Classical observables from quantum scattering amplitudes

UCLA Mani L. Bhaumik Institute for Theoretical Physics

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Quantum Gravity, RU, July 11 2023

Based on work in collaboration w/ [Bern, Herrmann, Parra-Martinez, Roiban, A. Smirnov, V. Smirnov, Solon, Shen, Zeng]

Outline

No new result for quantum observables…

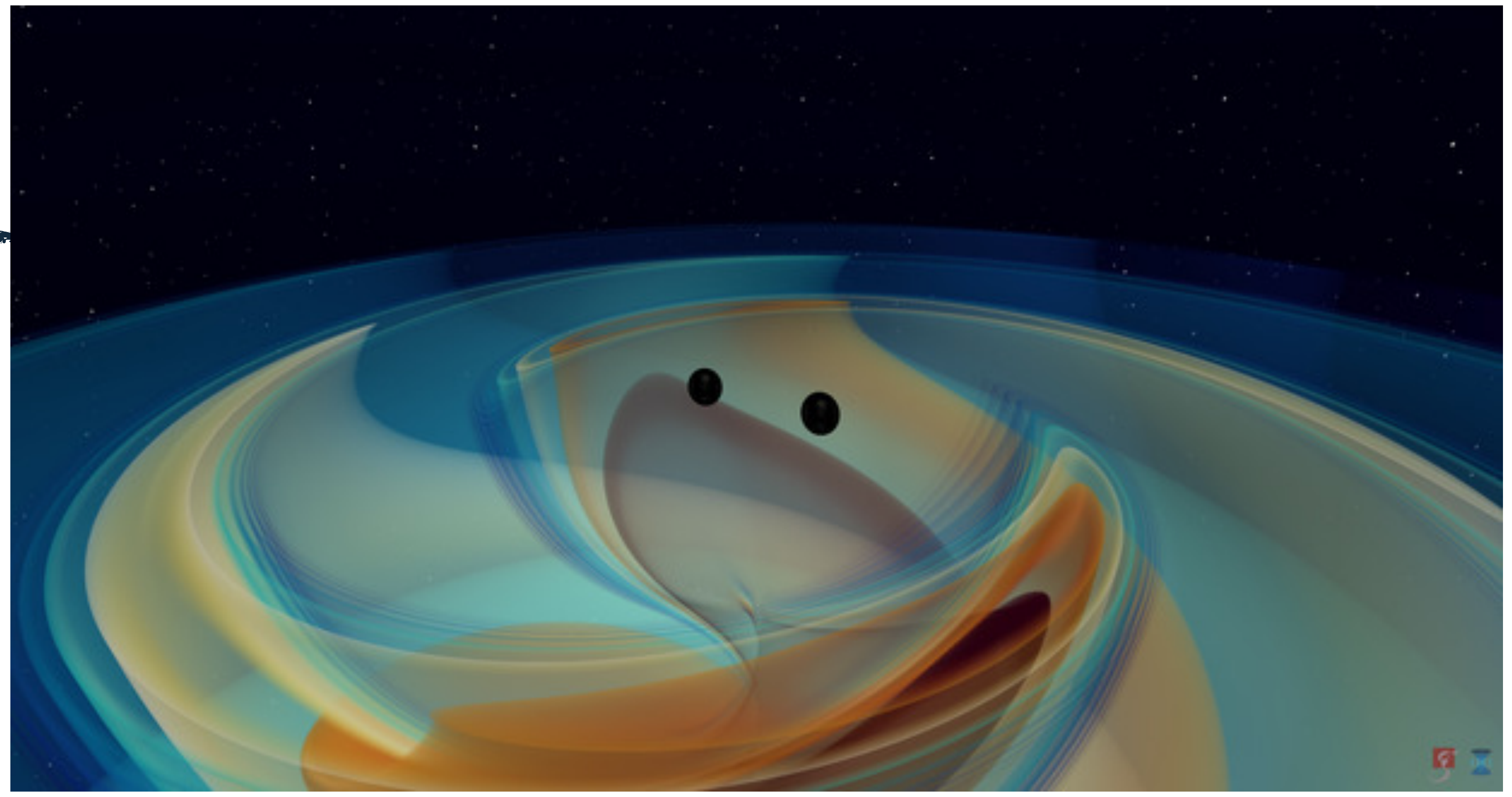
But:

- Quantum observables will feature prominently as proxy to classical observables
- QFT methods help in streamlining classical computations
- Amplitudes-based program and the observables the program is concerned with

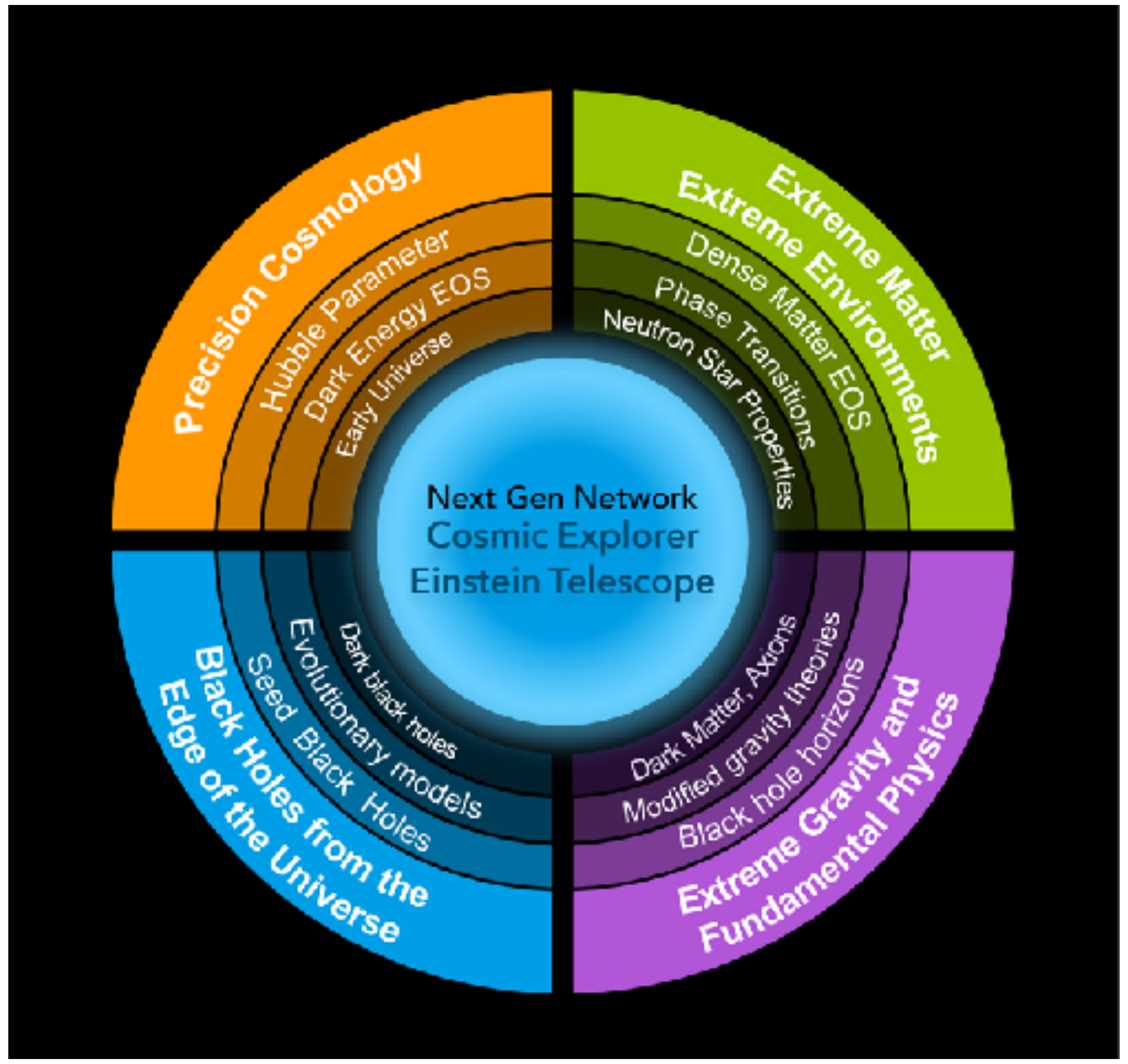
Quantum viewpoint extremely helpful for classical physics!

Motivation

- GW detected 2015 after 100 years, Nobel prize 2017
- Systems of interest: compact binary systems
- Physics goals:
  - Strong-field tests of GR, new physics
  - BH properties, abundance etc.
  - Ultra-dense matter (neutron star equation of state)
  - Multi-messenger astronomy
  - …

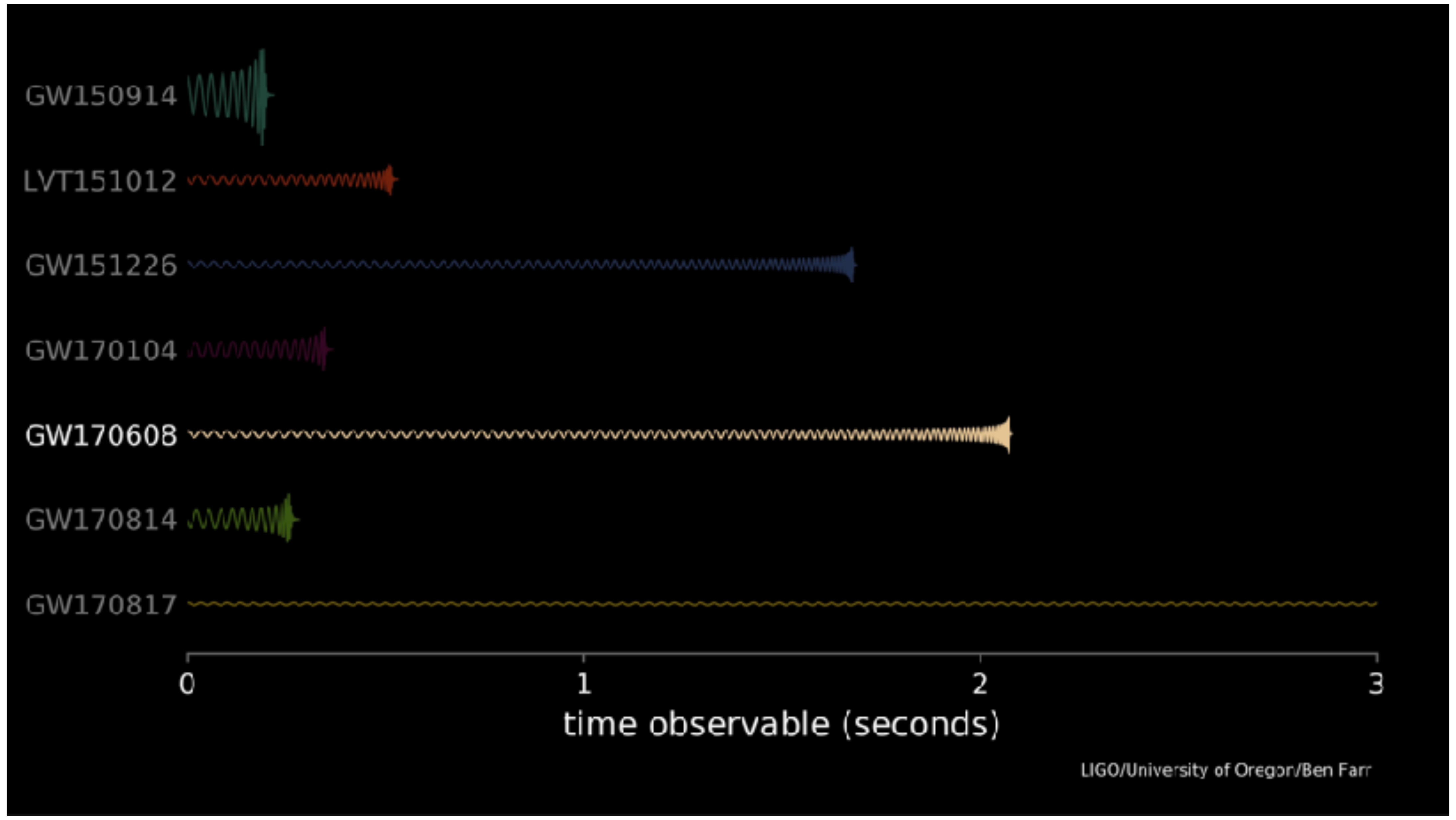


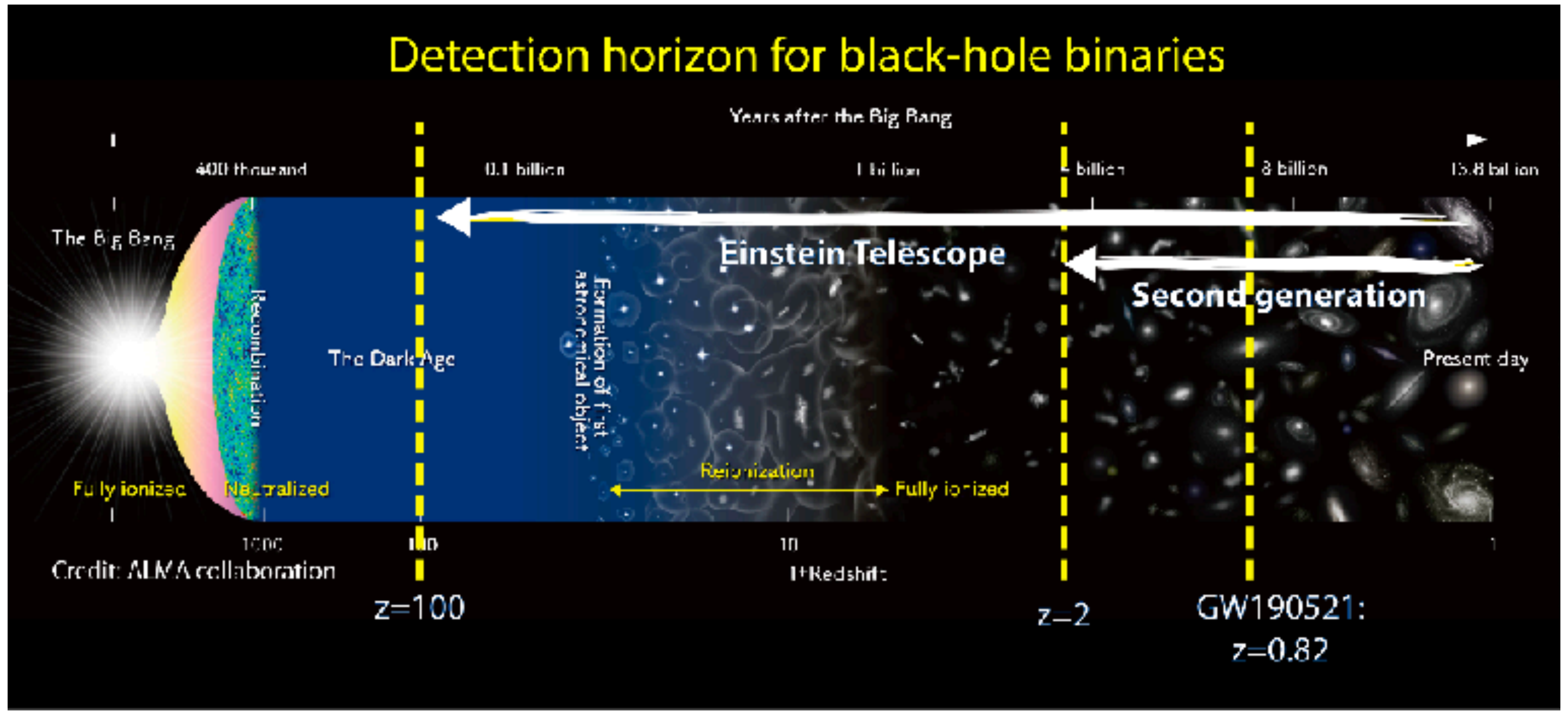
[GW190521, LIGO]

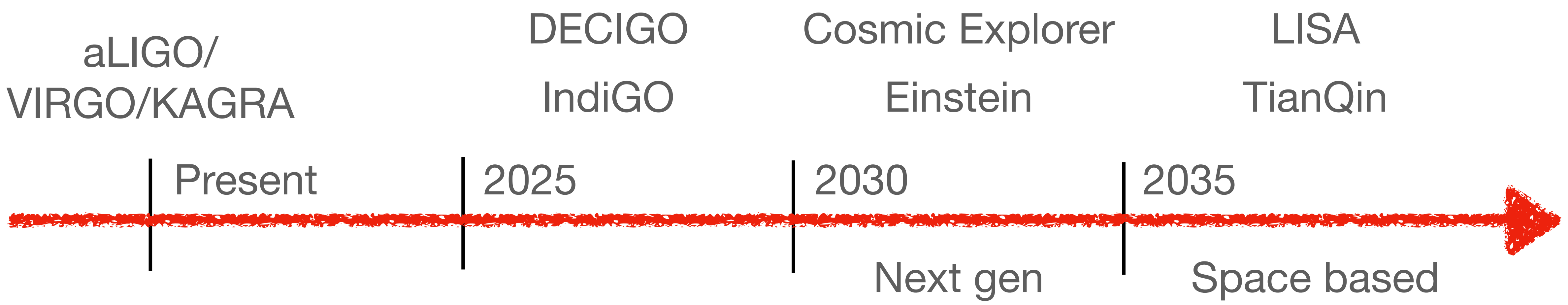


Motivation

- Present detectors (LIGO/VIRGO/KAGRA):
  - O(100) events. O4: 1 event/2.5 days
  - Good control over experimental errors
  - Relative length changes $\Delta\ell/\ell \sim 10^{-20}$
  - Observations over up to 10^3 cycles
- Next-gen. experiments (ca. 2035):
  - More precision 10-100x improved S/N
  - More data (bigger reach + sensitivity)
  - Extreme corners of parameter space, e.g. EMRI



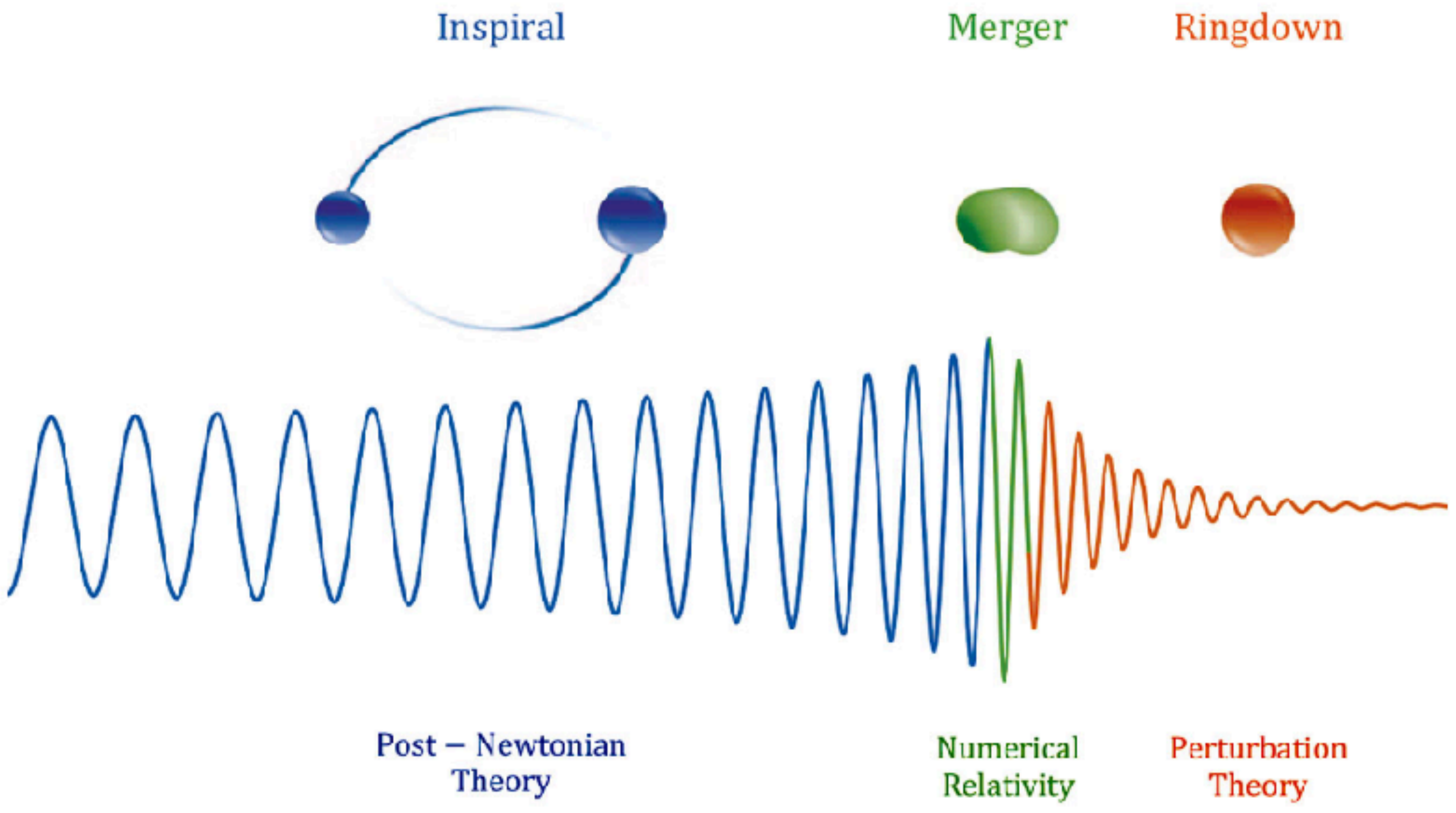




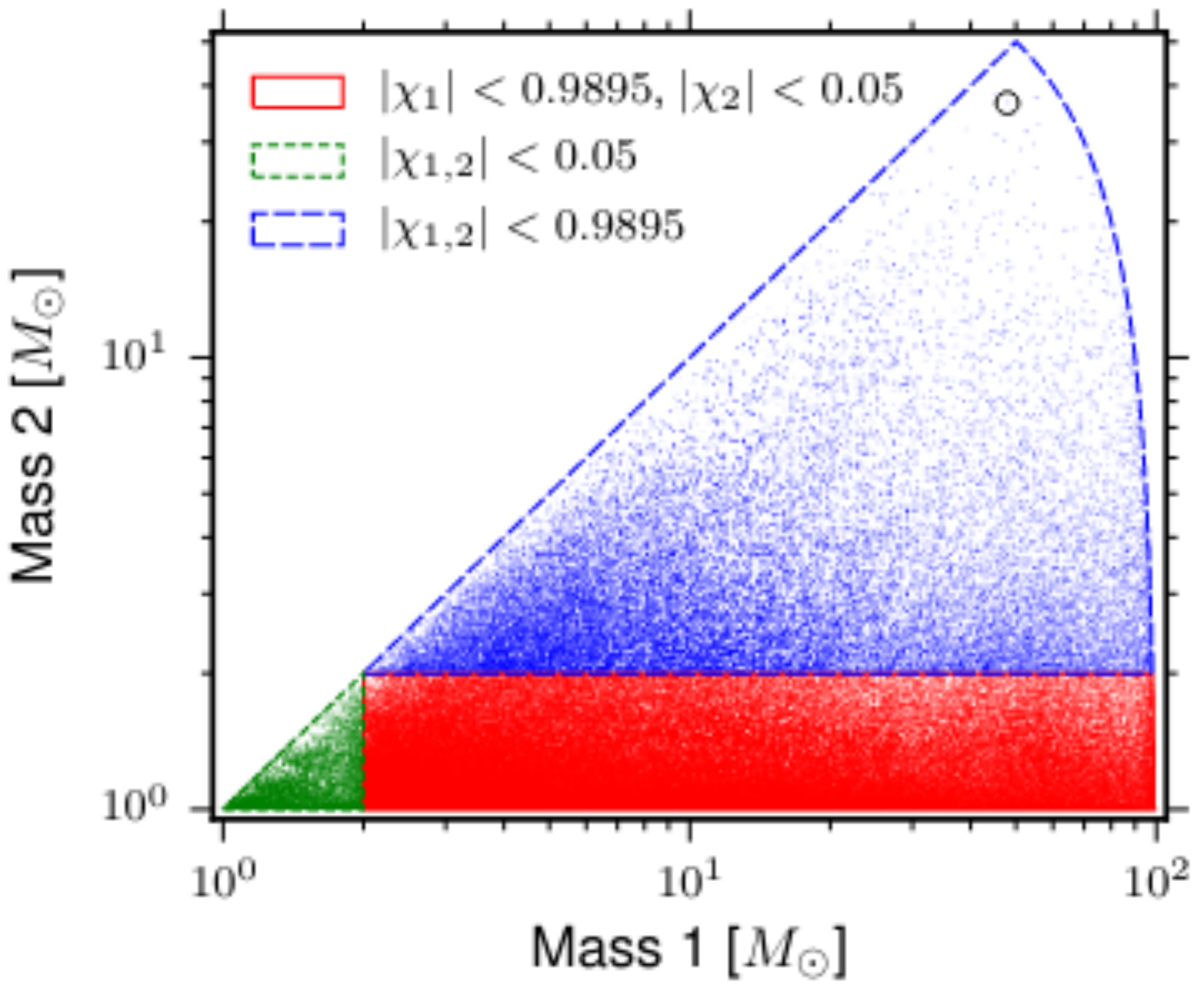
Motivation

- Waveform from Einstein Eqn $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$
- Significant resource requirements:
  - $O(10^5)$ CPU h/ NR template
  - GW150914: 250k templates
  - Challenging in PS-corners: $m_1 \ll m_2, v \to c, |\vec{L}|/m$
- Solution: analytic and hybrid models:
  (GW150914: post-Newtonian + effective-one-body)
- Corrections to Newton's potential to high orders

$$V(r) = -\frac{G\mu M}{r} + \frac{1}{c^2}\left[-\frac{3G\mu M \boldsymbol{v}^2}{2r} + \frac{G^2M^2}{r^2}\right] + \ldots$$



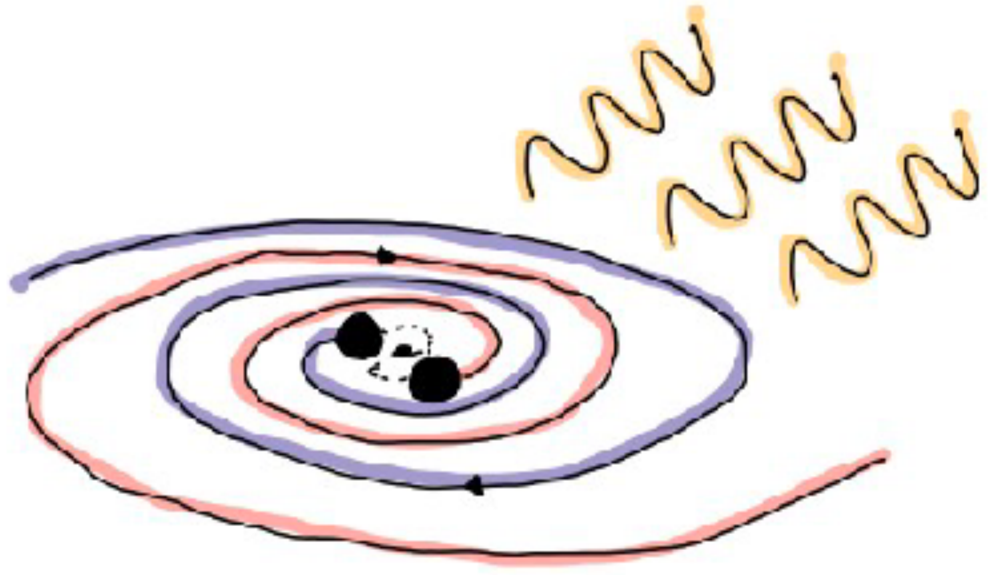
[1610.03567]



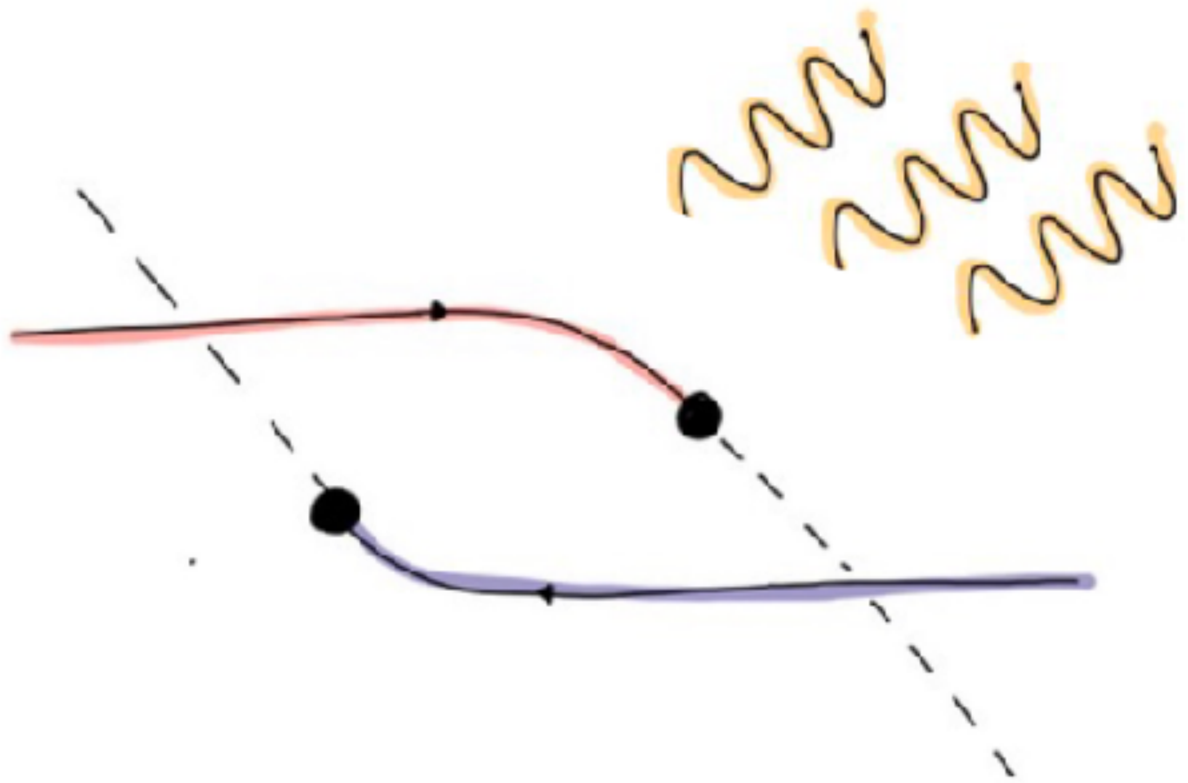
[GW150914, LIGO]

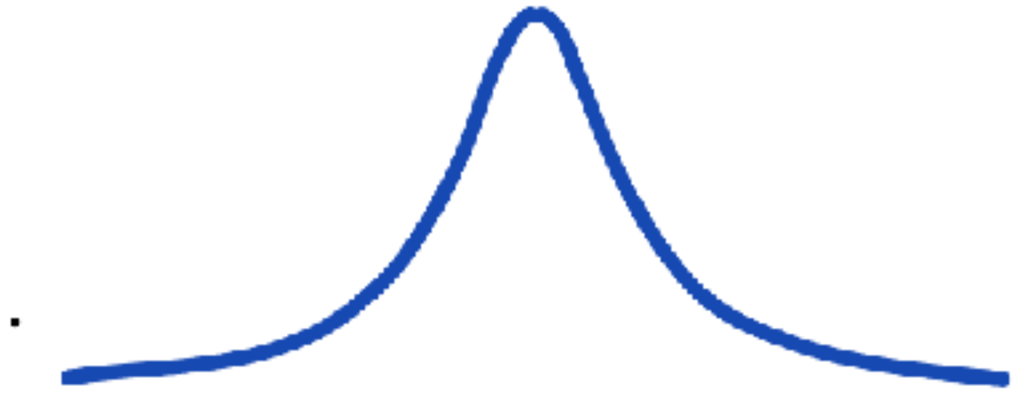
Gravitational Scattering

Process of interest: scattering of compact massive objects



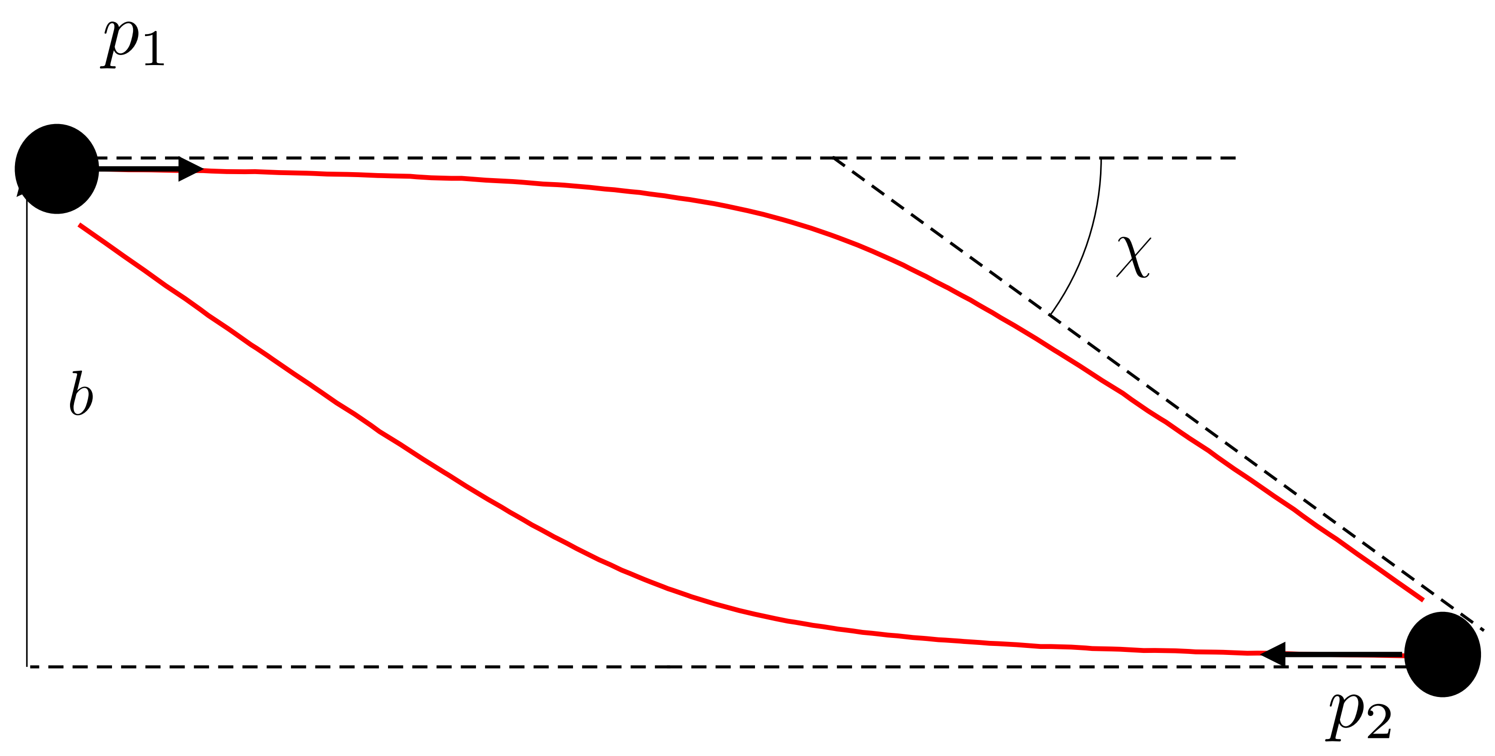
vs





Unlikely to be observed soon. Why bother?

Gravitational Scattering



Why bother?

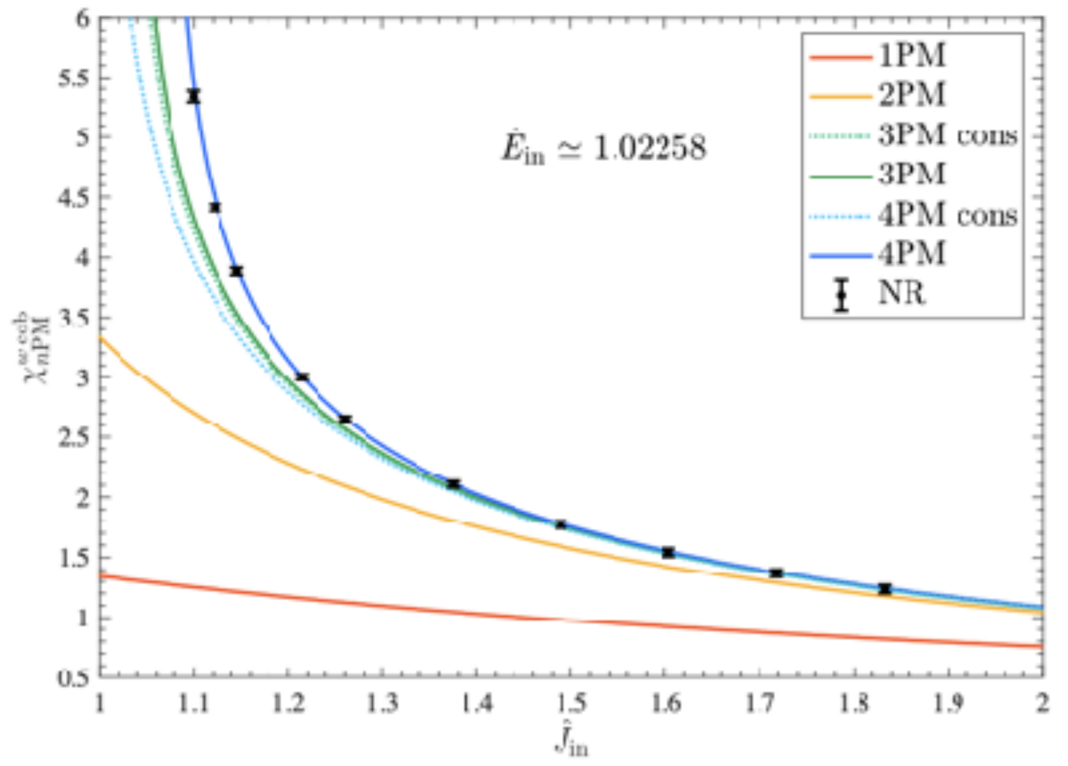
1. Arguable simplest process, determined by initial data $\{p_1, p_2, b\}$
   - Gauge/coordinate invariant approach
   - Large separations: perturbative, no merger
   - Benefits for numerical and analytic approach
2. Connection to the bound problem
   - Key subtleties (e.g. hereditary effects) are present
   - Universal information (e.g. instantaneous potentials)

$$V(r) = -\frac{G\mu M}{r} + \frac{1}{c^2}\left[-\frac{3G\mu M \boldsymbol{v}^2}{2r} + \frac{G^2 M^2}{r^2}\right]$$

3. Meshes well with the amplitudes program

Relativistic treatment exposes additional structures, e.g. mass polynomiality

Gravitational Scattering



[Damour, Rettegno]

- The GR community is very interested in scattering
  [Barack, Berti, Bini, Buonanno, Cardoso, Damour, East, Geralico, Gralla, Guercilena, Hinder, Hinderer, Hopper, Khalil, Lobo, Long, Nagar, Pfeiffer, Pretorious, Pretorius, Rettegno, Rezzolla, Sperhake, Steinhoff, Vines, Whittall, Yunes,. . .]

- Other communities are interested as well! Knorr's talk

- New results e.g from numerical relativity, more promised

Self-force correction to the deflection angle in black-hole scattering: a scalar charge toy model

Leor Barack[1] and Oliver Long[1]

[1] *Mathematical Sciences, University of Southampton, Southampton SO17 1BJ, United Kingdom*
(Dated: November 16, 2022)

Self-force effects in post-Minkowskian scattering

Samuel E. Gralla and Kunal Lobo

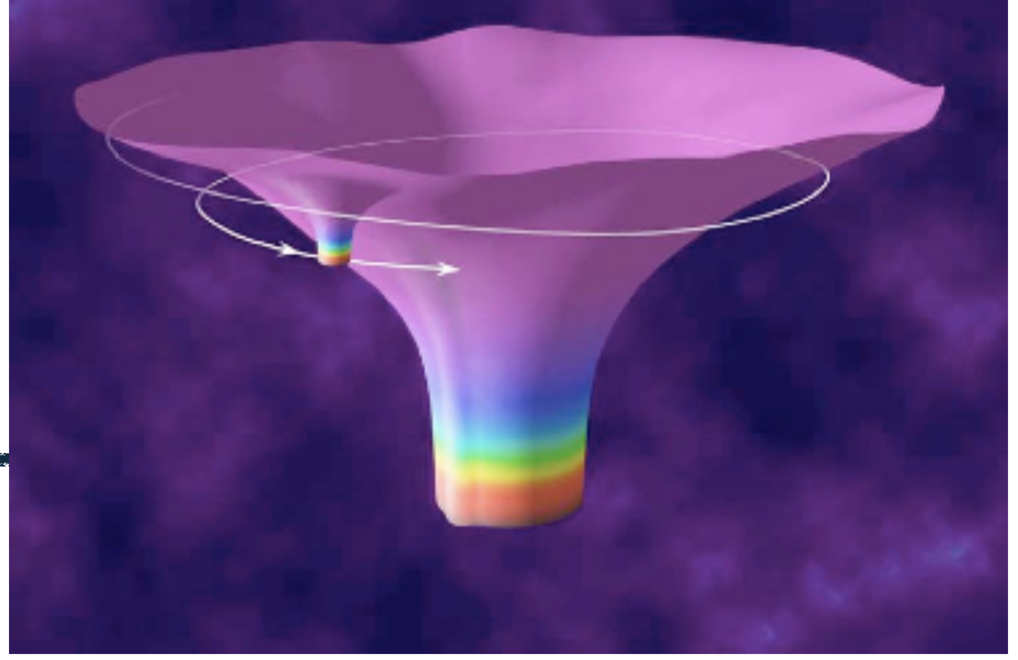
Department of Physics, University of Arizona, Tucson, Arizona 85721, USA

Strong-field scattering of two black holes: Numerical Relativity meets Post-Minkowskian gravity

Thibault Damour[1] and Piero Rettegno[2,3]

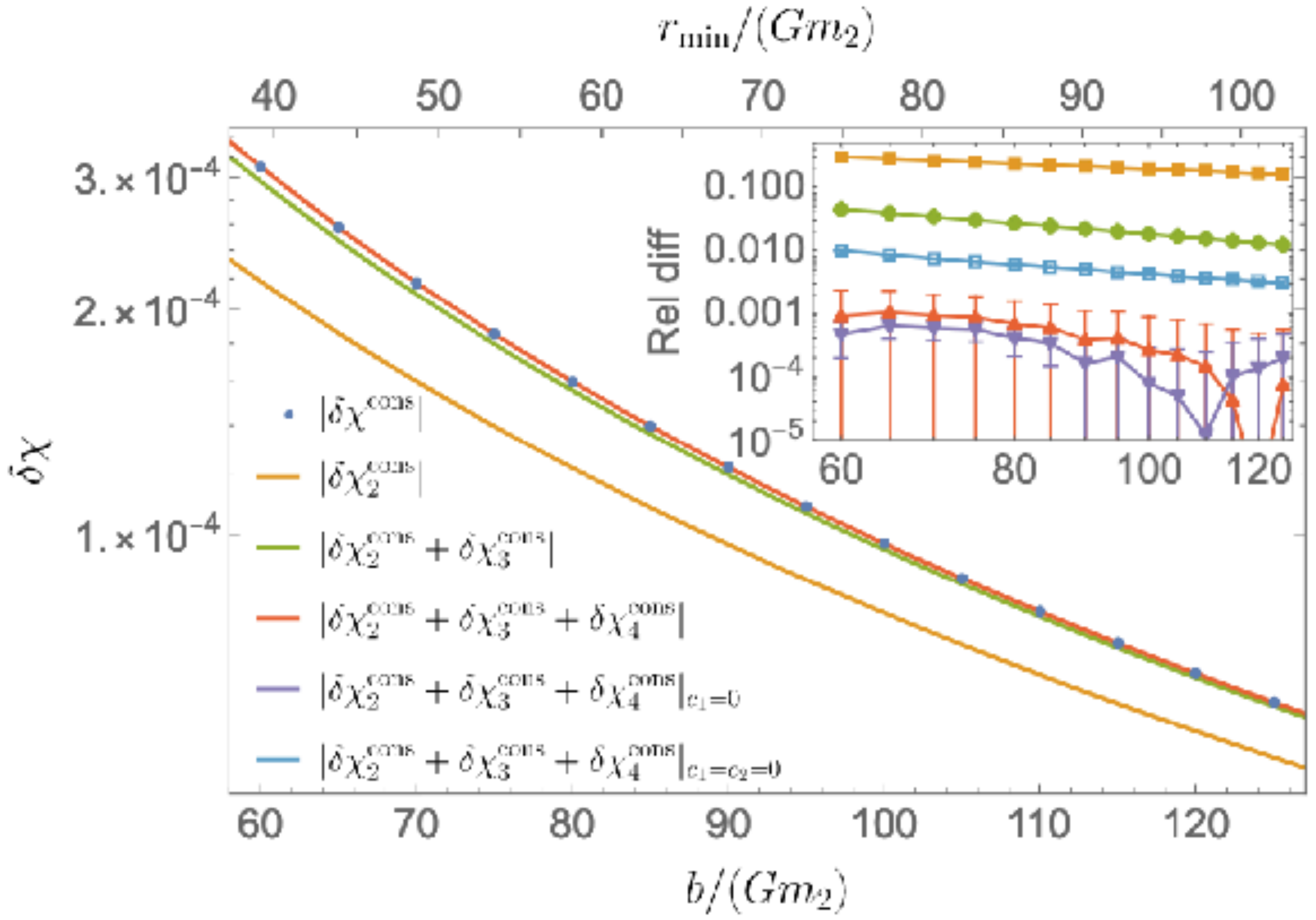
[1] *Institut des Hautes Etudes Scientifiques, 91440 Bures-sur-Yvette, France*
[2] *INFN Sezione di Torino, Via P. Giuria 1, 10125 Torino, Italy and*
[3] *Dipartimento di Fisica, Università di Torino, via P. Giuria 1, 10125 Torino, Italy*

Comparing to GR self-force



- GR self-force/post-Schwarzschild expansion $m_1/m_2 \sim 10^{-4}$, $G = \mathcal{O}(1)$
- Crucial for LISA: extreme mass ratio inspires (EMRI)
- Recent comparison between GR self-force and weak field expansion [Barack, Bern, Herrmann, Long, Parra-Martinez, Roiban, **MSR**, Shen, Solon, Teng, Zeng]
- Complementary:
  - Weak fields: perturbation theory
  - Stronger fields: self-force
- Toy model: part of the gravitational force is scalar
  Hopefully study Einstein gravity for soon!

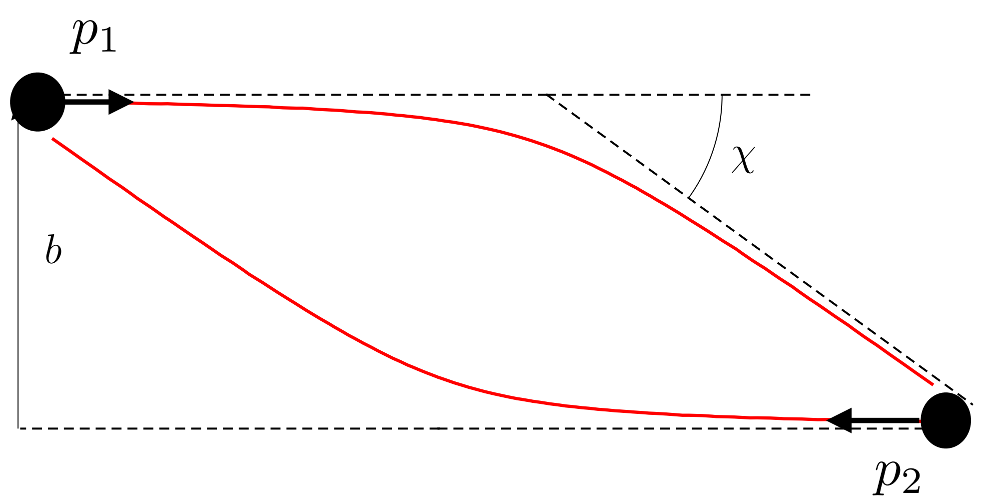
$$S = \int d^D x \sqrt{-g}\left[\frac{R}{16\pi G} + \frac{1}{2}\phi_1(\Box - m_1^2)\phi_1 + \frac{1}{2}\phi_2(\Box - m_2^2)\phi_2 + \frac{1}{2}\psi\Box\psi - 2\sqrt{\pi}\, m_1 Q\, \psi\phi_1^2\right].$$

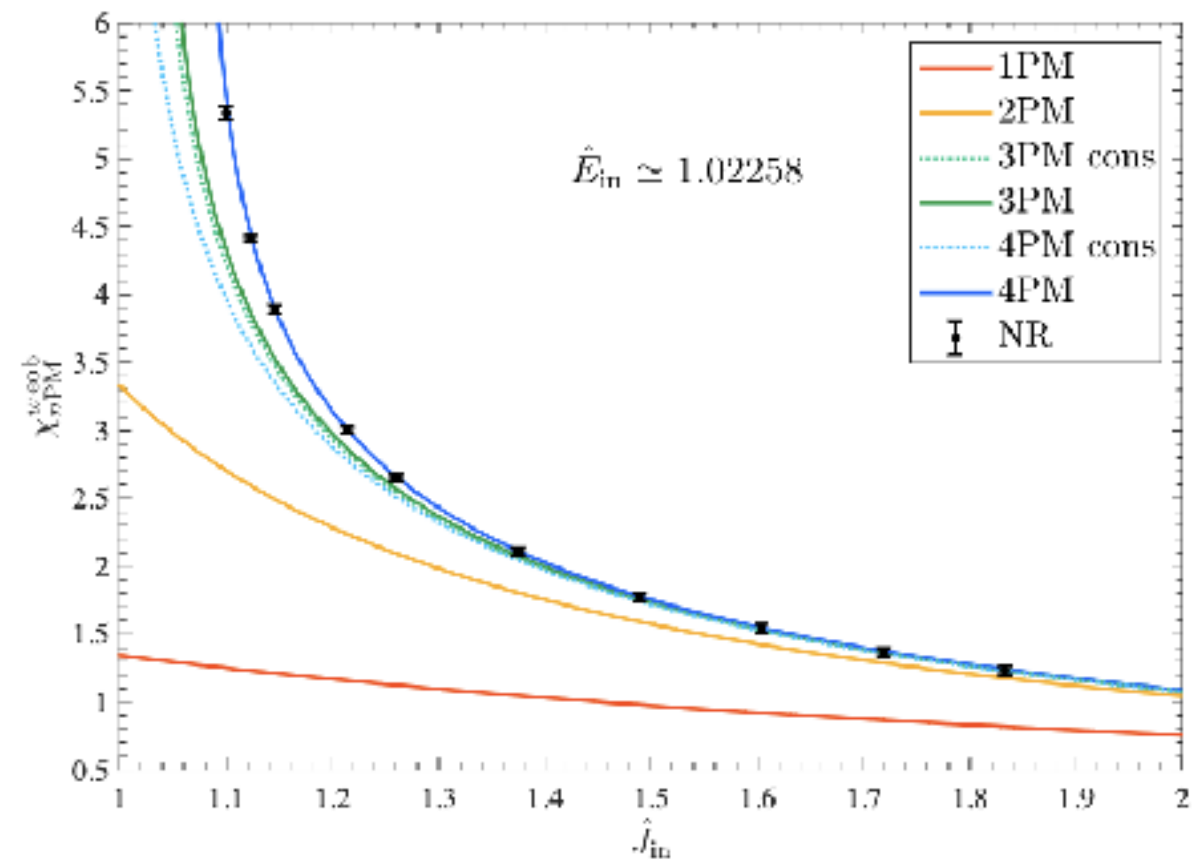


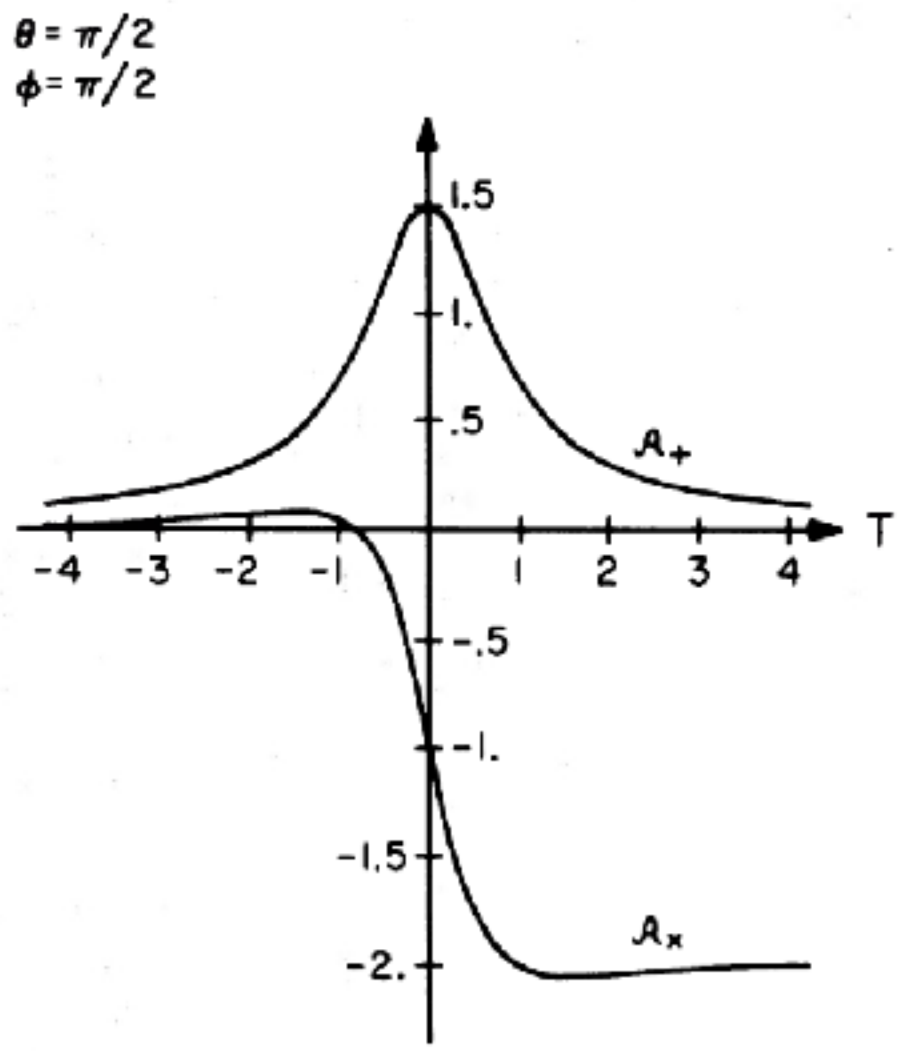
Percent-level agreement!

A Zoo of Scattering Observables

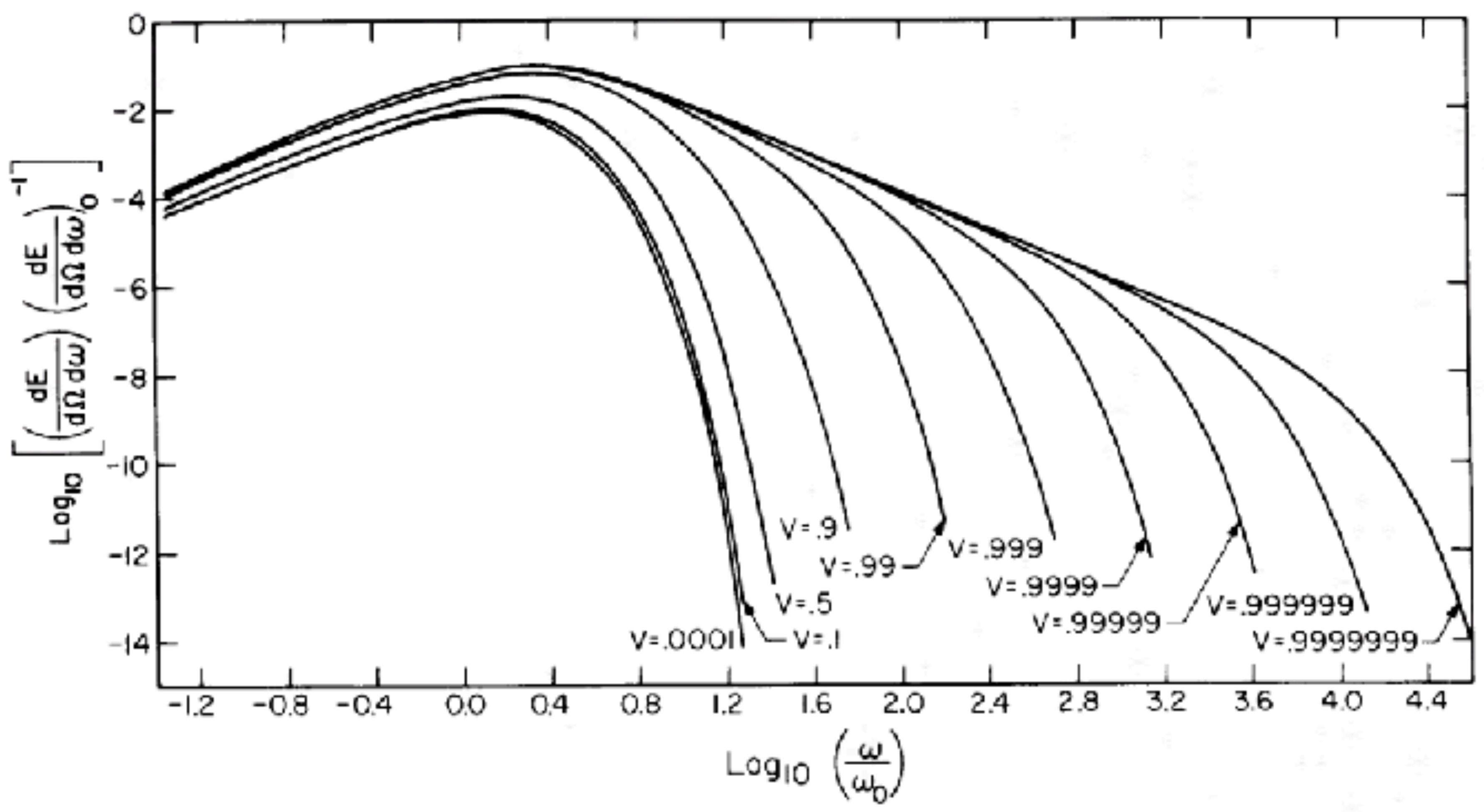
- Waveform $h_{\mu\nu}$







- Scattering angle χ
- Radiative losses $\Delta E, \Delta J$, spectra and differential quantities $\frac{\mathrm{d}E}{\mathrm{d}\omega}, \frac{\mathrm{d}E}{\mathrm{d}\Omega}, \ldots$
- "Spin-kick" ΔS_i
- Time delay Δt
- Collider-type observables like event shapes

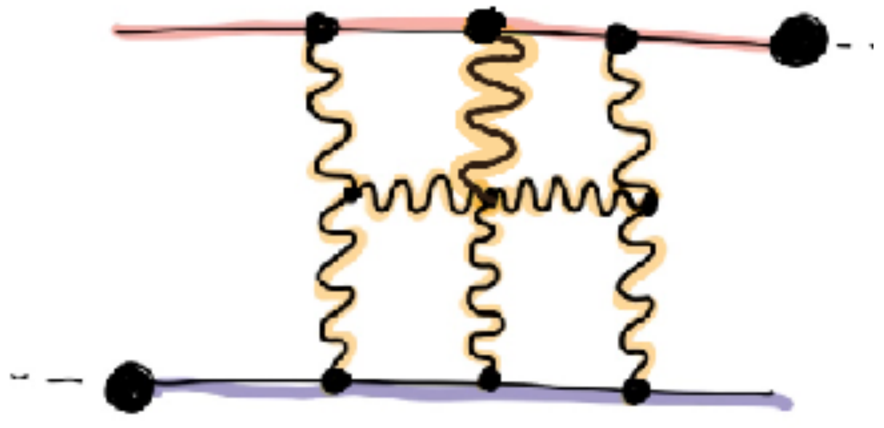


Post-Minkowskian (PM) Expansion

- Solution of Einsteins eq. around Minkowski space $G \to 0$

$$T^{\mu\nu} = \sum_{i=1}^{2} \int \mathrm{d}\sigma_i \frac{\delta^{(4)}(x - x_i(\sigma_i))}{\sqrt{-g}} \frac{\mathrm{d}x^\mu}{\mathrm{d}\sigma_i} \frac{\mathrm{d}x^\nu}{\mathrm{d}\sigma_i} \qquad g_{\mu\nu} = \eta_{\mu\nu} + \ldots\,, \quad x_i^\mu = x_{i,0}^\mu + v_i^\mu \sigma + \ldots\,.$$

- Feynman-type diagrams, algebraic complexity, gauge/coordinate dependence,…



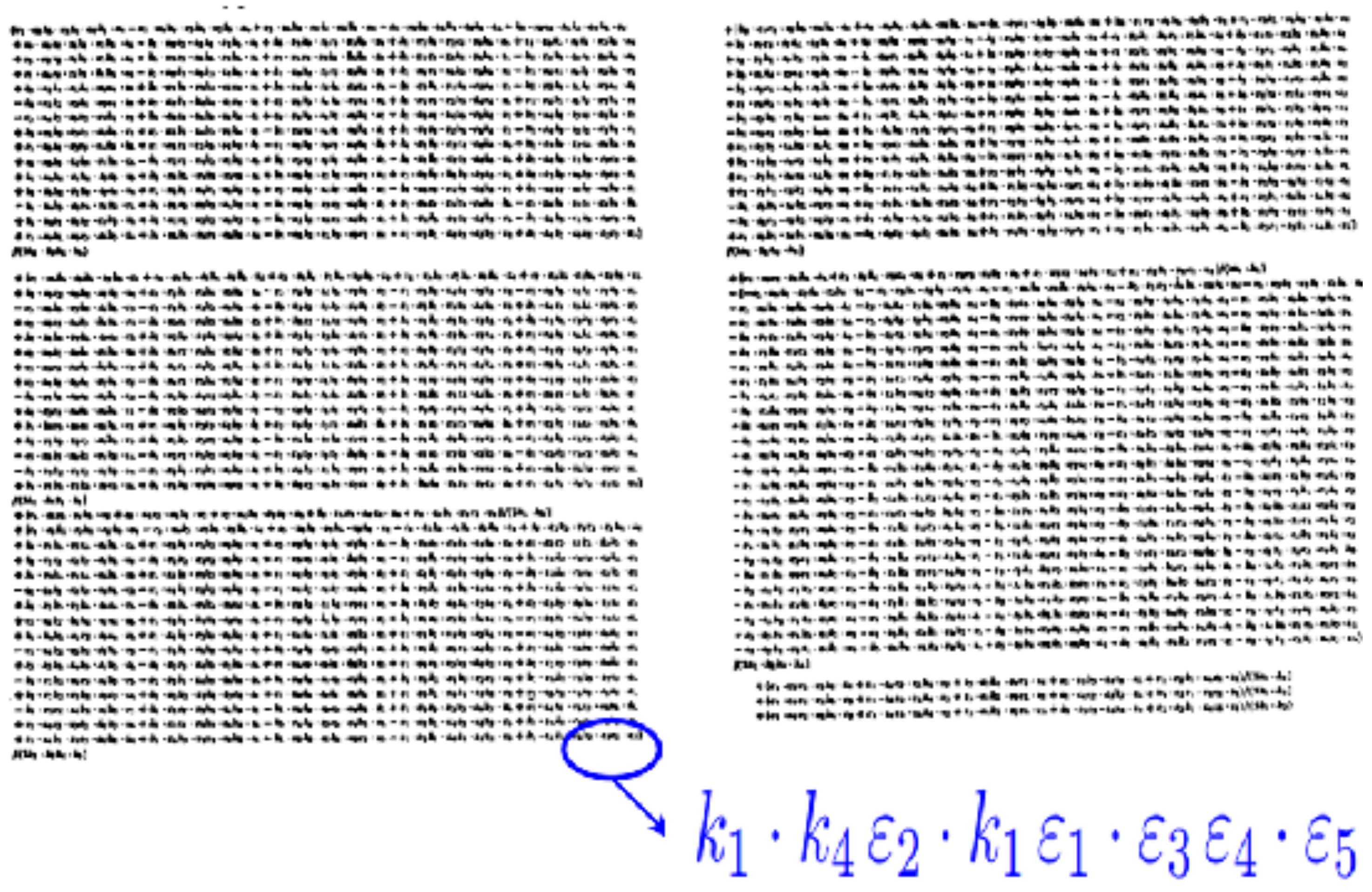
~10^{13} terms

$$\chi^{3\mathrm{PM}} = -\left[\frac{1}{12}\left(\frac{2(2\sigma^2-1)}{\sqrt{\sigma^2-1}}\right)^3 - \frac{2}{\pi}\left(\frac{2(2\sigma^2-1)}{\sqrt{\sigma^2-1}}\right)\left(\frac{3\pi}{4}\frac{5\sigma^2-1}{\sqrt{1+2\nu(\sigma-1)}}\right. \right.$$
$$+ 4\frac{\sqrt{\sigma^2-1}}{2(\sigma-1)\nu+1}\left(-\frac{3\nu\left(1-5\sigma^2\right)\left(1-2\sigma^2\right)\left(2\nu(\sigma-1)+1\right)}{2(\sigma+1)\left(2\nu(\sigma-1)+\sqrt{2\nu(\sigma-1)+1}+1\right)}\right.$$
$$- \frac{4\nu\left(-4\sigma^4+12\sigma^2+3\right)}{\sqrt{\sigma^2-1}}\operatorname{arcsinh}\left(\frac{\sqrt{\sigma-1}}{\sqrt{2}}\right)$$
$$\left.\left.\left. + \frac{1}{12}\left(4\nu\sigma^3+108\nu\sigma^2+206\nu\sigma-54\sigma^2-6\nu+3\right)\right)\right)\right]\frac{\nu^3 m^6 G^3}{J^3}$$

- Final results are simpler.
  Very familiar problem in collider phenomenology

On-shell simplicity

- Feynman diagrams contain un-physical information
- On-shell amplitudes simpler. E.g. $gg \to gggg$: 14, pages to 1 line
- Highly constrained: symmetries, analytic properties, kinematic limits,…
- Sometimes more structure than expected
- All-order results are possible!



$$|\mathcal{A}(1^-,2^-,3^+,4^+,5^+)|^2 = \frac{s_{12}^3}{s_{23}s_{45}s_{51}}$$

$$\mathcal{A}(1^-,2^+,3^+,4^+,5^+) = \mathcal{A}(1^+,2^+,3^+,4^+,5^+) = 0$$

The amplitudes-program for GW physics

- 50s: S-matrix (quantum) has all information on classical scattering!

The Two-body Problem in the Theory of the Quantized Gravitational Field †

By E. CORINALDESI ‡
Dublin Institute for Advanced Studies

Communicated by L. Rosenfeld ; MS. received 7th June 1955 and in amended form 17th November 1955

Abstract. The equations of the two-body problem of general relativity are derived by a Hamiltonian method based on an expansion of the general covariant Lagrangian in powers of the gravitational constant and by employing the techniques and the viewpoint of quantum field theory. It is found that, within the approximation in which they have so far been calculated, the equations could have been obtained identically from a linear theory of gravitation.

Fourth-Order Gravitational Potential Based on Quantum Field Theory.

Y. Iwasaki
Research Institute for Fundamental Physics, Kyoto University - Kyoto

(ricevuto l'1 Marzo 1971)

There have been many attempts (1) to understand the gravitational interaction in terms of quantum field theory in flat Minkowskian space-time in analogy to the electromagnetic interaction. Since in the case of the electromagnetic interaction there is excellent agreement between the quantized theory and experiment (2), we also believe that the gravitational interaction can be and should be understood by means of quantum field theory. This is the starting point of our discussions.

- Work on Bremsstrahlung [Feynman, Barker, Gupta, Kaskas, …]
- Great idea but hardly competitive, recomputed subleading two-body potential
- To convince people compute something new!

The amplitudes-program for GW physics

- Important developments:
  - Double copy, Gravity = (Gauge)^2 [e.g. Kawai Lewellen Tye; Bern, Carrasco, Johansson] 1985+
  - Generalized unitarity [Bern, Dixon, Dunbar, Kosower; Britto, Cachazo and Feng] 1998+
  - Improved EFT understanding [Beneke, Smirnov; Goldberger, Rothstein] 1997+
  - Integration tools (IBP, DE etc.) [Laporta; Tkachov; Chetyrkin; Kotikov; Remiddi, Gehrmann; Henn, Anastasiou, Melnikov, …] 1981+
  - Improvements in computing power
- Clear encouragement from GR community to revive the program!

High-energy gravitational scattering and the general relativistic two-body problem

Thibault Damour*

Institut des Hautes Etudes Scientifiques, 35 route de Chartres, 91440 Bures-sur-Yvette, France

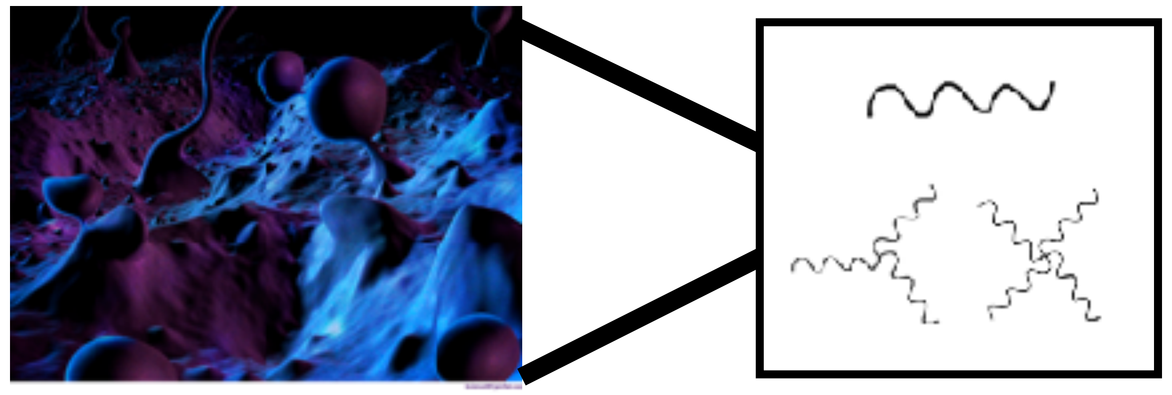
(Dated: October 31, 2017)

[...] tum gravitational scattering amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian

Effective field theory

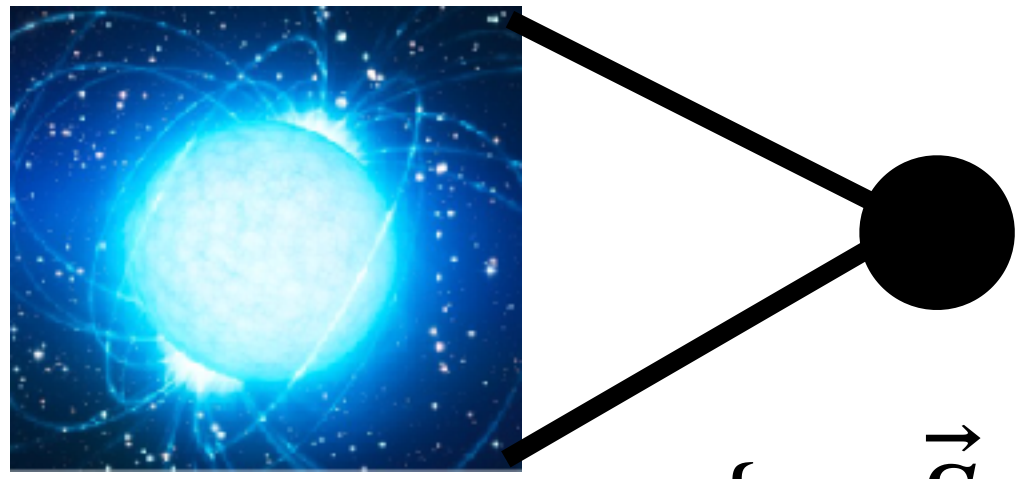
- Quantum/UV in higher-dimension operators [Donoghue '94]

$$L_{\text{grav}} = -\frac{1}{16\pi G}R + \frac{\alpha_1}{M_{\text{p}}^2} C_{\mu\nu}{}^{\rho\sigma} C_{\rho\sigma}{}^{\delta\lambda} C_{\delta\lambda}{}^{\mu\nu} + \ldots$$



- Finite size in non-minimal couplings [Goldberger, Rothstein '05]

$$L_{\text{matter}} = \sum_i \frac{1}{2}(\partial_\mu \phi_i)^2 - \frac{1}{2} m_i^2 \phi_i^2 + \lambda_i R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \phi_i^2 + \ldots$$



$\{m, \vec{S}, \lambda_i\}$

- Point-particle description breaks down at higher orders

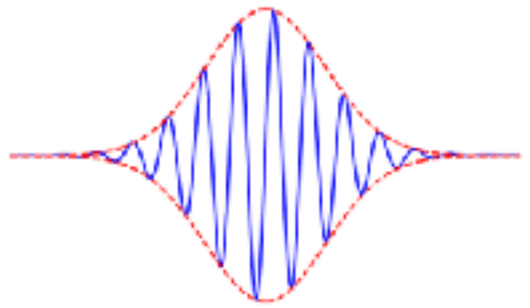
- Describing spin possible [Vaidya '14, Bern et. al. '20], introduces issues (higher-spin fields)

Effective field theory

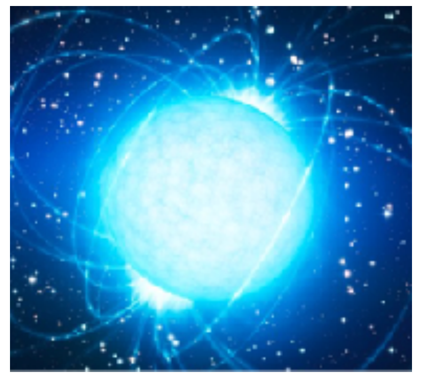
- Correspondence principle : Classical limit = large charges

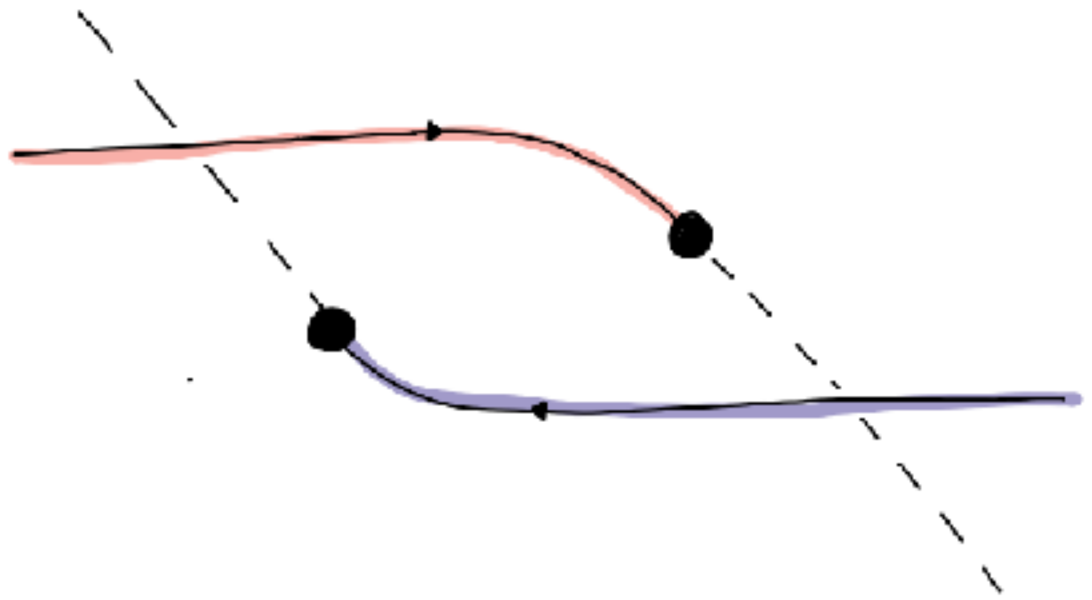
$$\hbar \ll J \sim pb \sim \mathcal{O}(10^{40}\hbar)$$

- Hierarchy of scales:

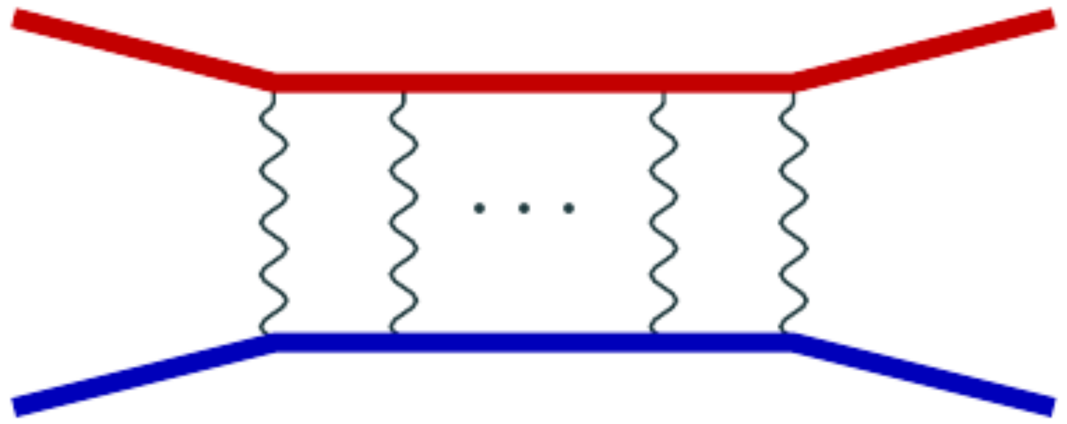


$$l_{\text{compton}} \ll R_{\text{S}} \sim GM \ll b$$





- Long distance physics ↔ soft graviton exchanges $q \sim \dfrac{1}{b} \ll p$



- Interested in non-analytic contributions:

$$\text{FT}[1/q^2] \sim \frac{1}{b}, \quad \text{FT}[\log(q)] \sim \frac{1}{b^3}, \quad \text{FT}[q^2] \sim \delta^{(4)}(b)$$

$$\hbar \times \frac{1}{\hbar} = \mathcal{O}(1)$$

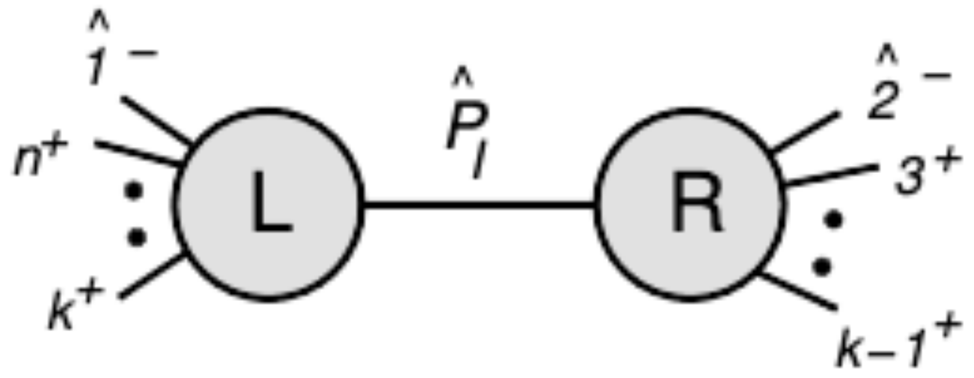
Tree amplitudes

- BCJ and KLT relations (string theory): Gravity from QCD trees   JJ's talk
  - $\mathscr{M}_4(1234) = -s_{12}\mathscr{A}_4(1234)\mathscr{A}_4(1243)\ \ldots$
- Computing QCD trees efficient
  - Closed form expressions

    $$\mathscr{A}(1^+, \ldots, i^-, \ldots, j^-, \ldots, n^+) = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

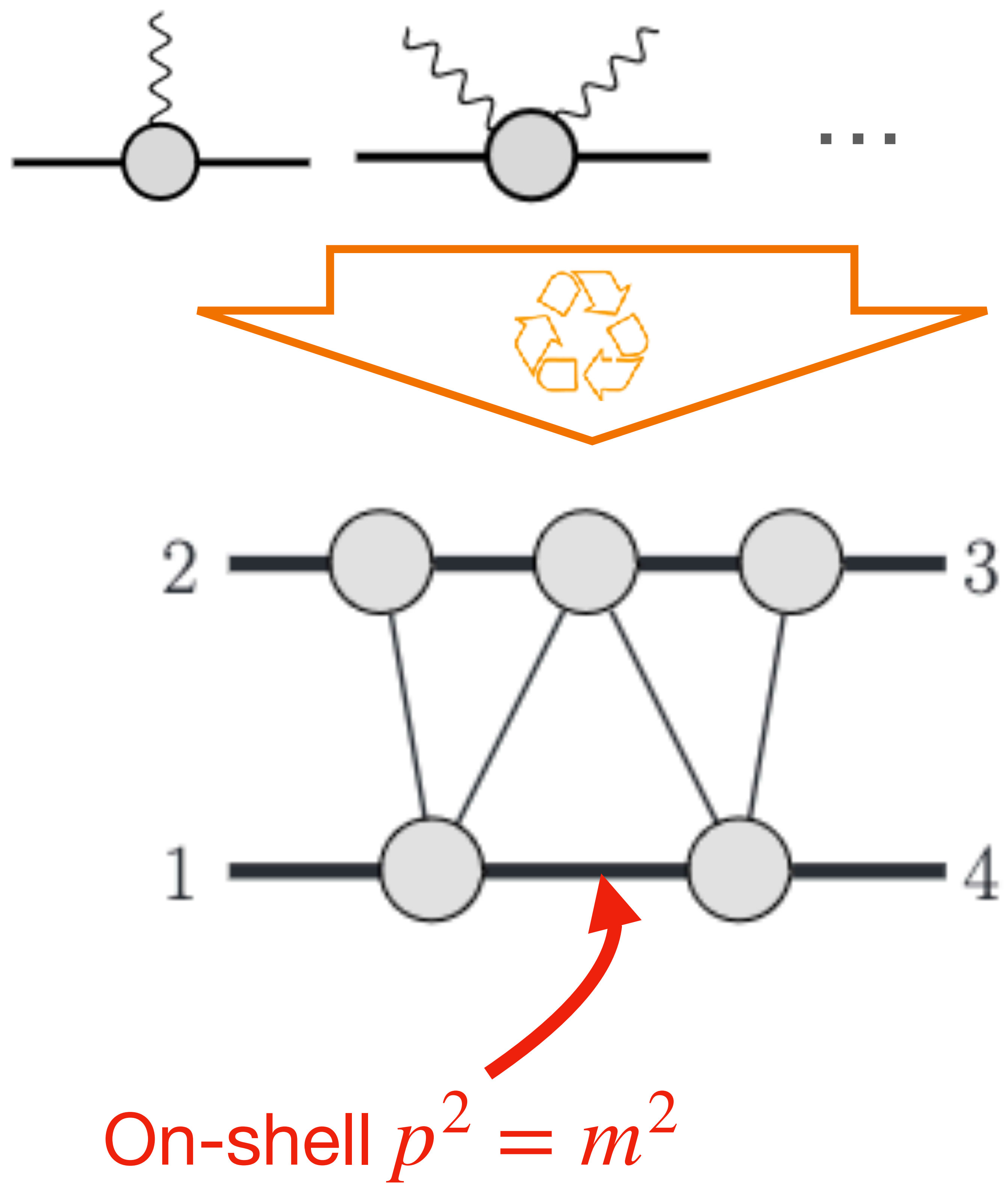
  - Recursion relations: BCFW, Berends-Giele,…
- Computing tree amplitudes is simple!

$$A_n[1^-2^-3^+\ldots n^+] = \sum_{k=4}^{n}$$



Generalized unitarity

- Loops from multi-particle cuts (residues)
- Recycle products of trees into loop amplitude
- Very powerful: sample computations
  - W+4 partons with 2 loops [Abreu et. al.]
  - 4 gravitons in Einstein gravity at 2 loops [Abreu et. al.]
  - 4 gravitons $\mathcal{N} = 8$ supergravity at 5 loops [Bern et. al.]
  - Classical gravity at 3 loops [Bern et. al.]
- Classical physics: Not all parts necessary
- Integrand construction possible to high orders!



Integration

- Loops introduce Feynman-type integrals

$$I = \int \mathrm{d}^D \ell \, \frac{1}{(\ell + p)^2 - m^2} \times \ldots$$

- Two-step procedure:
  1. Integral reduction (integration by parts, IBP)
  2. Evaluation of $\mathcal{O}(10^3)$ "master" integrals

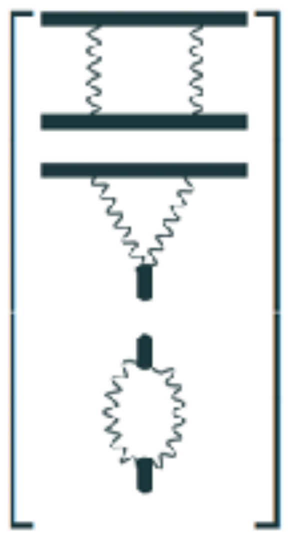
$$0 = \int \mathrm{d}^D \ell \, \frac{\partial}{\partial \ell^\mu} \frac{1}{(\ell + p)^2 - m^2} \ldots$$

- IBP: solve large linear systems [Laporta]
- Well studied problem
- Automated (e.g. FIRE6 [Smirnov, Chukharev]), make use of computers

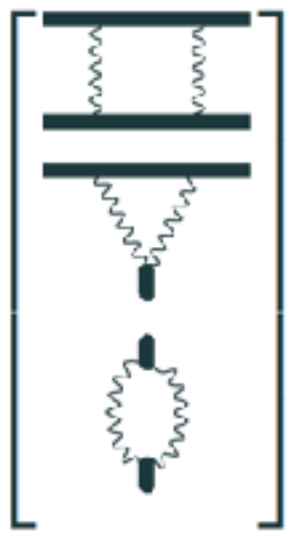
Integration

- Fuchsian diff. eqns. for master integrals

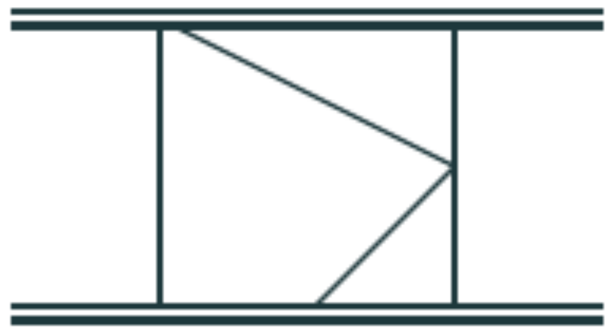
$$\frac{\partial}{\partial y}$$



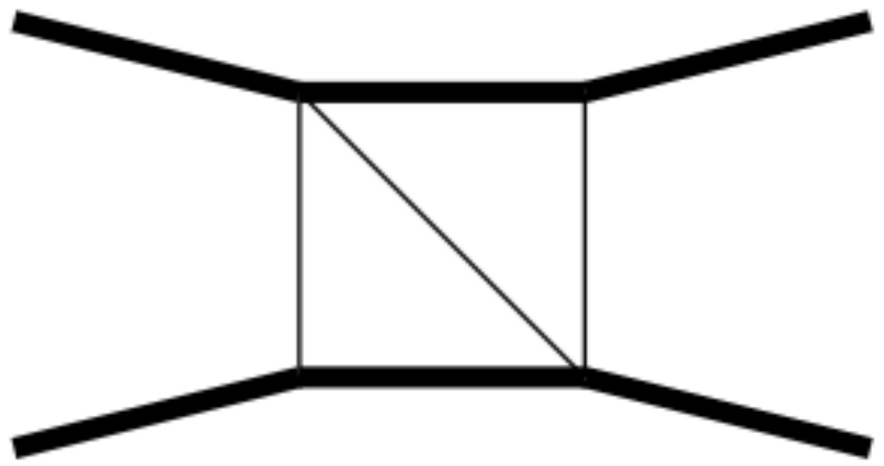
$$= \epsilon \frac{\partial}{\partial y} \underbrace{\log\left(\frac{y+\sqrt{y^2+1}}{y-\sqrt{y^2+1}}\right)}_{=2\,\mathrm{arcosh}\, y} \begin{bmatrix} 0 & 0 & 1/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



- Standard procedure: vast literature in collider physics
- Solutions in terms of iterated integrals: $\log, \mathrm{Li}_n, K, E, \ldots$
- Classical limit crucial:
  - Fewer master integrals
  - Single-variable problem! $y = 1/\sqrt{1-v^2}$
  - Simpler functions



$$= \frac{1}{\epsilon^2}\frac{8}{\sigma+1}K^2\left(\tfrac{\sigma-1}{\sigma+1}\right) + \mathcal{O}(\epsilon^{-1}),$$



$$\sim$$

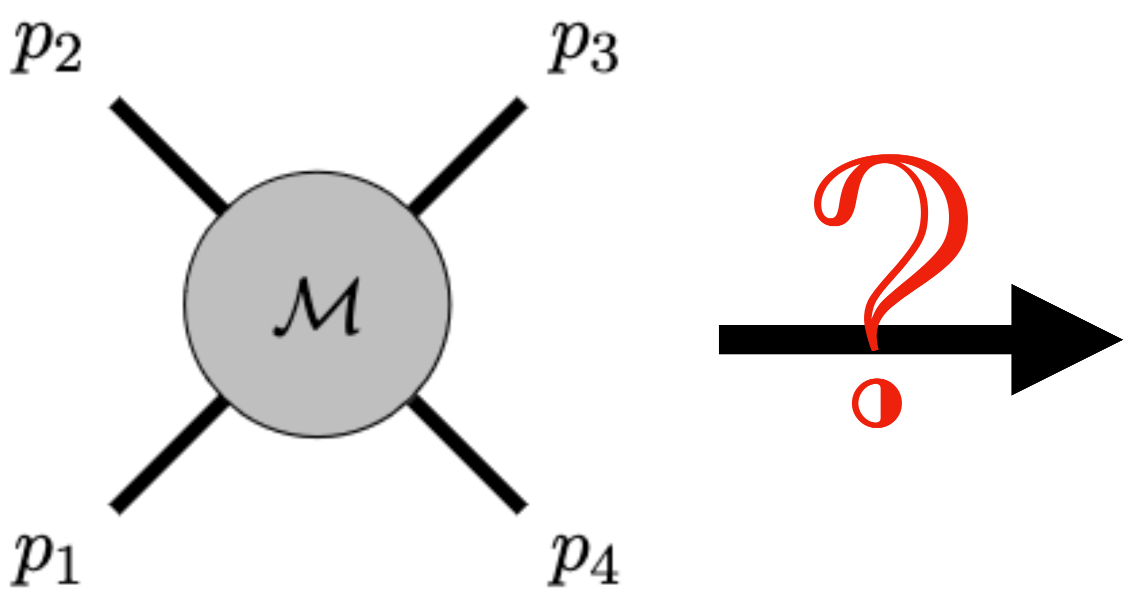
$$\begin{aligned}
&= 16\log\frac{-t}{m^2}\Big[\mathcal{E}_4\left(\begin{smallmatrix}-1 & 1 & 1\\ 0 & 1+1/y & 1\end{smallmatrix};\bar{x},\vec{a}\right) - \mathcal{E}_4\left(\begin{smallmatrix}-1 & 1 & 1\\ 0 & 1+y & 1\end{smallmatrix};\bar{x},\vec{a}\right) \qquad (5.8)\\
&+ \mathcal{E}_4\left(\begin{smallmatrix}-1 & 1 & 1\\ \infty & 1 & 1\end{smallmatrix};\bar{x},\vec{a}\right) + \mathcal{E}_4\left(\begin{smallmatrix}-1 & 1 & 1\\ 1 & 1 & 1\end{smallmatrix};\bar{x},\vec{a}\right) + \zeta_2\,\mathcal{E}_4\left(\begin{smallmatrix}-1\\ \infty\end{smallmatrix};\bar{x},\vec{a}\right) + \zeta_2\,\mathcal{E}_4\left(\begin{smallmatrix}-1\\ 1\end{smallmatrix};\bar{x},\vec{a}\right)\Big]\\
&- 8\left(8\zeta_2 + 4\mathrm{Li}_2(y) + \log^2 y\right)\Big[\mathcal{E}_4\left(\begin{smallmatrix}-1 & 1\\ 0 & 1+1/y\end{smallmatrix};\bar{x},\vec{a}\right) + \mathcal{E}_4\left(\begin{smallmatrix}-1 & 1\\ 0 & 1+y\end{smallmatrix};\bar{x},\vec{a}\right) - \mathcal{E}_4\left(\begin{smallmatrix}-1 & 1\\ 0 & 1\end{smallmatrix};\bar{x},\vec{a}\right)\Big]\\
&- 32\,\zeta_2\left[\mathcal{E}_4\left(\begin{smallmatrix}-1 & 1\\ \infty & 1\end{smallmatrix};\bar{x},\vec{a}\right) - \mathcal{E}_4\left(\begin{smallmatrix}-1 & 1\\ 1 & 1\end{smallmatrix};\bar{x},\vec{a}\right)\right] + 16\,\mathcal{E}_4\left(\begin{smallmatrix}-1 & 1 & 1 & 1\\ 0 & 1+1/y & 1 & 1\end{smallmatrix};\bar{x},\vec{a}\right)\\
&- 32\,\mathcal{E}_4\left(\begin{smallmatrix}-1 & 1 & 1 & 1\\ 0 & 1+1/y & 2 & 1\end{smallmatrix};\bar{x},\vec{a}\right) - 16\,\mathcal{E}_4\left(\begin{smallmatrix}-1 & 1 & 1 & 1\\ 0 & 1+y & 1 & 1\end{smallmatrix};\bar{x},\vec{a}\right) + 32\,\mathcal{E}_4\left(\begin{smallmatrix}-1 & 1 & 1 & 1\\ 0 & 1+y & 2 & 1\end{smallmatrix};\bar{x},\vec{a}\right)\\
&+ 16\,\mathcal{E}_4\left(\begin{smallmatrix}-1 & 1 & 1 & 1\\ \infty & 0 & 1 & 1\end{smallmatrix};\bar{x},\vec{a}\right) - 24\,\mathcal{E}_4\left(\begin{smallmatrix}-1 & 1 & 1 & 1\\ \infty & 1 & 1 & 1\end{smallmatrix};\bar{x},\vec{a}\right) - 32\,\mathcal{E}_4\left(\begin{smallmatrix}-1 & 1 & 1 & 1\\ \infty & 1 & 2 & 1\end{smallmatrix};\bar{x},\vec{a}\right)\\
&+ 16\,\mathcal{E}_4\left(\begin{smallmatrix}-1 & 1 & 1 & 1\\ 1 & 0 & 1 & 1\end{smallmatrix};\bar{x},\vec{a}\right) + 40\,\mathcal{E}_4\left(\begin{smallmatrix}-1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1\end{smallmatrix};\bar{x},\vec{a}\right) - 32\,\mathcal{E}_4\left(\begin{smallmatrix}-1 & 1 & 1 & 1\\ 1 & 1 & 2 & 1\end{smallmatrix};\bar{x},\vec{a}\right)\\
&+ \frac{4}{3}\left(12\mathrm{Li}_3(y) + 24\zeta_2\log y + \log^3 y\right)\left[\mathcal{E}_4\left(\begin{smallmatrix}-1\\ \infty\end{smallmatrix};\bar{x},\vec{a}\right) + \mathcal{E}_4\left(\begin{smallmatrix}-1\\ 1\end{smallmatrix};\bar{x},\vec{a}\right)\right]\\
&+ 64\zeta_4 - 32\zeta_2\mathrm{Li}_2(y) + 16\mathrm{Li}_4(y) + 8\zeta_2\log^2 y + \frac{1}{3}\log^4 y\,.
\end{aligned}$$

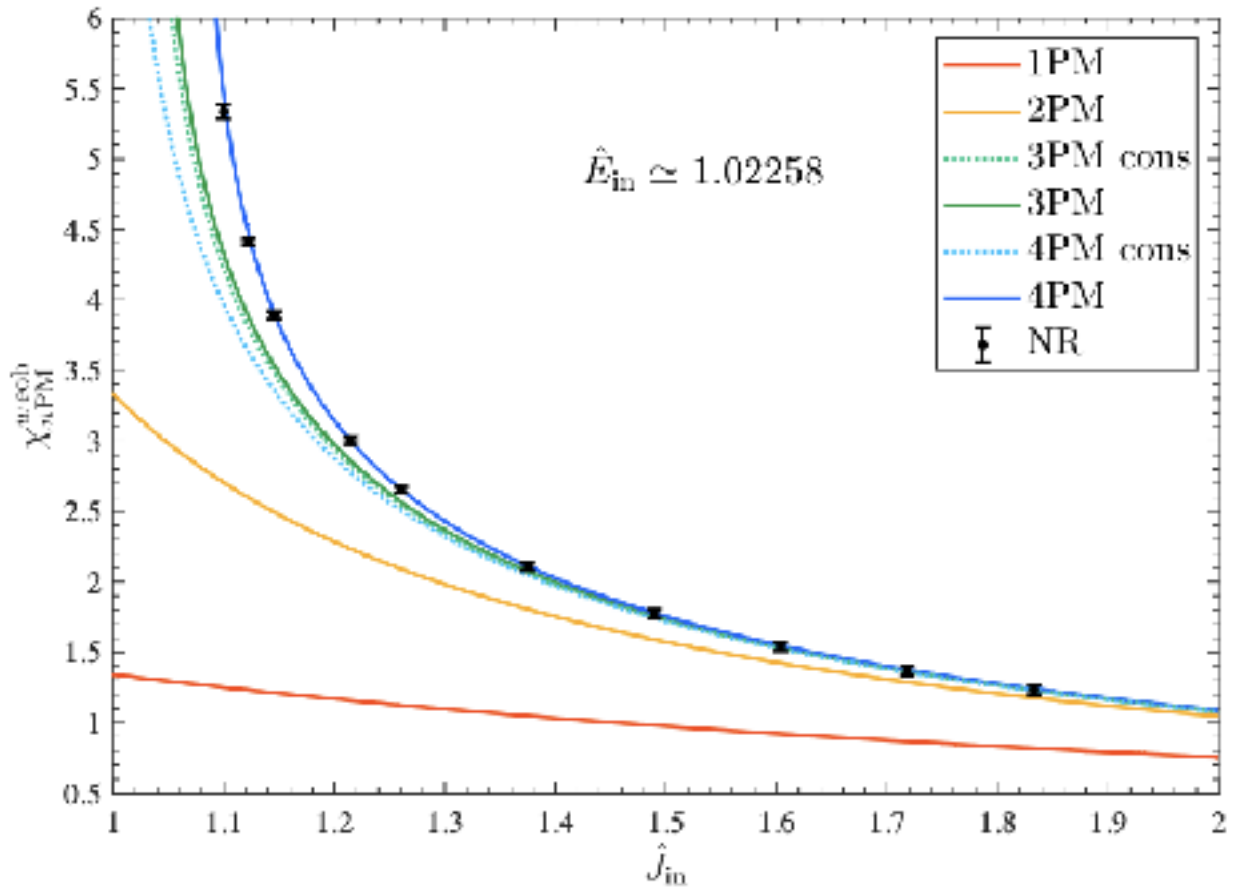
[Broedel, Dulat, Duhr, Penante, Tancredi]

$\rightarrow$ $\mathrm{arccosh}(y)\log(-t)$

Extracting classical physics from amplitudes

- From amplitude, how to extract classical physics?
- Amplitudes are not observables
- IR divergences, no well-defined semi-classical expansion

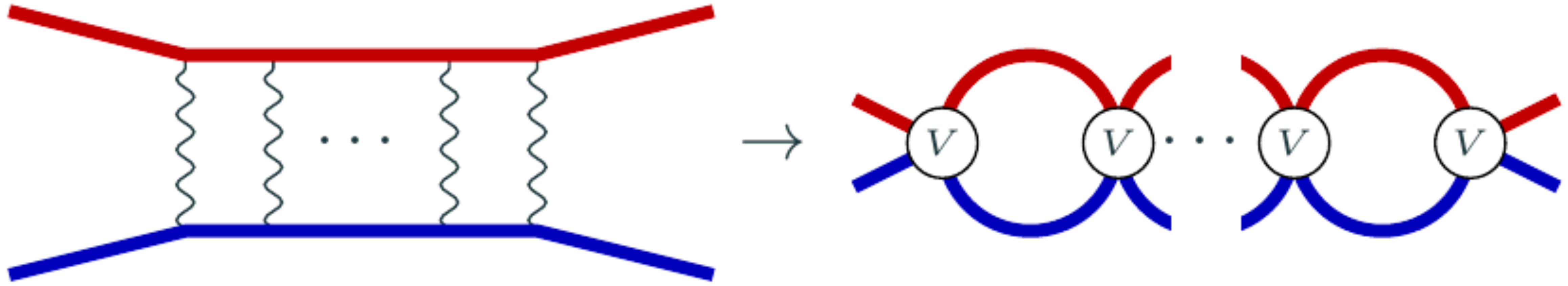




$$\mathscr{M}_{\text{tree}} \sim \frac{1}{\hbar}, \mathscr{M}_{L\text{-loop}} \sim \frac{1}{\hbar^{L+1}}$$

- Similar to collider problems:
  differential cross sections are well-defined

Matching computations



- EFT methods inspired by non-relativistic QED/QCD
- Schrödinger/Lippmann-Schwinger equation

$$\mathcal{M}(p,p') = \langle p|V|p'\rangle + \int \frac{d^3k}{(2\pi)^3}\frac{\langle p|V|k\rangle \mathcal{M}(k,p')}{E_p - E_k + i\epsilon}$$

- Divergences match
- Potential not observable, compute e.g. bound state energy spectrum
- Leading quantum corrections computed [Kirilin,Khriplovich; Akhundov,Bellucci,Shiekh; Bjerrum-Bohr, Donoghue, Holstein]

$$V(r) = -\frac{Gm_1m_2}{r} + G\hbar\alpha\delta(r) + \beta\frac{G^2\hbar m_1 m_2}{r^2} + \dots$$

- $V(r)$ coordinate dependent, but used in LIGO/VIRGO/KAGRA analysis

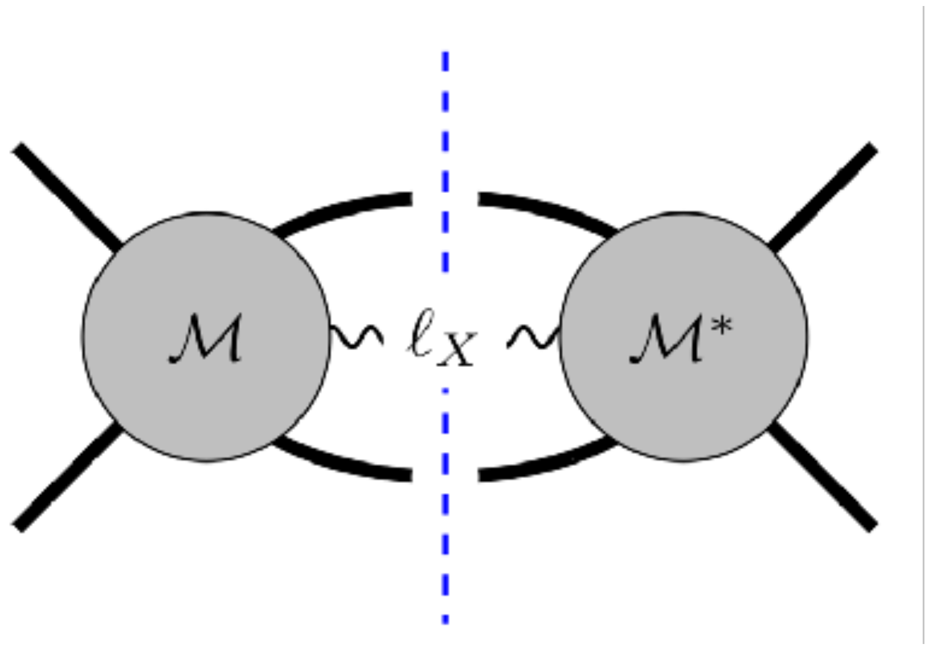
The “scenic” route [Kosower, Maybee, O’Connell]

- Directly compute observables

$$\Delta O = \langle \text{out} | \mathbb{O} | \text{out} \rangle - \langle \text{in} | \mathbb{O} | \text{in} \rangle \qquad | \text{out} \rangle = S | \text{in} \rangle$$

- Amplitudes enter through $S = 1 + \mathrm{i}T$

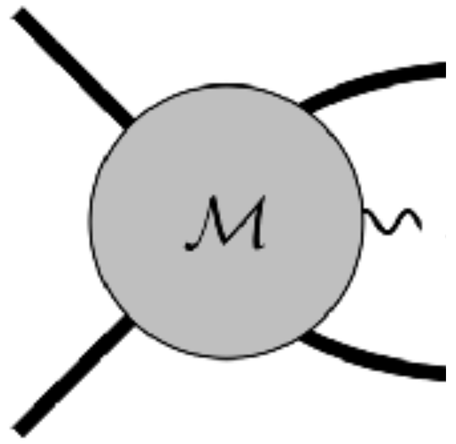
$\Delta E \sim$



- Divergences cancel

- Independent of $| \text{in} \rangle$ as long as sufficiently localized

$h_{\mu\nu} \sim$



- Similar to collider observables

$$| \text{in} \rangle = \int d\Phi(p_1) d\Phi(p_2)\; \phi_1(p_1)\phi_2(p_2)\; e^{ib\cdot p_1/\hbar} |p_1 p_2\rangle_{\text{in}}\,.$$

The "scenic" route [Kosower, Maybee, O'Connell]

- Not just scenic, many new results:
  - Radiated energy [Herrmann, Parra-Martinez, **MSR**, Zeng] to $\mathcal{O}(G^3)$
  - Radiated angular momentum [Manohar, Shen, Ridgway] to $\mathcal{O}(G^3)$ including soft modes
  - Subleading waveforms [Herdershee et. al.; Brandhuber et. al.; Elkhidir et. al.] including memory effect
  - Observables for Kerr BH [Febres Cordero, Kraus, Lin, **MSR**, Zeng]
- Possible extensions: Rad. energy spectrum, angular distribution,…

Generating functionals

- Unitarity: divergences exponentiate. e.g. eikonal

$$\mathrm{i}\mathscr{M}(E,b) = e^{\frac{2\mathrm{i}\delta(E,b)}{\hbar}} - 1 = \frac{2\mathrm{i}\delta}{\hbar} - \frac{4\delta^2}{\hbar^2} + \ldots$$

- New information in classical part $\sim \hbar^{-1}$, divergent pieces predicted
- New "Amplitude-action relation" [Bern et. al.] relating gauge invariant objects

$$\mathcal{M} = \mathrm{i}\int_J (e^{\mathrm{i}I_r(J)/\hbar} - 1)\,, \quad I_r(J,E) = \int_{\mathrm{trajectory}} p_r(J,E)\mathrm{d}r$$

$$\mathcal{M}_{\mathrm{tree}} = \boxed{\frac{G}{\hbar} I_r^0}\,, \mathcal{M}_{1-\mathrm{loop}} = \frac{G^2}{\hbar^2} I_r^0 \star I_r^0 + \boxed{\frac{G^2}{\hbar} I_r^1}\,, \ldots$$

To order G^5 and beyond

- Amplitude-based approach is competitive
- State-of-the-art results, to be used in LIGO/VIRGO/KAGRA pipeline
- Very active field. Hundreds of papers in past 4 years
- More precision $\rightarrow$ more loops. State of the art: $O(G^4)$/ 3 loops [Bern et. al.]
- Recently also spin [Jakobsen et. al.] (worldline QFT) and radiation [Damgaard et. al.][Dlapa et. al.] (classical)

To order G^5 (and beyond)

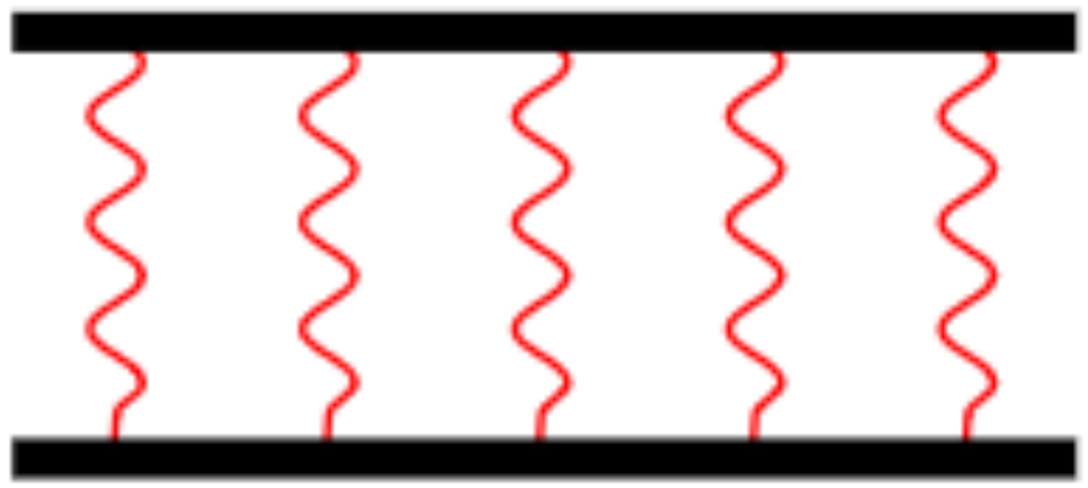
- Main goal: compute high orders in perturbation theory
- $O(G^5)$ will have new features $\mathcal{M} = \mathcal{M}_0 + \mathcal{M}_1\nu + \mathcal{M}_2\nu^2$
  - Second order self-force (important for EMRI/LISA)
  - Contributions related to memory effect

$$\nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

- Exponential complexity growth: 3 loop easy, 4 loop (very) hard
- Bottleneck: integration
  - Large systems of equations
  - Millions of integrals
  - Book keeping
  - Master integrals, special functions, etc.
- New ideas are needed!

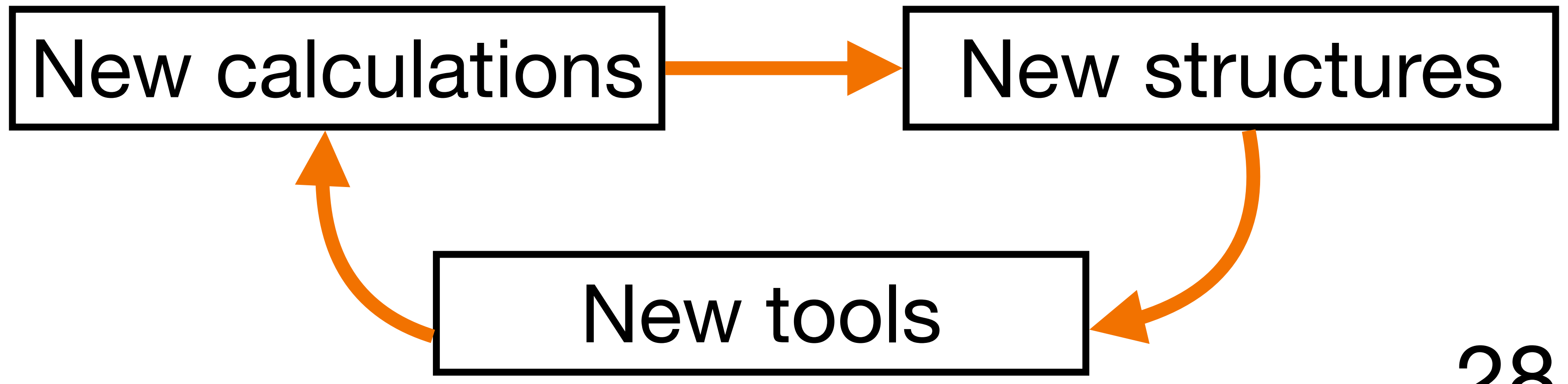
To order G^5 (and beyond)

- Virtuous circle: new computations help improving methods
- Consider simpler models
  - Supersymmetry → HE limit
  - Scalar force → comparison to self force
  - Scalar electrodynamics, …



- Scattering angle in QED $\mathcal{O}(\alpha^5)$ [Bern et. al.]→ 4 loop computations are possible!
- Confident about near term progress on $\mathcal{O}(G^5)$

$$\chi^{5\mathrm{PL}}_{\mathrm{pot}} = \frac{\alpha^5 M^4}{30 J^5 E^4 (\sigma^2 - 1)^{5/2}} \times \left[r_0^{(0)} + \sum_{k=1}^{12} \left(\nu r_k^{(1)} + \nu^2 r_k^{(2)} \right) f_k \right]$$



Conclusion

- Scattering of heavy particles simple processes in gravity
  - Clean environment to clear conceptual issues
  - Many observables
- Classical Observables obtained from quantum scattering amplitudes
- Problem is best treated on-shell and through asymptotic data
  - Avoid algebraic complexity and gauge dependence
- Many new results, program instrumental in progress in perturbative classical gravity
- Key insights from collider physics, can we learn from other approaches to the quantum problem?