Holographic Descriptions of Spacetime

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OUTLINE

- The Holographic Principle
- The AdS/CFT correspondence
- Black Holes in AdS
- Puzzle 1: Euclidean Wormholes
- Puzzle 2: The information paradox
- Puzzle 3: The infalling observer
- A connection between 3d gravity and a matrix/tensor model
- Holographic Spacetime Outlook

The Holographic Principle

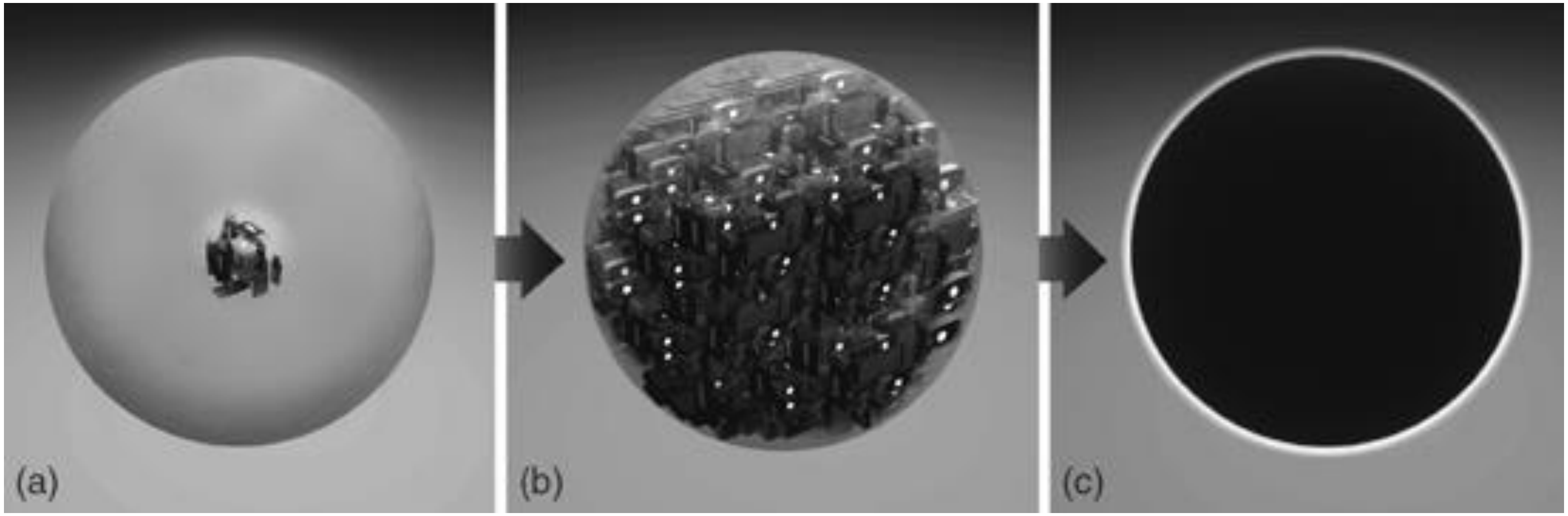
(will not discuss various entropy bounds)

Lessons from the 1970's (Bekenstein, Hawking, Carter, Bardeen,…)

- Black holes have "no hair"
- Black holes carry an "entropy" proportional to the area of the horizon (S=A/4G)
- Black holes have a temperature
- Black holes emit Hawking radiation and can evaporate

Black holes behave like thermodynamic or hydrodynamical systems.

An important lesson from black holes: the maximal amount of information that can be stored is proportional to the area, not the volume.

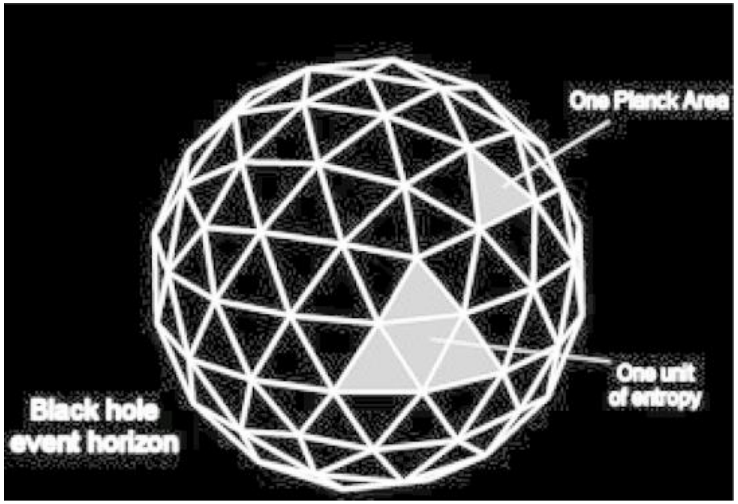


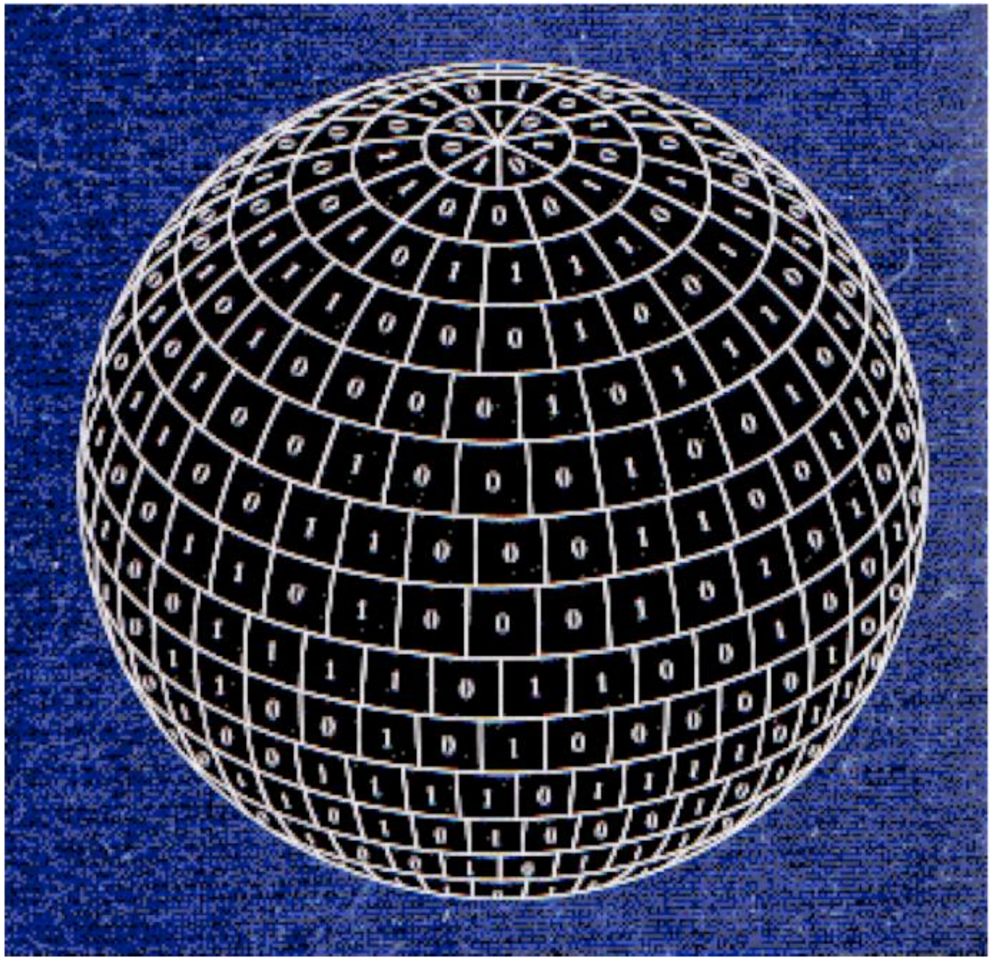
The “holographic principe:”

As far as entropy/information goes, gravity behaves like a local (extensive) quantum system with one less dimension,

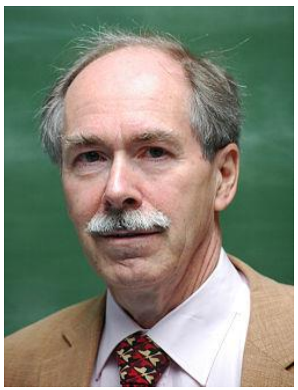
The Holographic Principle makes quantum gravity very different from a standard quantum field theory.

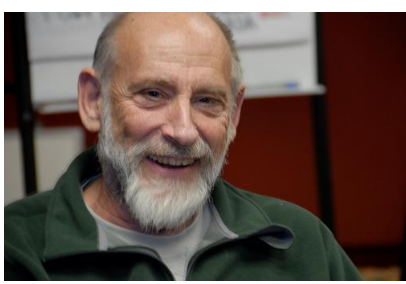
Information can be mapped to bits on a surface.

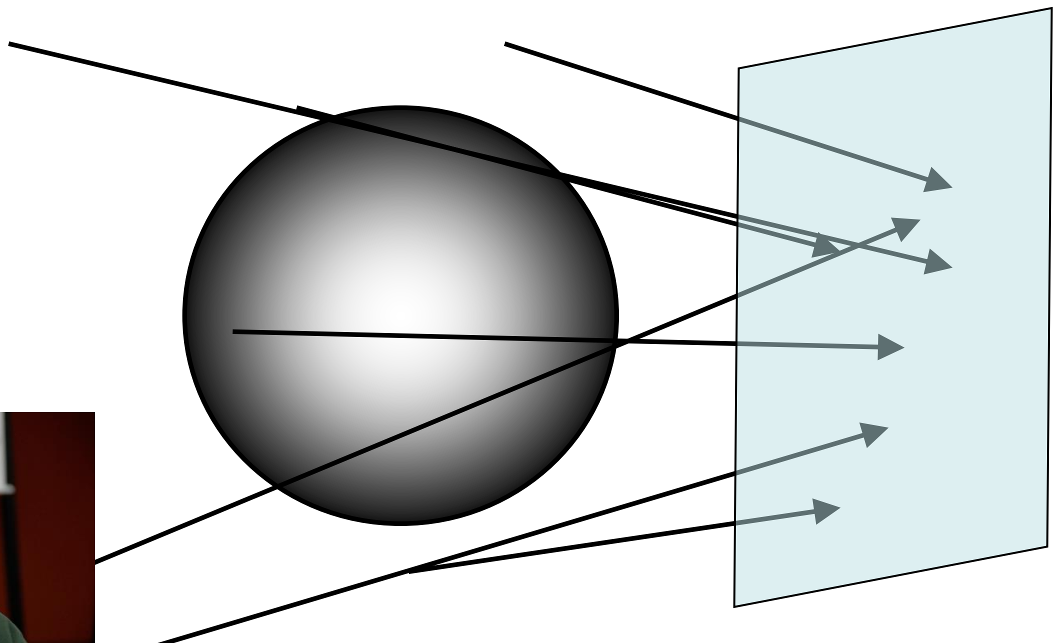




The Holographic Principle







Holographic screen

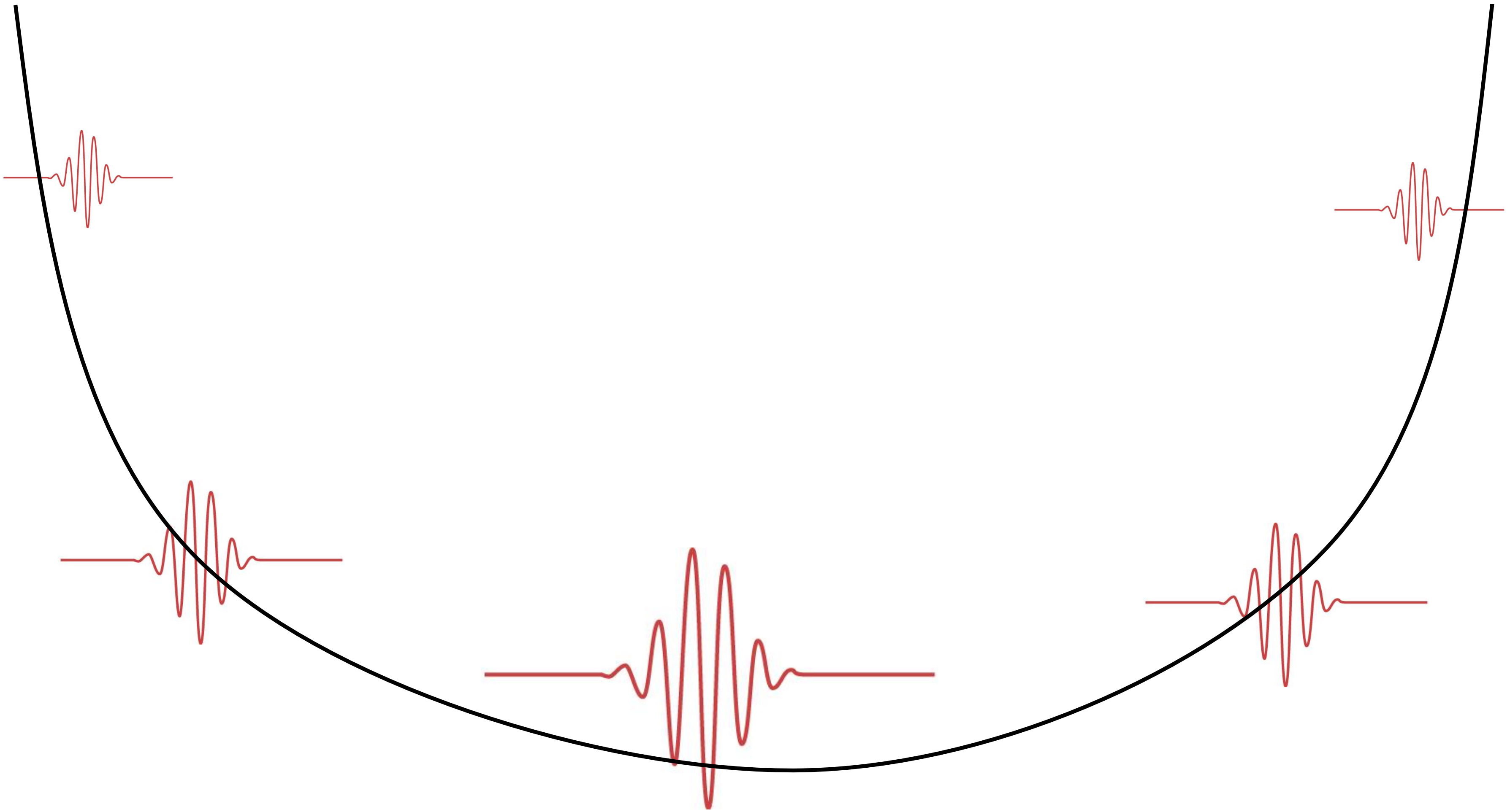
Many questions:

Where to put the holographic screen? Anywhere? Boundary of spacetime? How to figure out what “lives” on the holographic screen? What happens if something moves through the holographic screen? Must the screen be timelike?

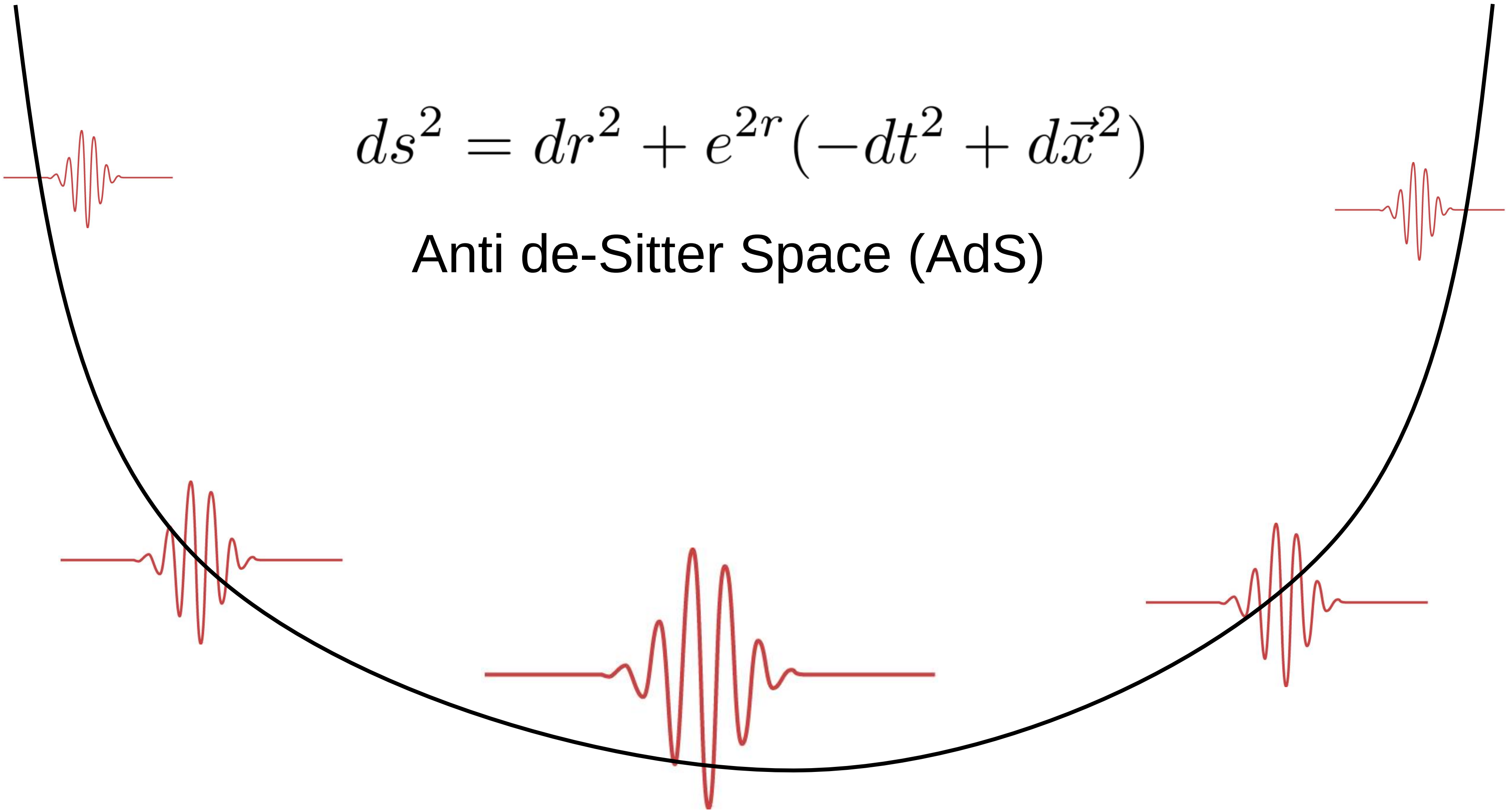
The AdS/CFT correspondence

How to describe a theory where spacetime is quantum?

Idea: put gravity in a box

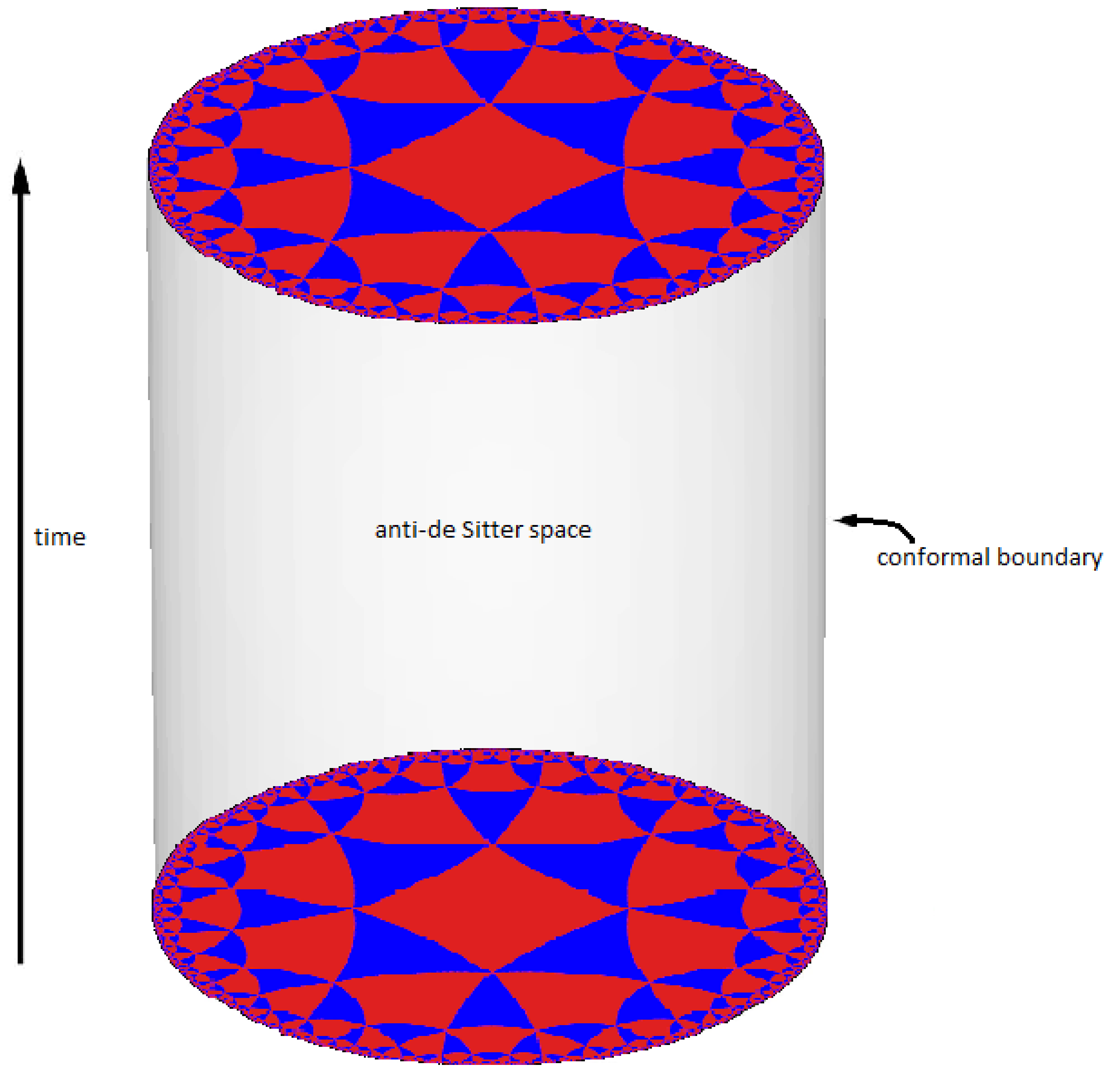


Quantum fluctuations near the boundary are suppressed. Time and space at the boundary are well-defined.



$$ds^2 = dr^2 + e^{2r}(-dt^2 + d\vec{x}^2)$$

Anti de-Sitter Space (AdS)



from wikipedia

One could imagine that a more conventional quantum system "lives" at the holographic screen at the conformal boundary.

It appears that we lost a dimension, but that is a feature, not a bug, as we discussed before. Moreover

- The Hamiltonian of general relativity is of the form

$$H = 0 + \text{boundary terms}$$

On general semi-classical grounds one can argue that the quantum system

- must be strongly coupled.
- must have a large number of degrees of freedom ("large N theories")
- must not have simple low-energy operators of spin larger than two.
- must have only a few simple low-energy operators.
- must be scale and conformally invariant.

Do such quantum systems exist? Yes, some examples were originally identified in string theory and were part of the original AdS/CFT correspondence proposal.

These QFT's are local, unitary strongly coupled large N gauge theories of various types.

Maldacena 1997

No similar statements are currently available on other space-times like flat space, de Sitter space, etc.

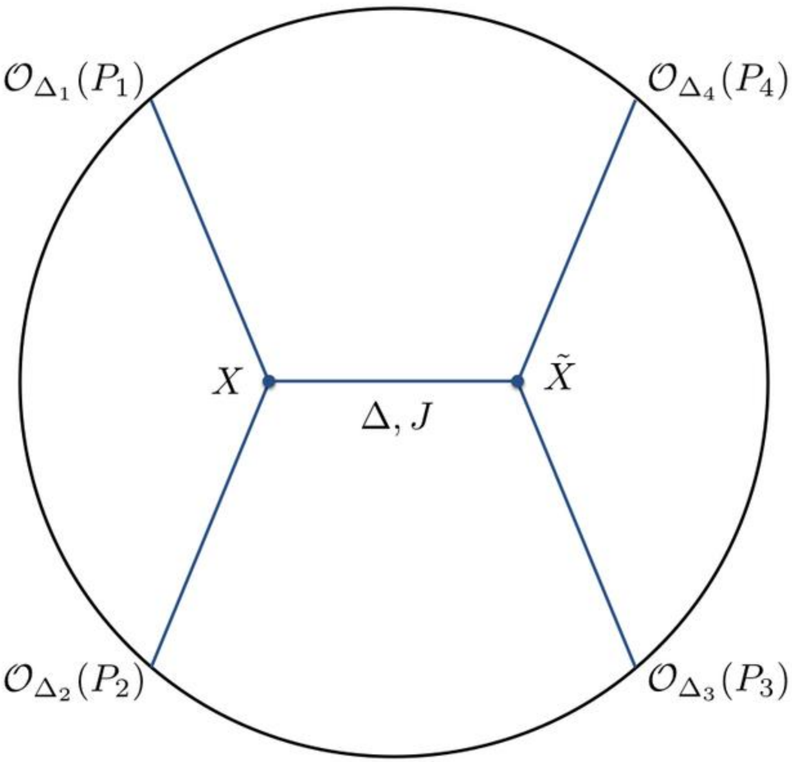
One can think of AdS as an IR regulator of flat space, but there is currently no top-down description of a de Sitter bubble in AdS.

One may ask why one cannot put boundaries elsewhere in general spacetimes and play the same game. Main issue is *locality* of the boundary theory.

In AdS, the AdS lightcone and the boundary lightcone align. For general boundaries this is not the case.

Only well-defined observables “live” on the boundary.

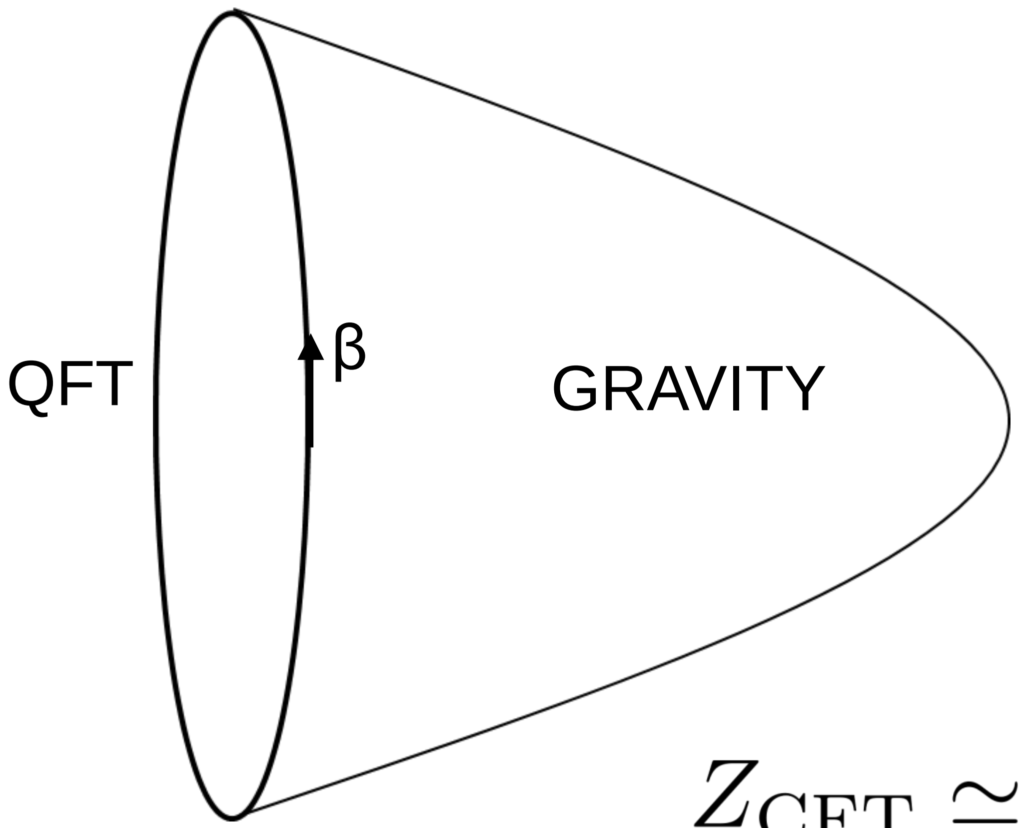
Correlation functions in CFT = Green’s functions (coordinate space Feynman diagrams) in AdS



To study spacetime is somewhat similar to medical imaging

Partition function of the CFT = Partition function of gravity theory with the right boundary condition ≈

Exp[-classical gravitational action evaluated on-shell]



$$Z_{\mathrm{CFT}} \simeq e^{-S_{\mathrm{grav}}}$$

A few conceptual observations

- Time on the boundary is well defined, time in the interior is only approximately defined. On the boundary the usual rules of quantum mechanics apply.

- The ground state can be prepared with a Euclidean no-boundary gravitational path integral

- The indefinite nature of the Euclidean path integral does not appear to be an issue

- The extra dimension geometrizes scale transformations:

JdB, Verlinde, Verlinde, '99

- space-time is "emergent"

- No obvious separate UV fixed point?

Quantum information

(Quantum information) theory appears to play an important and perhaps fundamental role in understanding quantum gravity.

Perhaps one reason is that gravity, as a low-energy effective field theory, resembles a thermodynamic system, which are best described in terms of coarse grained quantities such as entropy.

Information theoretic quantities quantify, among other, ignorance, and in the case of quantum gravity, they appear to quantify ignorance about the microscopic details of the UV complete description.

The Role of Quantum Information

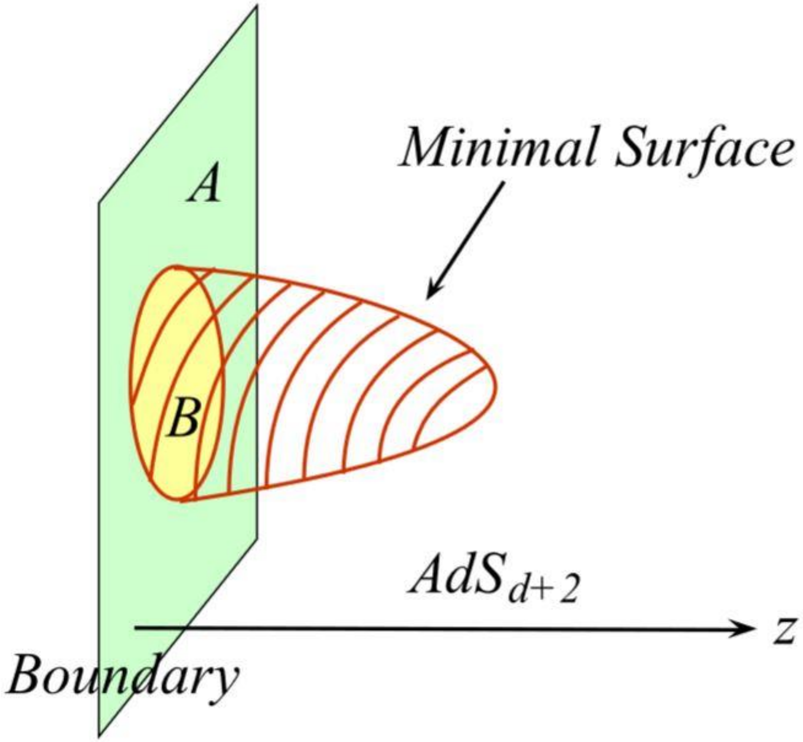
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Bell (or EPR) pair, entanglement: measurements are correlated.

$$|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$$

Entanglement entropy S_A ~ number of Bell pairs that entangle A and B.

Ryu-Takayanagi ('06): entanglement entropy in CFT = area of minimal surface in gravity (S_A=area/4G).

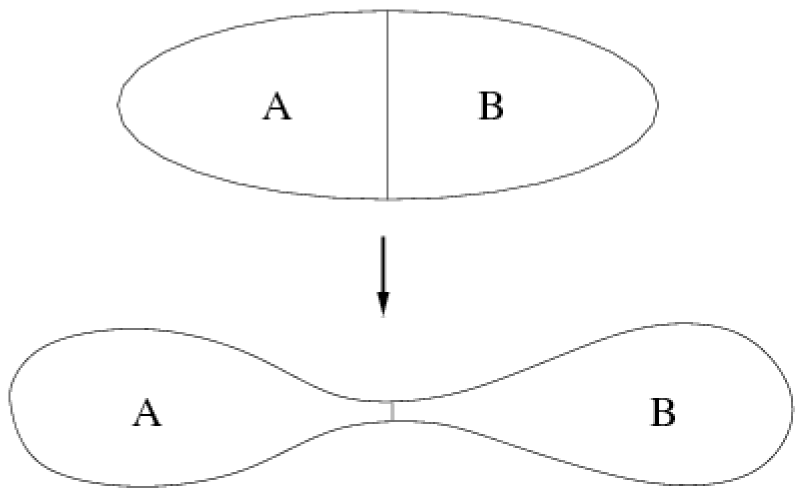


Postulating that entanglement entropy is computed by minimal area surfaces implies the linearized Einstein equations. Faulkner, Guica, Hartman, Myers, van Raamsdonk `13

Entanglement is needed to build op a connected spacetime (van Raamsdonk '10).

The correlations in entangled states are reproduced by making spacetime connected.

Exactly which types of entanglement have smooth geometric representations is still not entirely clear.



Amazingly, many quantum information theoretic concepts have a gravitational interpretation:

- quantum error correction
- entanglement of purification
- various protocols
- differential entropy
- quantum teleportation
- relative entropy
- Renyi entropy
- mutual information
- entropy inequalities like strong subadditivity

which led to the idea that perhaps quantum gravity can be formulated purely in information theoretic terms (but this has not been achieved yet).

Take home message

- Quantum gravity in AdS is equivalent to a putative quantum system living on the boundary of AdS.
- Many properties of such a putative quantum system can be determined using semi-classical reasoning.
- A more detailed and precise statement can be obtained from string theory, agreeing with all semi-classical expectations.
- Precise observables are only defined on the boundary of spacetime.
- It is therefore not straightforward to examine local properties of the gravitational theory (like the fate of an infalling observer).
- Gravity geometrizes quantum information
- Smooth connected geometries correspond to particular entanglement patterns but the precise map is unknown ("ER=EPR")

Black Holes in AdS

Black Hole in AdS = CFT at finite temperature

At low temperatures, a thermal gas of particles in AdS corresponds to a thermal gas of (confined) excitations in the CFT.

At higher temperatures, the thermal gas collapses into a black hole. In the CFT, the theory deconfines and one obtains a deconfined plasma.

Bañados, Teitelboim & Zanelli '92

A test of AdS/CFT: the BTZ black hole

$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2} dt^2 + \frac{r^2}{(r^2 - r_+^2)(r^2 - r_-^2)} + r^2 \left(d\phi - \frac{r_+ r_-}{r^2} dt \right)^2$$

$$S = \frac{\pi}{2G} r_+ = \frac{\pi}{2G} (\sqrt{M+J} + \sqrt{M-J})$$

Cardy formula of 2d CFT

$$S = 2\pi \sqrt{\frac{c}{6}(L_0 - \frac{c}{24})} + 2\pi \sqrt{\frac{c}{6}(\bar{L}_0 - \frac{c}{24})}$$

Perfect agreement!

Black Hole in AdS = CFT at finite temperature

Subset of Einstein Field Equations = equations of hydrodynamics for CFT plasma (gravity somehow knows about the right variables for hydrodynamics)

Bhattacharyya, Hubeny, Minwalla, Rangamani, ‘08

Falling into the black hole = dissipation

Black hole creation = thermalization

Gravitational predictions:

- Hydrodynamics has very low viscosity Kovtun, Son, Starinets,’01
- Thermalization proceeds maximally fast

Balasubramanian, Bernamonti, JdB et al, ‘11

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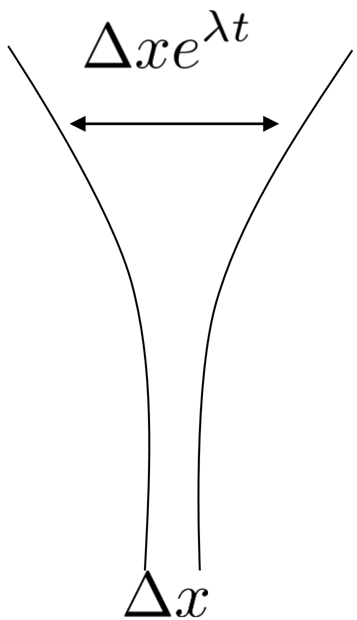
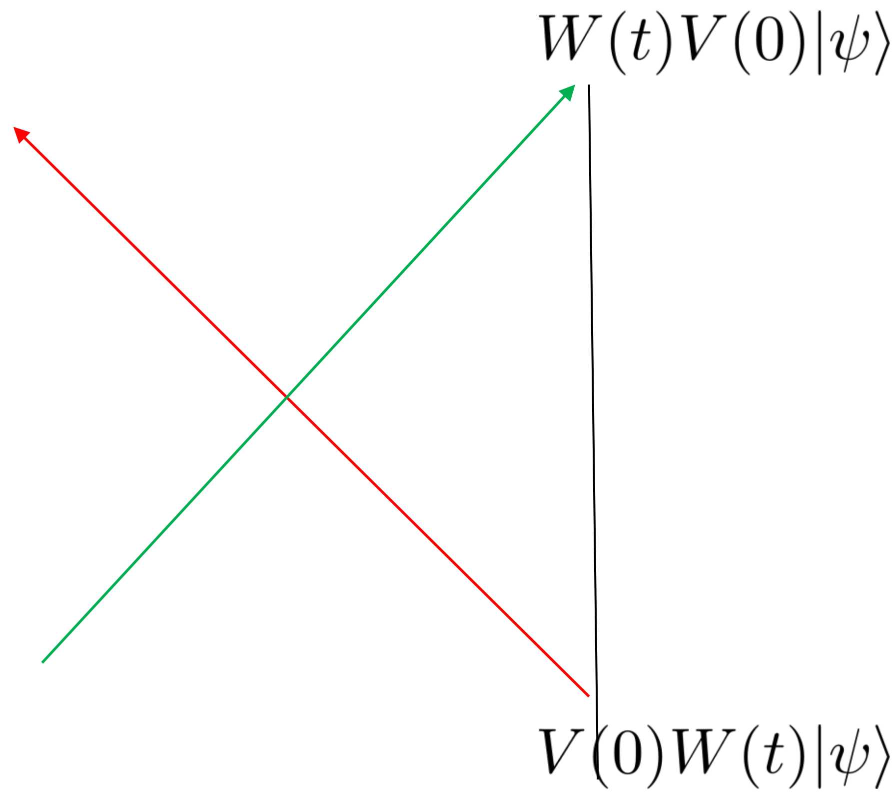
Black holes and chaos





$$\left(\frac{\partial q(t)}{\partial q(0)}\right)^2 \sim$$

$$\{q(t), p(0)\}^2 \sim$$

$$[q(t), p(0)]^2$$

$$\langle V(0)W(t)V(0)W(t)\rangle_\beta \sim c_0 - c_1 N^{-2} e^{\lambda t}$$

$$\lambda \leq \frac{2\pi}{\beta}$$

For black holes: $\lambda = \frac{2\pi}{\beta}$

Maldacena, Shenker, Stanford, '15

If you throw something into a black hole, after roughly a scrambling time $t_{\rm scr} \sim \beta \log S$ it will be a Planck distance away from the horizon.

There is no low-energy computation that will be able to diagnose this and the black hole looks just like a stationary black hole from that moment forward.

Are there any diagnostic tools which see anything interesting happen after the scrambling time?

Yes: complexity

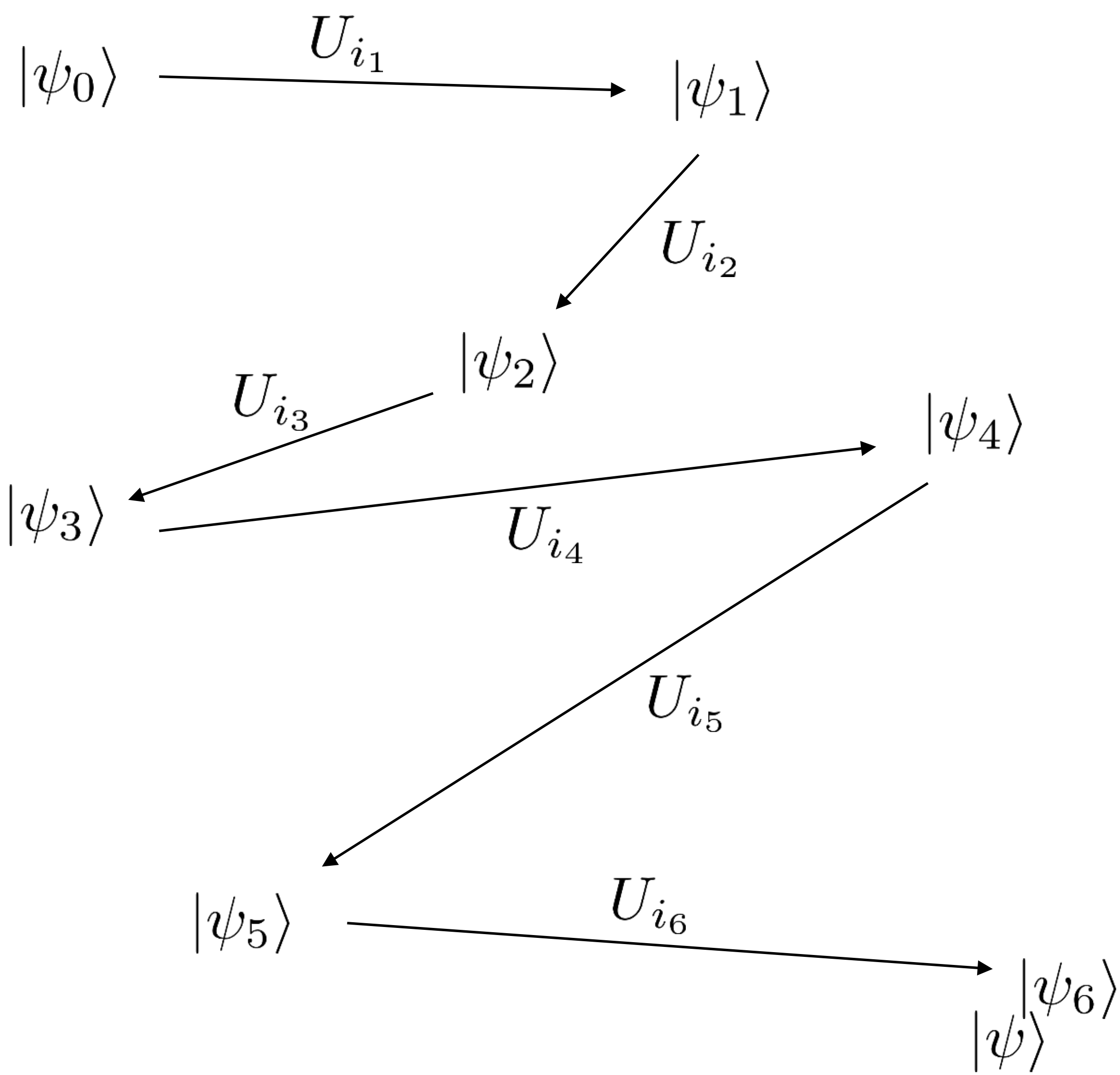
Complexity is defined as follows.

Given a reference state $|\psi_0\rangle$, a set of unitary gates U_i , and an error ϵ, the complexity of a state $|\psi\rangle$ is defined as

$$C(|\psi\rangle) = \min_N \left| \prod_{k=1}^{N} U_{i_k} |\psi_0\rangle - |\psi\rangle \right| < \epsilon$$

It is not clear what a natural definition of complexity in a quantum field theory is supposed to be and what a good choice of gates would be.

But on general grounds one expects complexity to keep on growing for a long time even after the system has effectively thermalized.



$|\psi_0\rangle$
U_{i_1}
$|\psi_1\rangle$
U_{i_2}
$|\psi_2\rangle$
U_{i_3}
$|\psi_4\rangle$
$|\psi_3\rangle$
U_{i_4}
U_{i_5}
$|\psi_5\rangle$
U_{i_6}
$|\psi_6\rangle$
$|\psi\rangle$

In black hole backgrounds, there is also something which keeps on growing for a long time, which is the volume of a constant time slice.

This led to the conjecture “complexity equals volume”.

Stanford, Susskind ‘14

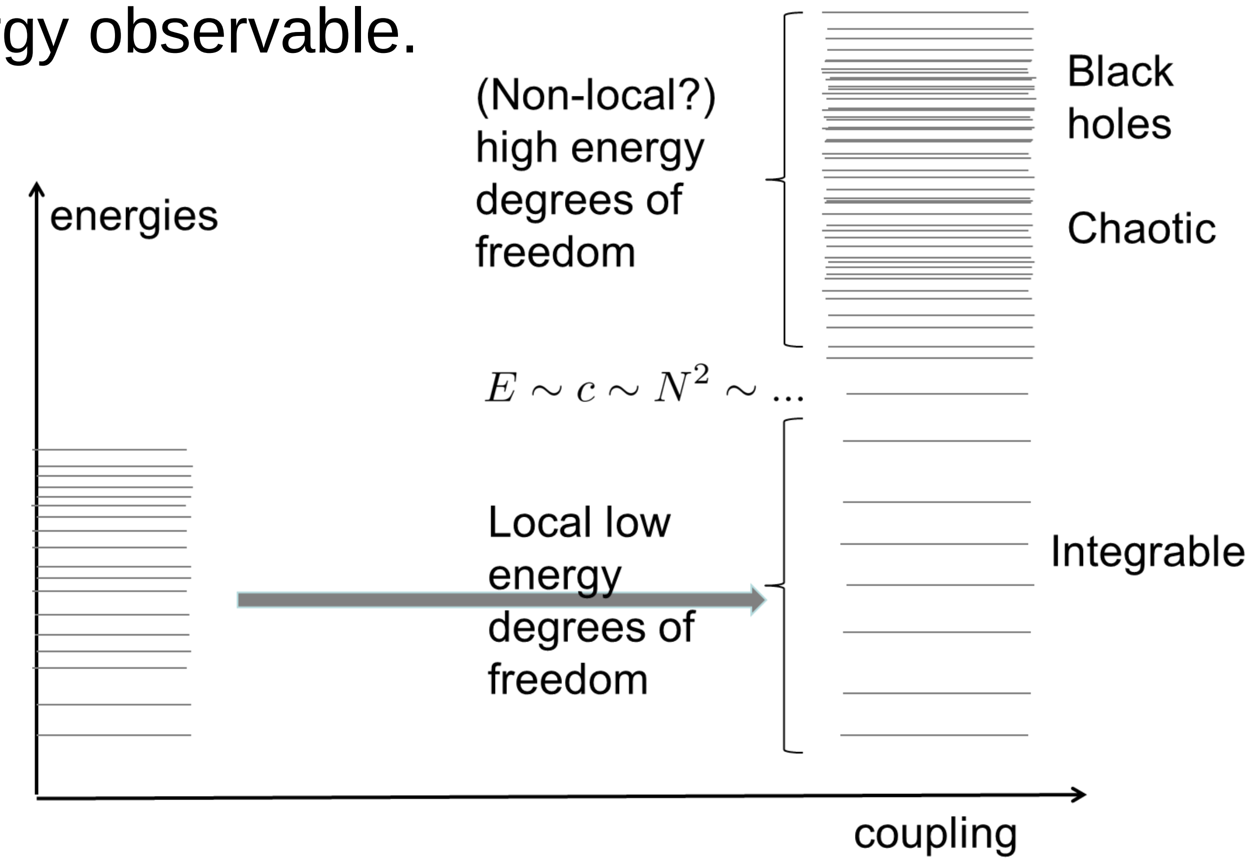
More generally, one can try to interpret arbitrary space-time regions as quantum circuits and try to define a notion of complexity for such circuits.

Chandra, JdB, Flory, Heller, Hörtner, Rolph ’21 ‘22

However, no one has so far been able to make this mathematically precise.

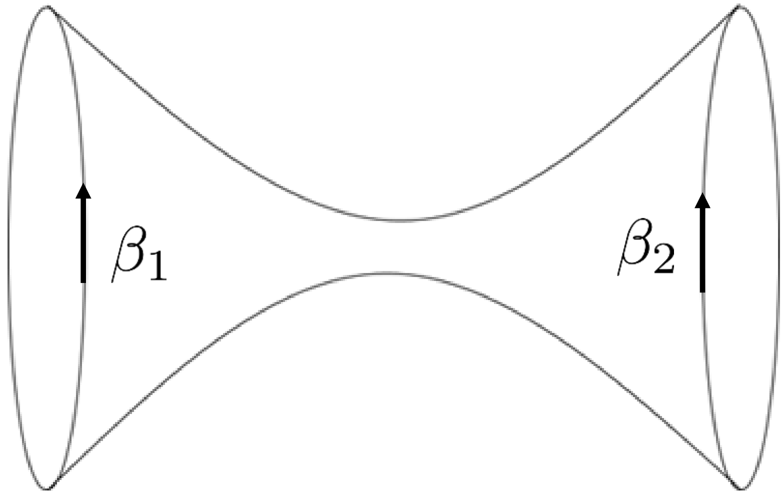
Take home message:

- Black holes are extremely chaotic.
- The high-energy spectrum of the QFT must therefore also be very dense and highly chaotic.
- It is very difficult to probe black hole features after the scrambling time – complexity might be such a probe but it is not a low-energy observable.



Puzzle 1: Euclidean Wormholes

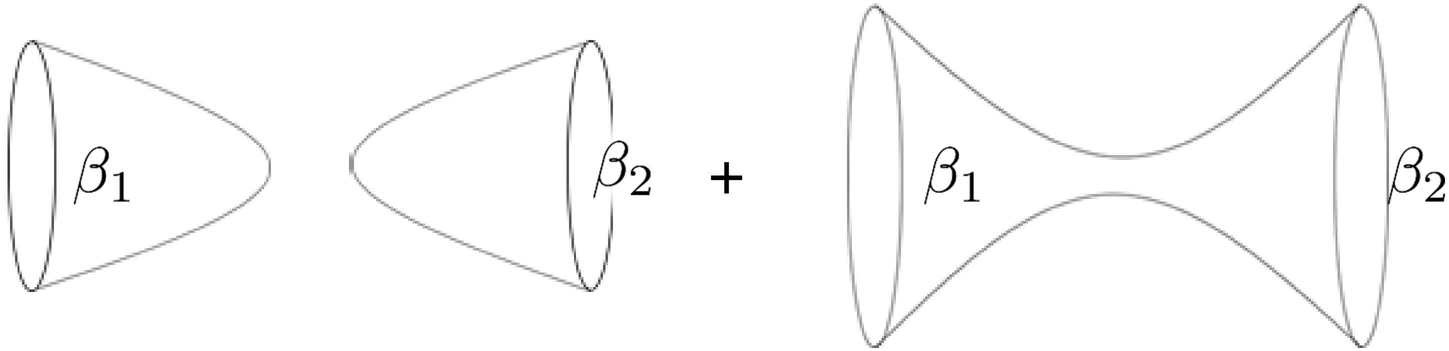
Semi-classical gravity seems to give rise to correlations between multiple holographic screens (copies of the same theory) due to the existence of Euclidean wormhole solutions



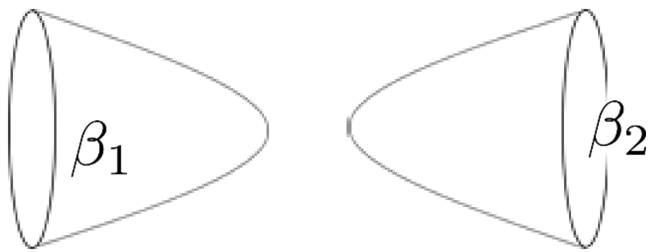
Such correlations (lack of factorization) could arise due to disorder averages but in standard AdS/CFT there was no need for (or a sign of) disorder.

"factorization puzzle"

$\langle Z(\beta_1)Z(\beta_2)\rangle =$



$\langle Z(\beta_1)\rangle\langle Z(\beta_2)\rangle =$



$$\implies \langle Z(\beta_1)Z(\beta_2)\rangle \neq \langle Z(\beta_1)\rangle\langle Z(\beta_2)\rangle$$

This would be fine if the theory would carry additional parameters that we need to average over because then

$$\sum_{\alpha} \langle Z_\alpha(\beta_1) Z_\alpha(\beta_2) \rangle \neq \sum_{\alpha} \langle Z_\alpha(\beta_1) \rangle \sum_{\alpha'} \langle Z_{\alpha'}(\beta_2) \rangle$$

Does gravity at a fundamental level involve some sort of averaging over parameters?

This has been suggested before in the context of “baby universes”.

Coleman ‘88
Giddings Strominger ‘88

A different perspective:

Semi-classical gravity has no access to detailed features of black holes or the chaotic high-energy part of the spectrum.

In the spirit of statistical physics, the best description is one where we maximize ignorance (=entropy) while being compatible with low-energy observations.

Wigner ‘55

This gives rise to a “matrix model” which describes an ensemble of Hamiltonians rather than a single Hamiltonian.

Such a matrix model has correlations with agree with wormhole computations.

Semi-classical gravity $\Rightarrow \int dH\, \mu[H]$

Integral over all Hamiltonians which are indistinguishable for low-energy observers

Wormhole =

$$\int dH\, \mu[H] \mathrm{Tr}(e^{-\beta_1 H}) \mathrm{Tr}(e^{-\beta_2 H}) - \int dH\, \mu[H] \mathrm{Tr}(e^{-\beta_1 H}) \int dH\, \mu[H] \mathrm{Tr}(e^{-\beta_2 H})$$

Two predictions:

-Euclidean wormholes contain no new information

-Their contribution should disappear once we UV complete the theory.

In a series of papers, we collected a lot of evidence for this picture:

Alex Belin, JdB, arXiv:2006.05499
Alex Belin, JdB, Pranyal Nayak, Julian Sonner, arXiv:2012.07875
Alex Belin, JdB, Diego Liska, arXiv:2110.14649
Alex Belin, JdB, Pranyal Nayak, Julian Sonner, arXiv:2111.06373
Tarek Anous, Alex Belin, JdB, Diego Liska, arXiv:2112.09143
+ various to appear

A lot of related work has appeared, e.g:

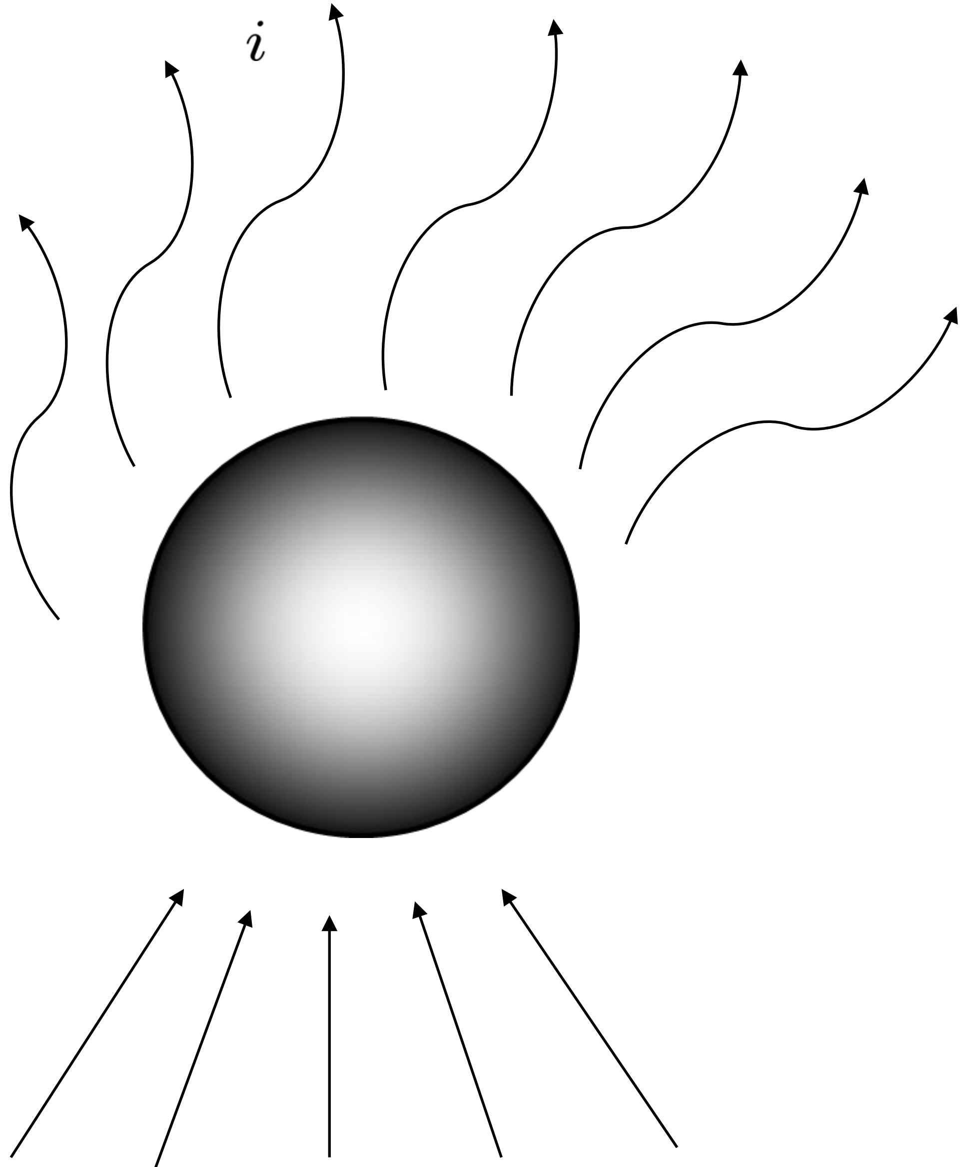
Pollack, Rozali, Sully, Wakeham ‘20
Liu, Vardhan ‘20
Altland, Sonner ‘20
Janssen, Mirbabayi, Zograf ’21
Sasieta ‘21
Altland, Bagrets, Nayak, Sonner, Vielma ‘21
Freivogel, Nikolakopoulou, Rotundo ’21
Schlenker, Witten ‘22
Chandra, Collier, Hartman, Maloney ‘22

Take home message

- Semi-classical gravity only contains a statistical theory of the chaotic sector of the theory.
- The lack of access to microscopically detailed information of the chaotic sector is what is responsible for the appearance of wormholes.
- There is no need to interpret semiclassical gravity fundamentally as an averaged theory.
- The contribution from euclidean wormholes should disappear in a full UV-complete description.
- This statistical perspective could possibly lead to a combinatorial description of gravity and establish a connection with “discrete approaches” (end of talk)

Puzzle 2: The information paradox

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$



Information paradox: conflict between

- Locality
- Unitarity
- Equivalence Principle

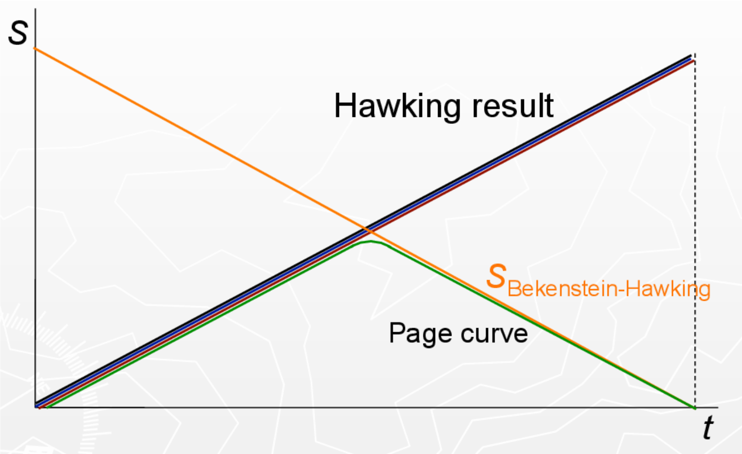
$$\rho = |\psi\rangle\langle\psi|$$

The AdS/CFT correspondence seems to respect unitarity, approximate locality, and the equivalence principle.

So how does it resolve the information loss paradox?

Penington ’19
Almheiri, Engelhardt, Marolf, Maxfield ‘19
Penington, Shenker, Stanford, Yang ‘19
Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini ‘19

Couple the boundary of AdS to an external, large system, which captures all the radiation. Use AdS/CFT technology to compute the entropy contained in that radiation.



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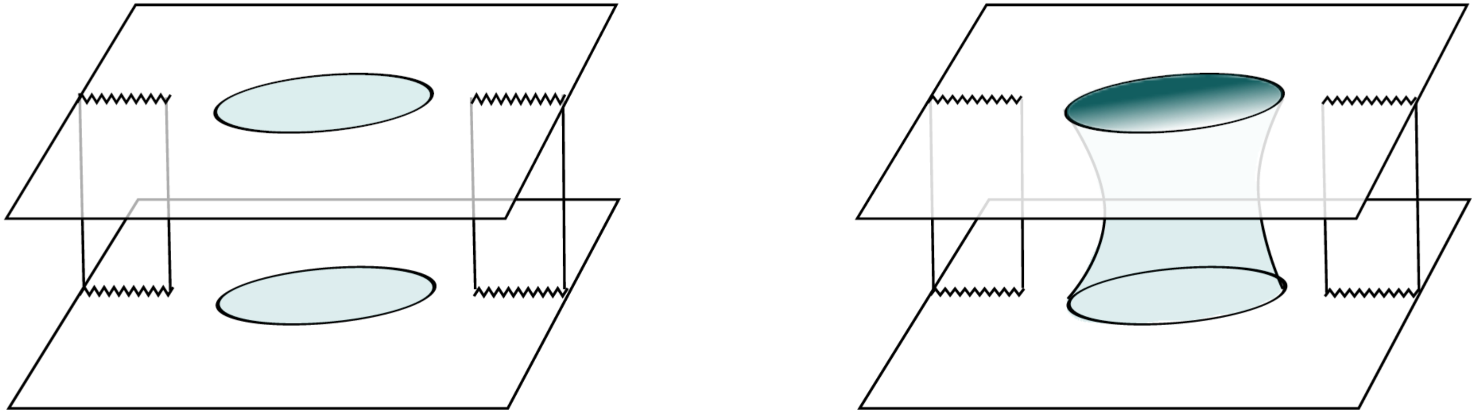


(a) (b)

In the computation different semiclassical configurations appear, depending on whether or not spacetime is connected by a wormhole. For late times the latter dominates the computation.

The page curve is reproduced.

It is possible to write down very simple models which capure this behavior (JdB, Hollander, Rolph, to appear)

Comments:

- The computation shows that the radiation carries information about part of the black hole interior ("islands")
- To uncover the information, extremely complex measurements have to be made. The relevant apparatus will create a substantial backreaction on the black hole geometry.
- The computation only relies on semi-classical general relativity (not on string theory per se).
- The computation does not admit a direct translation in the language of effective field theory.
- The computation does not elucidate the nature of the individual microstates which make up the black hole.
- Very small but very non-local effects seem to be key.

$$\begin{pmatrix} e^{-S} & 0 & 0 & 0 & 0 \\ 0 & e^{-S} & 0 & 0 & 0 \\ 0 & 0 & e^{-S} & 0 & 0 \\ 0 & 0 & 0 & e^{-S} & 0 \\ 0 & 0 & 0 & 0 & e^{-S} \end{pmatrix} \Longrightarrow \begin{pmatrix} e^{-S} & e^{-S} & e^{-S} & e^{-S} & e^{-S} \\ e^{-S} & e^{-S} & e^{-S} & e^{-S} & e^{-S} \\ e^{-S} & e^{-S} & e^{-S} & e^{-S} & e^{-S} \\ e^{-S} & e^{-S} & e^{-S} & e^{-S} & e^{-S} \\ e^{-S} & e^{-S} & e^{-S} & e^{-S} & e^{-S} \end{pmatrix}$$

Mixed state

Pure state

$$e^{-S} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Take home message

- By coupling AdS/CFT to an external system we can precisely factorize the Hilbert space in “black hole” and “radiation”.
- The computation of the entropy of the radiation can be formulated as a suitable Euclidean path integral question.
- A Euclidean wormhole solution dominates the computation at late times precisely reproducing the Page curve.
- One sees that one has to give up strict locality.
- To measure the detailed state of the Hawking radiation requires a very complex measurement beyond what a low-energy observer can achieve.

Puzzle 3: The infalling observer

It is very hard to find something which probes the geometry of a black hole behind the horizon.

One can construct operators which describe physics behind the horizon but it is very hard to determine whether these are local operators.

The problem is roughly that if U is a complicated non-local unitary it is still true that

$$\langle \Psi | \mathcal{O}_1 \ldots \mathcal{O}_n | \Psi \rangle = \langle U \Psi | U \mathcal{O}_1 U^\dagger \ldots U \mathcal{O}_n U^\dagger | U \Psi \rangle$$

so correlation functions cannot unambiguously distinguish local from non-local operators.

In JdB, Jafferis, Lamprou ’22 we showed that this unitary ambiguity can be resolved using a combination of

- modeling an observer as a small black hole
- using the fact that a suitable “modular operator” from the theory of von Neumann algebras can be used to construct proper time near the probe black hole
- this modular operator can also be used to propagate local fields forward along world-lines
- in this way we can construct a basis of local operators along a world-line
- outside the black hole the modular operator can be found by minimizing a suitable set of correlation functions
- inside the black hole the modular operator can be found by minimizing a suitable notion of complexity

This provides an in principle framework to study the fate of the infalling observer.

It does not assume an a priori geometrical description of the interior.

By consider scaling with the Newton constant of various quantities, we can also distinguish geometric from non-geometric interiors, and find a diagnostic for the presence of the singularity.

For very old black holes which saturate complexity, one might be inclined to conclude that roughly half have no geometric interior?

Stanford, Yang '22

Take home message

- We constructed an in principle framework to study the physics behind the horizon and the fate of the infalling observer.
- It seems plausible that for generic collapsing matter, the resulting horizon will not be a special place and an infalling observer will not see anything that violates the equivalence principle.
- Due to the chaotic nature of black hole, it appears inevitable to use the concept of complexity.
- To make things mathematically more rigorous, a better understanding of the right definition of complexity in quantum field theory is needed.

Take home message:

Any theory of quantum gravity should ideally

- Explain the entropy of the black hole in terms of microscopic degrees of freedom of the theory. ✓
- Resolve the information paradox. ✓
- Explain what happens to an infalling observer (firewall?) ✓
- Be compatible with the holographic principle. ✓
- Explain what happens at the black hole singularity ✓
- Have interesting observational implications (or not)? ✗
- Apply to different spacetime geometries ✗

A connection between 3d gravity and a matrix/tensor model

A connection between 3d gravity and a matrix/tensor model.

Belin, JdB, Jafferis, Nayak, Sonner, to appear soon

Recall:

Semi-classical gravity $\Rightarrow \int dH \, \mu[H]$

↓

Integral over all Hamiltonians which are indistinguishable for low-energy observers

For 3d gravity, the dual description is a 2d CFT.

So instead of Hamiltonians, it is more accurate to average over all 2d theories which are very close to an actual CFT ("approximate CFT's) so that they are indistinguishable for low-energy observers.

Data of a 2d CFT:

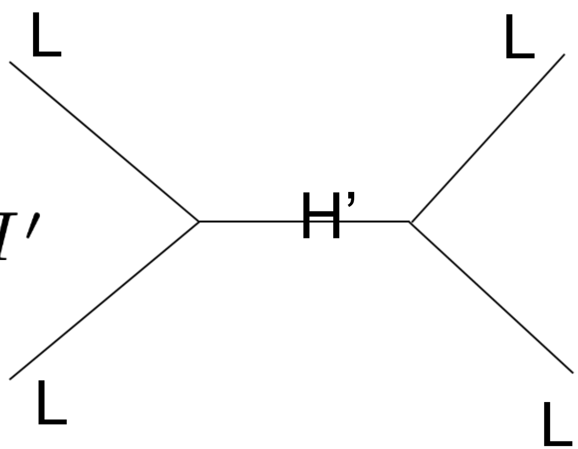
Scaling dimensions: Δ_i

Operator Product Expansion Coefficients: C_{ijk}

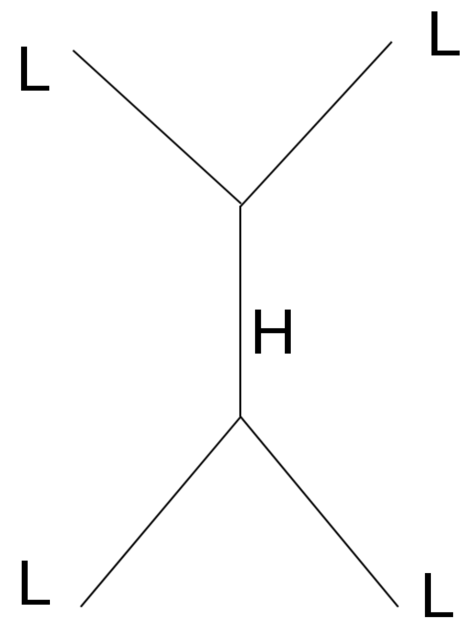
Together with a set of consistency conditions (Moore-Seiberg axioms).

One important relation: crossing symmetry

$\sum_{H'}$



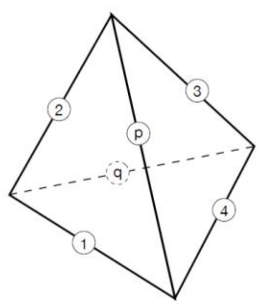
$= \sum_{H}$



The idea is now to average over all CFT2 data with a spectrum which is very close to that of 3d gravity, and with a weight schematically of the form

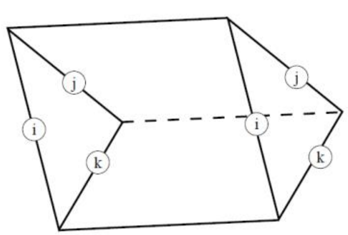
$$P(\Delta_i, C_{ijk}) \sim \exp\left(-a \sum (\text{axioms})^2\right)$$

Result is a quartic tensor model with Feynman rules



$$= \begin{Bmatrix} \mathcal{O}_q & \mathcal{O}_2 & \mathcal{O}_1 \\ \mathcal{O}_p & \mathcal{O}_4 & \mathcal{O}_3 \end{Bmatrix}$$

Virasoro 6j symbol



$$= C_0\left(P_i, P_j, P_k\right) C_0\left(\bar{P}_i, \bar{P}_j, \bar{P}_k\right)$$

This is reminiscent of various other discrete descriptions of 3d gravity.

Regge ’61; Boulatov ‘92; Turaev Viro ‘92

It also seems to be related to the so-called Teichmüller TQFT (Andersen, Kishaev ‘11 ’13) which was recently connected to 3d gravity (Collier, Eberhardt, Zhang ’23)

To be continued….

Holographic Spacetime Outlook

1. The role of quantum information

There is an amazing interplay between concepts of quantum information theory and gravitational quantities including the Einstein equations.

To what extent can gravity be reformulated in purely information theoretic terms?

2. The role of (computational) complexity

Computational complexity plays a role in understanding black holes, infalling observers, and wormholes.

Are limitations on computational and observational power of fundamental or foundational importance in quantum gravity?

Is there a preferred notion of computational complexity which is relevant for quantum gravity?

Can almost anything happen as long as it is too complex to measure?

3. Gravity is a non-local theory

Non-locality was crucial in order to resolve the information paradox.

What are the rules related to non-locality?

4. The role of the observer

We tend to ask meta observer questions – how important is it that we ourselves are actual observers and part of the system?

Is the description of quantum gravity observer dependent?

What is the right language to approach this question?

5. Is there a description of quantum gravity in flat space?

Current focus is on “celestial holography”.

What are the right observables? The (non)perturbative S-matrix in flat space?

Can we get the right answer from AdS by removing the box?

Could there be a holographic dual living at past/future null infinity? What kind of object could this possibly be?

6. Is there a description of quantum gravity in closed universes such de Sitter space?

Is quantum gravity in de Sitter space UV completeable?

What are the right observables? (Non)-perturbative cosmological correlators?

Is it important to take an observer-centric point of view?

Can we put a bubble of de Sitter in AdS?

What could a possible holographic dual look like? A finite dimensional quantum mechanical system?

What determines the initial conditions in the past? Of should we impose boundary conditions in the future?

7. Where are the microscopic degrees of freedom which make up a black hole?

Do they have interesting gravitational features (multipole moments)? Can they act as black hole mimickers?

Or are they indistinguishable for low-energy observers as maximal chaos and statistical physics intuition would suggest?

8. Are there observational implications of a theory of quantum gravity?

Show that the graviton has a wave-function? Just tests semi-classical gravity not quantum gravity?

Is UV completeability an important constraint on observations (the “swampland” program)?

Is there something like quantum gravitational noise which is observable?

What role do observations and decoherence play for possible observable implications?