UNIVERSITY OF ALBERTA

The problem of time in quantum cosmology with a quantum clock

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Outline

The Problem of Time

- 1st class constraints, e.g., $\mathcal{C}(q, p) = 0$, generate gauge transformations

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- In the quantum regime: $\hat{H}\Psi=0 \Rightarrow$ no Schrodinger equation with $\frac{\partial\Psi}{\partial t}$ on the RHS

Conditional Probability Interpretation

- Consider quantities T **(clock)** and Q **(interested to find its evolution)**: conditional probability of $Q = Q_0$ happening given that $T = T_0$ has happened is [Myers, 2006]

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- **Wrong propagator** of a single-particle: the Page-Wooters formalism yields **no motion**! [Kuchar, 2011]

$$\langle \psi_{x,T} \mid \psi_{x',T'} \rangle = \left| \delta \left(T - T' \right) \delta \left(x - x' \right) \right|^2$$

instead of

$$\langle \psi_{x,T} \mid \psi_{x',T'} \rangle = \left[\frac{2\pi i \left(T - T' \right)}{m} \right]^{-1/2} \exp \frac{im \left(x - x' \right)^2}{2 \left(T - T' \right)}$$

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- **Violation of the Constraints**: $\hat{\mathcal{P}}_{Q_0}$, $\hat{\mathcal{P}}_{T_0}$ do not commute with $H \Rightarrow$ leaving constraint surface after acting

Modified Conditional Probability Interpretation

- Choose Q, T to be **evolving constants of motion**, instead of values of fields which in a totally constrained systems are not physically observable [Gambini, Porto, 2001; Gambini, Porto, Pullin, Torterolo, 2009]

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- Notice the **difference in numerator**!

$$\hat{\mathcal{P}}_{m_T}(t)\hat{\rho}\hat{\mathcal{P}}_{e_2}(t)\hat{\mathcal{P}}_{m_T}(t) \quad \text{and not} \quad \hat{\mathcal{P}}_{e_2}(t)\hat{\mathcal{P}}_{m_T}(t)\hat{\rho}\hat{\mathcal{P}}_{m_T}(t)$$

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- Remedies the issues of Page-Wooters formalism

Outline

The Model [Gambini, Rastgoo, Roberts, *to appear in 2023*]

FLRW cosmology

$$ds^2 = -dt^2 + a^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right)$$

with two scalar fields ϕ_i, $i = 1, 2$ with Hamiltonian constraint

$$\mathcal{H} = -\frac{6}{\gamma^2}c^2\sqrt{|p|} + \frac{8\pi G}{|p|^{\frac{3}{2}}}\sum_{i=1}^{2} p_{\phi_i}^2$$

where

$$c = \gamma\dot{a}, \qquad |p| = a^2$$

Dirac Observables

From two of EoM

$$\dot{p}_{\phi_i} = \{p_{\phi_i}, N\mathcal{H}\} = 0, \qquad i = 1, 2.$$

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Defining

$$\Pi_1 = -\,\phi_1, \qquad\qquad \Pi_2 = -\,\phi_2$$

leads to a 4D phase space

$$\{O_i, \Pi_j\} = \delta_{ij}, \qquad i, j = 1, 2$$

on which we can define two more Dirac obervables...

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and another ECM (the one measured against the clock)

$$E_2(t) := p_{\phi_1} p_{\phi_2} \ln\left(|p|\right) = \beta\sqrt{O_1^2 + O_2^2}\left(O_2\Pi_1 + O_1 O_2 t\right)$$

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Schrödinger representation

$$T \rightarrow \hat{T}, \qquad E_2 \rightarrow \hat{E}$$

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  - Also issues with defining the conditional probability

Self-adjointness

- **Unbounded symmetric** operator $\hat{A}$ is **self-adjoint iff** its **deficiency indices**

$$n_{\pm}\left(\hat{A}\right) := \dim\left[\ker\left(\hat{A}^{*} \pm i\hat{I}\right)\right]$$

obey

$$n_{+} = 0 = n_{-}$$

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- Turns out **both** $\hat{E}_1$, $\hat{E}_2$ **are self-adjoint**

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Conditional probability of $E_2 \in \left[e_2^{(1)} = e_2^{(0)} - \Delta e_2, e_2^{(2)} = e_2^{(0)} + \Delta e_2\right]$ given that $T = T_0$, for a density operator $\hat{\rho}$:

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The density operator $\hat{\rho} = |\psi_\rho\rangle\langle\psi_\rho|$ localized in configuration space

$$|\psi_\rho\rangle = \int_{-\infty}^{\infty} dO_1 \int_{-\infty}^{\infty} dO_2\, N_\rho\, \Theta\left(O_1 - o_1^{(1)}\right)\Theta\left(o_1^{(2)} - O_1\right) \times \\ \Theta\left(O_2 - o_2^{(1)}\right)\Theta\left(o_2^{(2)} - O_2\right)|O_1, O_2\rangle$$

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At the moment

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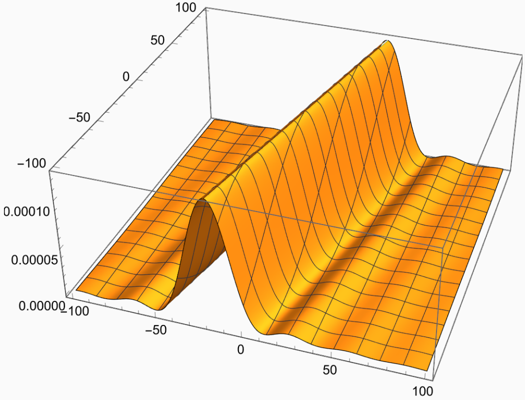
- Numerator obtained it under certain approximations (under construction)

The Probability

The **very preliminary!** and aproximate result:



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- Stay tuned for the paper coming out soon!