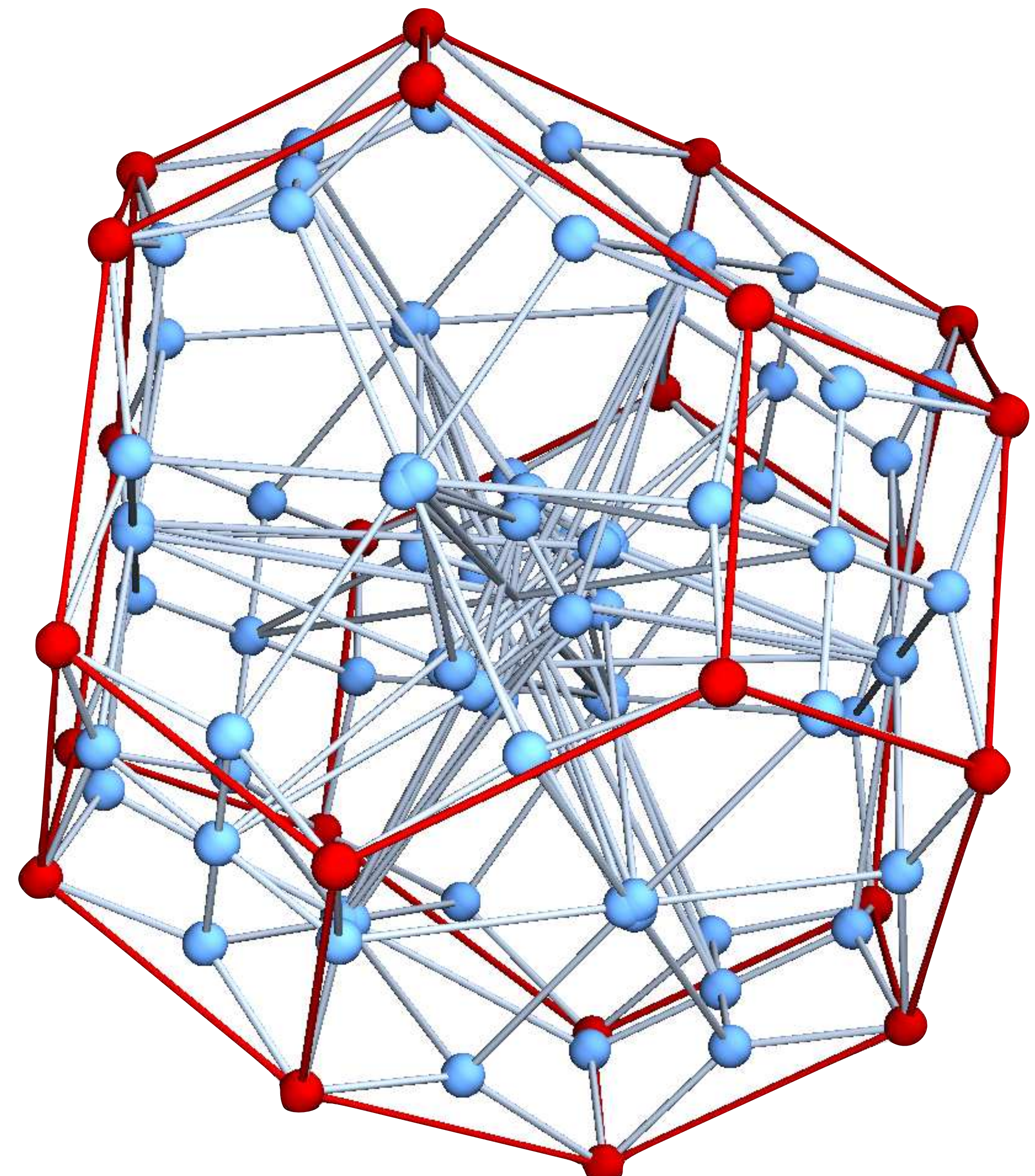


set of masters

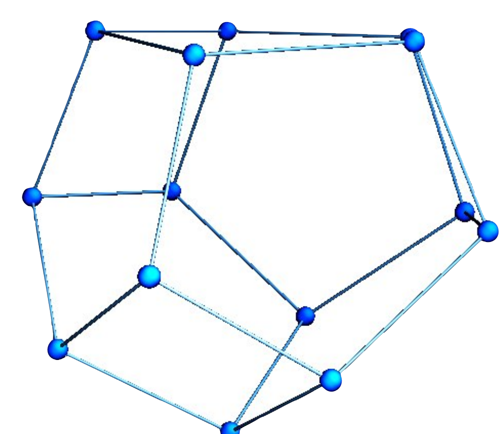




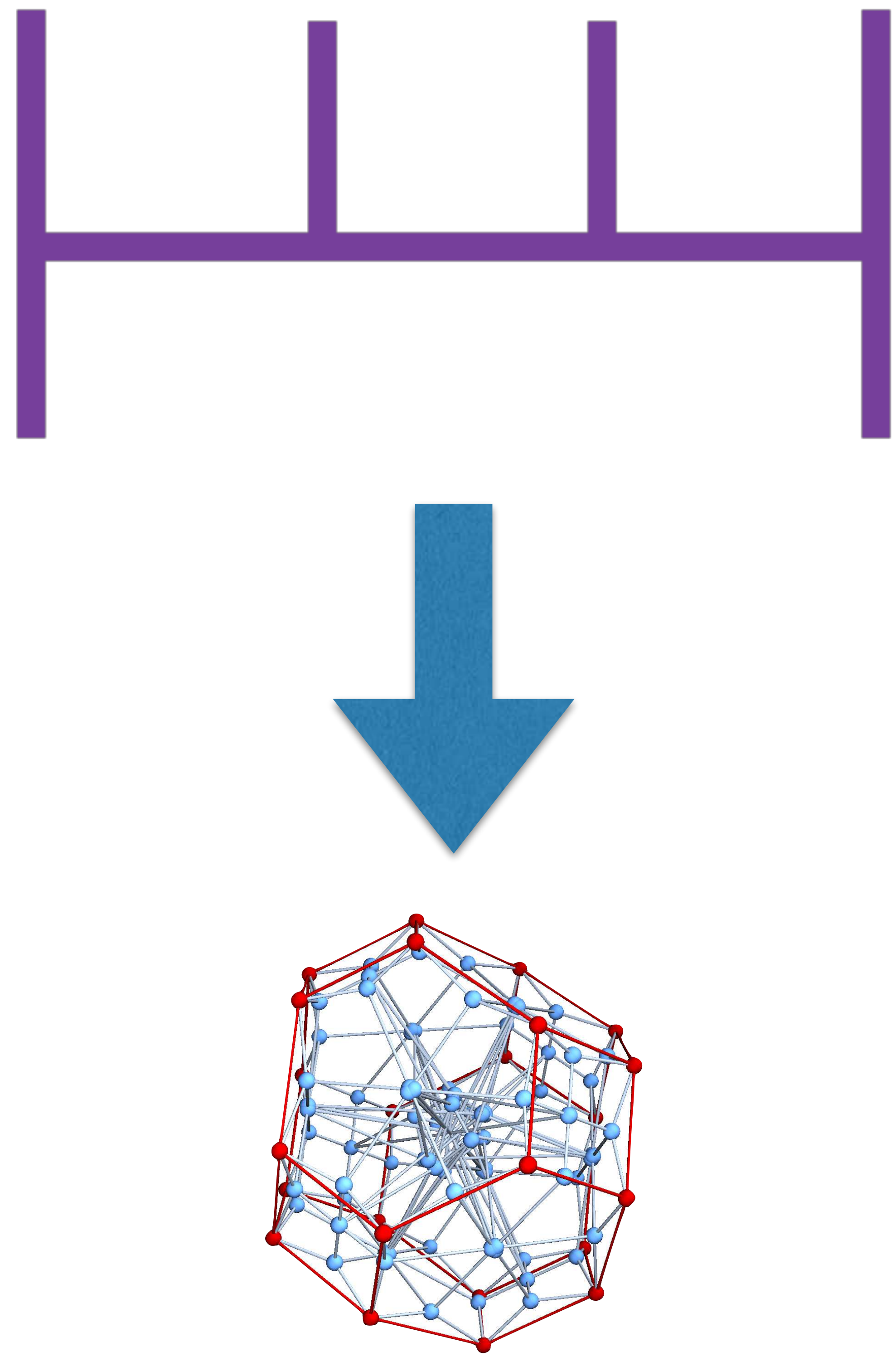
full amplitude



masters fixed by 6



JJMC



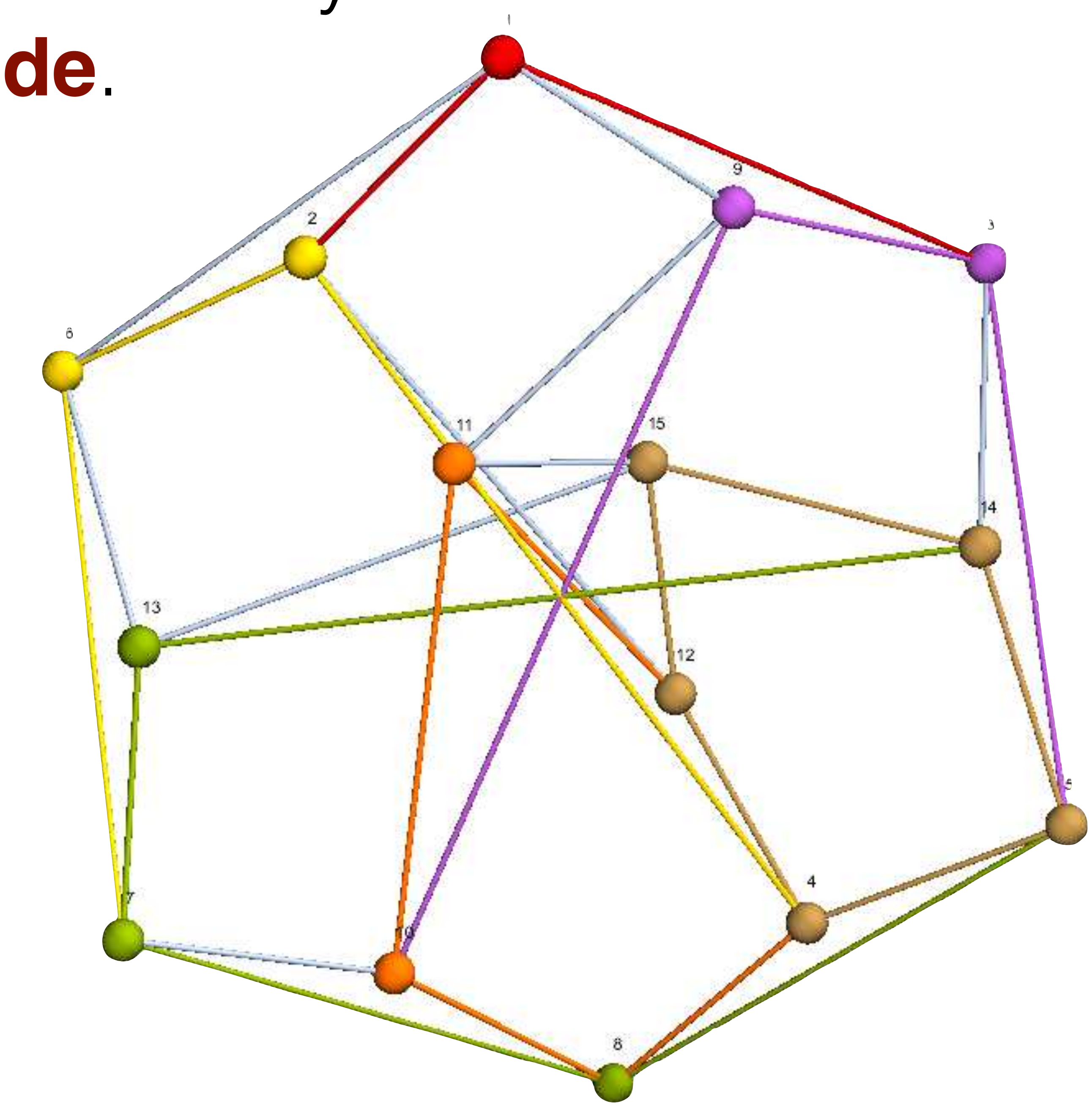
Exploiting Double-Copy to extract predictions.

II. Loop Level

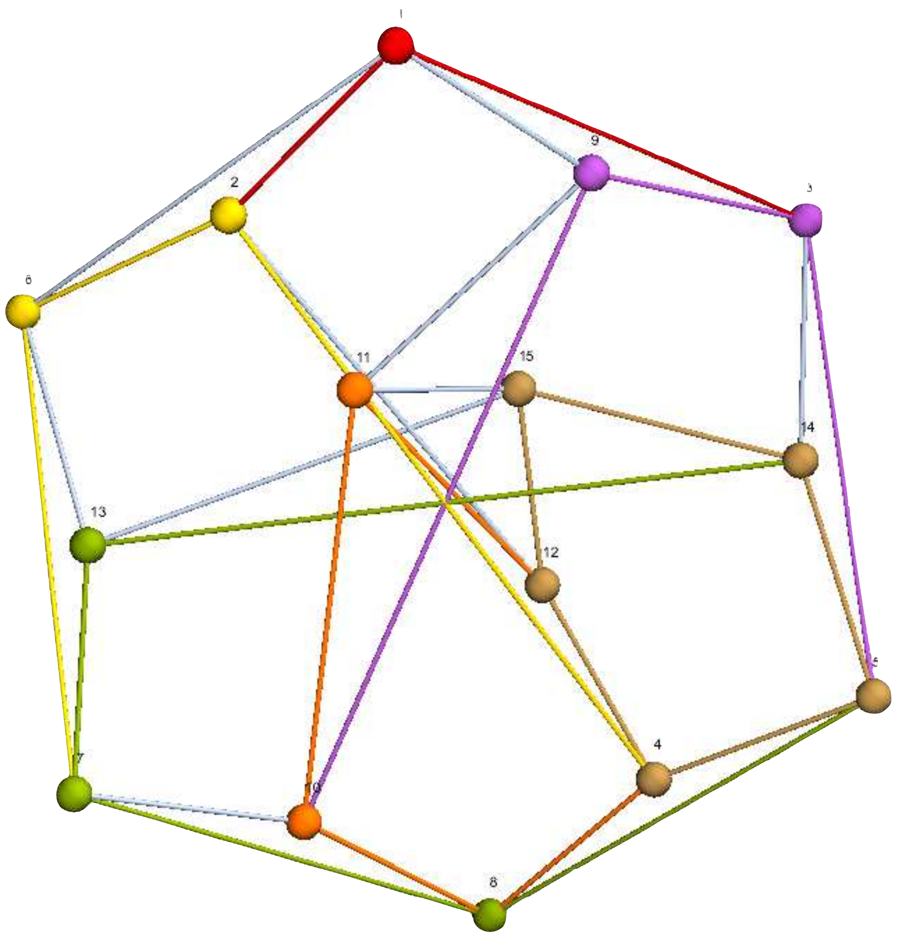
Physics = Geometry

(the best polytopes are built on graphs of graphs!)

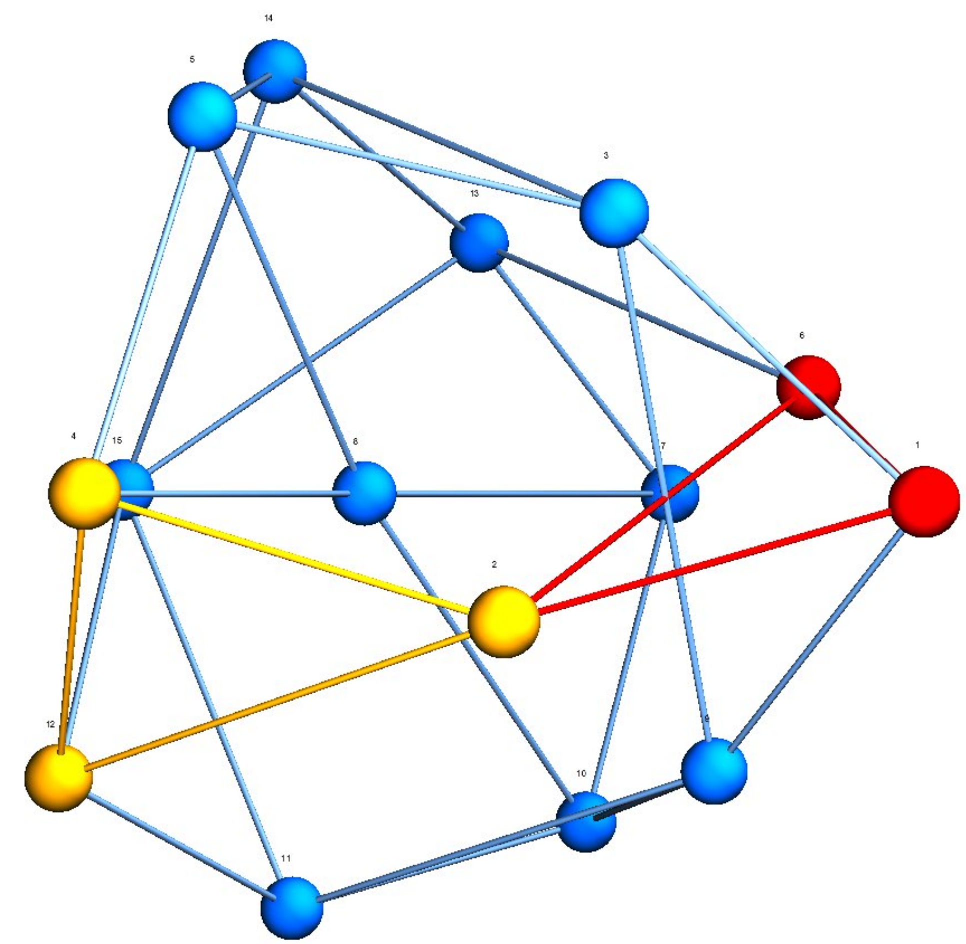
Given a generic (non color-dual) representation for a **gauge amplitude**, and all you want is the related **gravity amplitude**.

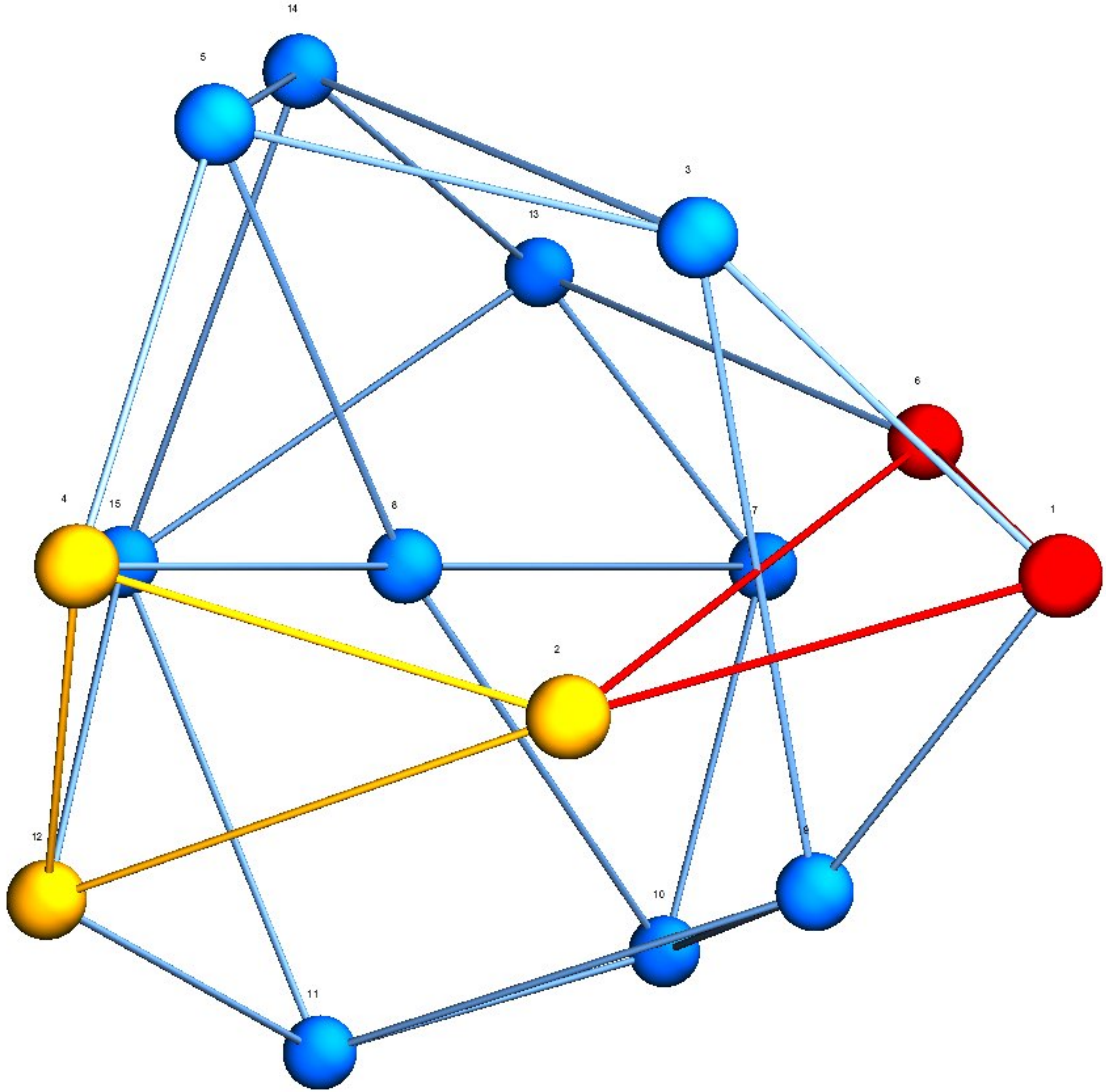


Is there a simple path forward?

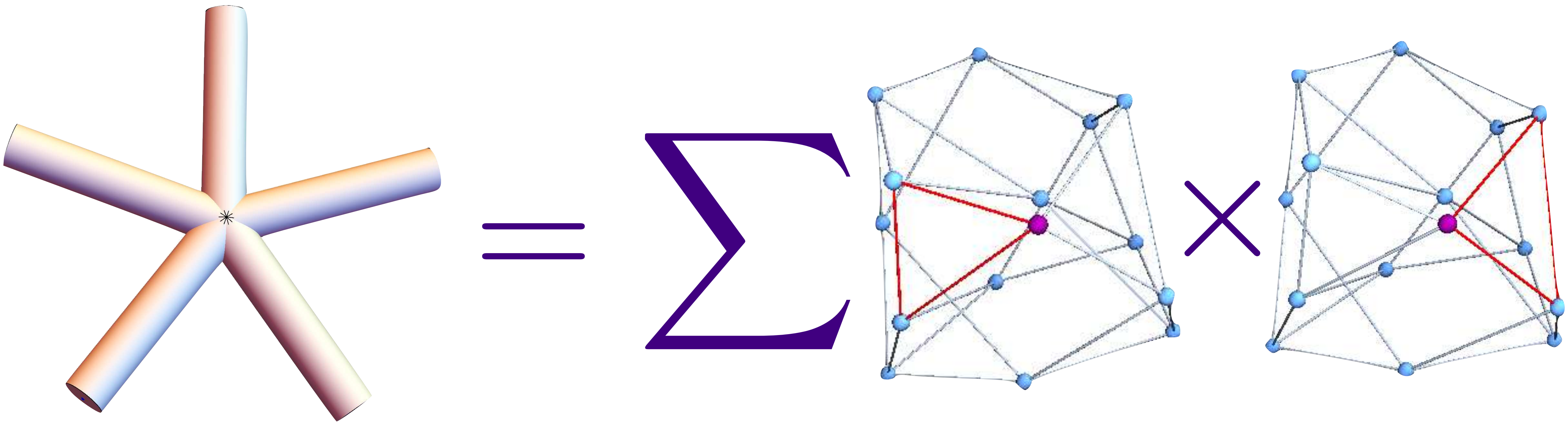


YES.




14
5
3
13
6
4
15
8
7
1
2
12
9
10
11

The idea is natural: take all non-vanishing kinematic-Jacobi combinations (the triangles), double-copy them with each other, use this information to ***define*** *off-shell* contact graphs in the double-copy theory.



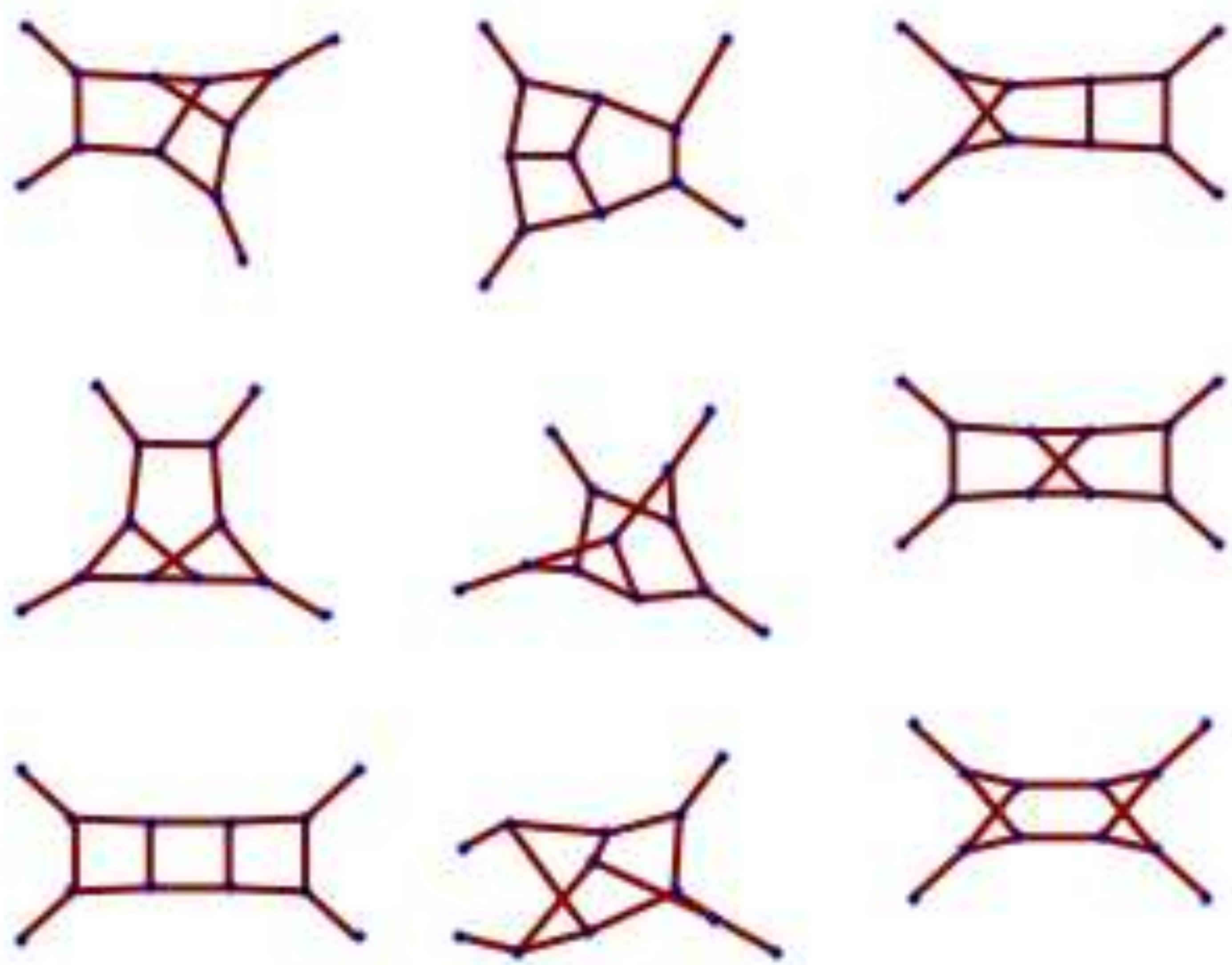
with Bern, Chen, Johansson, Roiban

How does this come together for a full multi-loop integrand?

Full 3-loop Example

3-loop cubic graphs

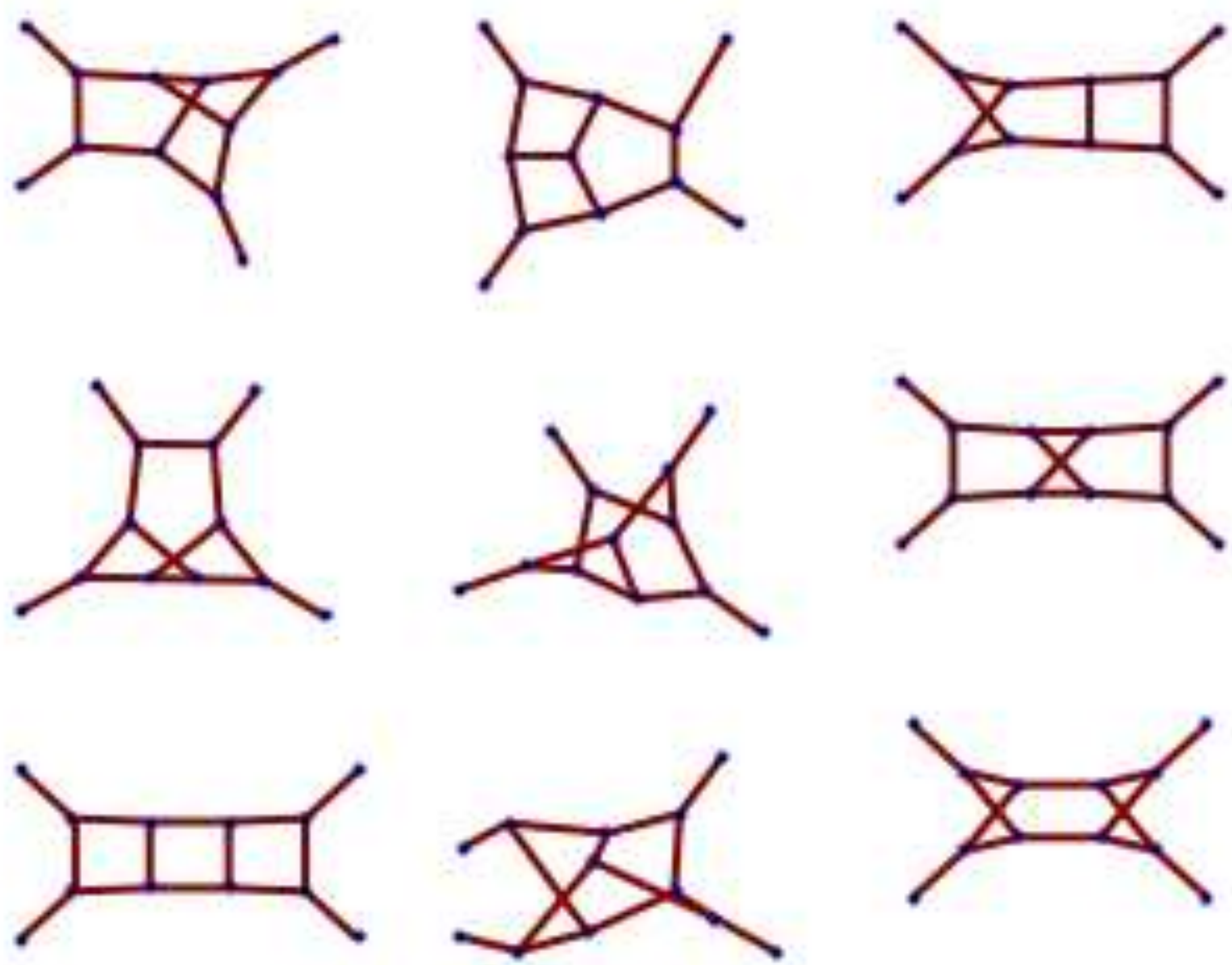
Graph	$\mathcal{N}=4$ sYM numerators.
(a)-(d)	s^2
(e)-(g)	$s(p_5^2+\tau_{45})$
(h)	$s(\tau_{26}+\tau_{36})-t(\tau_{17}+\tau_{27})+st$
(i)	$s(p_5^2+\tau_{45})-t(p_5^2+\tau_{56}+p_6^2)-(s-t)p_6^2/3$



Bern, JJMC, Chen, Johansson, Roiban

ASSIGN square of 3-loop cubic graphs to N=8 SG

Graph	$\mathcal{N}=8$ SG cubic numerators.
(a)-(d)	$\left[s^2\right]^2$
(e)-(g)	$\left[s(p_5^2+\tau_{45})\right]^2$
(h)	$\left[s(\tau_{26}+\tau_{36})-t(\tau_{17}+\tau_{27})+st\right]^2$
(i)	$\left[s(p_5^2+\tau_{45})-t(p_5^2+\tau_{56}+p_6^2)-(s-t)p_6^2/3\right]^2$

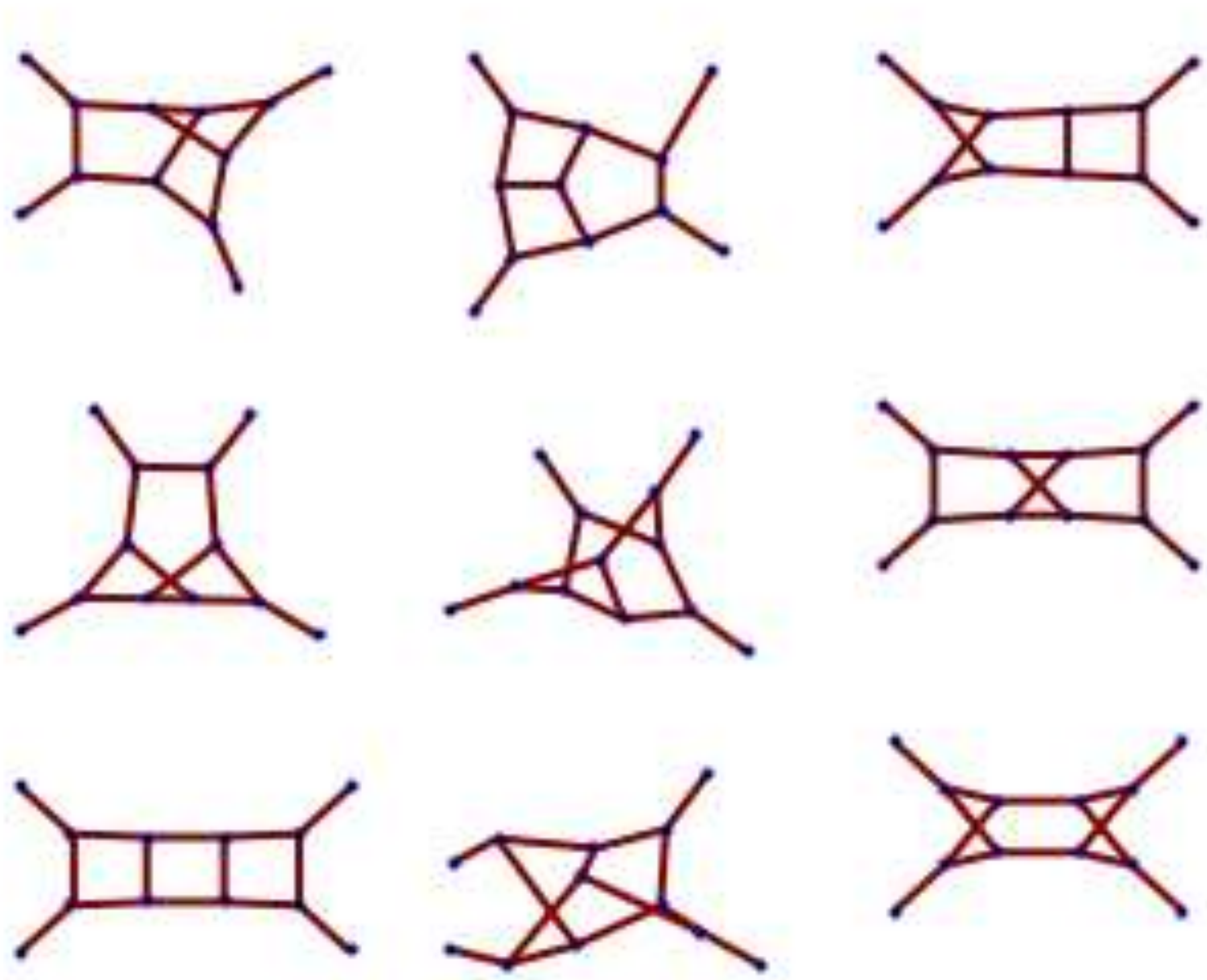


Bern, JJMC, Chen, Johansson, Roiban

ASSIGN square of 3-loop cubic graphs to N=8 SG

This is just the starting point.

Graph	$\mathcal{N}=8$ SG cubic numerators.
(a)-(d)	$\left[s^2\right]^2$
(e)-(g)	$\left[s(p_5^2+\tau_{45})\right]^2$
(h)	$\left[s(\tau_{26}+\tau_{36})-t(\tau_{17}+\tau_{27})+st\right]^2$
(i)	$\left[s(p_5^2+\tau_{45})-t(p_5^2+\tau_{56}+p_6^2)-(s-t)p_6^2/3\right]^2$

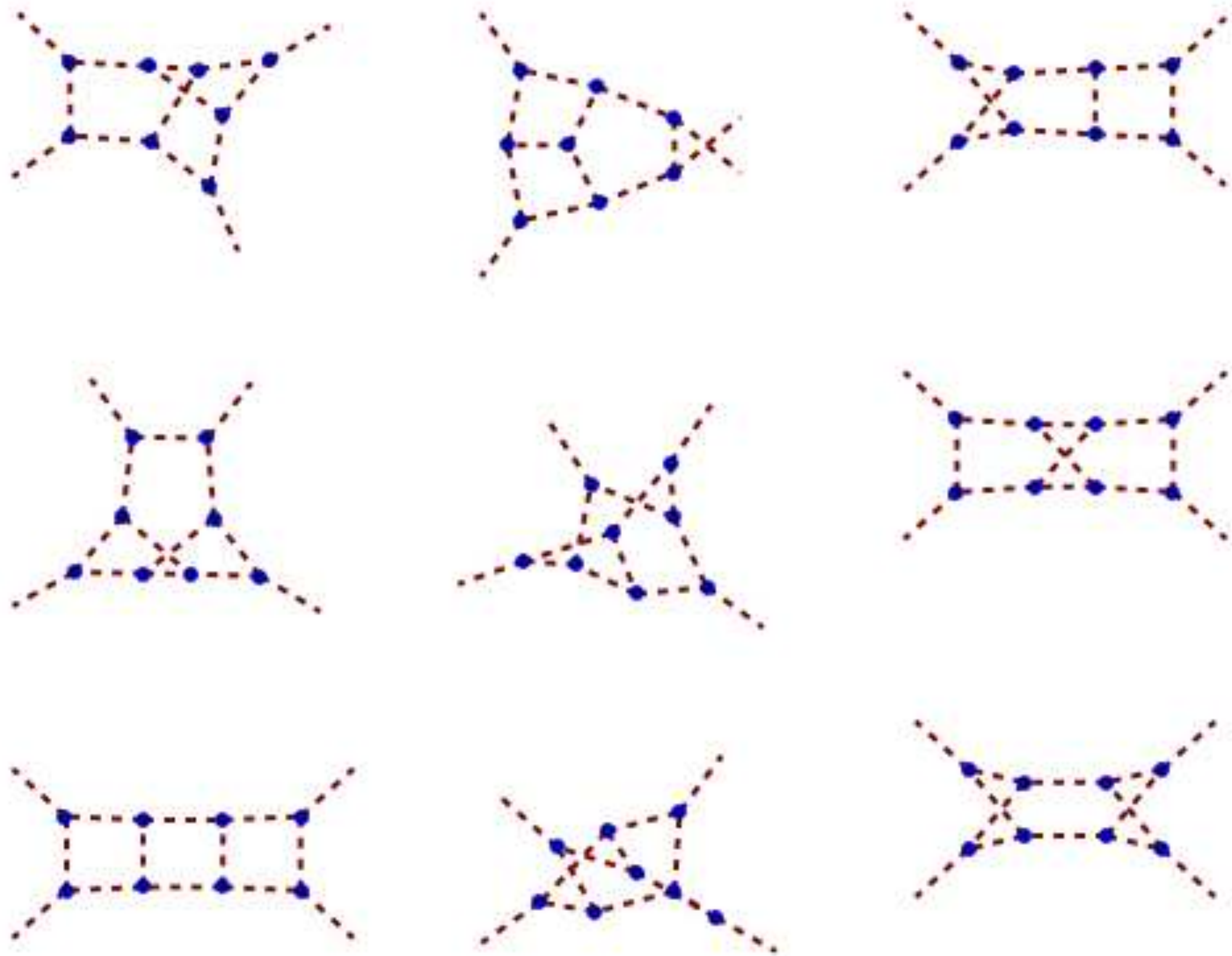


Bern, JJMC, Chen, Johansson, Roiban

Those cubic gravity dressings

automatically satisfy all of these cuts

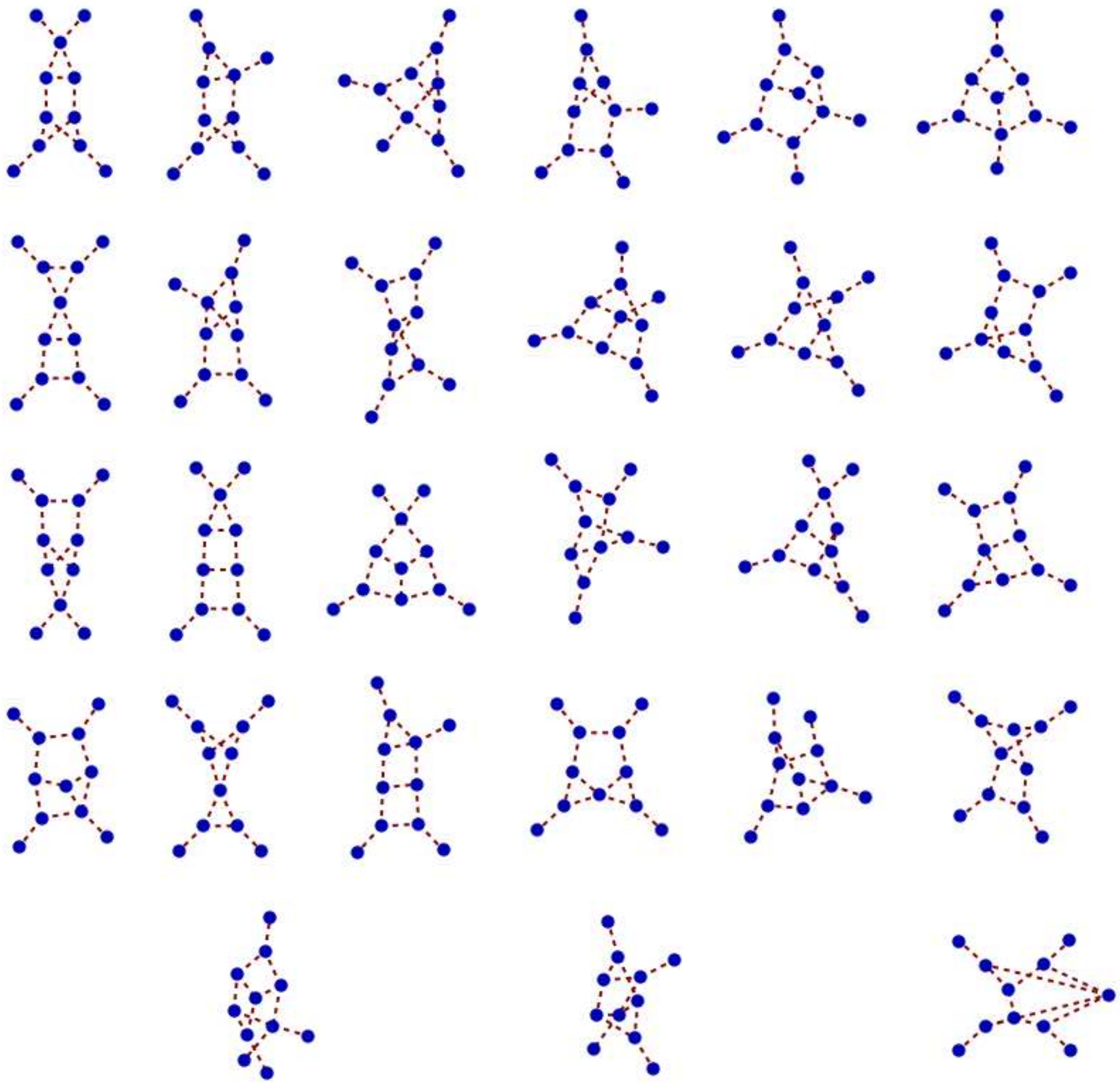
N^0 cut



Bern, JJMC, Chen, Johansson, Roiban

Those cubic gravity dressings

automatically satisfy all of these cuts too



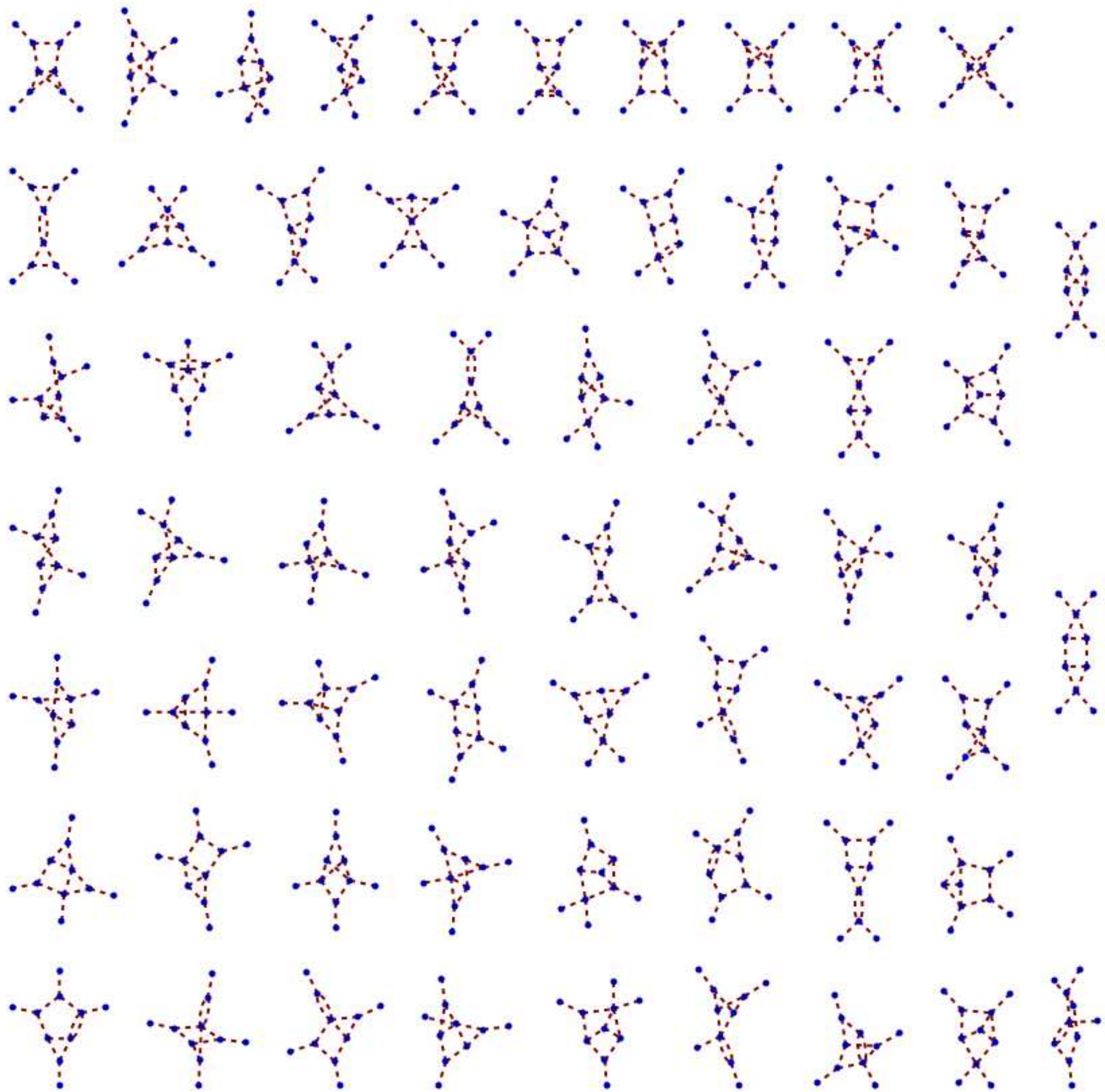
N^1 cut

Bern, JJMC, Chen, Johansson, Roiban

Those cubic gravity dressings

satisfy most of these cuts!

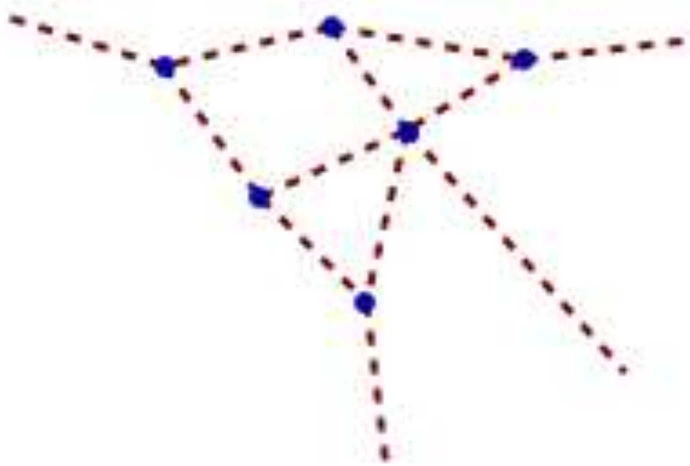
N^2 cut

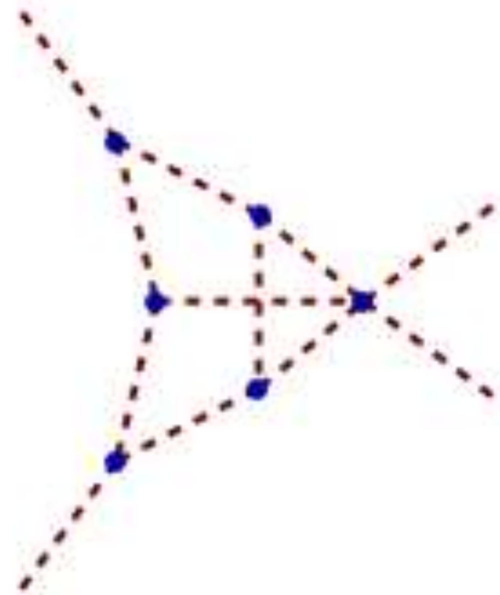


Bern, JJMC, Chen, Johansson, Roiban

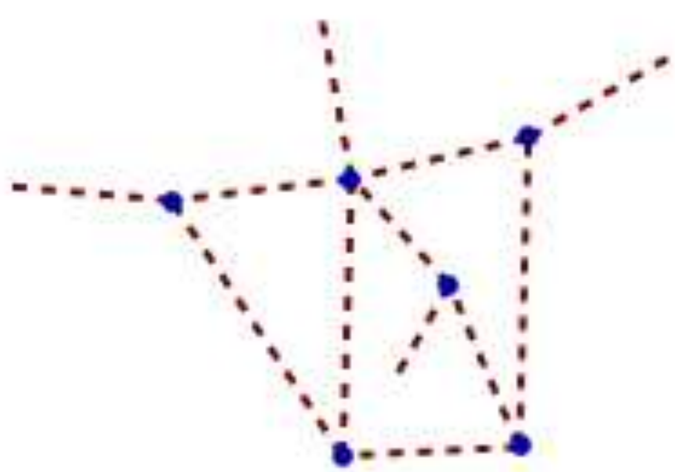
Only 4 non-vanishing cuts

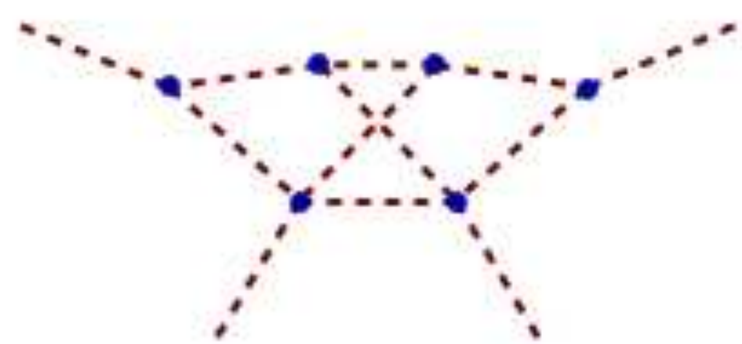
N^2 cut





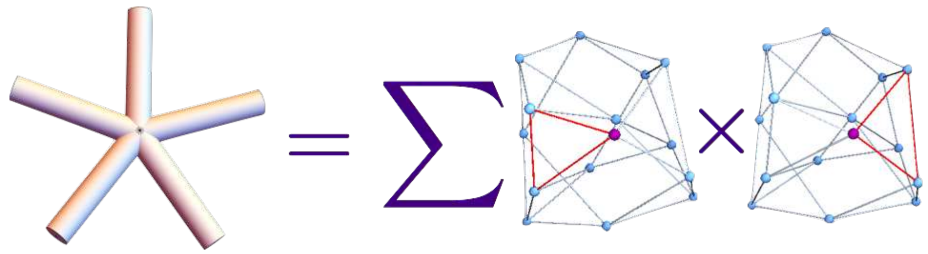
Need to add 4 "contact" contributions



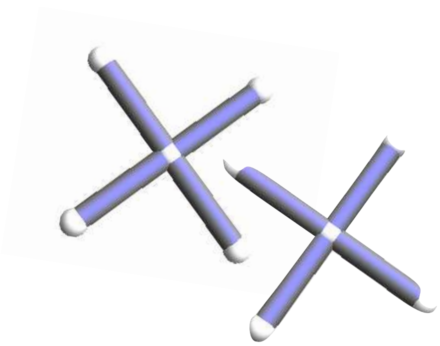


Bern, JJMC, Chen, Johansson, Roiban

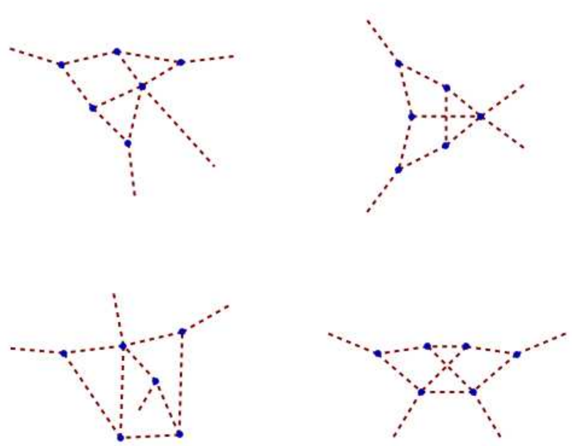
Need to add 4 "contact" contributions



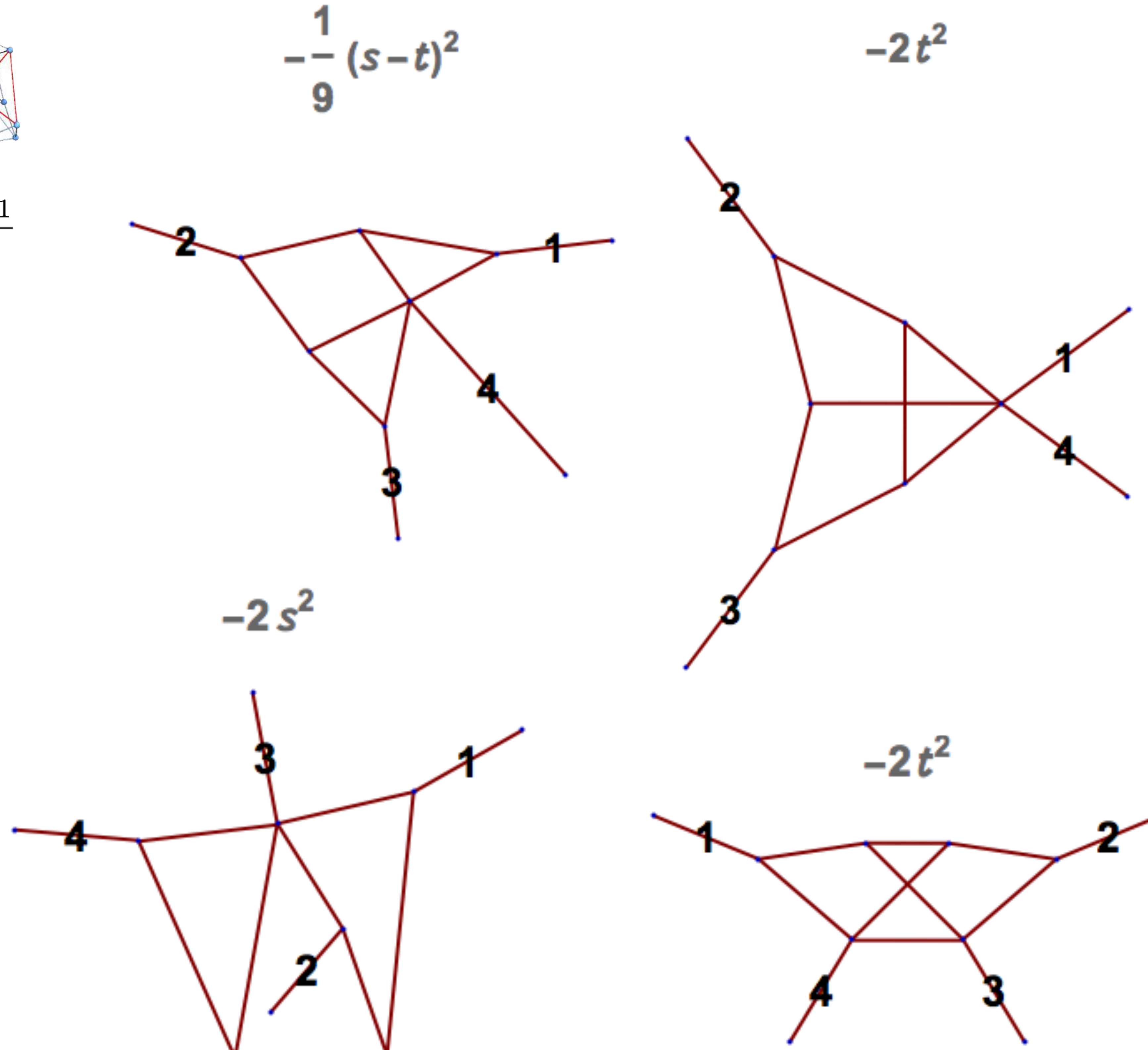
$$= -\frac{1}{6}\sum_i \frac{J_{i,1}J'_{i,2} + J_{i,2}J'_{i,1}}{d^{(1)}_{i,1}d^{(1)}_{i,2}}$$



$$= -\frac{1}{9}\sum_i \frac{J_{i,1}J'_{i,2} + J_{i,2}J'_{i,1}}{d^{(1)}_{i,1}d^{(1)}_{i,2}}$$



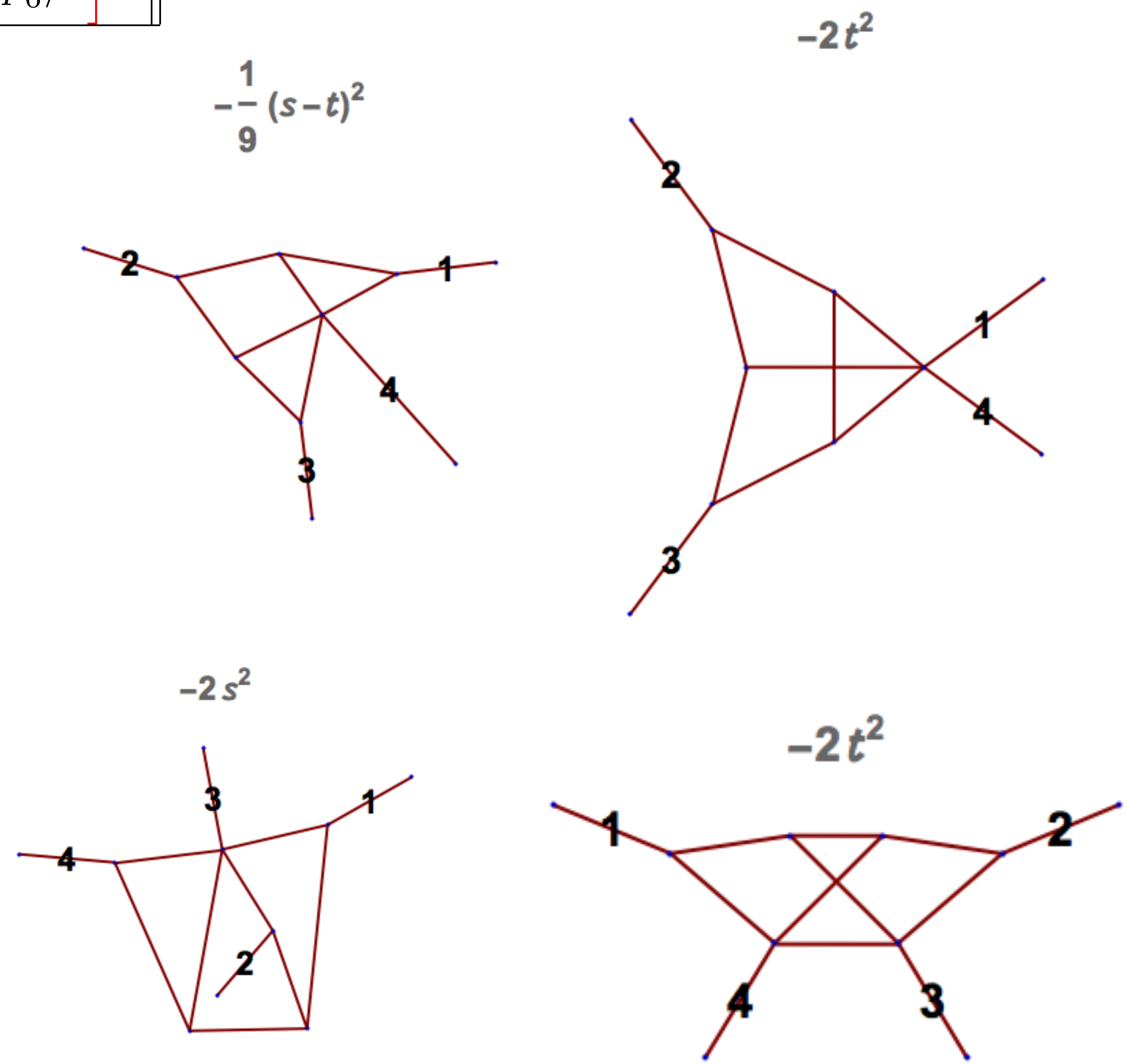
….but you just write them down



Bern, JJMC, Chen, Johansson, Roiban

Graph	$\mathcal{N}=8$ SG cubic numerators.
(a)-(d)	$\left[s^2\right]^2$
(e)-(g)	$\left[s(p_5^2+\tau_{45})\right]^2$
(h)	$\left[s(\tau_{26}+\tau_{36})-t(\tau_{17}+\tau_{27})+st\right]^2$
(i)	$\left[s(p_5^2+\tau_{45})-t(p_5^2+\tau_{56}+p_6^2)-(s-t)p_6^2/3\right]^2$

Complete 3-loop integrand for N=8 SG

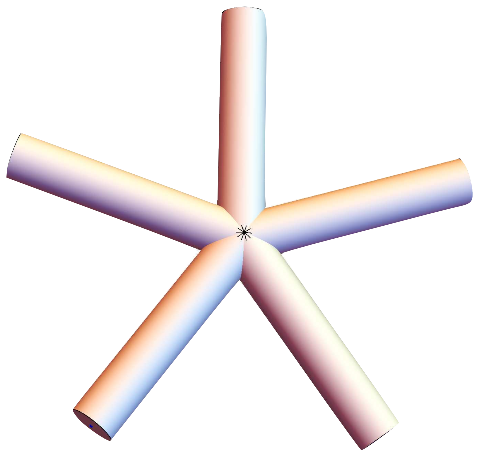


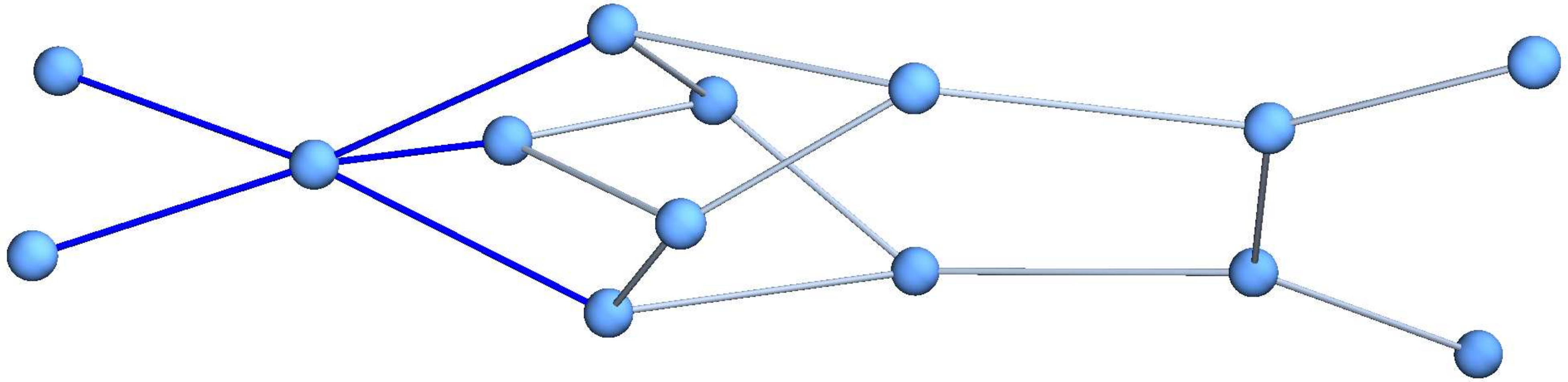
Bern, JJMC, Chen, Johansson, Roiban

Some more examples

Some 5-loop examples

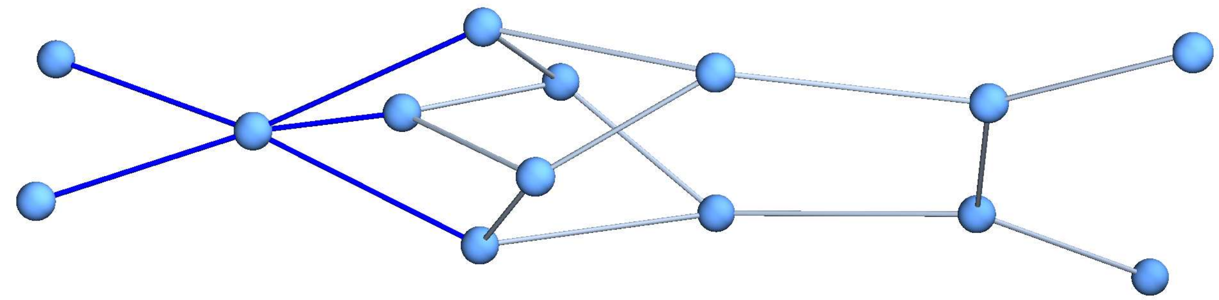
5-loops, potential N^2 contact



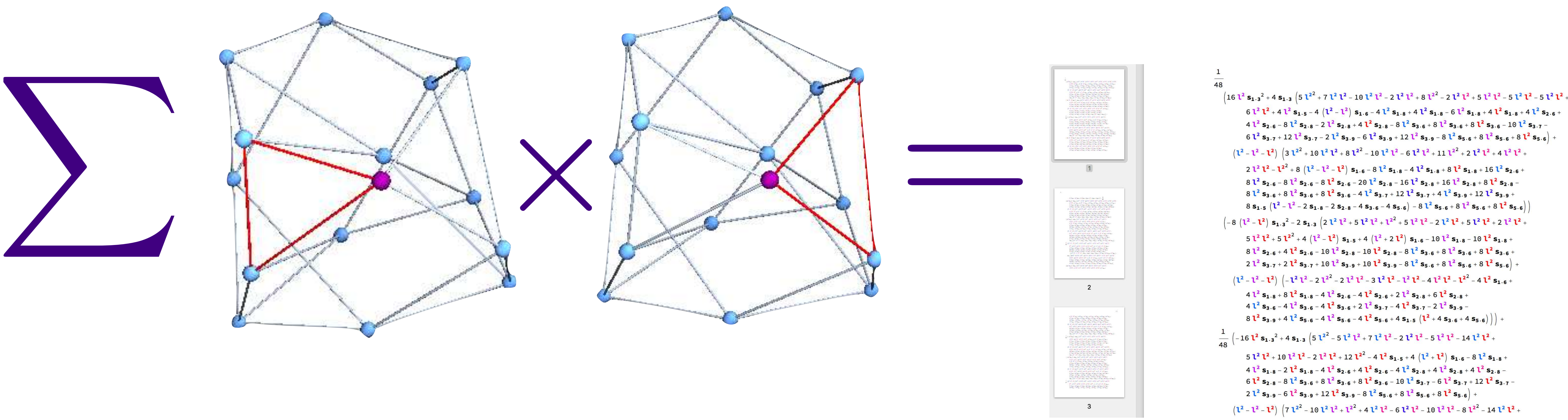


This is a serious example.

5-loops, potential N^2 contact



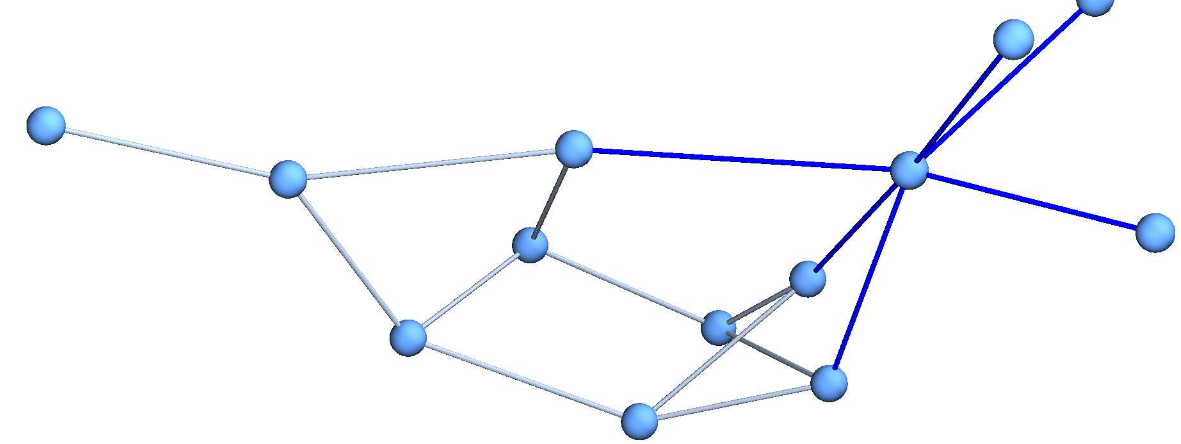
Contact / Missing Information you can just write down:

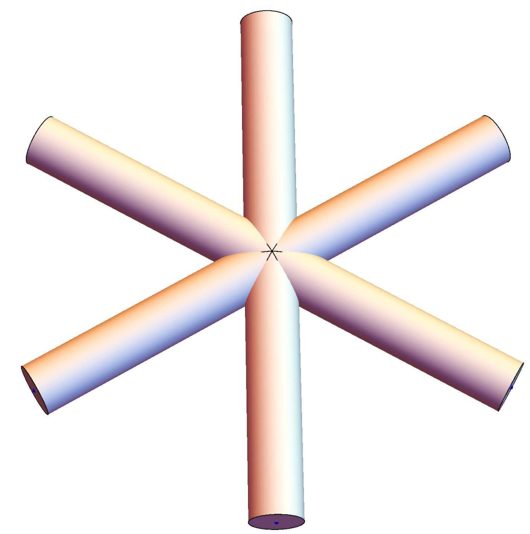


8 pages, local

5-loops isn't for the faint of heart.

5-loops, potential N^3 contact

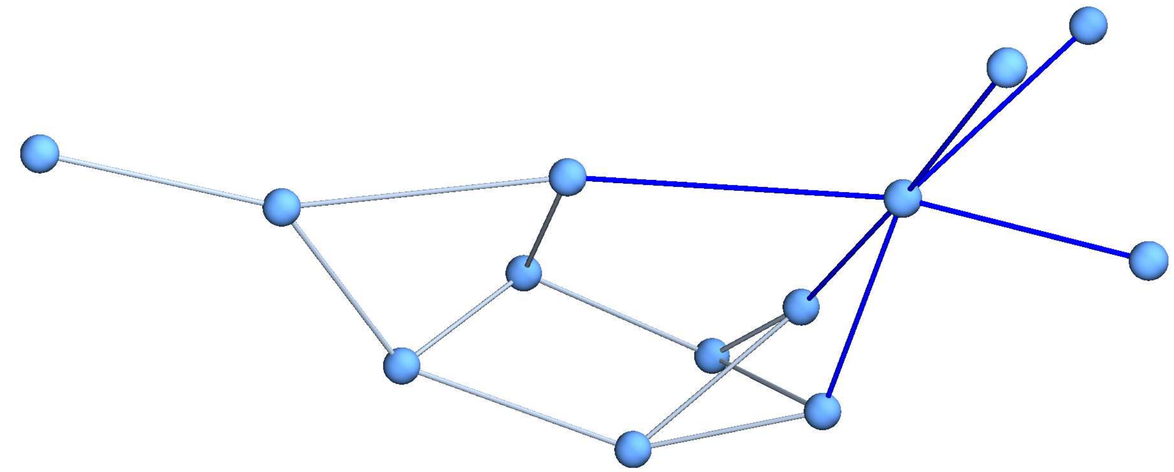




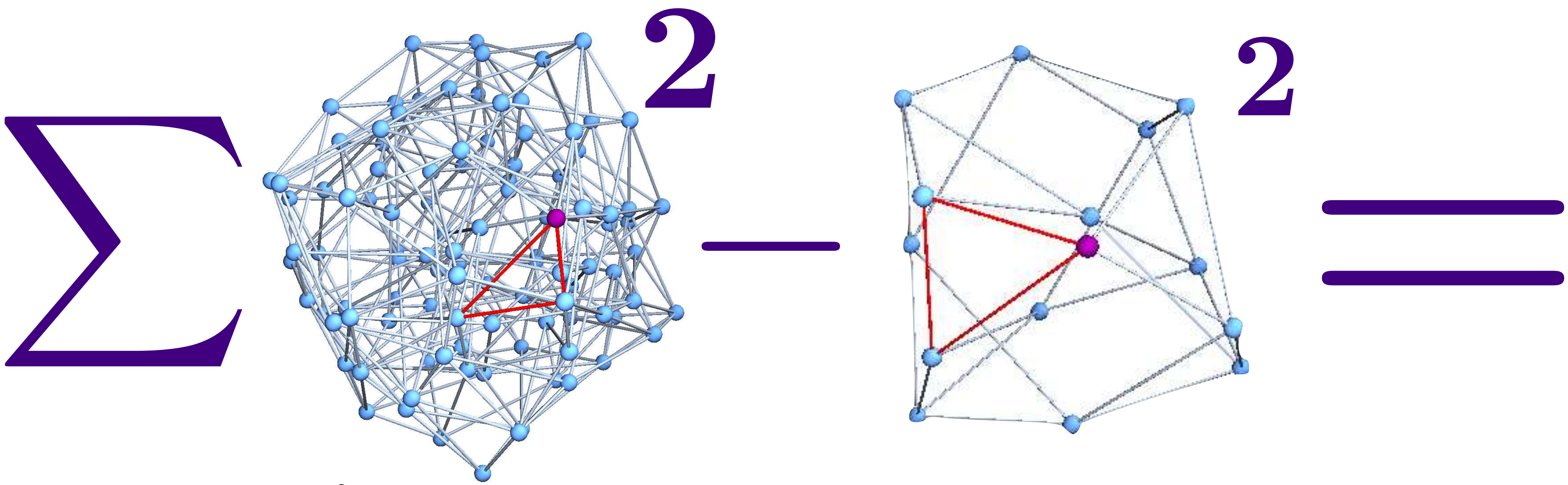
$$N^3\text{-contact} = \text{off shell}\left[\left(\text{truth} - \sum_{\text{g}\in\text{cut}} \frac{{\text{n}_\text{g}}^2}{\text{d}_\text{g}} - \sum_{\text{g}\in\text{N}^2\text{contacts}} \frac{\text{N}_\text{g}}{\text{d}_\text{g}}\right)\Bigg|_{\text{cut}}\right]$$

$$= \text{off shell}\left[\left(\sum_{g\in\text{cut}} \frac{{\mathring{n}_g}^2}{d_g} - \sum_{g\in\text{cut}} \frac{{n_g}^2}{d_g} - \sum_{g\in N^2\text{contacts}} \frac{N_g}{d_g}\right)\Bigg|_{\text{cut}}\right]$$

5-loops, potential N^3 contact



Contact / Missing Information you just write down:



$$-\left(\ell^2-\ell^2-\ell^2+\ell^2\right)^2$$

$$\Big(4\,{\ell^2}^2\,\ell^2-10\,\ell^2\,{\ell^2}^2+4\,{\ell^2}^3+{\ell^2}^2\,\ell^2+4\,\ell^2\,\ell^2\,\ell^2-5\,{\ell^2}^2\,\ell^2+2\,\ell^2\,{\ell^2}^2+{\ell^2}^3+{\ell^2}^2\,\ell^2+4\,\ell^2\,\ell^2\,\ell^2-5\,{\ell^2}^2\,\ell^2+$$

$$4\,\ell^2\,\ell^2\,\ell^2+3\,{\ell^2}^2\,\ell^2+2\,\ell^2\,{\ell^2}^2+3\,\ell^2\,{\ell^2}^2+{\ell^2}^3+{\ell^2}^2\left(2\,\ell^2+\ell^2+\ell^2-2\,\ell^2\right)+{\ell^2}^2\left(2\,\ell^2+\ell^2+\ell^2-2\,\ell^2\right)-$$

$$2\left({\ell^2}^2-{\ell^2}^2+3\,\ell^2\left(\ell^2+\ell^2\right)-2\,\ell^2\left(\ell^2+\ell^2\right)+2\left(\ell^2+\ell^2\right)^2\right)\ell^2+$$

$$\left(4\,\ell^2-4\,\ell^2+5\left(\ell^2+\ell^2\right)\right){\ell^2}^2-2\,{\ell^2}^3+\ell^2\Big(7\,{\ell^2}^2-2\,\ell^2\,\ell^2-2\,{\ell^2}^2-2\,\ell^2\,\ell^2-4\,\ell^2\,\ell^2-2\,{\ell^2}^2+$$

$$2\,\ell^2\left(2\,\ell^2+\ell^2+\ell^2-2\,\ell^2\right)-2\,\ell^2\left(3\,\ell^2+\ell^2+\ell^2-2\,\ell^2\right)-2\left(\ell^2-3\left(\ell^2+\ell^2\right)\right)\ell^2-4\,{\ell^2}^2\Big)-$$

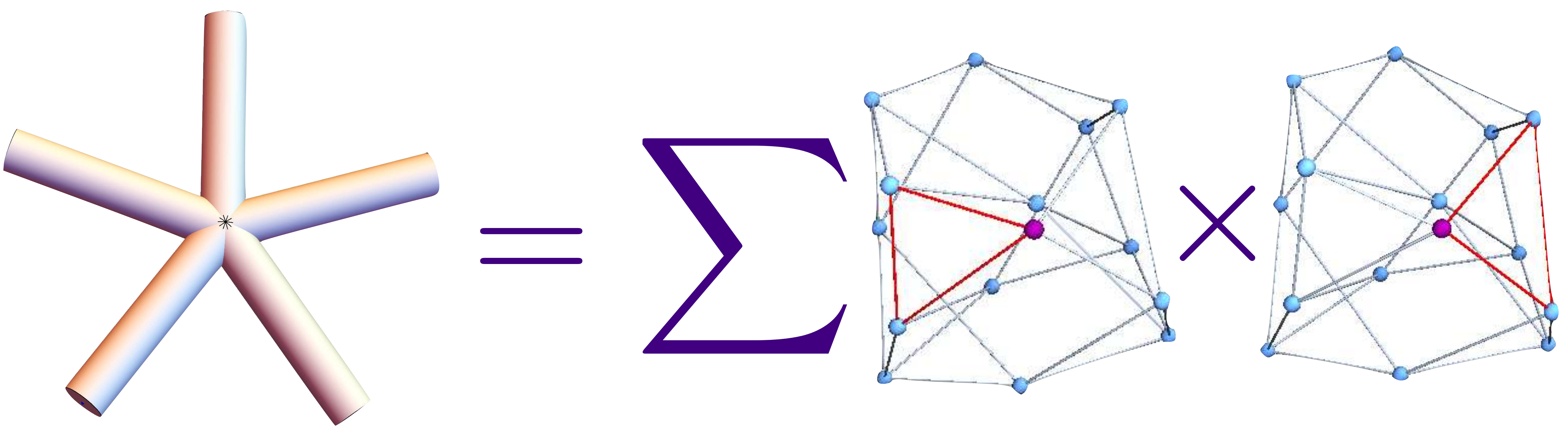
$$\ell^2\left(-7\,{\ell^2}^2+2\,\ell^2\left(3\,\ell^2+\ell^2+\ell^2-2\,\ell^2\right)+2\left(\ell^2+\ell^2-2\,\ell^2\right)\left(\ell^2+\ell^2-\ell^2\right)+2\,\ell^2\left(\ell^2+\ell^2+\ell^2\right)\right)\Big)$$

Summary

c/k + gen. gauge transforms →

can directly **double-copy** **non-c/k representations**
resulting in add'l ***local*** higher-point contact terms

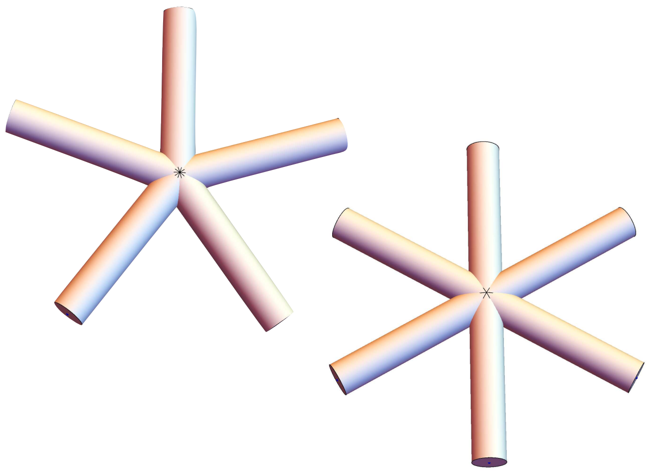
(something you can figure out more or less from tree-level considerations)



Gen. Double Copy Summary

- **Control through 5-pt => all N^2 cuts**
- **Control through 6-pt => all N^3 cuts**

… and so on



Multiplicity and loop-order independent!

works for any double-copy theory b/c of single-copy properties (sYM/NLSM/Z-theory/…)

provides a *simple* path forward for tough to crack multi loop double-copy constructions where both copies involve kinematic weights from known theories…

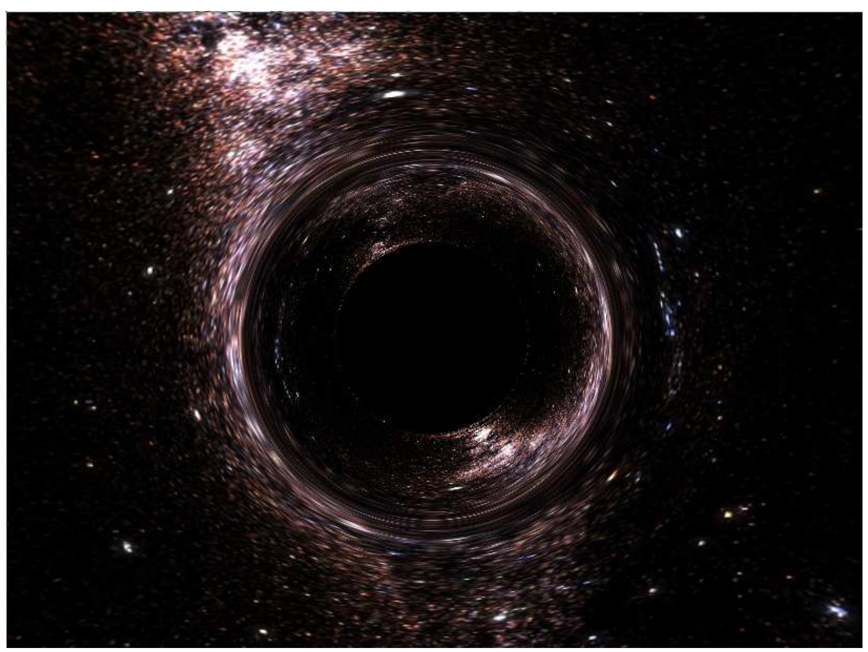
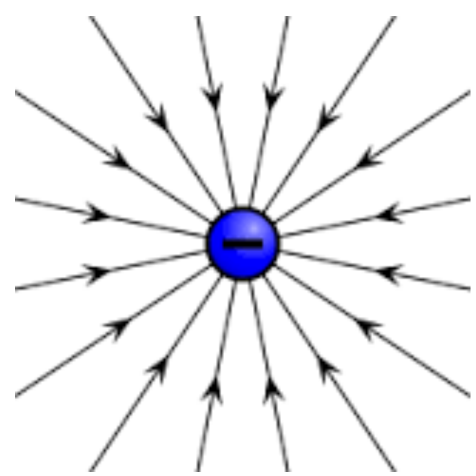
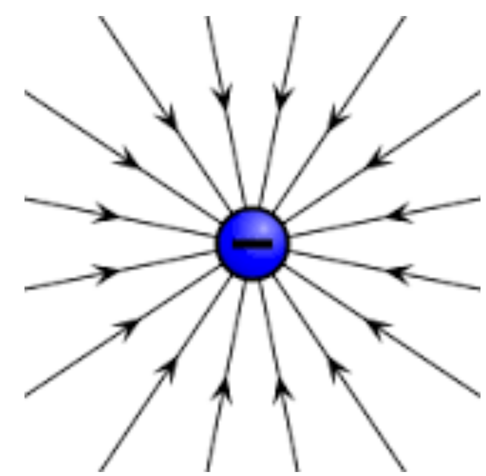
Classical Solutions

Do classical solutions double-copy?

(See also work of Saotome & Akhoury, and combinations of Anastasiou, Borsten, Duff, Hughes, Nagy)

Monteiro, O'Connell, and White began a program amassing evidence that the answer could be **yes**, at least for certain classes of solutions.

Monteiro, O'Connell, White '14
Luna, Monteiro, O'Connell, White '15
Luna, Monteiro, Nicholson, O'Connell, White '16
Luna, Monteiro, Nicholson, O'Connell'18
Lee '18
Berman, Cacón, Luna, White '19
Godazgar, Godazgar, Monteiro, Viega, Pope '21
Gonzáles, Momeni, Rumbutus '21

 $=$  $\otimes$ 

Geodesics and Event Horizons?

Gonzo, Shi '21
Chawla, Keeler '23
Easson, Herczeg, Manton, Pezzelle '23
Ball, Benke, Chen, Volovich '23

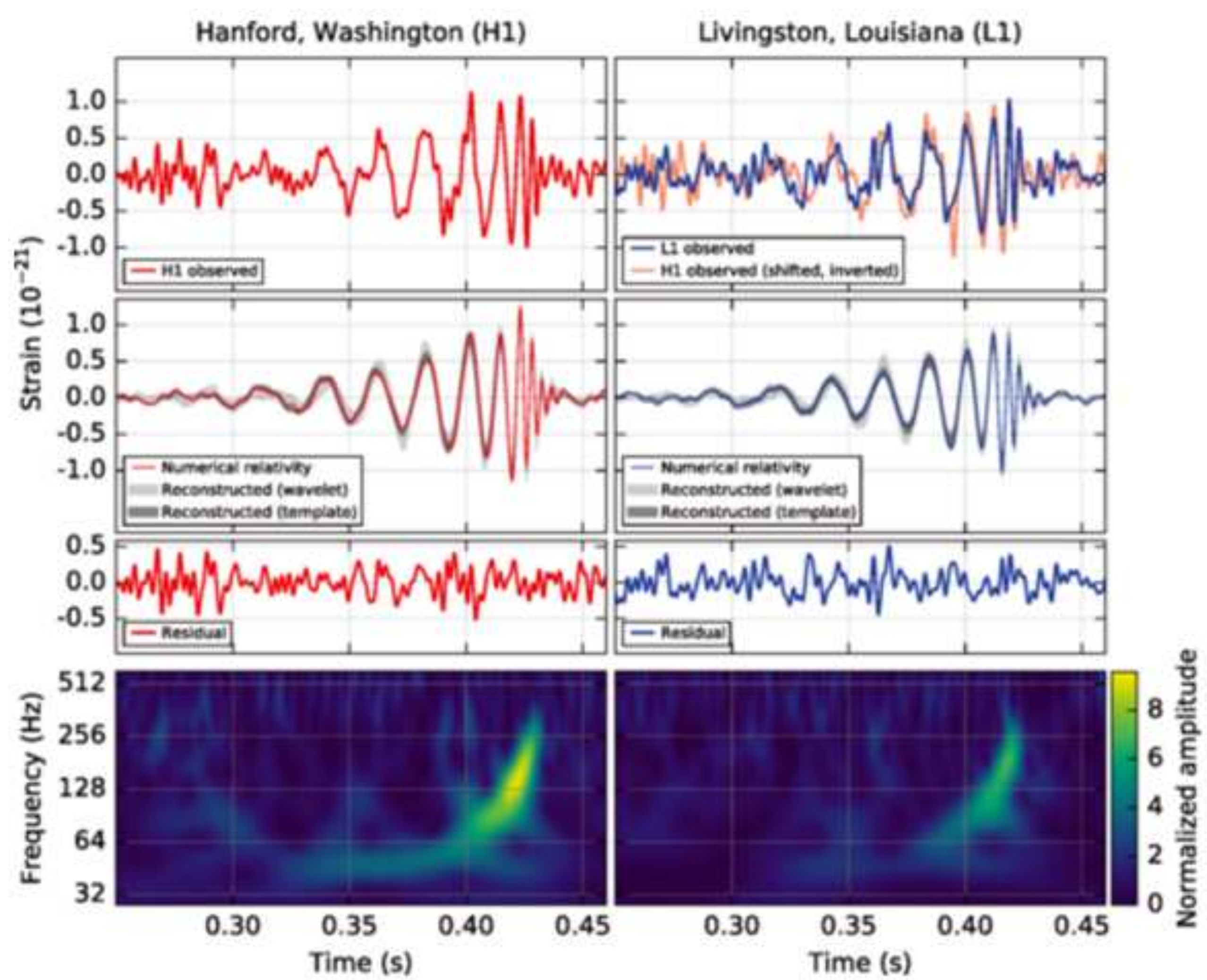
Classical gravity is a Double Copy?

Remind you of some of the double-copy positives:

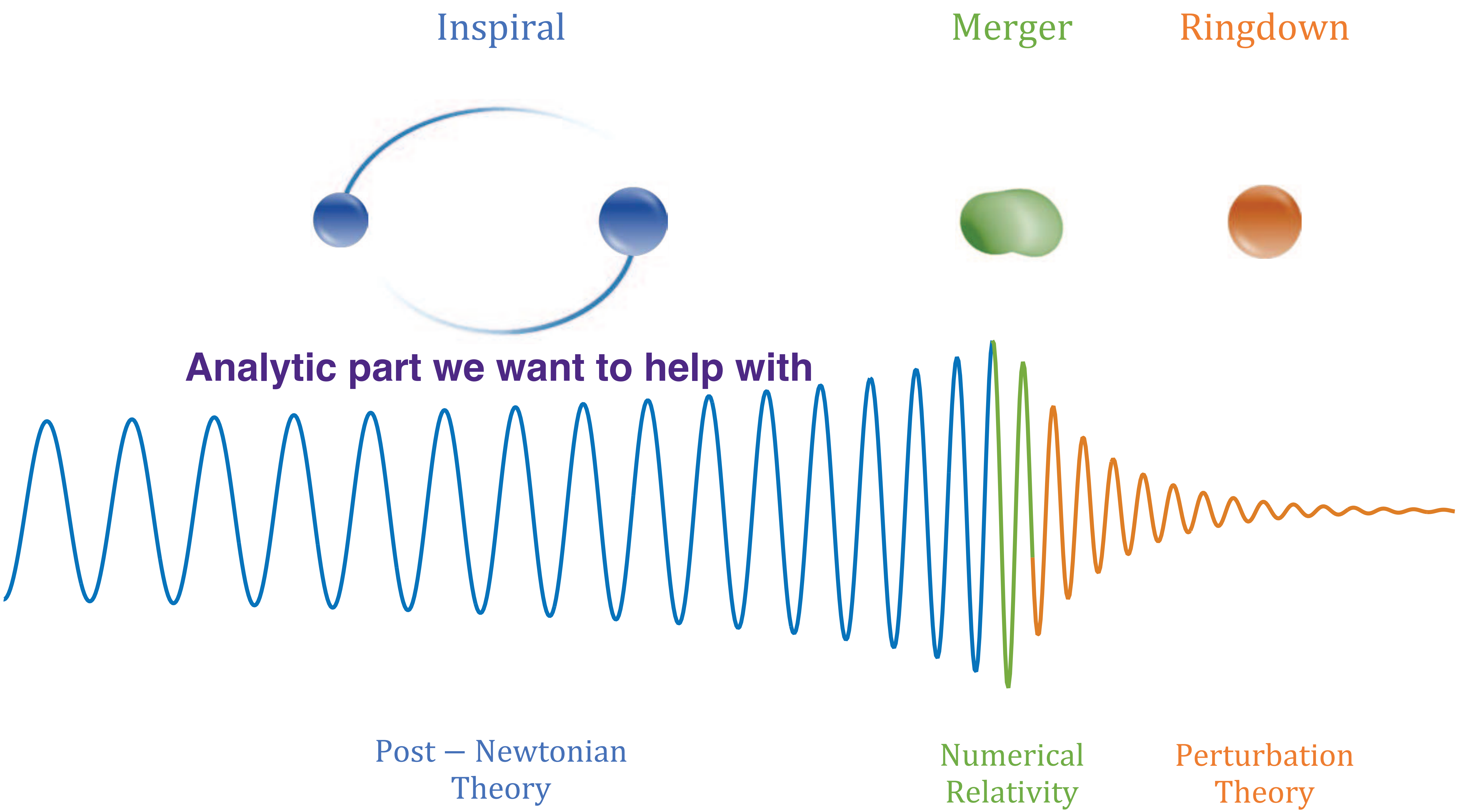
+ Constrained solutions => can exploit for technical simplicity in prediction

+ Unifying web of relationships between theories

Open question: how far can this go?



From Antelis and Morena, arXiv: 1610.03567



High-energy gravitational scattering and the general relativistic two-body problem

Thibault Damour*
Institut des Hautes Etudes Scientifiques, 35 route de Chartres, 91440 Bures-sur-Yvette, France
(Dated: October 31, 2017)

A technique for translating the classical scattering function of two gravitationally interacting bod-

... amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian

"… and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian."

Hard to resist an invitation with this kind of clarity!

G^3 corrections to Newton's Potential from GR

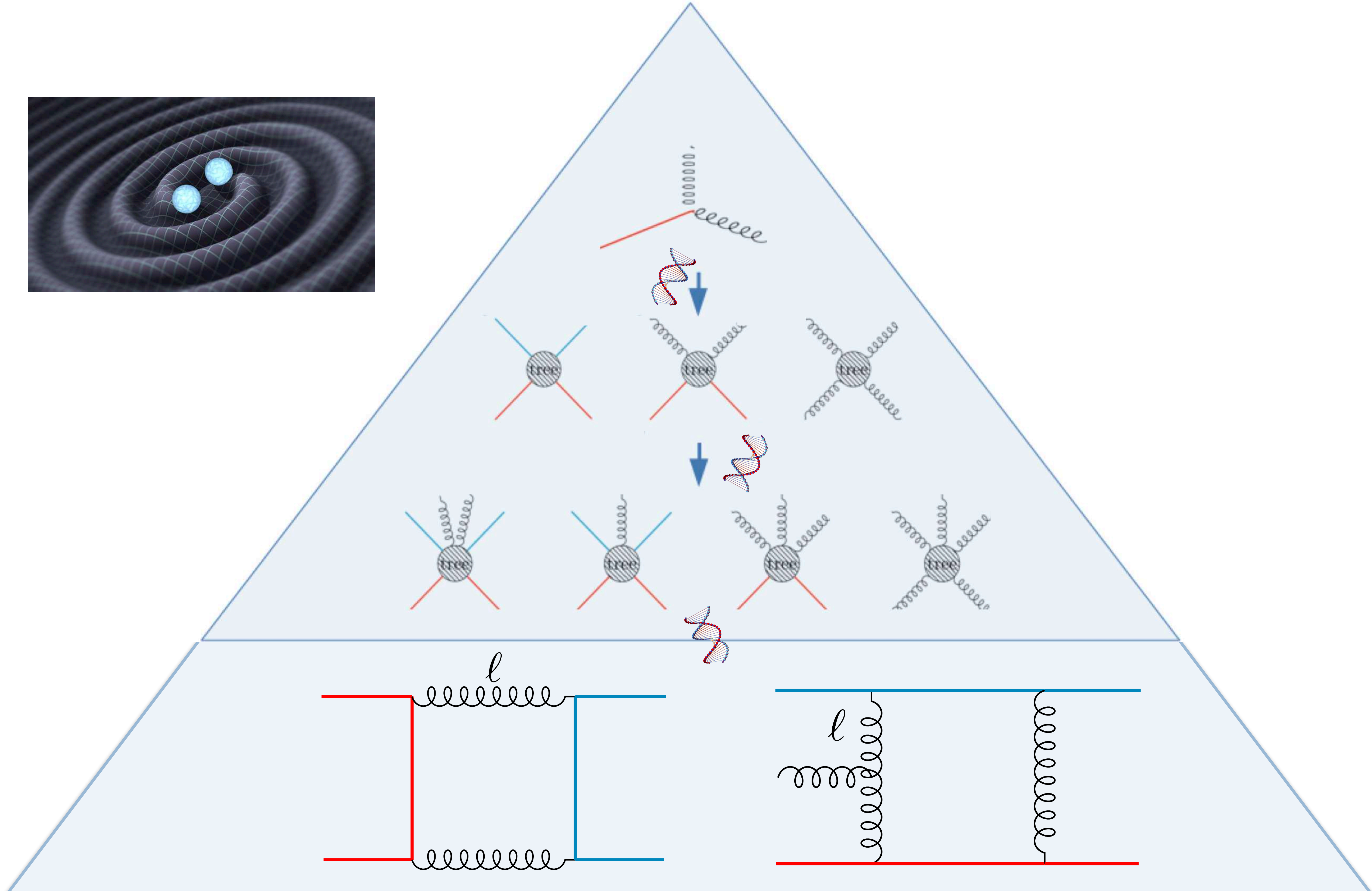
Bern, Cheung, Roiban, Shen, Solon, Zeng (2019)

- **Very difficult using standard methods.**
- **Of direct importance to LIGO/Virgo theorists.**
- **Can in principle enter LIGO/Virgo analysis pipeline.**

arXiv:1710.10599v1 [gr-qc] 29 Oct 2017

Bootstrap to Radiative Effects

JJMC, Holm '20, '21



Elkhidir, O'Connell, Pergola, **Holm** '23
Georgoudis, Heisenberg, **Holm** '23

Brandhuber, Brown, Chen, De Angelis, Gowdy, Travaglini '23

Herderschee, Roiban, Teng '23

Closing

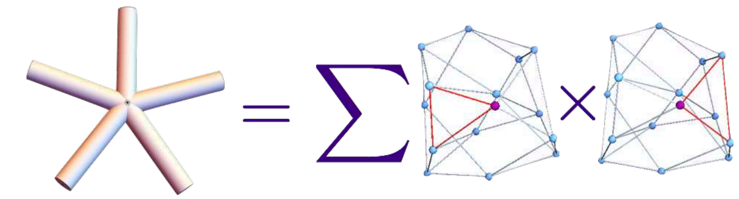
Key Point:

MANY Theories are field theoretic Double Copies

Ingredients: **and we can systematically go deep**

color

α'



spin 0,1/2,1

For all these theories:

Bi-Adjoint Scalar	(S) YM (...(S) QCD...)	(S) Gr (...(S) Einstein-YM...)
NLSM	(S) Born-Infeld	Special Galileon
Z-theory	**Open String**	**Closed String**

Tons of problems to explore and learn from!