Observables and dynamical frames in gravity

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based on: Goeller, PH, Kirklin 2206.01198; Carrozza, Eccles, PH 2205.00913

The challenge of observables in gravity

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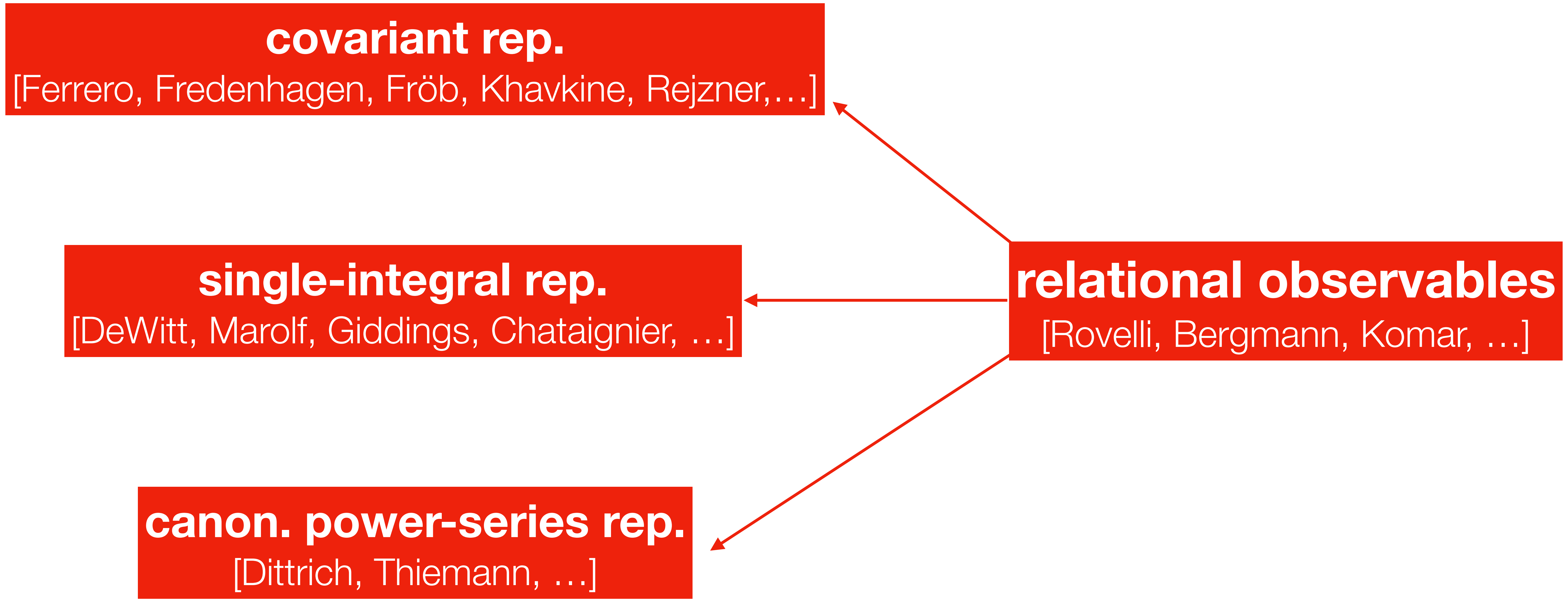

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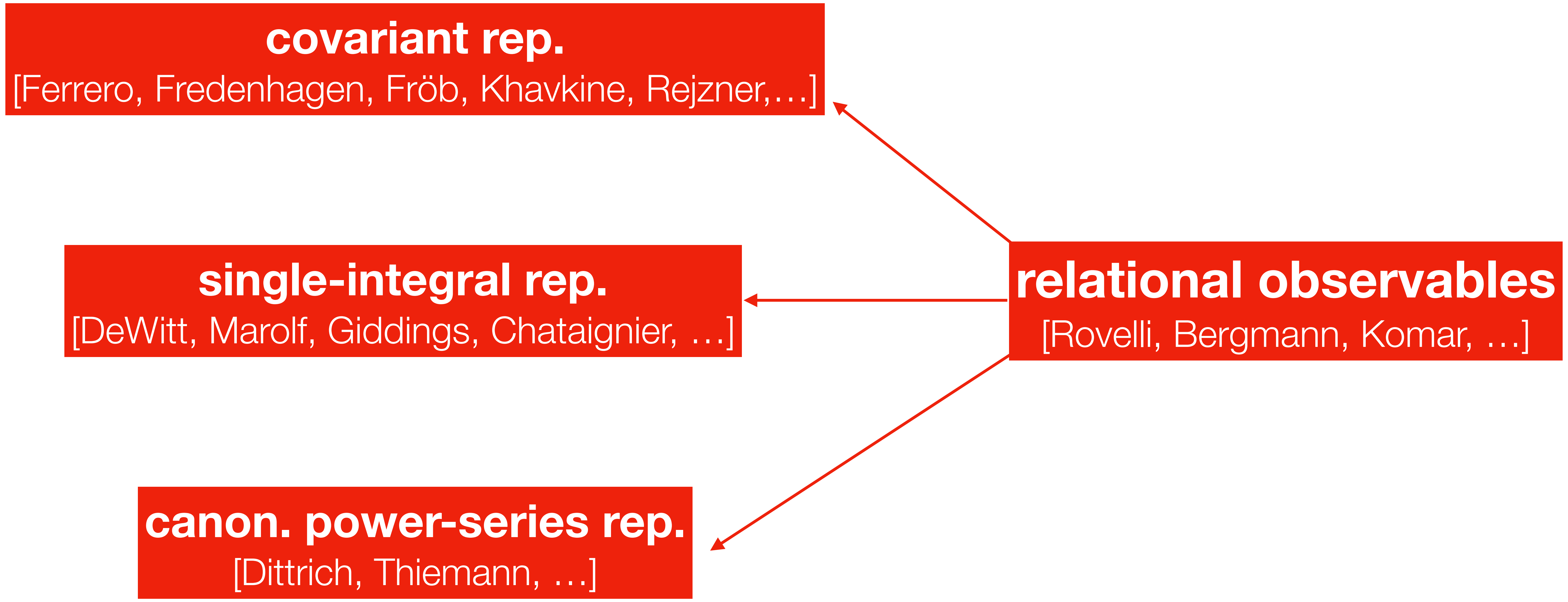

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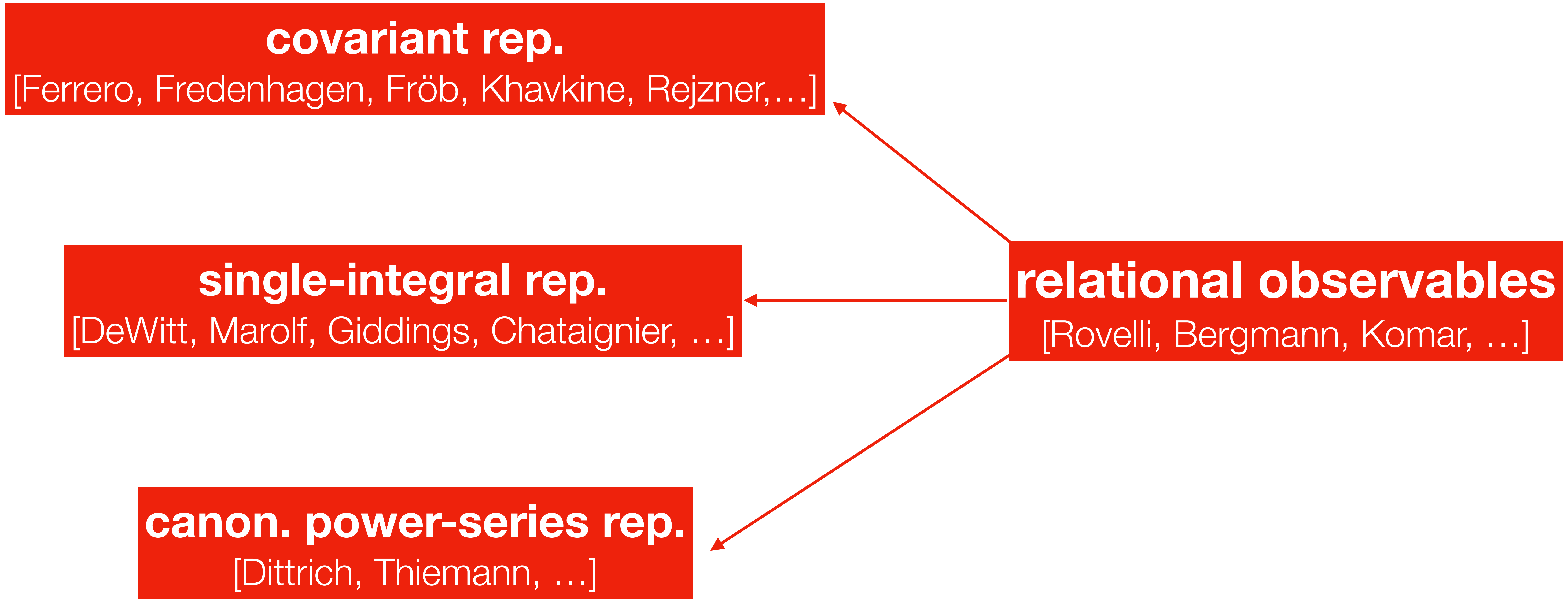

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Relation? ⇒ One approach for all?

- **relational ideas involved ⇒ can one clarify link to dynamical/quantum frame program?**

⇒ can we formulate general covariance in terms of observables?

Gauge-invariant observables & locality in gravity

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will not give up gauge-invariance, but adjust notion of locality

⇒ notion of locality that fails is one based on fixed, non-dynamical — and hence unphysical — reference frames

Dynamical reference frames in gravity

> “The theory.... introduces two kinds of physical things, i.e., (1) measuring rods and clocks, (2) all other things, e.g., the electro-magnetic field, the material point, etc. This, in a certain sense, is inconsistent; strictly speaking measuring rods and clocks would have to be represented as solutions of the basic equations..., not, as it were, as theoretically self-sufficient entities...”

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- **internal frames (tetrad) in SR: different in- and output spaces:**

Minkowski

$$e^\mu_a : \mathscr{M} \to \mathscr{O}$$

frame orientation space

$e^\mu_a \in \mathrm{O}(3,1)$ **group valued frame**

  - **“gauge transformations”:** $\Lambda^\mu{}_\nu \, e^\nu_a$ $\quad \Lambda^\mu{}_\nu \in \mathrm{SO}_+(3,1)$

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spacetime

- **dynamical/internal frames in gravity: different in- and output spaces** $\mathscr{R}^{-1}[\phi] : \mathscr{M} \to \mathscr{O}$ ← frame orientation space

  $\mathscr{R}^{-1}[\phi] \in \mathrm{Diff}(\mathscr{M}, \mathscr{O})$ **may be “group valued frame”** **dynamical coord. system**

  - **gauge transformations:** $\mathscr{R}^{-1}[f_*\phi] = \mathscr{R}^{-1}[\phi] \circ f^{-1}$ $f \in \mathrm{Diff}(\mathscr{M})$ **frame a set of field-dep. scalars**

Dressed observables in a nutshell

[Giddings, Donnelly, Harlow, Shenker, Stanford, …]

Aim: given some “naked” (non-invariant) field quantity, “dress” it with suitable DoFs to composite operator that *is* invariant

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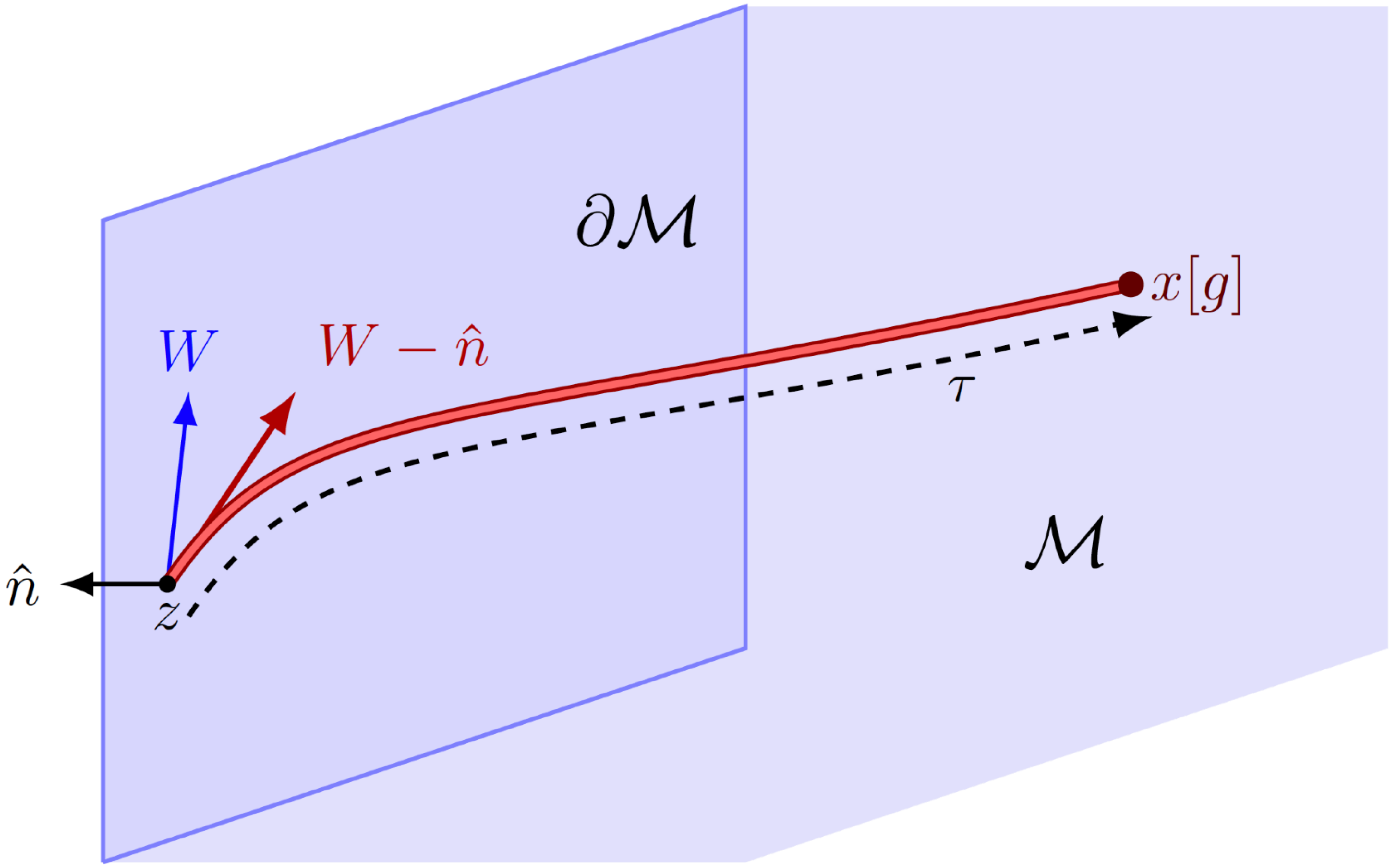

⇒ E.g. shoot geodesic in from bdry: $x = x_{\tau,z,W}[g]$

⇒ transforms covariantly: $x_{\tau,z,W}[f_*g] = f(x_{\tau,z,W}[g])$

so, e.g. $O_{\varphi,x}[\phi] = \varphi(x_{\tau,z,W}[g])$ **is gauge-inv.**

dressed observable: what's the value of scalar at end of geodesic?

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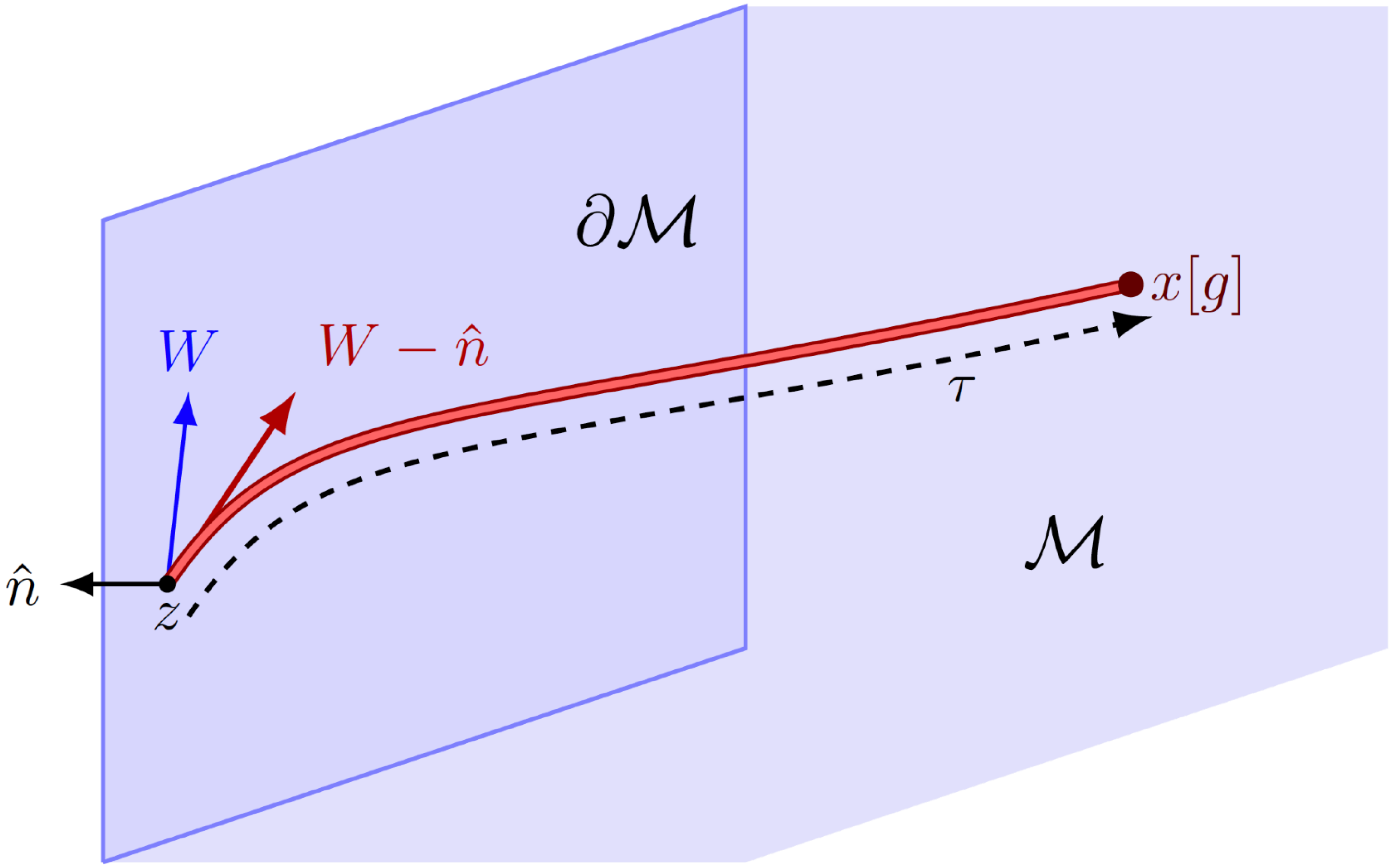

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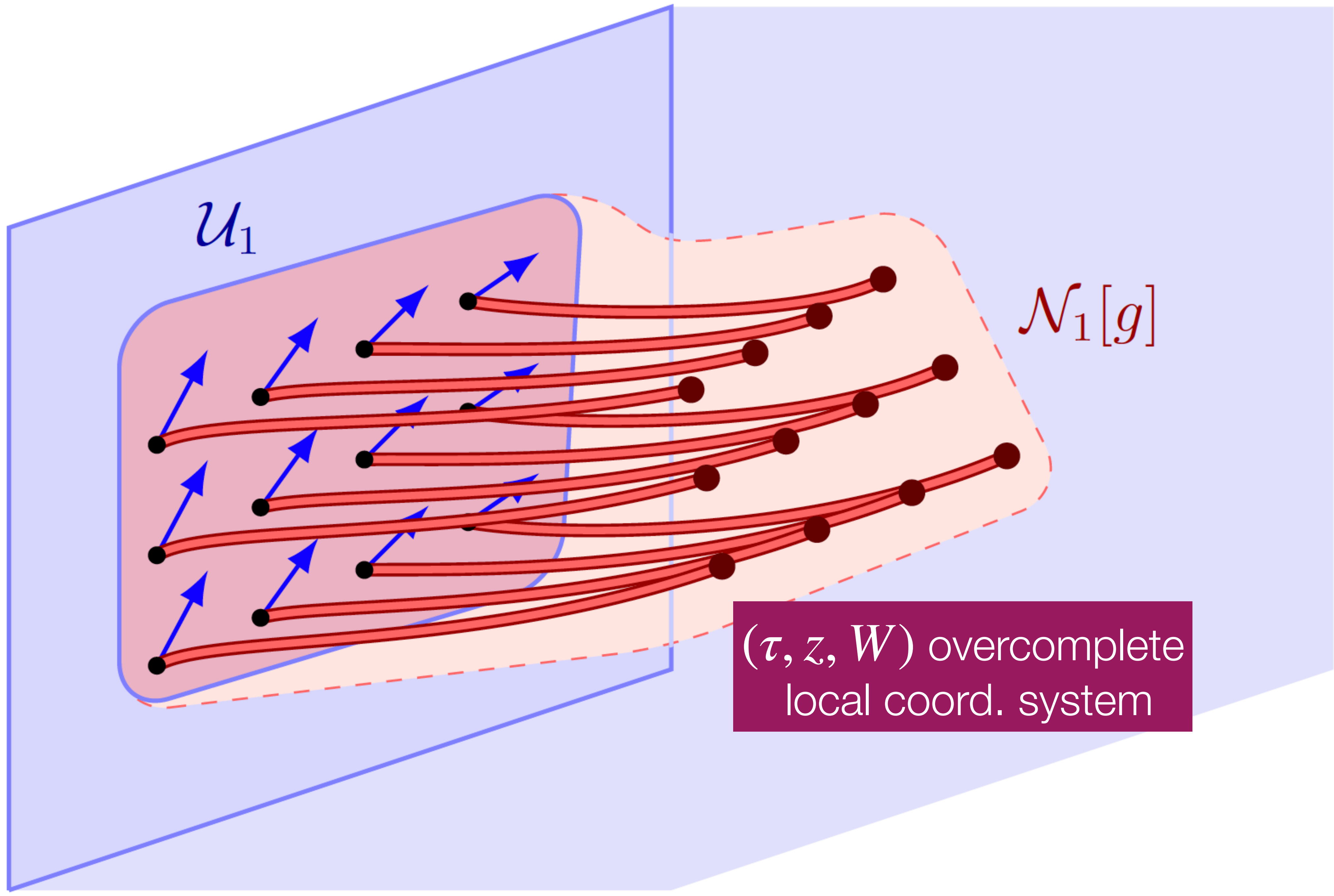

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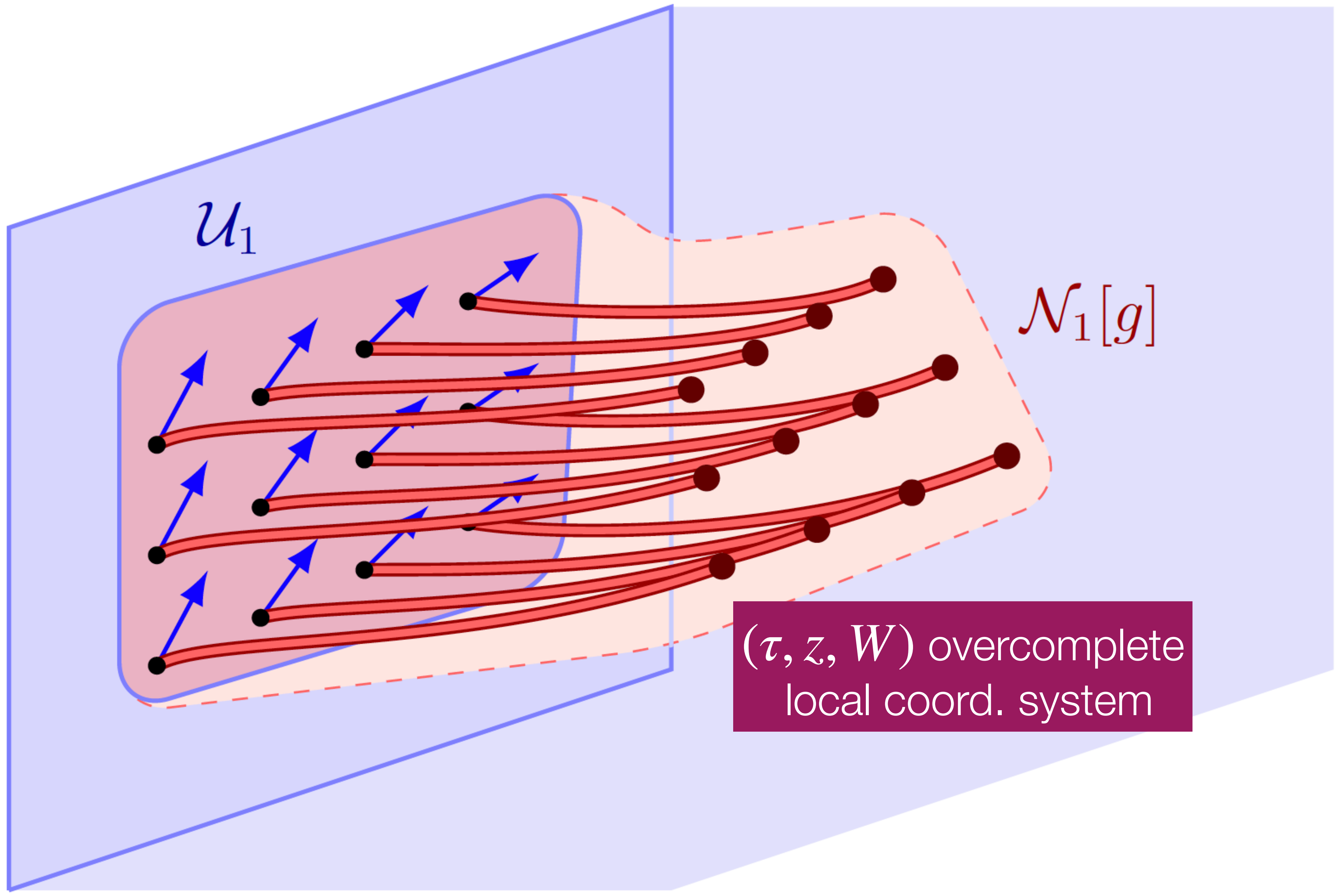

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⇒ restrict to $\mathcal{O}_1 \subset \mathcal{O}$ s.t. injective (e.g. fix bdry vector field W)

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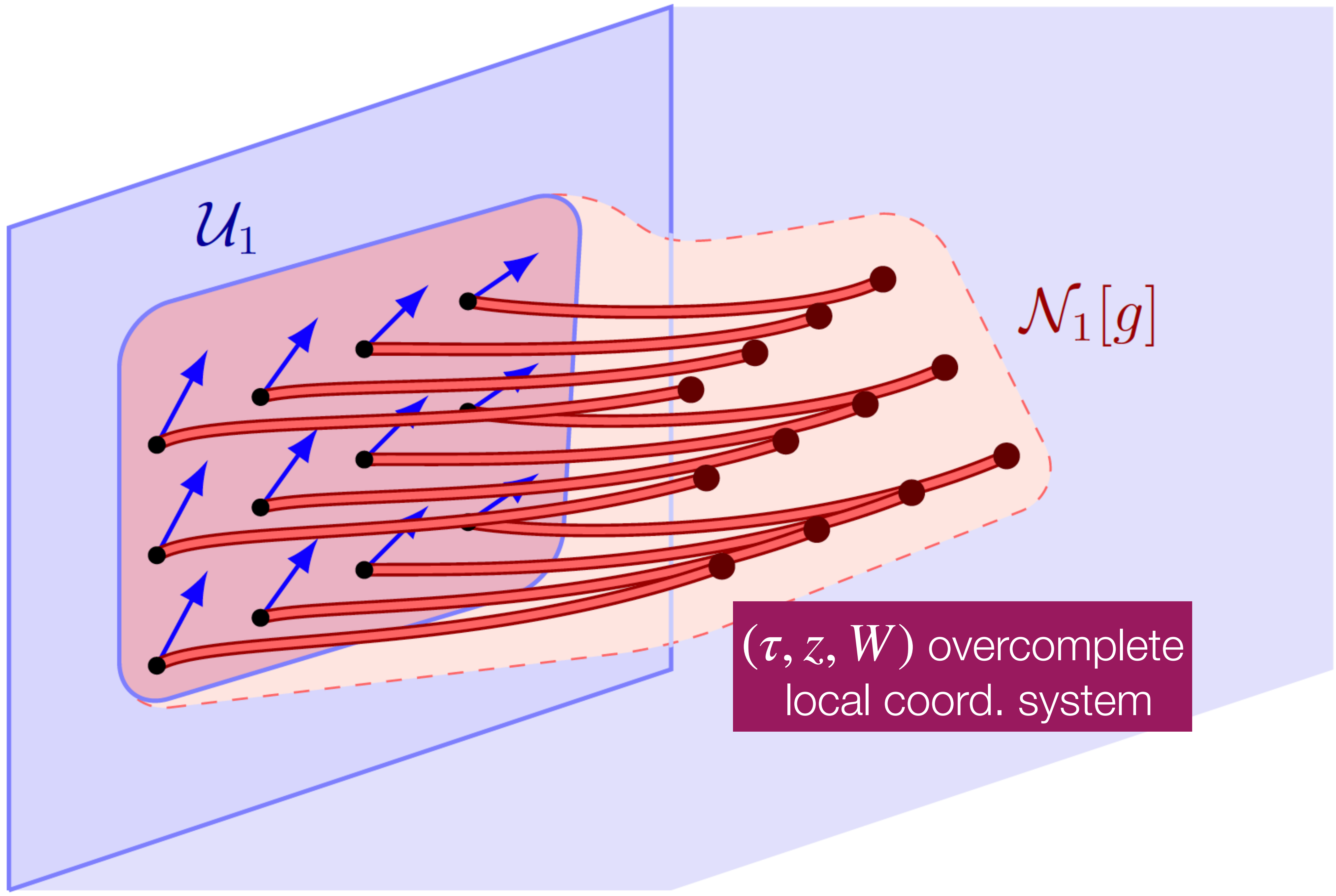

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frame field $\mathscr{R}_1^{-1}[g] : \mathcal{N}_1[g] \subset \mathscr{M} \to \mathscr{O}_1$ $x \mapsto (T(x), Z^k(x))$ **reference scalars/dyn. coords**

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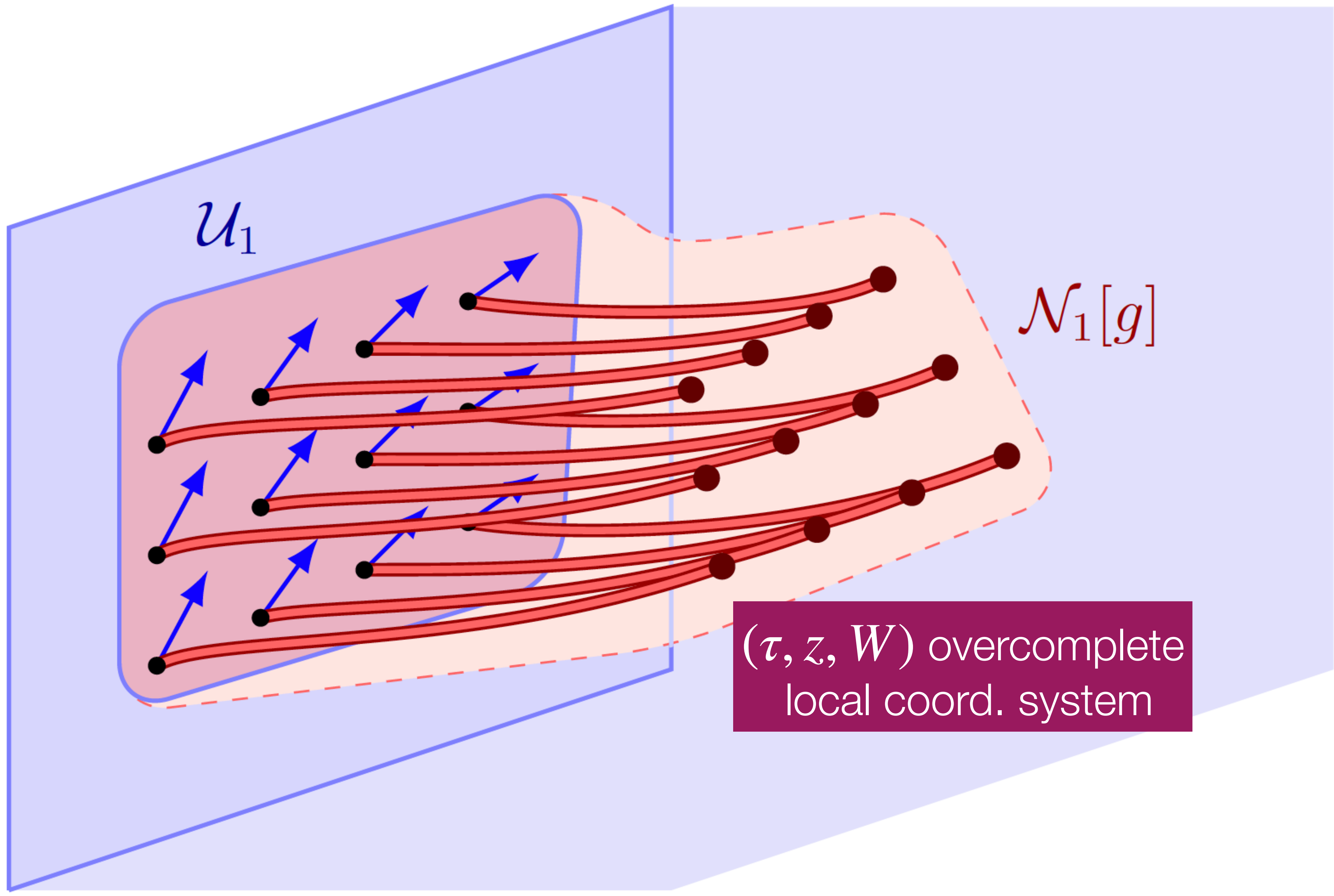

(τ, z, W) overcomplete local coord. system

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dressed observable: what's the value of scalar at end of geodesic?

=

rel. obs.: what's the value of φ where frame in local orientation τ, z?

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Covariant relational observables

[Goeller, PH, Kirklin ’22]

Aim: localize non-inv. quantities relative to reference scalar fields built from field content

$\Rightarrow$ **some gauge cov. frame** $\mathscr{R}^{-1}[f_*\phi] = \mathscr{R}^{-1}[\phi] \circ f^{-1}$ **(typically locally built from matter)**

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observable on the local frame orientation space $\mathscr{O}$

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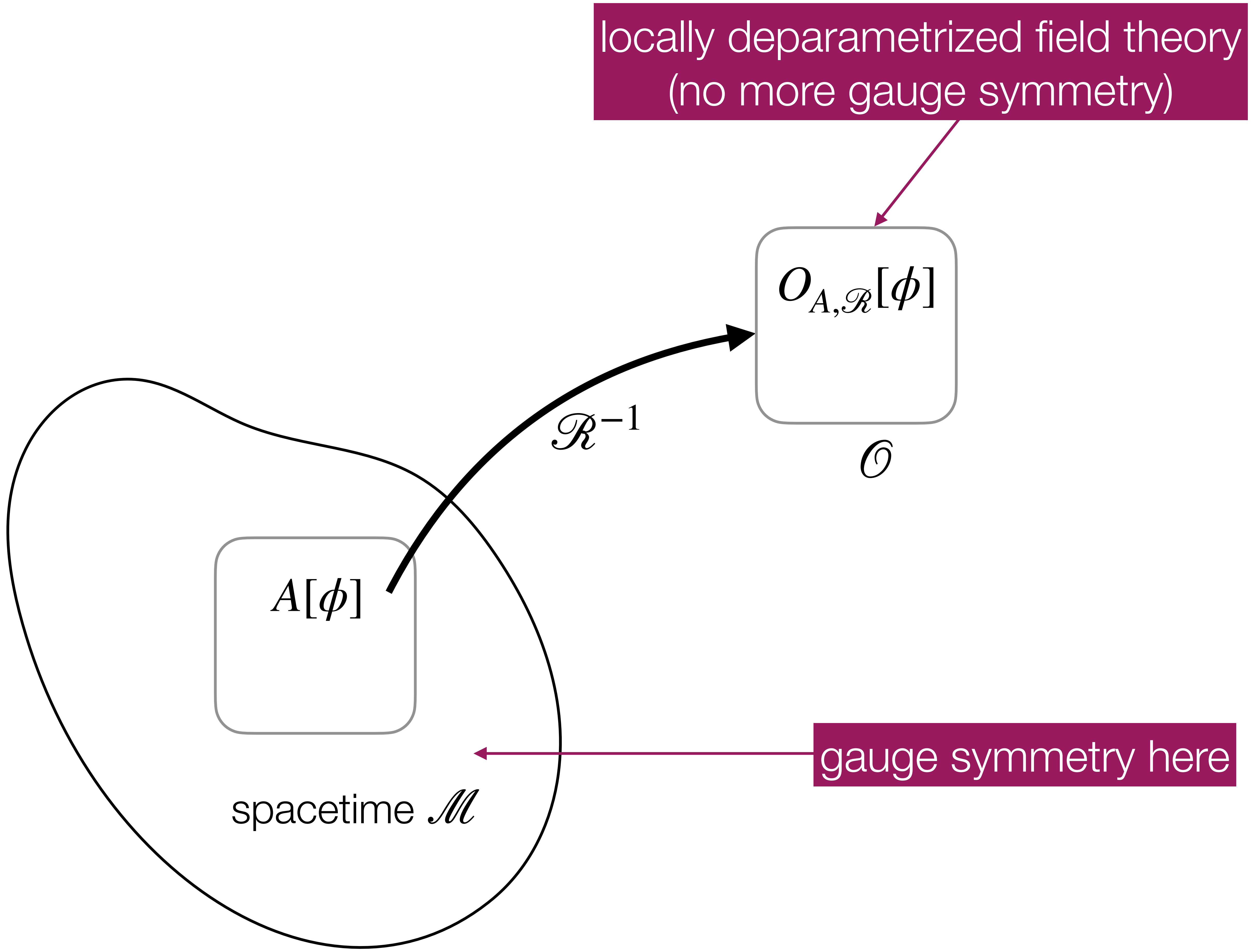
relational observable

answers "what is the value of (certain component of) A at the event in spacetime, where the frame field $\mathscr{R}^{-1}$ is in local orientation $o \in \mathscr{O}$?"



locally deparametrized field theory (no more gauge symmetry)

gauge symmetry here

dressed = covariant relational observables

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dressed and cov. rel. obs are equivalent/unified
if frame (scalar) fields allowed to be general
(so allowed to be built locally or non-locally from matter or metric)
⇒ equips dressed observable with clear interpretation

Single-integral relational observables

[DeWitt, Marolf, Giddings, Chataignier, …]

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Toy example: Z**-model** [Giddings, Marolf, Hartle '06] **4 scalar reference fields** Z^k **parametrizing spacetime**

$$O_{\varphi,x}[\phi] = \int_{\mathscr{M}} d^4y \sqrt{|g|}\, \varphi(y)\, \delta^4(Z^k(y) - \xi^k) \left| \frac{\partial Z}{\partial y} \right|$$

relational observable

answers "what is the value of φ at the event $x[\phi]$ in spacetime, where the reference fields take values ξ^k?"

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relation to cov. rep.?

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relation to cov. rep.?

set $\mathscr{R}^{-1}[\phi] = Z$ **and** $\xi = o$

Single-integral relational observables

[DeWitt, Marolf, Giddings, Chataignier, …]

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equivalent to our construction

rel. observable "what's the value of scalar at event where frame field is in orientation o?"

[Goeller, PH, Kirklin '22]

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single-integral and covariant relational observables equivalent

Power series representation of relational observables

[Dittrich, Thiemann, …]

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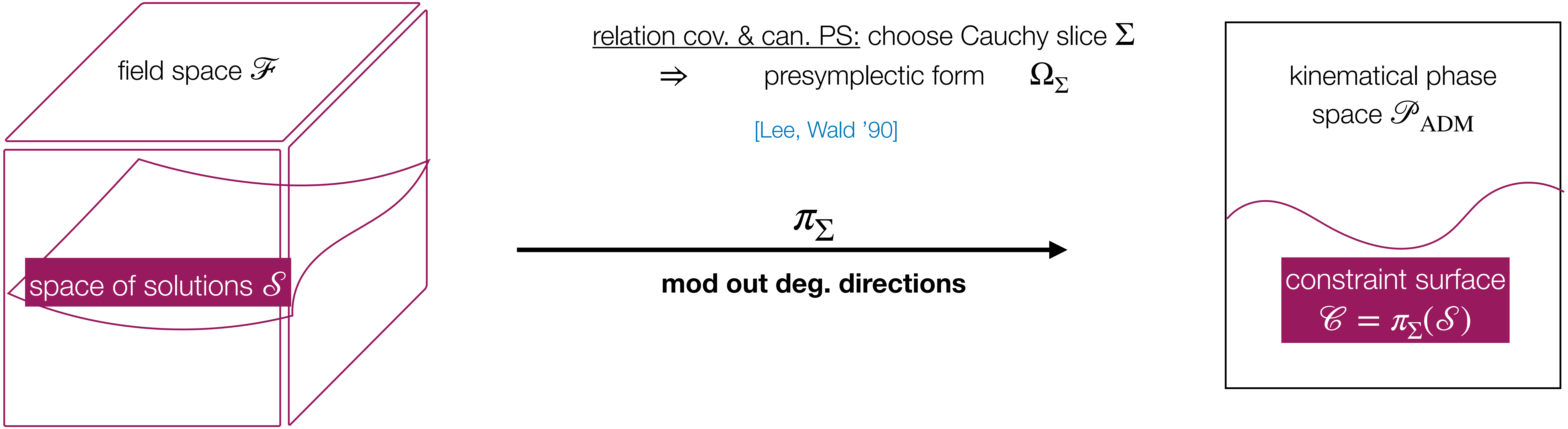

field space $\mathscr{F}$

space of solutions $\mathscr{S}$

relation cov. & can. PS: choose Cauchy slice Σ

$\Rightarrow$ presymplectic form Ω_Σ

[Lee, Wald '90]

$$\xrightarrow{\pi_\Sigma}$$

mod out deg. directions

kinematical phase space $\mathscr{P}_{\text{ADM}}$

constraint surface $\mathscr{C} = \pi_\Sigma(\mathscr{S})$

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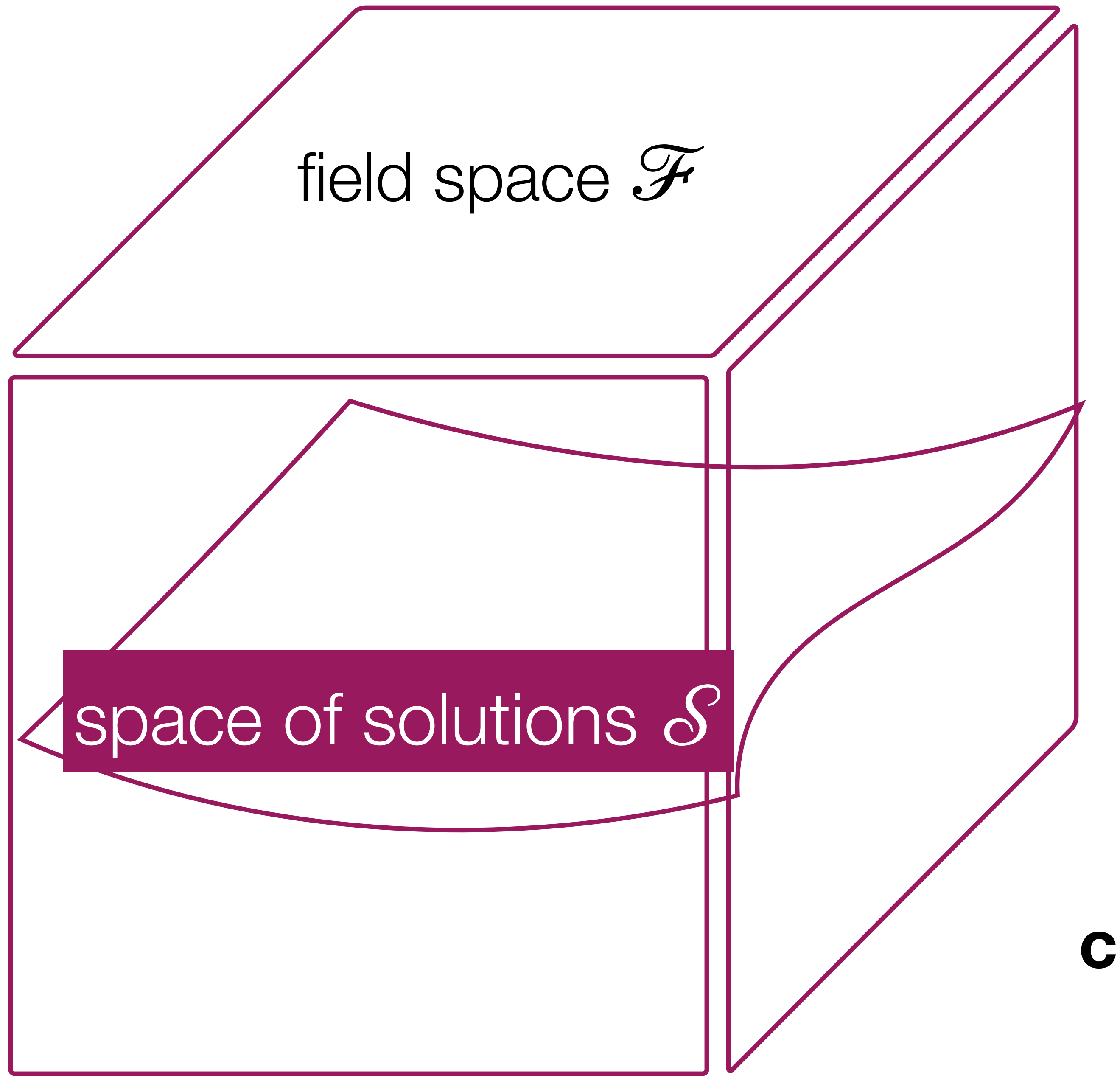

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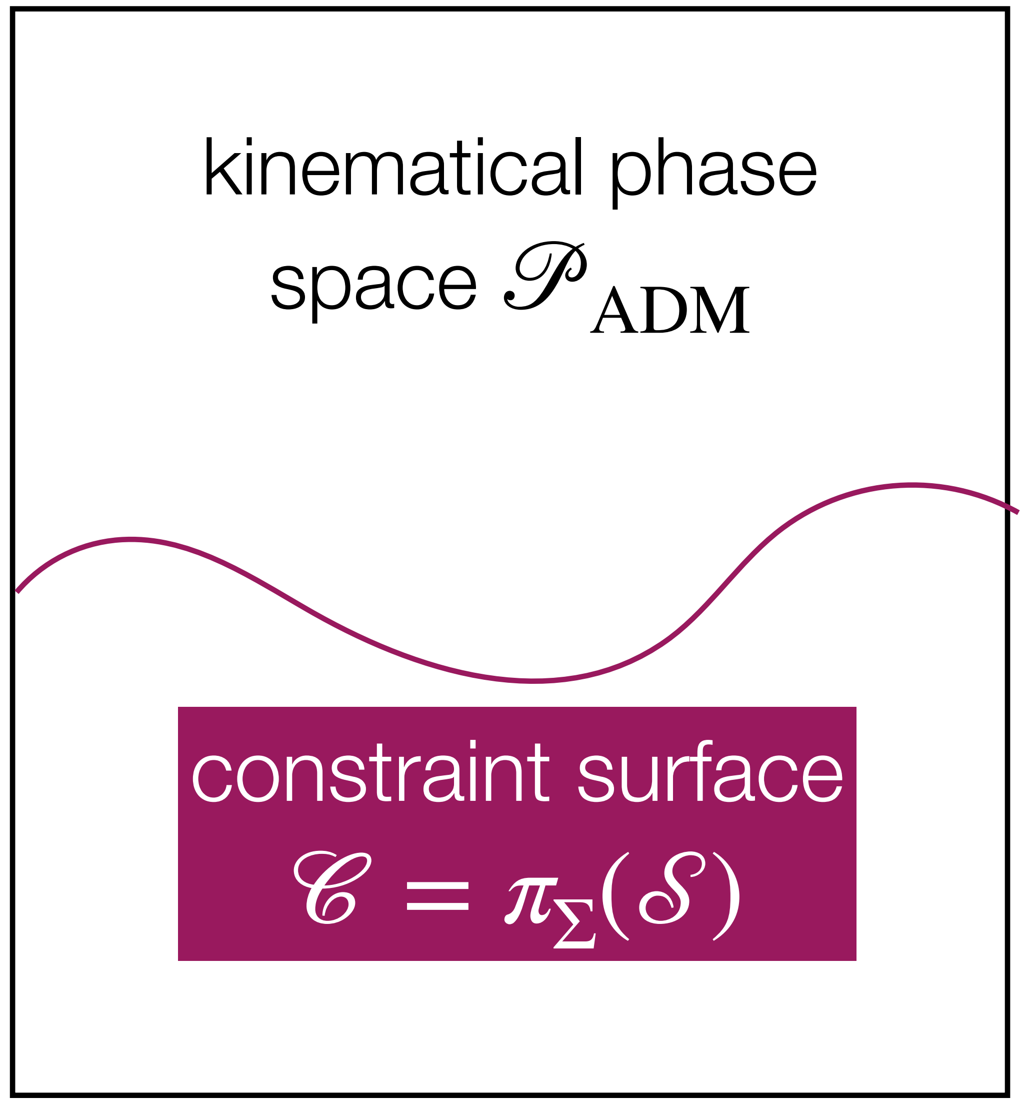

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$$O_{A,R}[\phi] = \tilde{O}_{A,R} \circ \pi_\Sigma[\phi]$$

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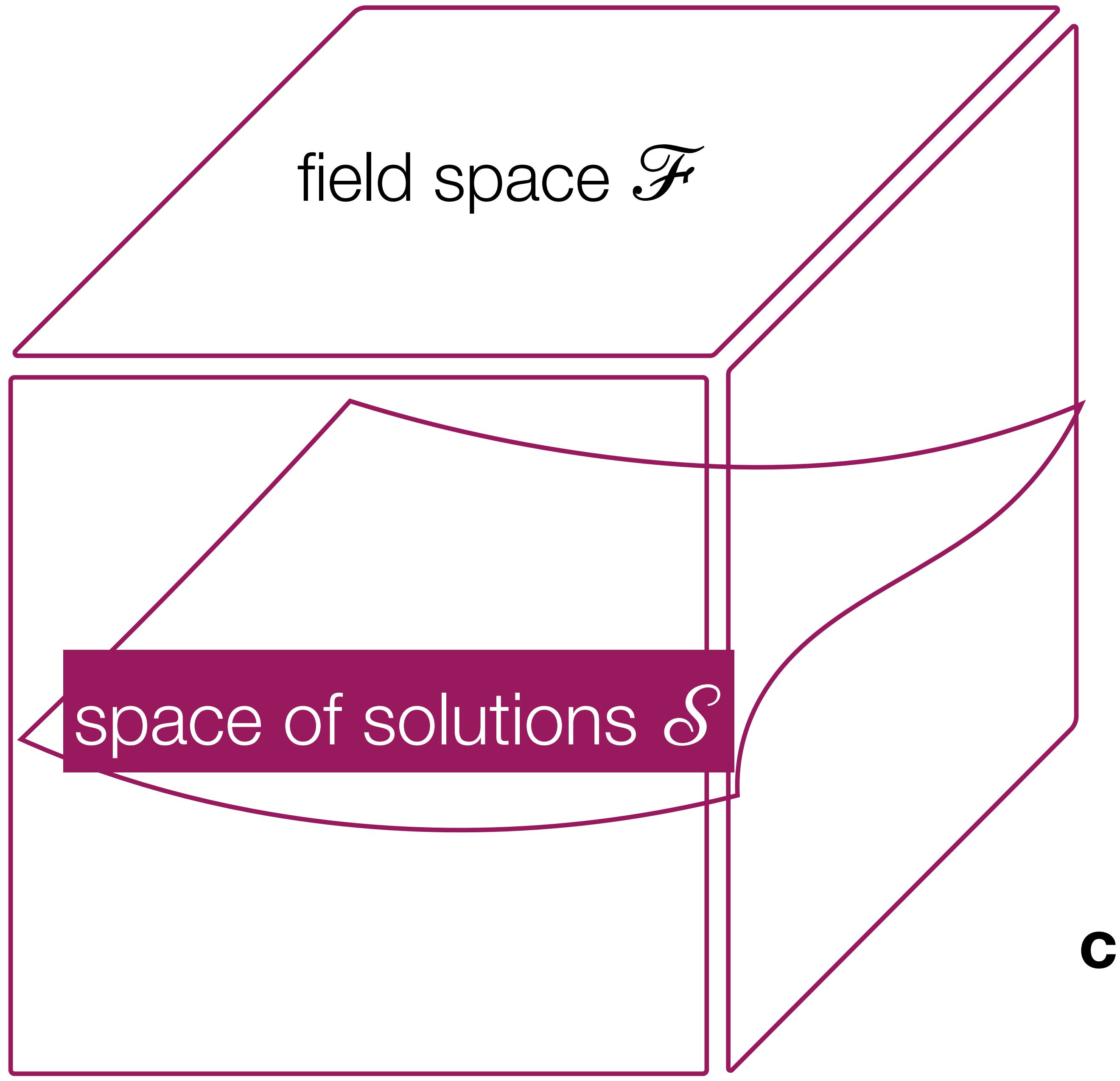

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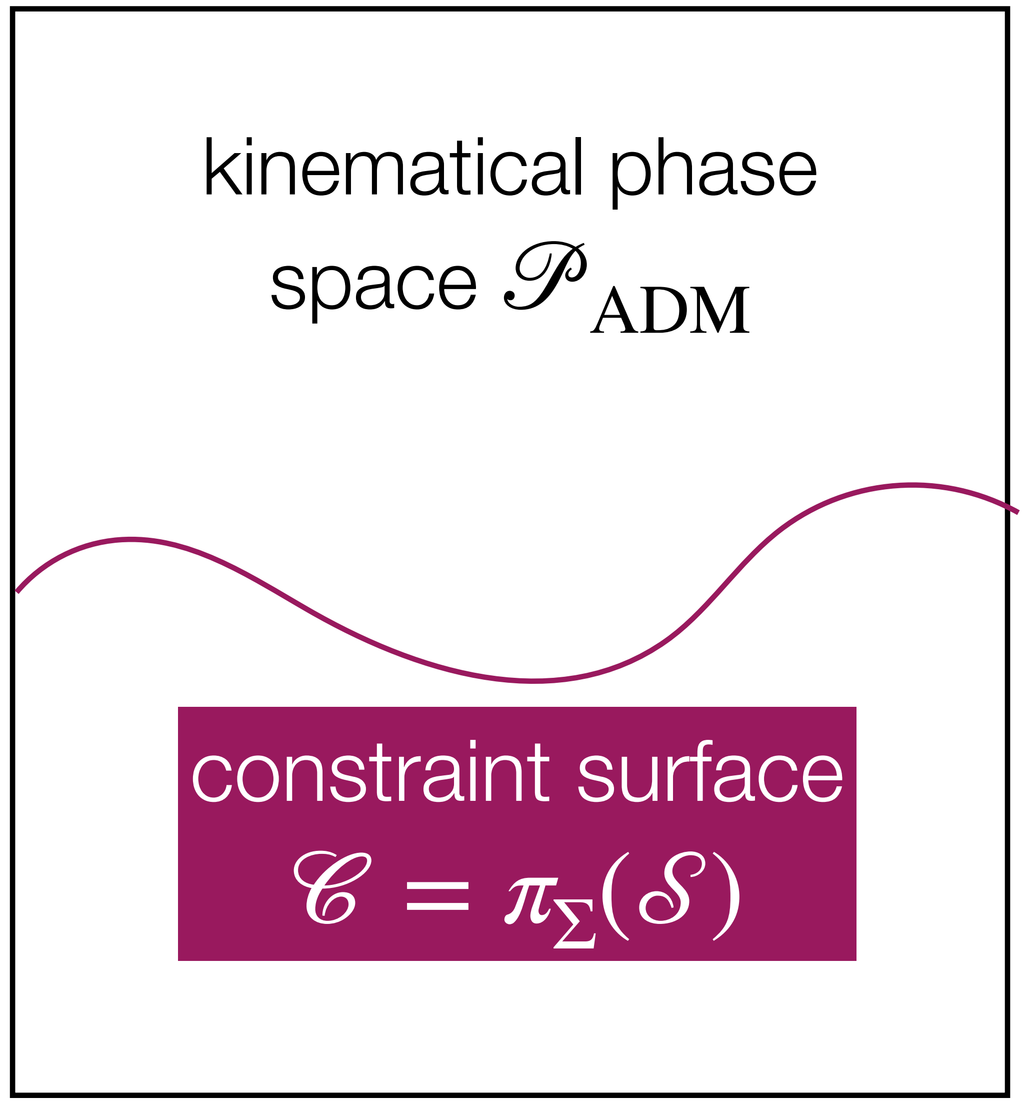

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under certain restrictions, canonical one can be written as power series:

$$\tilde{O}_{A,R} \approx \sum_{n=0}^{\infty} \frac{1}{n!} \{C[u], \tilde{A}\}_n$$

[Goeller, PH, Kirklin '22]

only on $\mathscr{C}$

smeared ADM constraints

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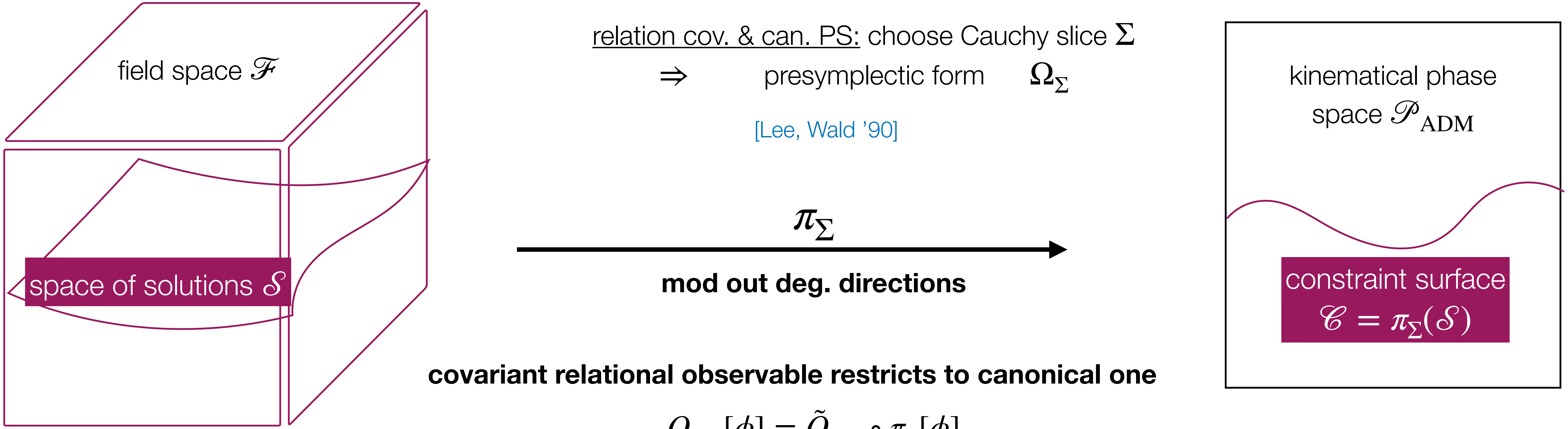

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cov. & power series reps equivalent (under certain restrictions)

Quasilocal generalizations

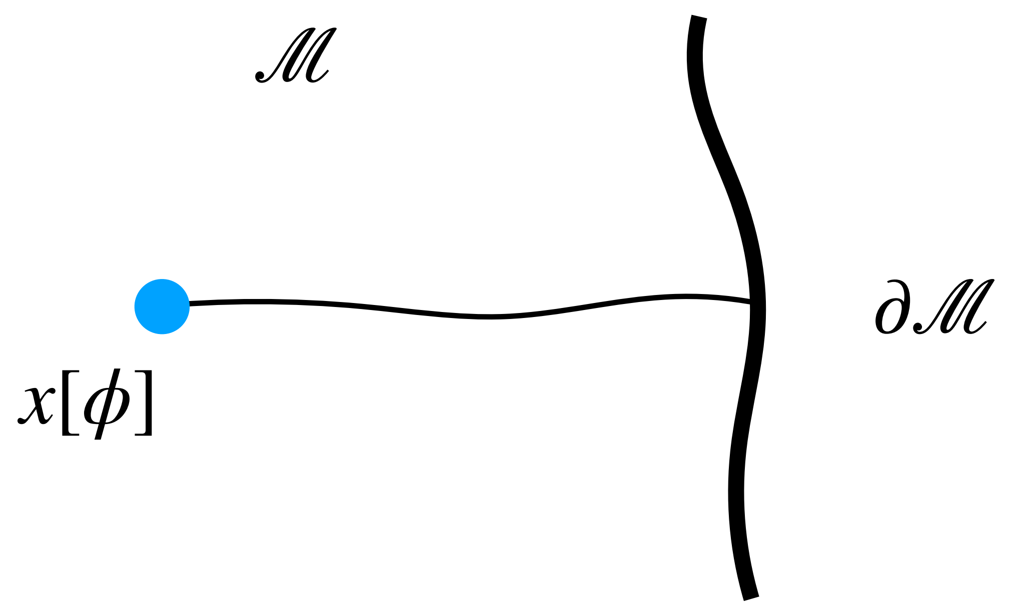
[Goeller, PH, Kirklin ’22]

space of solutions

so far dyn. frames for parametrizing events: $x : \mathscr{S} \to \mathscr{M}$

gauge covariance $x[f_* \phi] = f(x[\phi])$

$\Rightarrow$ **turn local cov. quantity into relational observable, e.g.** $\varphi(x[\phi])$



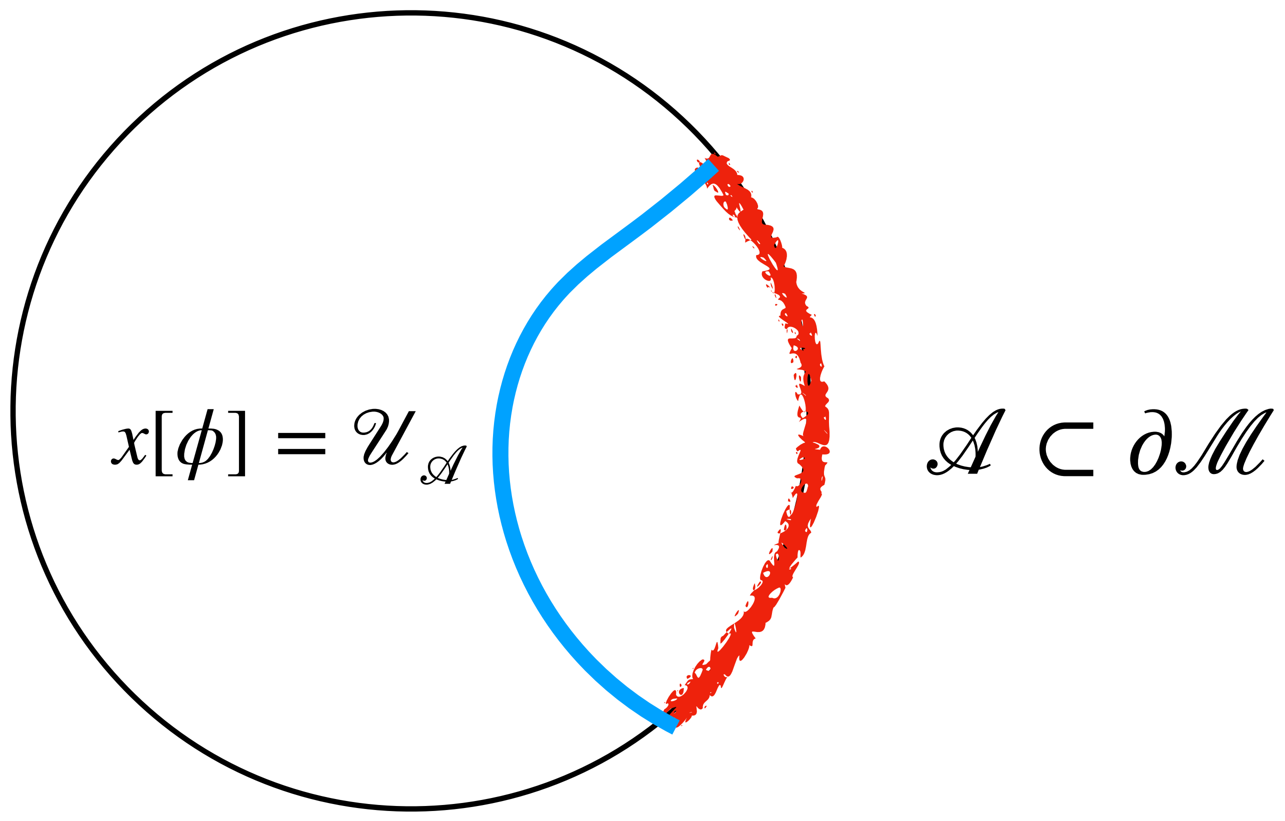
e.g. boundary anchored geodesic frame

can generalize to parametrizing extended objects: $x : \mathscr{S} \to \mathscr{K}$

e.g. space of d-dim submanifolds

$\Rightarrow$ **turn cov. quantities on** $\mathscr{K}$ **into relational observables, e.g.** $\mathrm{Vol}_{\mathscr{U}\subset\mathscr{M}}(\mathrm{x}[\phi])$

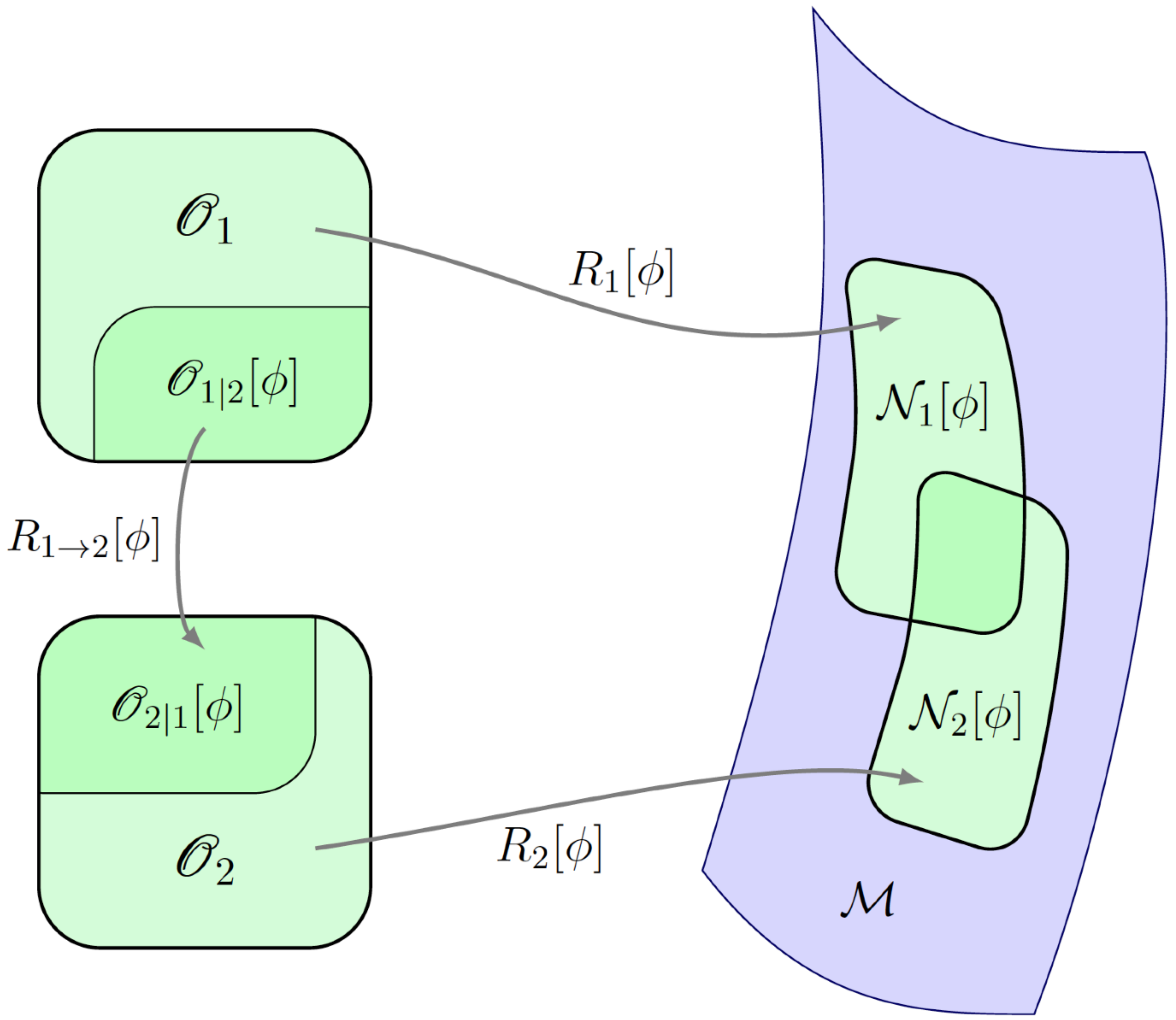
volumes of submanifolds as relational observables



e.g. minimal surfaces in holography

Internal frame changes

[Goeller, PH, Kirklin '22]



restrict to injective frames with overlapping images $\mathcal{N}_1[\phi] \cap \mathcal{N}_2[\phi] \neq \varnothing$

change of frame map: $\mathscr{R}_{1\to 2}[\phi] = \mathscr{R}_2^{-1}[\phi] \circ \mathscr{R}_1[\phi] : \mathscr{O}_1 \to \mathscr{O}_2$

dynamical coord. change

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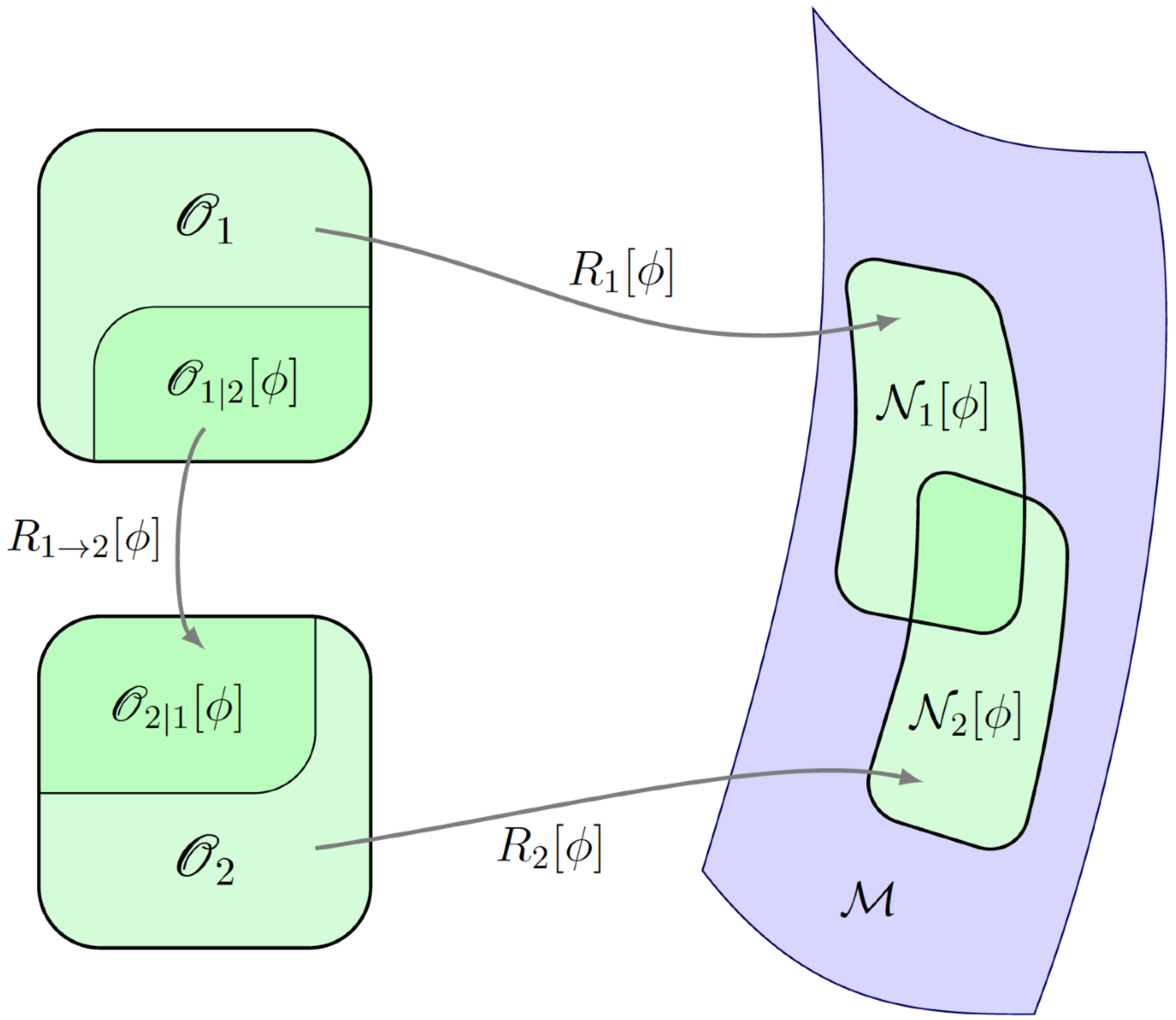

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is rel. observable describing 2nd frame rel. to 1st ⇒ gauge-inv.

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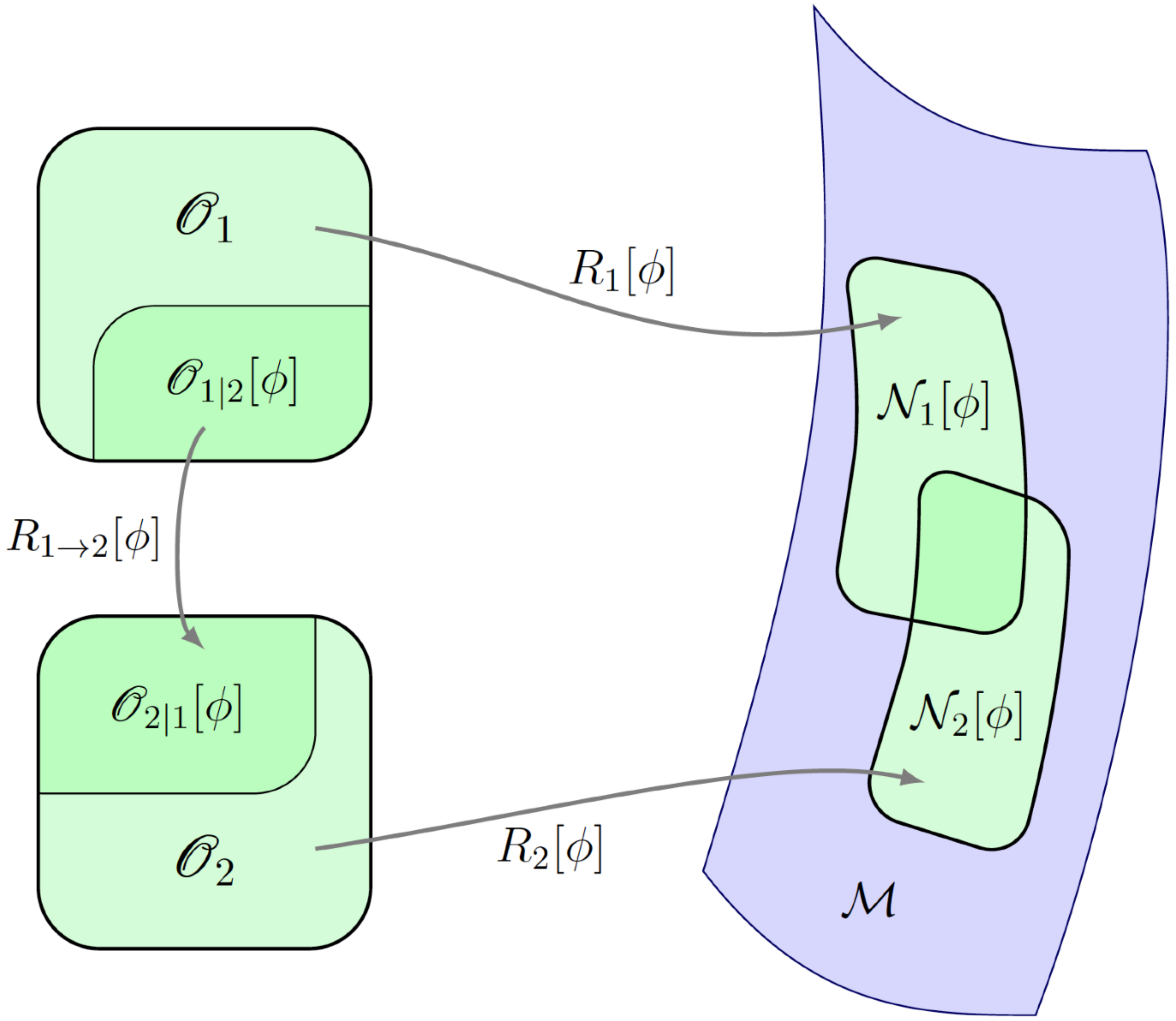

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⇒ **relational observables transform as** $O_{T,\mathscr{R}_2}[\phi] = (\mathscr{R}_{1\to2}[\phi])_* O_{T,\mathscr{R}_1}[\phi]$

change of gauge-inv. description of T from internal perspective of frame 1 into internal perspective of frame 2

Recall: general covariance

“All the laws of physics are the same in every reference frame.”

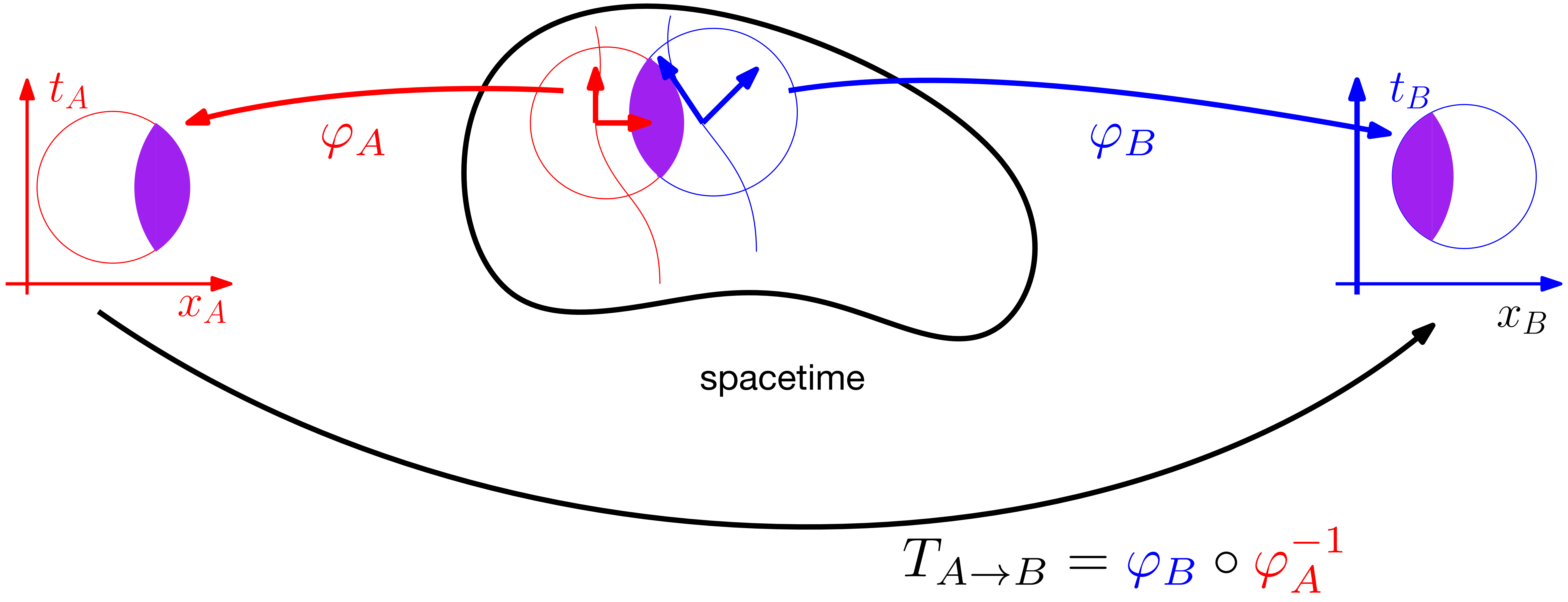
can only compare states and observables in the overlap of two fixed (non-dyn.) coordinate frames

$$E_A[\phi] = 0 \quad \Leftrightarrow \quad E_B[\phi] = 0$$

spaces of solutions (local phase spaces) for the overlap relative to A and B are the same

⇒ tension with gauge symmetry: colloquial statement of general covariance refers to quantities that are not gauge-invariant

coordinate transformation is change/reorientation of external background frame



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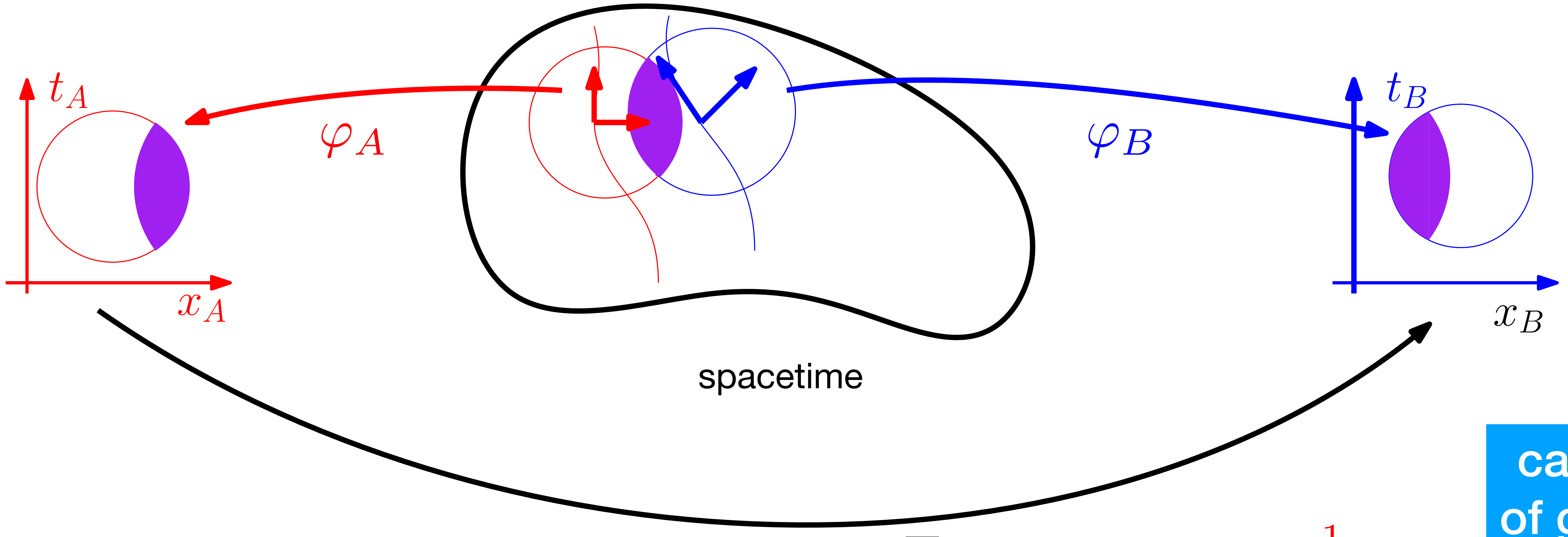

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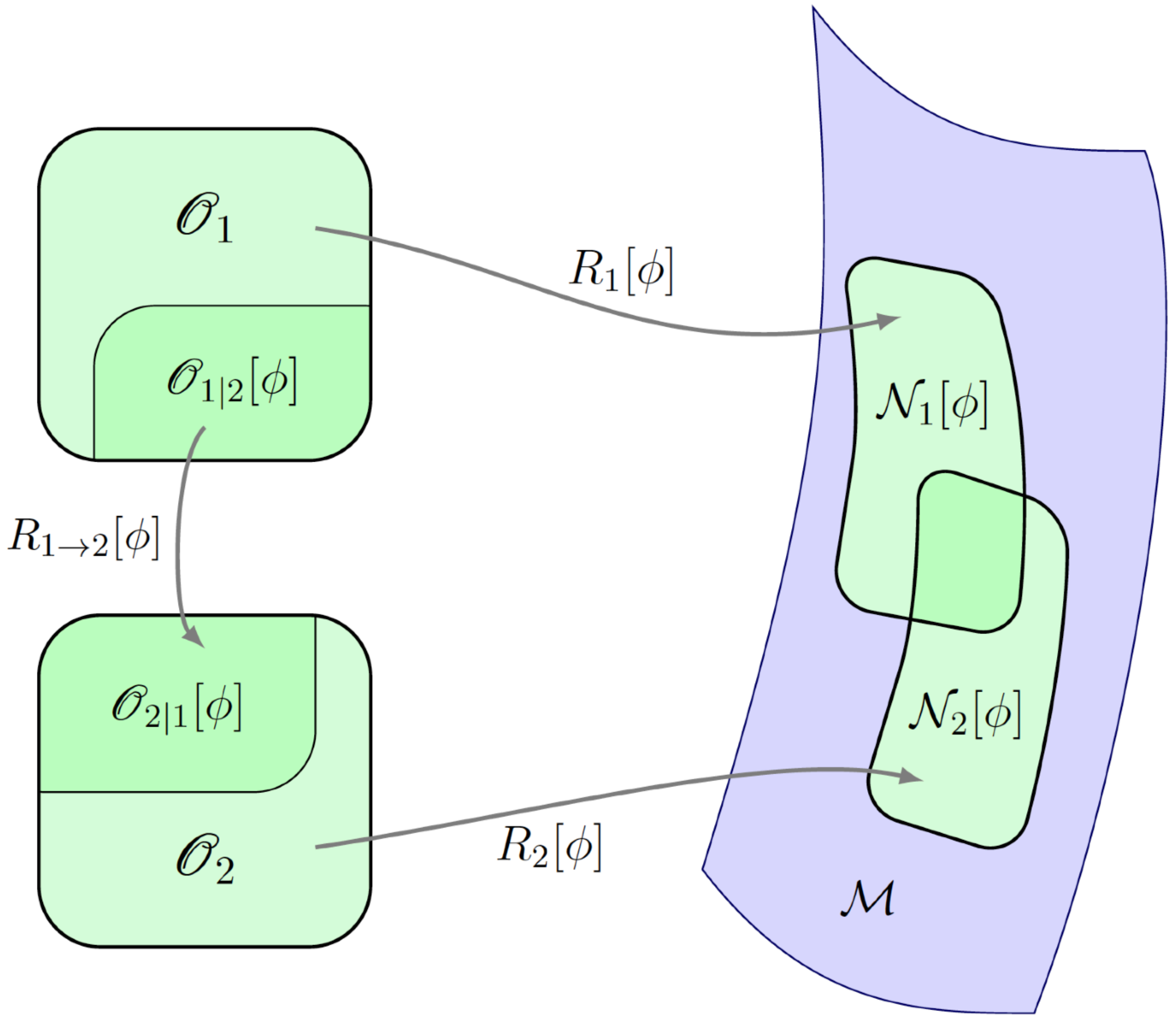
can we have a formulation of general frame covariance that *is* gauge-invariant?

Dynamical frame covariance: a relational update of general covariance

[Goeller, PH, Kirklin ’22]



$$\delta L[\phi] = E[\phi] + d\theta[\phi]$$

EoM term: $E \approx 0$

bdry term

⇒ can map EoM to orientation spaces

⇒ gauge-inv. EoMs for relational fields (in terms of relational observables)

⇒ can show: for gen. cov. Lagrangian $L[f_*\phi] = f_* L[\phi]$

$$E_1[\phi_s] = 0 \quad \Leftrightarrow \quad E_2[\phi_s] = 0$$

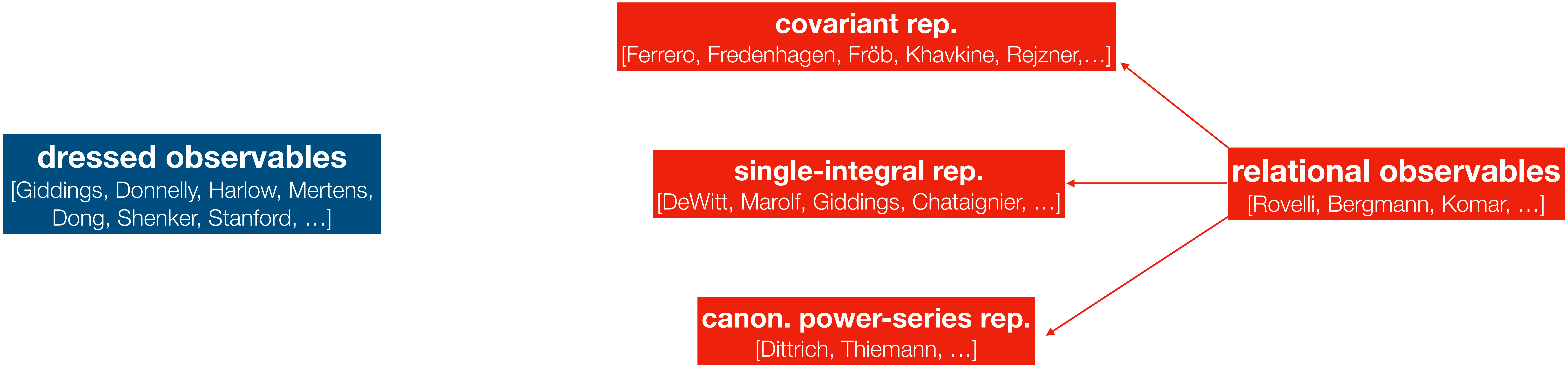
EoMs relative to frames $\mathscr{R}_1$ and $\mathscr{R}_2$

spaces of relational solutions (local physical phase spaces) for the overlap the same

“All the laws of physics are the same in every dynamical reference frame”

Summary

- **one approach for all: dynamical frames help to unify and generalize different approaches to observables in gravity**



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[Ferrero, Fredenhagen, Fröb, Khavkine, Rejzner,…]

dressed observables
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⇒ **suitably extended they are equivalent** (up to fine print for canonical approach)

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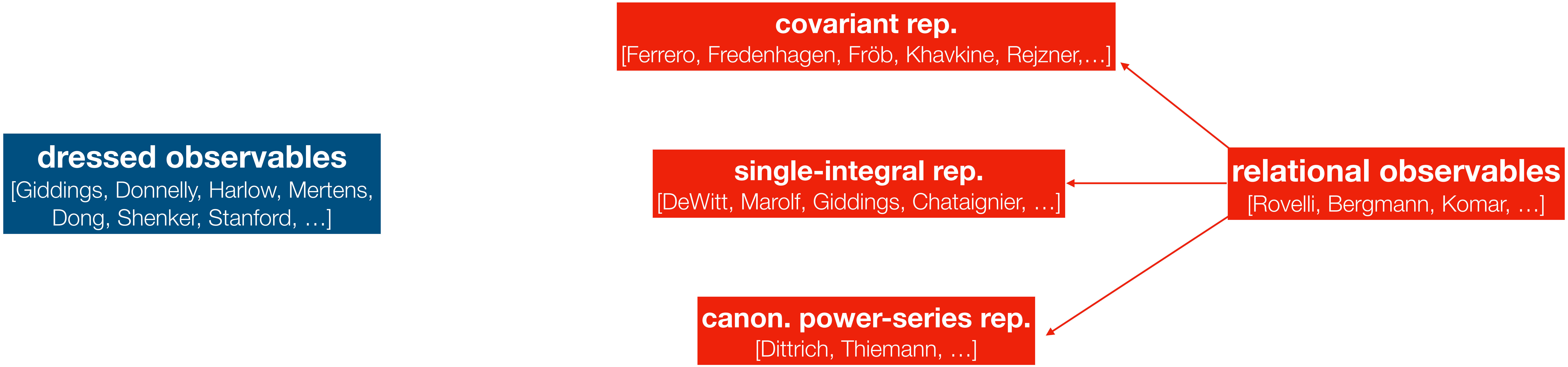

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- **relational observables in QT (depends on approach)**

for QRFs: PH, Smith, Lock ’21; de la Hamette, Galley, PH, Müller, Loveridge ’21
perturbative AQFT: Rejzner, Fröb, ….
asymptotic safety: Baldazzi, Falls, Ferrero ‘21