FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

Causality violations in Lorentzian path integrals for discrete gravity

Seth Kurankyi Asante, FSU Jena

arXiv:2112.15387

Bianca Dittrich, Sebastian Steinhaus, Hal Haggard, José Diogo Simão, José Padua-Argüelles

Quantum Gravity 2023

Radboud University, Nijmegen

July 11 , 2023

Alexander von Humboldt Stiftung / Foundation

Gravitational Path Integral

Non-perturbative quantum gravity

Many approaches-

$$\mathscr{Z} = \int_{M/\mathrm{Diff}(M)} [\mathscr{D}\mu(\mathrm{geom})]\, e^{-\mathrm{i}\, S[\mathrm{geom}]}$$

What are the fundamental degrees of freedom for QG ?

Gravitational Path Integral

Non-perturbative quantum gravity

Many approaches-

$$\mathscr{Z} = \int_{M/\mathrm{Diff}(M)} [\mathscr{D}\mu(\mathrm{geom})]\, e^{-\mathrm{i}\, S[\mathrm{geom}]}$$

What are the fundamental degrees of freedom for QG ?

Path Integral approaches

[*Bianca's talk*]

★ Computing Lorentzian path integrals

- Deal with convergence, high oscillatory integrals. Picard-Lefschetz methods
- Euclidean path integral via Wick rotation of limited usage

Gravitational Path Integral

Non-perturbative quantum gravity

Many approaches-

$$\mathscr{Z} = \int_{M/\mathrm{Diff}(M)} [\mathscr{D}\mu(\mathrm{geom})]\, e^{-\mathrm{i}\, S[\mathrm{geom}]}$$

What are the fundamental degrees of freedom for QG ?

Path Integral approaches

[Bianca's talk]

★ Computing Lorentzian path integrals

- Deal with convergence, high oscillatory integrals. Picard-Lefschetz methods
- Euclidean path integral via Wick rotation of limited usage

★ What configurations should be summed over in path integral ?

- impose causality conditions on geometries?
- Allow topology change? Picard-Lefschetz can inform

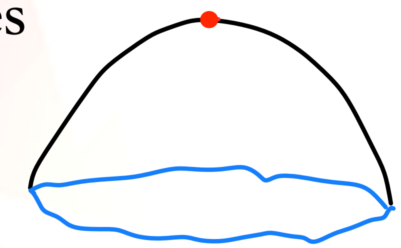
Causality Violations

2D gravity

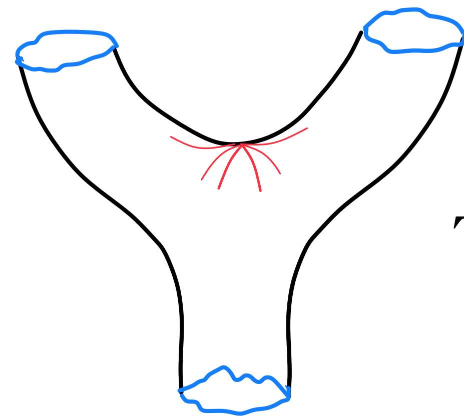
[Sorkin, Luoko]

Causality violating configurations gets imaginary contributions to the action

- Topology changes



Yarmulke



Trouser

Which configurations are enhanced or suppressed ?

[Sorkin, Luoko '97]

2D: Supress trouser-like configurations

[Dittrich, Padua-Argüelles, SKA]

4D: Enhancing Yarmulke configurations lead to non-sensible results

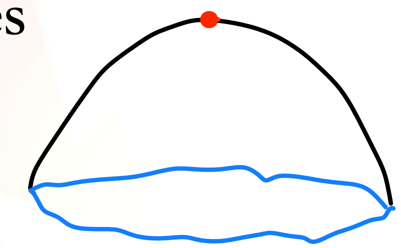
Causality Violations

2D gravity

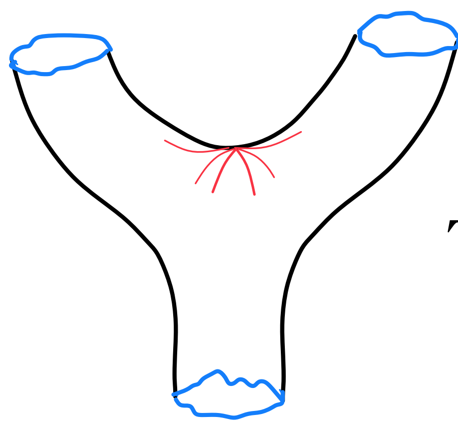
[Sorkin, Luoko]

Causality violating configurations gets imaginary contributions to the action

- Topology changes



Yarmulke



Trouser

Which configurations are enhanced or suppressed ?

[Sorkin, Luoko '97]

2D: Supress trouser-like configurations

[Dittrich, Padua-Argüelles, SKA]

4D: Enhancing Yarmulke configurations lead to non-sensible results

Is there a general mechanism to deal with causal violations?

Outline

Lorentzian Regge Calculus

- Lorentzian Angles
- Analytical Continuation

Path Integrals: Picard Lefschetz

- deSitter Cosmology examples
- Mechanism to suppress causal irregularities

Outline

Lorentzian Regge Calculus

- Lorentzian Angles

- Analytical Continuation

Path Integrals: Picard Lefschetz

- deSitter Cosmology examples

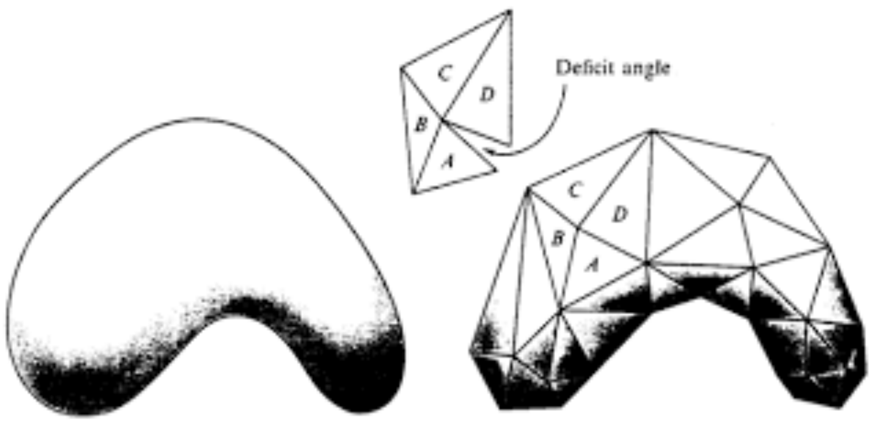
- Mechanism to suppress causal irregularities

Discrete gravity

Regge Calculus [Regge '61]

Discretization of Einstein Hilbert action (length/metric formulation)

$$S_{\text{EH}} = \int_{\mathcal{M}} d^D x \sqrt{|g|}(R - 2\Lambda) \quad \longleftrightarrow \quad S_{\text{Regge}}[\mathcal{T}] = \sum_{h:\text{hinge}} \text{Vol}_h \, \delta_h - \Lambda \sum_{\sigma:\text{simplex}} \text{Vol}_\sigma$$



Discrete gravity

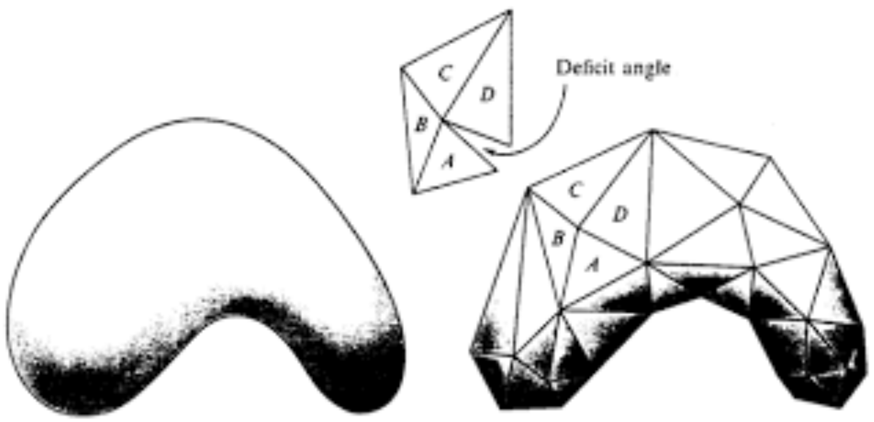
Regge Calculus

[Regge '61]

Discretization of Einstein Hilbert action (length/metric formulation)

$$S_{\mathrm{EH}} = \int_{\mathcal{M}} d^D x \sqrt{|g|}(R - 2\Lambda) \quad \longleftrightarrow \quad S_{\mathrm{Regge}}[\mathcal{T}] = \sum_{h:\mathrm{hinge}} \mathrm{Vol}_h \, \delta_h - \Lambda \sum_{\sigma:\mathrm{simplex}} \mathrm{Vol}_\sigma$$

- based on simplicial discretization
  - use Minkowski flat simplices: piecewise flat geometry

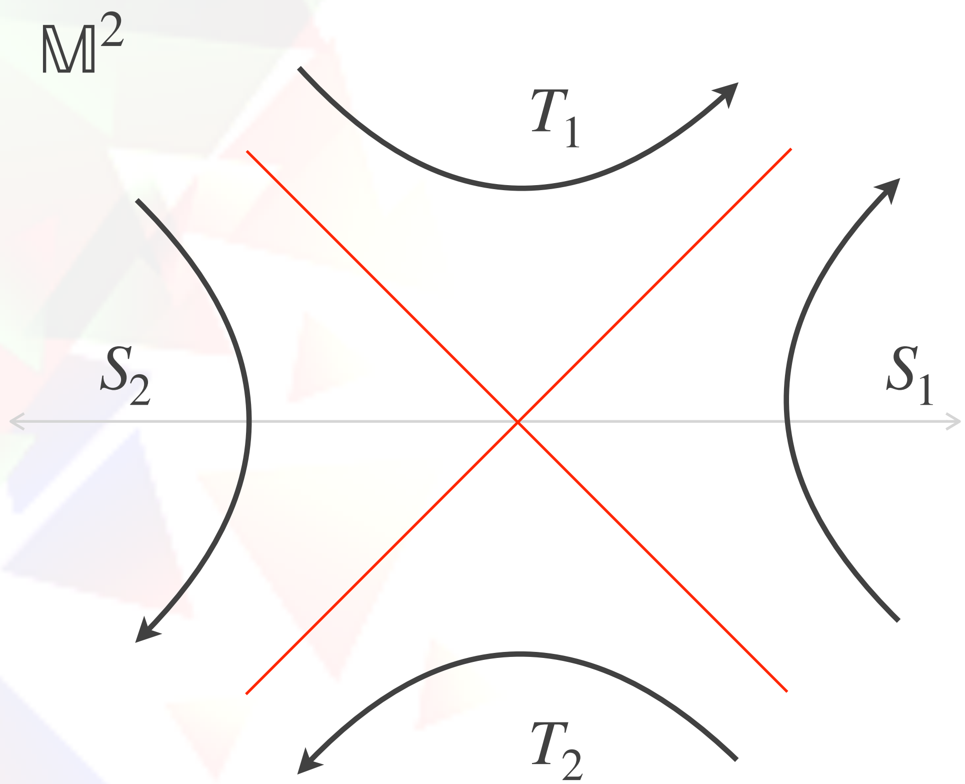


- curvature δ_h distributed on co-dim 2 surfaces
- dynamics: Regge equations of motion
  - variations of edge lengths
- other variables possible: area, area-angle Regge calculus

Lorentzian spacetimes

Lorentzian Angles

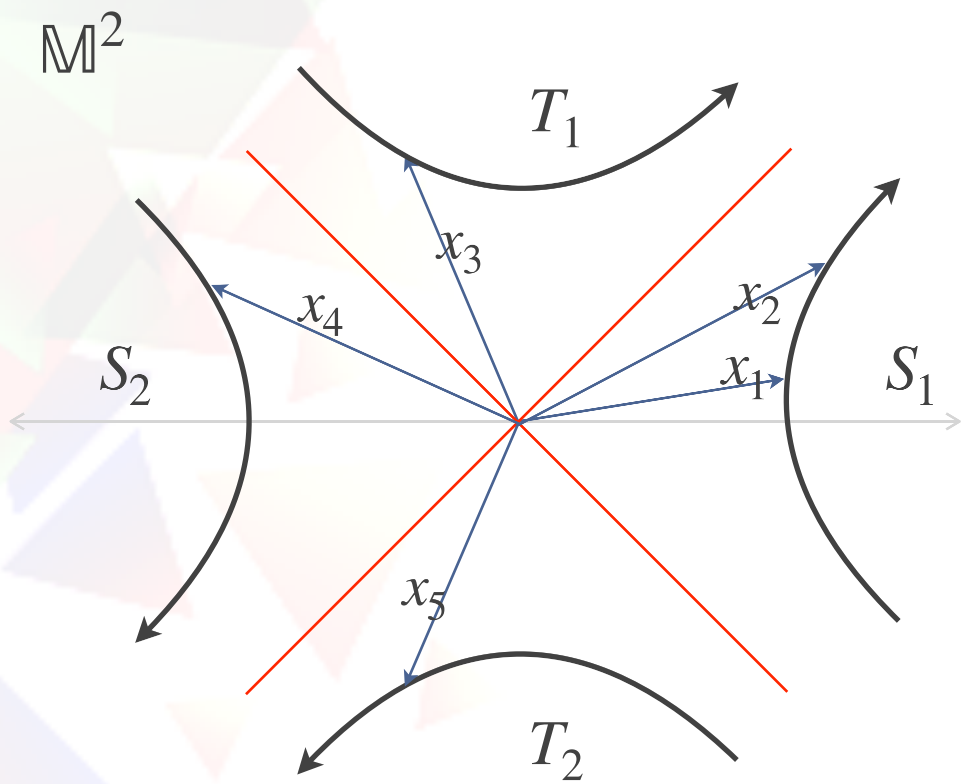
[Alexandrov ’01, Sorkin ’19, Jia ‘21, SKA, Dittrich, Padua-Argüelles ’21]



Lorentzian spacetimes

Lorentzian Angles

[*Alexandrov '01, Sorkin '19, Jia '21, SKA, Dittrich, Padua-Argüelles '21*]



$$\theta_{12} = \cosh^{-1}(x_1 \cdot x_2)$$

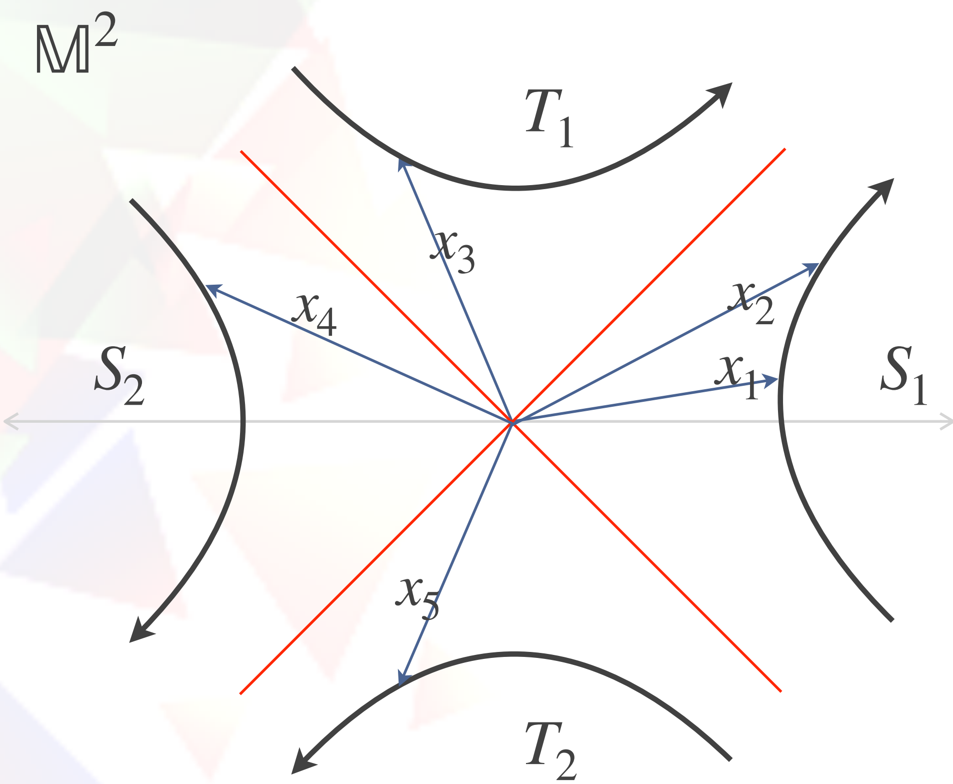
$$\theta_{13} = \sinh^{-1}(x_1 \cdot x_3) \mp \frac{\pi i}{2}$$

$$\theta_{14} = -\cosh^{-1}(-x_1 \cdot x_4) \mp \pi i$$

$$\theta_{35} = \cosh^{-1}(x_3 \cdot x_5) \mp \pi i$$

Lorentzian Angles

[Alexandrov ’01, Sorkin ’19, Jia ‘21, SKA, Dittrich, Padua-Argüelles ’21]



Choice of $\mp i\pi/2$ for light ray crossings

$$\theta_{12} = \cosh^{-1}(x_1 \cdot x_2)$$

$$\theta_{13} = \sinh^{-1}(x_1 \cdot x_3) \mp \frac{\pi i}{2}$$

$$\theta_{14} = -\cosh^{-1}(-x_1 \cdot x_4) \mp \pi i$$

$$\theta_{35} = \cosh^{-1}(x_3 \cdot x_5) \mp \pi i$$

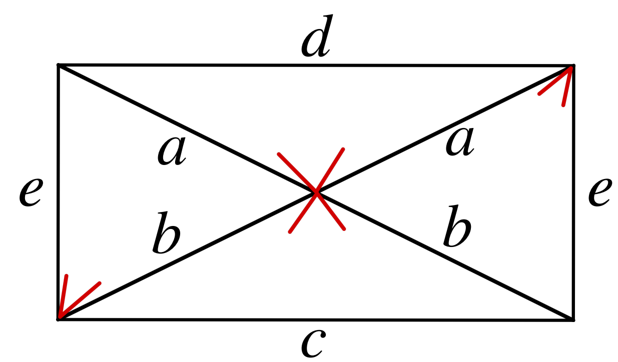
Two choices $L_\mp$ can either enhance or suppress irregular configurations

(Hinge) Causality

Causal violation generic in Regge Calculus

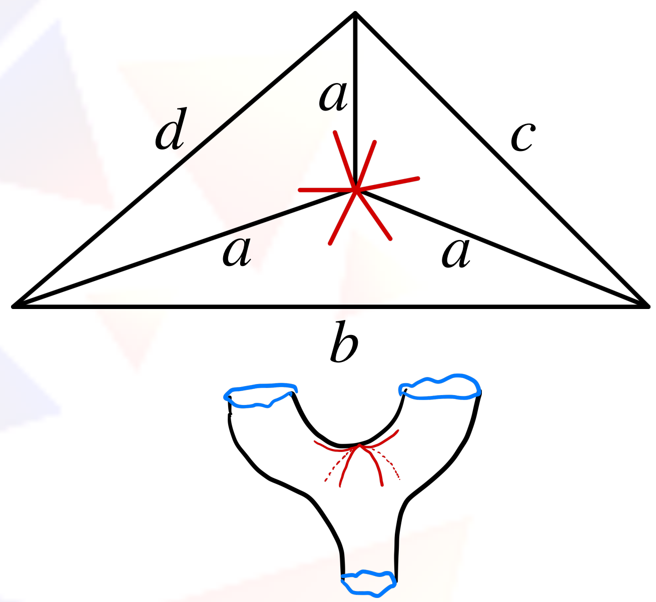
2D Triangulations

Regular configuration

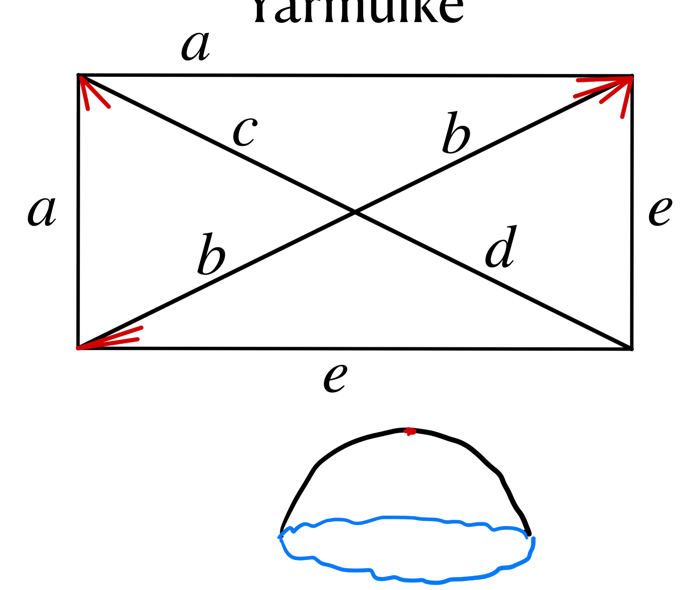


Irregular configurations:

Trouser-like



Yarmulke

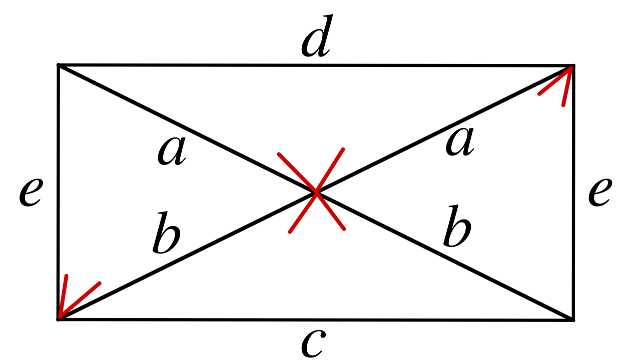


(Hinge) Causality

Causal violation generic in Regge Calculus

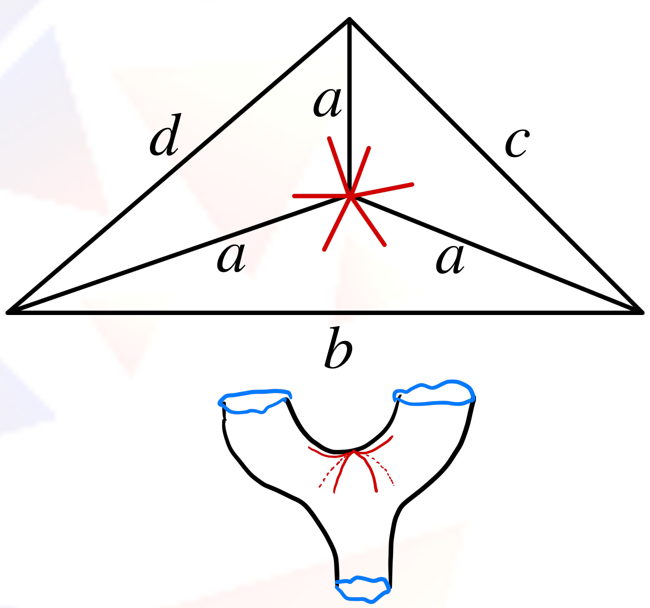
2D Triangulations

Regular configuration

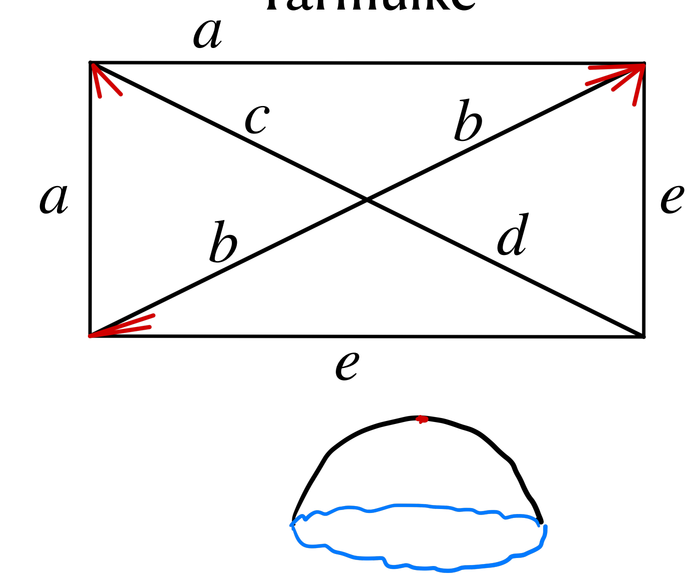


Irregular configurations:

Trouser-like



Yarmulke



[*Jordan, Loll* '13]

Higher Dimensions: Other causality conditions Edge causality, Vertex Causality

Complexification

[Dittrich, Padua-Argüelles, SKA]

Deform path integral into complex plane

Simple Complexification: $a \star b = a_0 b_0 \, e^{i\phi} + \sum_i a_i b_i$ $\quad |a|^2_\star = a \star a \quad a, b \in \mathbb{R}^n$

Complexification

[Dittrich, Padua-Argüelles, SKA]

Deform path integral into complex plane

Simple Complexification: $a \star b = a_0 b_0 \, e^{i\phi} + \sum_i a_i b_i$ $\quad |a|_\star^2 = a \star a \quad a, b \in \mathbb{R}^n$

Complex angles

$$\theta = -i \log \frac{a \star b + |a \wedge b|_\star}{|a|_\star |b|_\star}$$

$$\theta(\phi) = \begin{cases} \theta^+ & \text{for } \phi \in (0,\pi) \\ \theta^- & \text{for } \phi \in (-\pi,0) \end{cases}$$

Generalizes complex dihedral angles in Regge calculus

Complexification

[Dittrich, Padua-Argüelles, SKA]

Deform path integral into complex plane

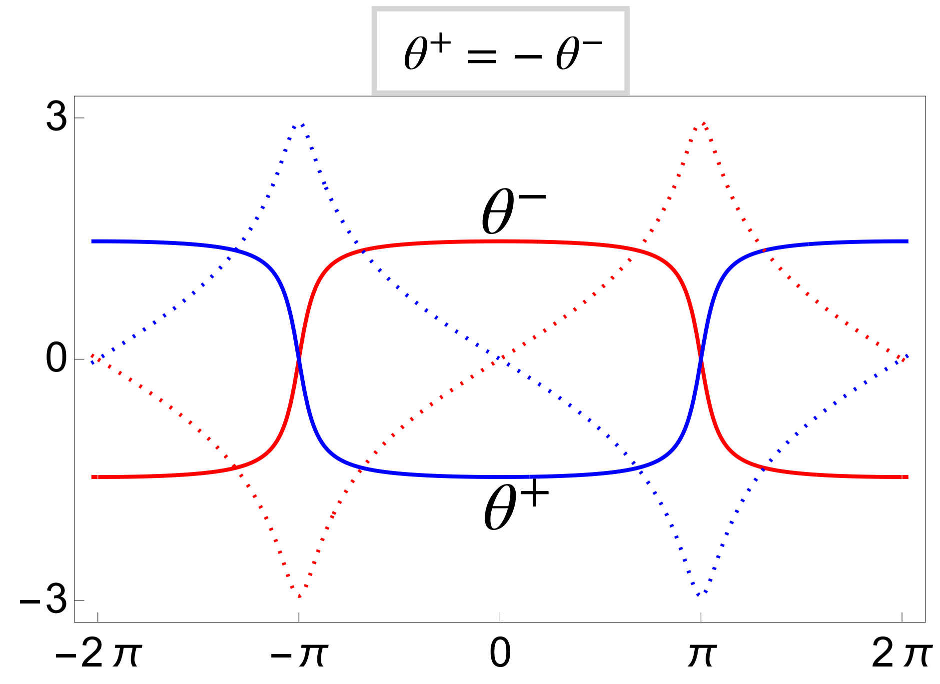
Simple Complexification: $a \star b = a_0 b_0 \, e^{i\phi} + \sum_i a_i b_i$ $\quad |a|^2_\star = a \star a \quad a, b \in \mathbb{R}^n$

Complex angles

$$\theta = -i \log \frac{a \star b + |a \wedge b|_\star}{|a|_\star |b|_\star}$$

$$\theta(\phi) = \begin{cases} \theta^+ & \text{for } \phi \in (0,\pi) \\ \theta^- & \text{for } \phi \in (-\pi,0) \end{cases}$$

Generalizes complex dihedral angles in Regge calculus



Analytical continuation

- extend $\theta(\phi)$ to the region $\phi \in (-2\pi, 2\pi]$
- 4π periodic

Complex Regge Calculus

[Dittrich, Padua-Argüelles, SKA]

Analytical continued dihedral angles

$$\delta^{\pm}(\phi) = 2\pi \pm \sum_{\sigma} \theta_{\sigma}^{\pm}(\phi)$$

$\delta^{+} = \delta^{-}$ regular configurations

action

$$S_{\mathrm{R}}[\mathcal{T}] = \sum_{\mathrm{hinges}} |\,\mathrm{Vol}_h\,|\,\delta_h - \Lambda \sum_{\sigma} |\,\mathrm{Vol}_\sigma\,|$$

Complex Regge Calculus

[Dittrich, Padua-Argüelles, SKA]

Analytical continued dihedral angles

$$\delta^{\pm}(\phi) = 2\pi \pm \sum_{\sigma} \theta^{\pm}_{\sigma}(\phi)$$

$\delta^+ = \delta^-$ regular configurations

action

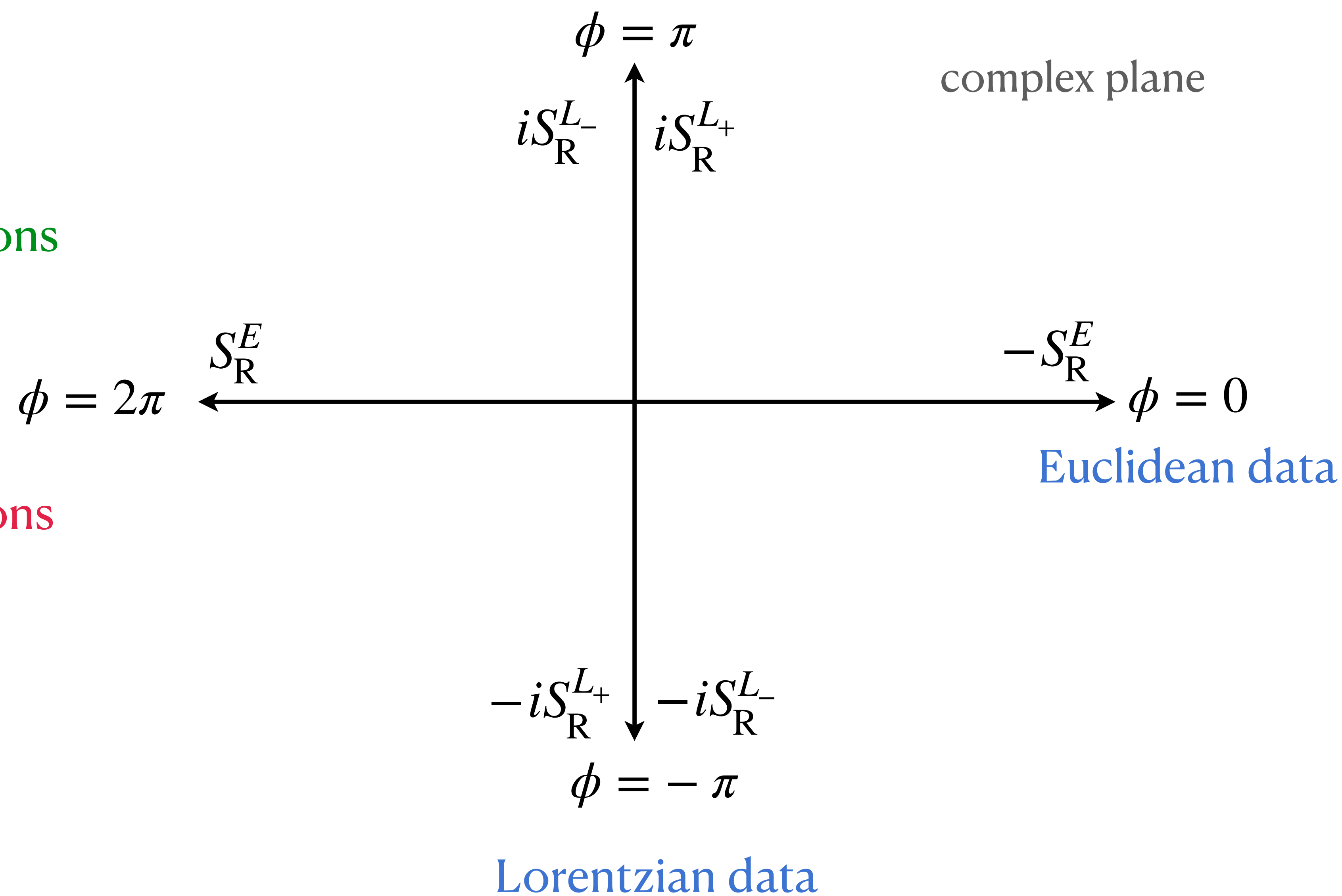
$$S_{\mathrm{R}}[\mathcal{T}] = \sum_{\text{hinges}} |\mathrm{Vol}_h|\, \delta_h - \Lambda \sum_{\sigma} |\mathrm{Vol}_\sigma|$$

Complex Regge action

No ambiguity for regular configurations

$$S^{L_+}_{\mathrm{R}} = S^{L_-}_{\mathrm{R}}$$

Branch cuts for irregular configurations



Outline

Lorentzian Regge Calculus

- Lorentzian Angles
- Analytical Continuation

Path Integrals: Picard Lefschetz

- deSitter Cosmology examples
- Mechanism to suppress causal irregularities

Picard-Lefschetz method

Idea:

[*Witten '01 '09, Vassiliev, Tanizaki, Koike,…*]

Converts oscillatory integrals into sum of integrations with exponentially fast convergence

$$Z = \int_X \mathrm{d}x \, e^{iS(x)/\hbar} \longrightarrow Z = \sum_\sigma n_\sigma \int_{\mathcal{J}_\sigma} \mathrm{d}x \, e^{iS(x)/\hbar}$$

Summary:

Picard-Lefschetz method

Idea:

[*Witten '01 '09, Vassiliev, Tanizaki, Koike,...*]

Converts oscillatory integrals into sum of integrations with exponentially fast convergence

$$Z = \int_X \mathrm{d}x\, e^{iS(x)/\hbar} \longrightarrow Z = \sum_\sigma n_\sigma \int_{\mathcal{J}_\sigma} \mathrm{d}x\, e^{iS(x)/\hbar}$$

Summary:

- Complexify integration domain $X \to X^{\mathbb{C}}$

Picard-Lefschetz method

Idea:

[Witten '01 '09, Vassiliev, Tanizaki, Koike,...]

Converts oscillatory integrals into sum of integrations with exponentially fast convergence

$$Z = \int_X \mathrm{d}x\, e^{iS(x)/\hbar} \longrightarrow Z = \sum_\sigma n_\sigma \int_{\mathcal{J}_\sigma} \mathrm{d}x\, e^{iS(x)/\hbar}$$

Summary:

- Complexify integration domain $X \to X^{\mathbb{C}}$
- Find critical points of action in the complex domain $p_\sigma \in X^{\mathbb{C}}$

Picard-Lefschetz method

Idea:

[Witten '01 '09, Vassiliev, Tanizaki, Koike,…]

Converts oscillatory integrals into sum of integrations with exponentially fast convergence

$$Z = \int_X \mathrm{d}x\, e^{iS(x)/\hbar} \longrightarrow Z = \sum_\sigma n_\sigma \int_{\mathcal{J}_\sigma} \mathrm{d}x\, e^{iS(x)/\hbar}$$

Summary:

- Complexify integration domain $X \to X^{\mathbb{C}}$
- Find critical points of action in the complex domain $p_\sigma \in X^{\mathbb{C}}$
- Use critical points in the complex domain to find integration cycles Lefschetz thimbles $\mathcal{J}_\sigma$

Along thimbles:

Real part of integrand decreases monotonically while imaginary part is constant

Picard-Lefschetz method

Idea:

[Witten ’01 ’09, Vassiliev, Tanizaki, Koike,...]

Converts oscillatory integrals into sum of integrations with exponentially fast convergence

$$Z = \int_X \mathrm{d}x\, e^{iS(x)/\hbar} \longrightarrow Z = \sum_\sigma n_\sigma \int_{\mathcal{J}_\sigma} \mathrm{d}x\, e^{iS(x)/\hbar}$$

Summary:

- Complexify integration domain $X \to X^{\mathbb{C}}$

- Find critical points of action in the complex domain $p_\sigma \in X^{\mathbb{C}}$

- Use critical points in the complex domain to find integration cycles Lefschetz thimbles $\mathcal{J}_\sigma$

  Along thimbles:

  Real part of integrand decreases monotonically while imaginary part is constant

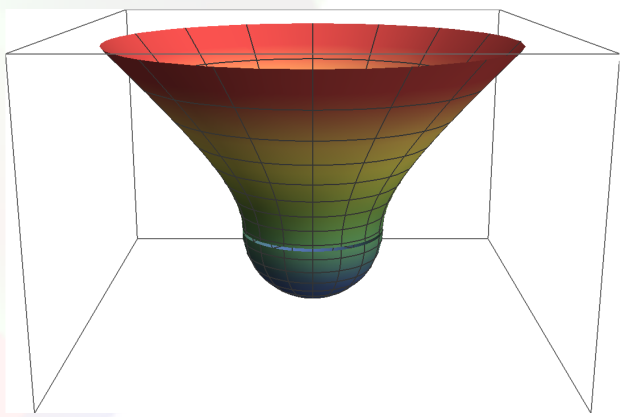
- Express real integration domain in terms of Lefschetz thimbles $X = \sum_\sigma n_\sigma \mathcal{J}_\sigma$

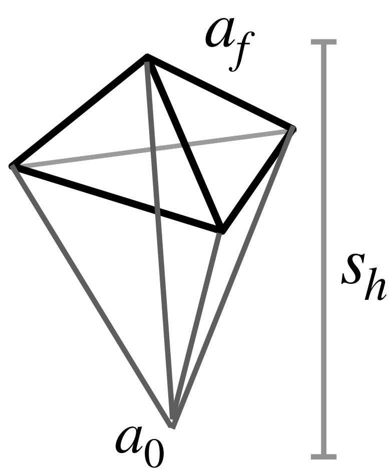
Applications: Cosmology

FLRW spacetimes

[Hartle, Hawking, Feldbrugge, Lehners, Turok, Williams, Lui, Collins, Dittrich, Gielen, Schander….]

No boundary proposal





Features:

Discrete: Ball model

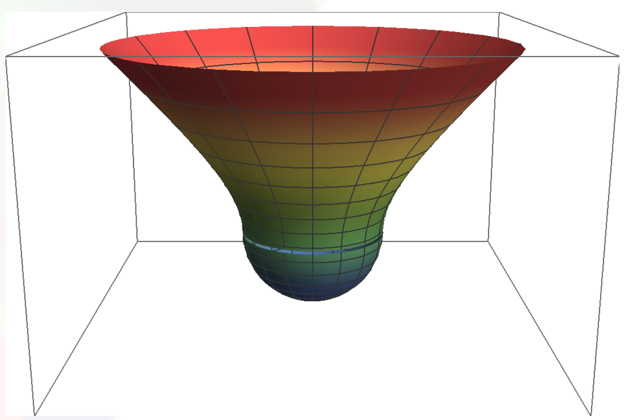
Topology changing configuration

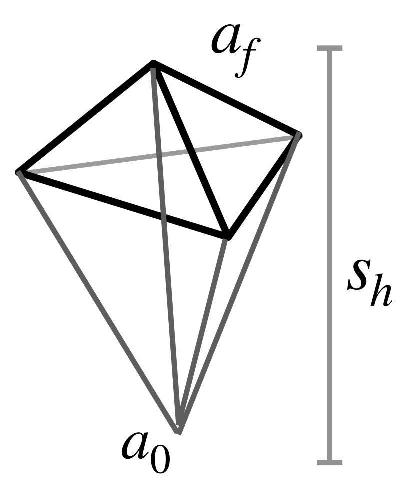
Applications: Cosmology

FLRW spacetimes

[Hartle, Hawking, Feldbrugge, Lehners, Turok, Williams, Lui, Collins, Dittrich, Gielen, Schander….]

No boundary proposal





Features:

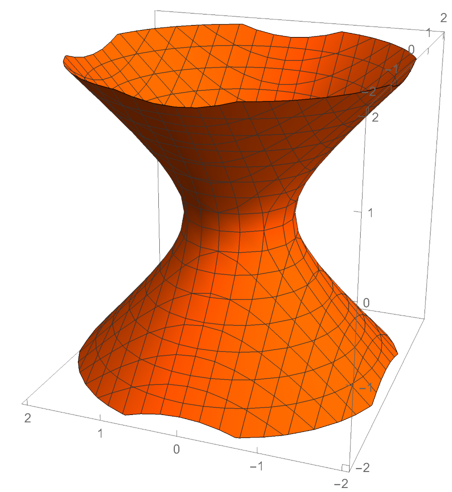
Discrete: Ball model

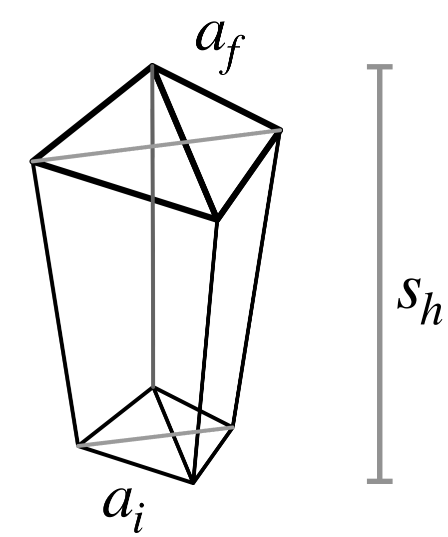
Topology changing configuration

Euclidean saddle points

For small a_f

deSitter cosmological spacetime





discretization

Discrete: Shell model

Causal violations at hinges

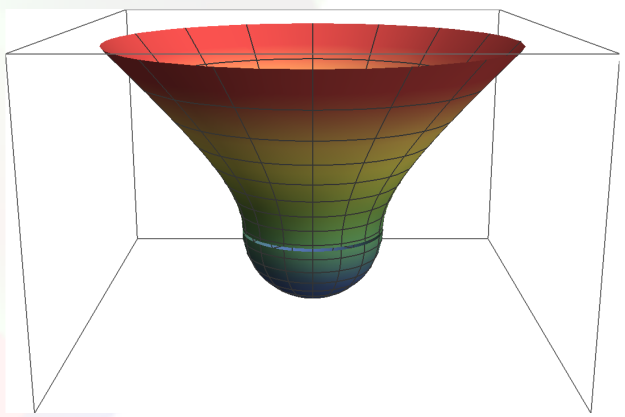
Lorentzian saddle points

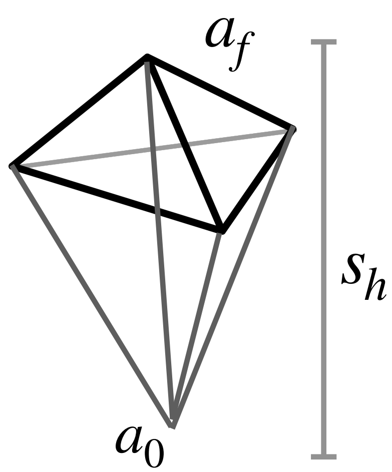
Applications: Cosmology

FLRW spacetimes

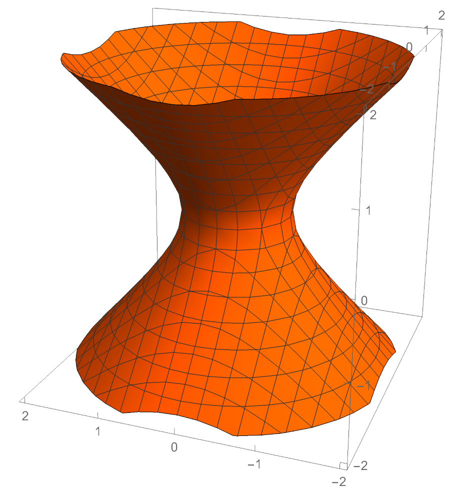
[Hartle, Hawking, Feldbrugge, Lehners, Turok, Williams, Lui, Collins, Dittrich, Gielen, Schander….]

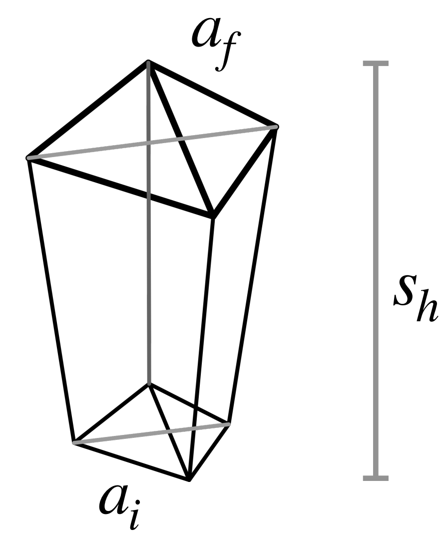
No boundary proposal





deSitter cosmological spacetime





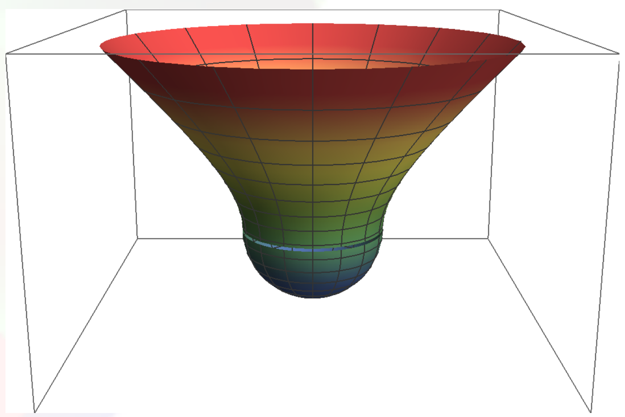
discretization

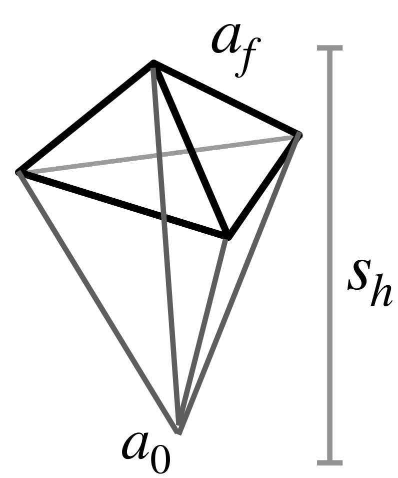
Applications: Cosmology

FLRW spacetimes

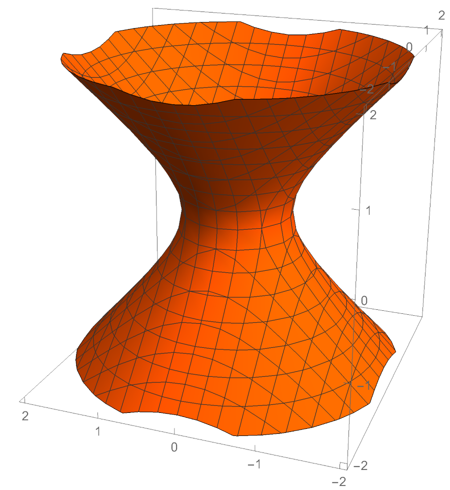
[Hartle, Hawking, Feldbrugge, Lehners, Turok, Williams, Lui, Collins, Dittrich, Gielen, Schander….]

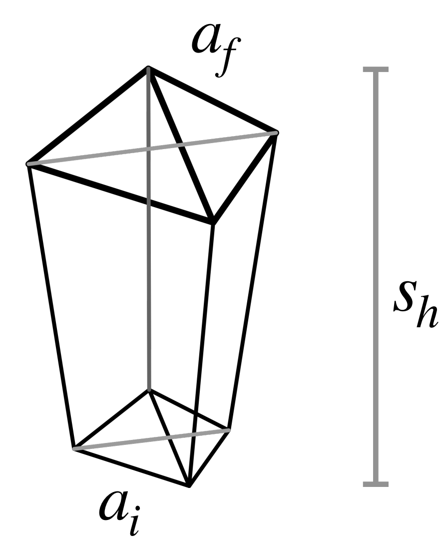
No boundary proposal





deSitter cosmological spacetime

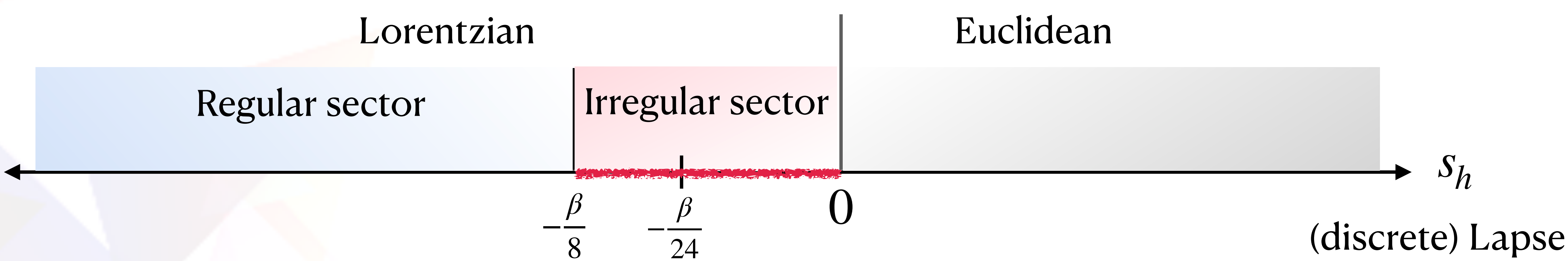




discretization

Hinge Causality-Violations

[Dittrich, Gielen, Schander, SKA, Padua-Argüelles ,..]



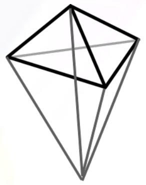
$\beta = (l_i + l_f)^2$

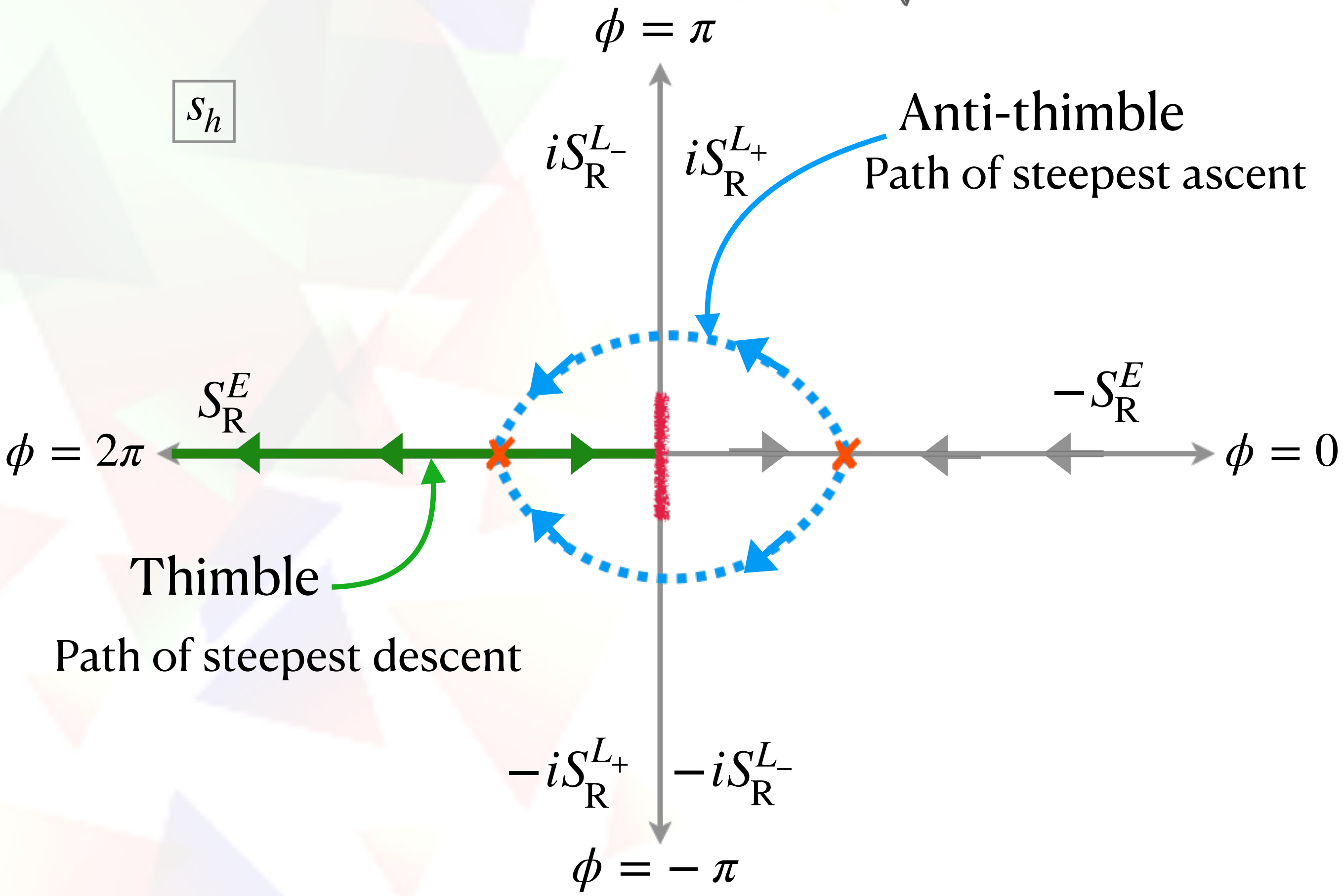
Cosmology Examples

Lefschetz thimbles

$\int \mathrm{d}s_h \, e^W$

Ball model





Real part of action decreases along thimbles $\mathrm{Re}(W) < 0$

Thimble along Euclidean axis at $\phi = 2\pi$ leads to e^{+S^E} Vilenkin choice

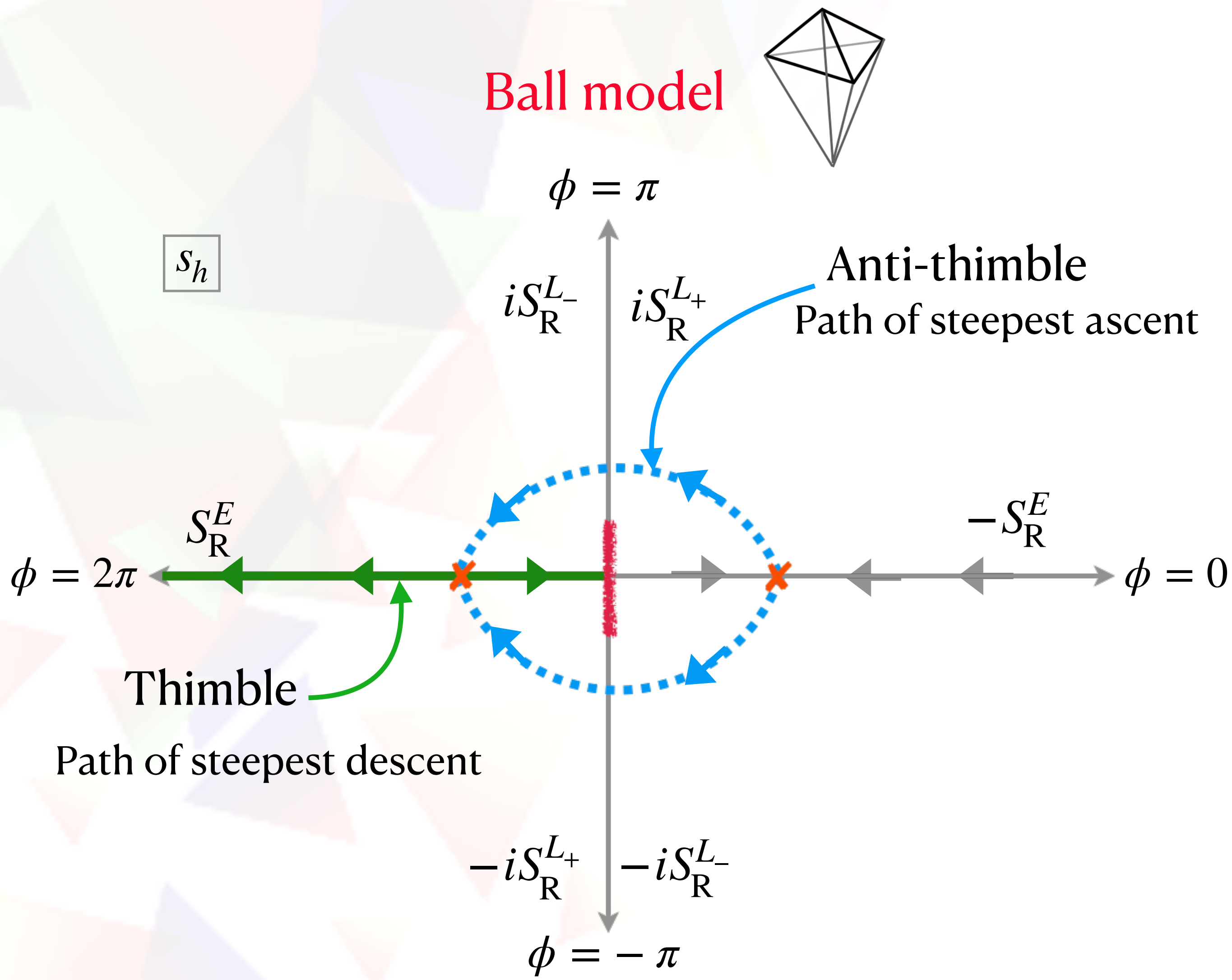
Feldbrugge et al..

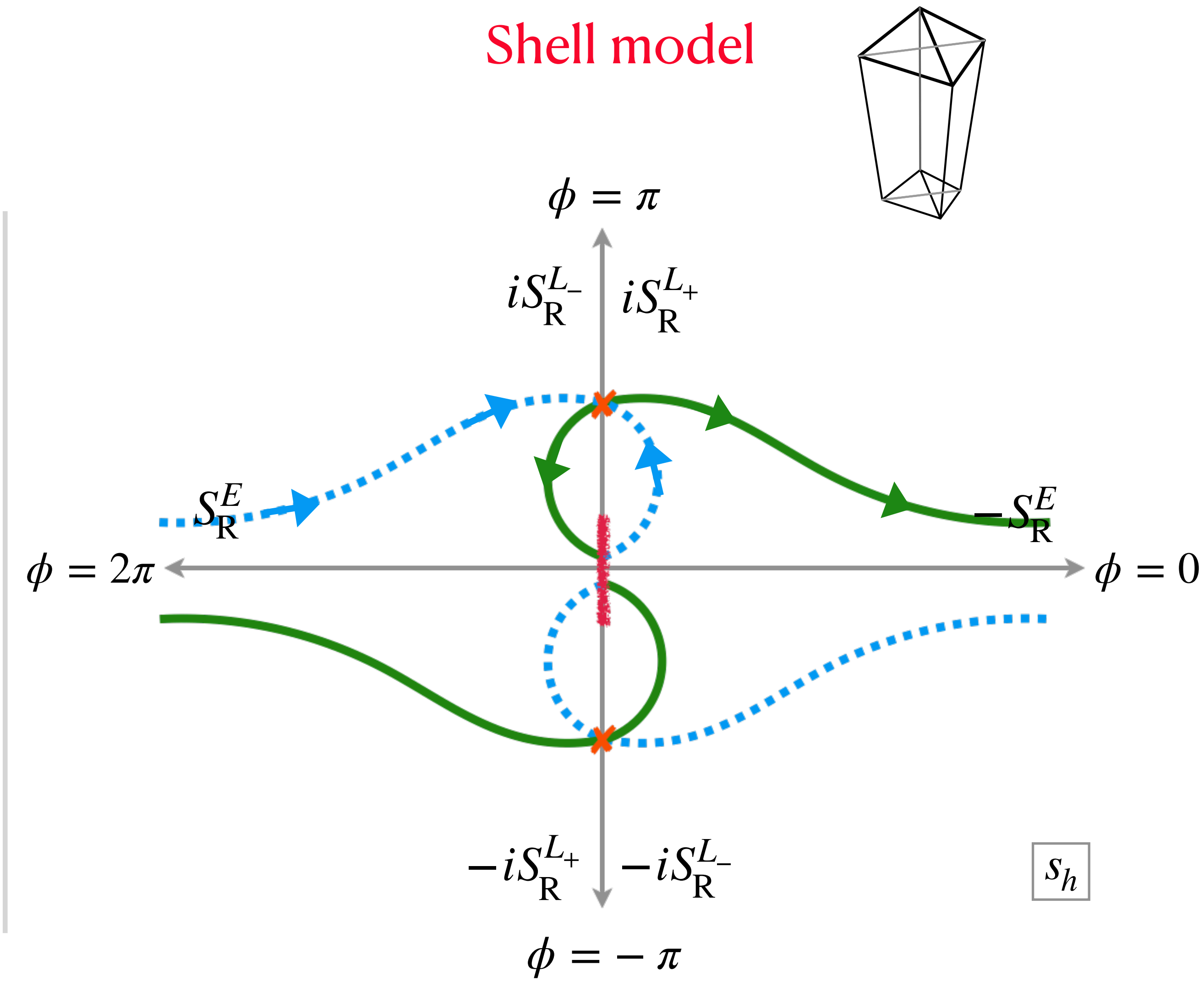
opposite Hartle-Hawking choice e^{-S^E}

Cosmology Examples

Lefschetz thimbles

$\int ds_h \, e^W$





Real part of action decreases along thimbles $\mathrm{Re}(W) < 0$

Thimble along Euclidean axis at $\phi = 2\pi$ leads to e^{+S^E}

Vilenkin choice

Feldbrugge et al..

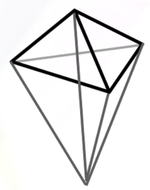
opposite Hartle-Hawking choice e^{-S^E}

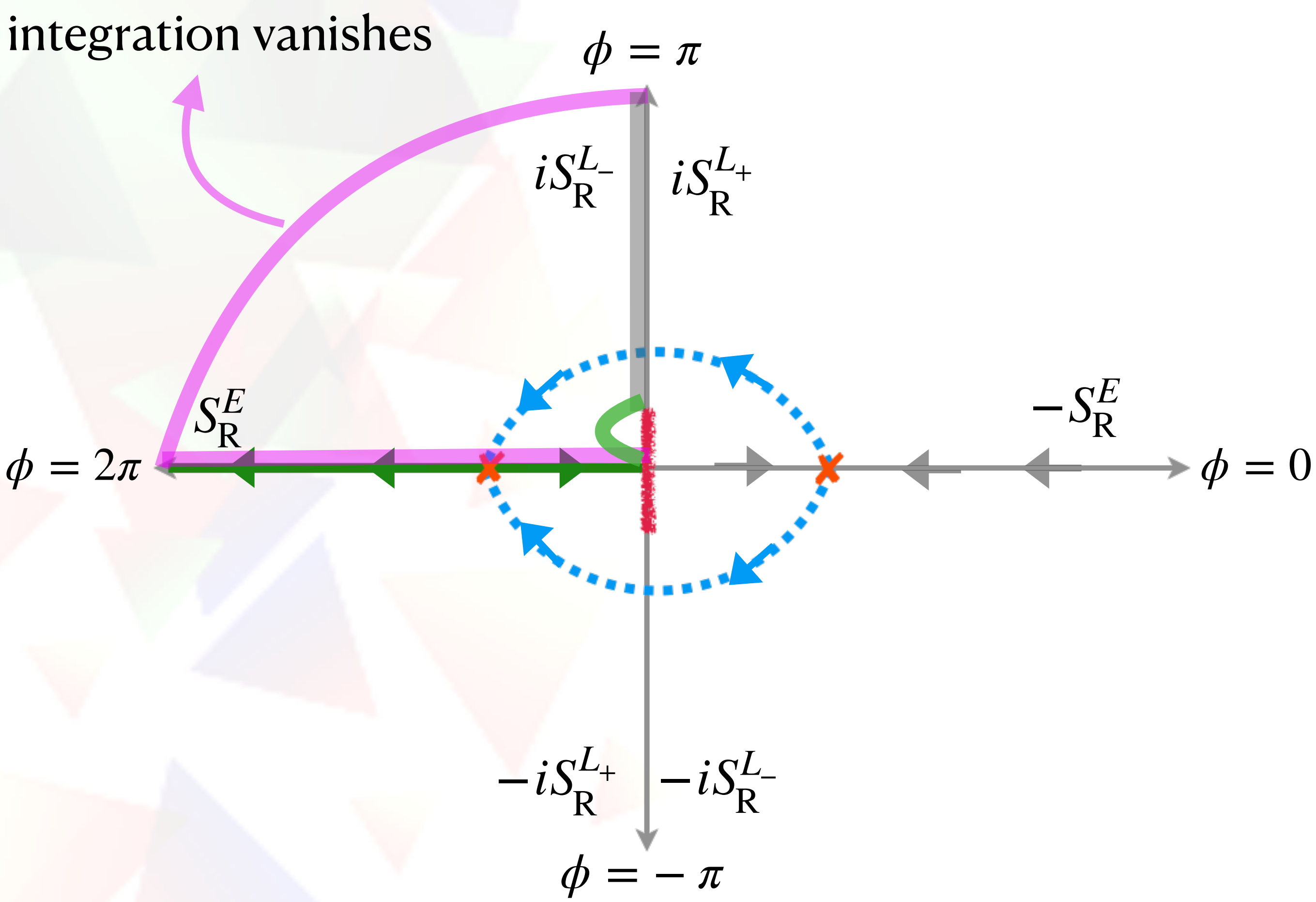
Cosmology Examples

Deform integration contours

$\int \mathrm{d}s_h \, e^W$

Ball model





Suppression from rotation of integration contour

Include configurations with irregular causalities

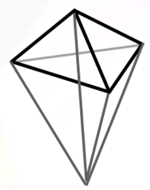
Lefschetz thimbles picks the suppressing side of the branch cut. $\mathrm{Re}(W) < 0$

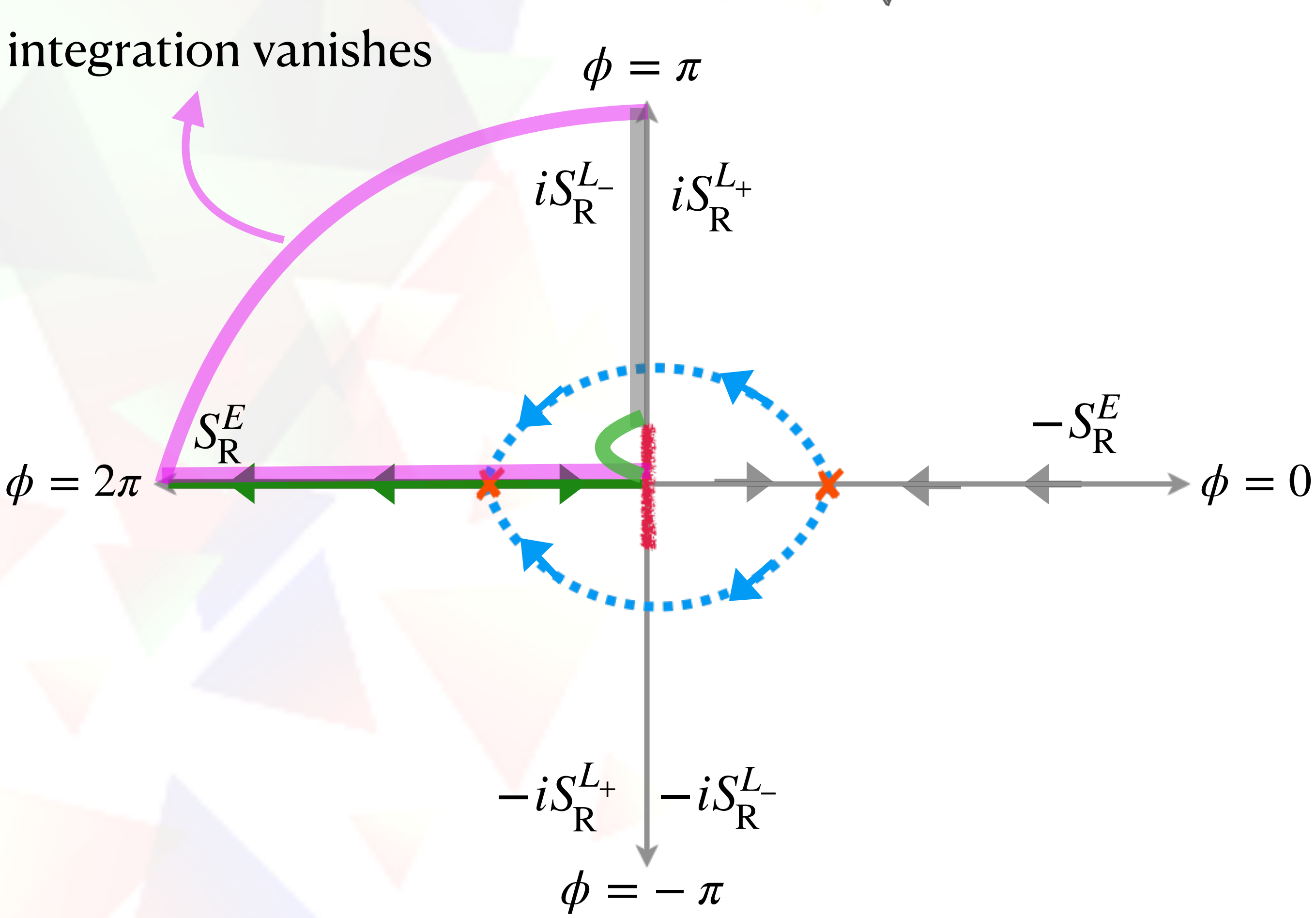
Cosmology Examples

Deform integration contours

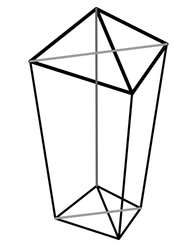
$\int \mathrm{d}s_h \, e^{W}$

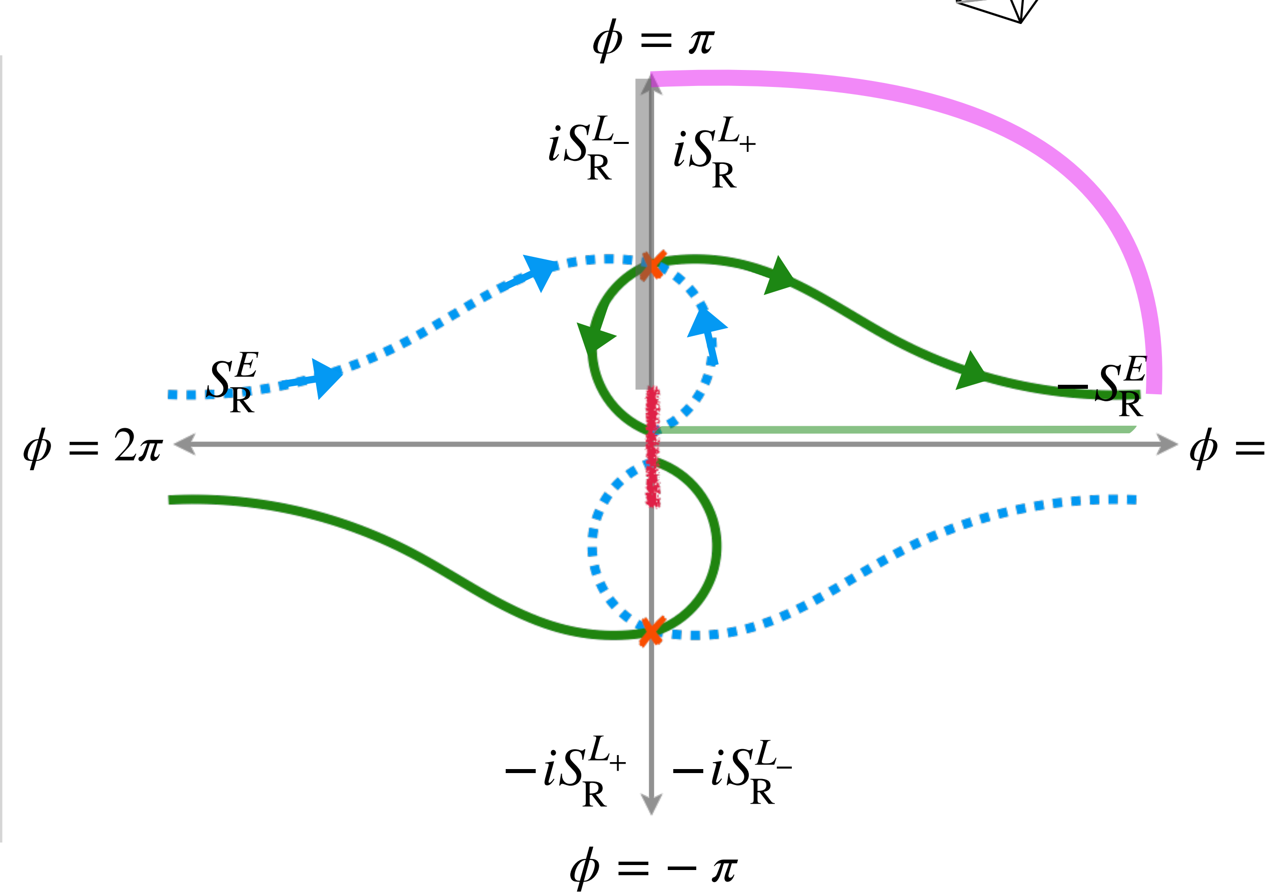
Ball model





Shell model





Suppression from rotation of integration contour

Include configurations with irregular causalities

Lefschetz thimbles picks the suppressing side of the branch cut. $\mathrm{Re}(W) < 0$

Concluding Remarks

- Progress in computing Lorentzian path integrals for discrete gravity
- (Hinge) causality-violating configurations have branch cuts for complex Regge action
- Wick rotation determined by dynamics: overcome conformal factor problem

Concluding Remarks

- Progress in computing Lorentzian path integrals for discrete gravity
- (Hinge) causality-violating configurations have branch cuts for complex Regge action
- Wick rotation determined by dynamics: overcome conformal factor problem
- Picard-Lefschetz: Fast convergence along Lefschetz thimbles
- Can allow for causal violations: Examples give suppression via Picard-Lefschetz formalism

Concluding Remarks

- Progress in computing Lorentzian path integrals for discrete gravity
- (Hinge) causality-violating configurations have branch cuts for complex Regge action
- Wick rotation determined by dynamics: overcome conformal factor problem
- Picard-Lefschetz: Fast convergence along Lefschetz thimbles
- Can allow for causal violations: Examples give suppression via Picard-Lefschetz formalism

Outlook:

- Test more features with examples: Higher dimensional integrals
- Application to effective spin foams *[Dittrich, Padua-Argüelles '23]*
- Causality violations in EPRL/FK spin foam models

Concluding Remarks

- Progress in computing Lorentzian path integrals for discrete gravity
- (Hinge) causality-violating configurations have branch cuts for complex Regge action
- Wick rotation determined by dynamics: overcome conformal factor problem
- Picard-Lefschetz: Fast convergence along Lefschetz thimbles
- Can allow for causal violations: Examples give suppression via Picard-Lefschetz formalism

Outlook:

- Test more features with examples: Higher dimensional integrals
- Application to effective spin foams *[Dittrich, Padua-Argüelles '23]*
- Causality violations in EPRL/FK spin foam models

THANK YOU !