Impact of matter fields on quantum geometry

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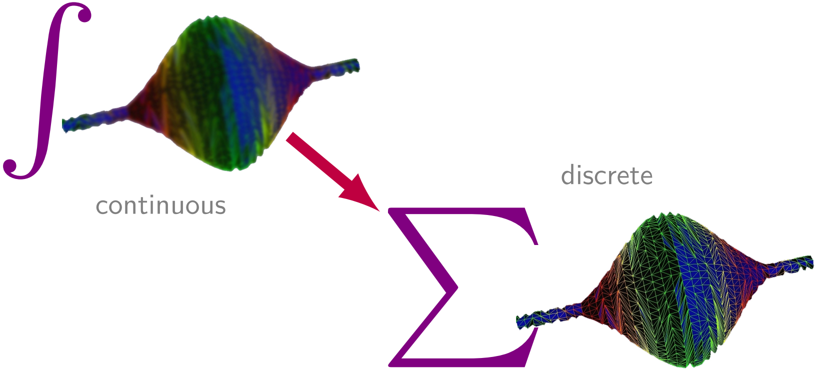
Introduction to Causal Dynamical Triangulations

What is Causal Dynamical Triangulations?

***C**ausal **D**ynamical **T**riangulations* is a background-independent and diffeomorphism-invariant approach to quantum gravity. It provides a lattice regularization of the formal gravitational path integral via a sum over causal triangulations.



Causal Dynamical Triangulations

The partition function and action

$$\int \mathcal{D}[g] e^{iS^{EH}[g]} \longrightarrow \sum_{\mathcal{T}} e^{-S^{R}[\mathcal{T}]}$$

The Regge action $S^R[\mathcal{T}]$ is equal to the Einstein-Hilbert action S^{EH} evaluated on a triangulation T built of *four-simplices*.

- Global time foliation of the spacetime manifold (**Causal** DT)
- Fixed spatial and global topology ($\mathcal{M} = T^3 \times T^1$)
- Monte Carlo simulations - expectation values of observables

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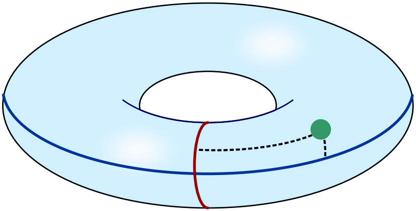
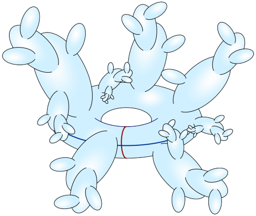




Dynamical scalar fields

- We introduce *dynamical (backreacting)* scalar fields $\phi_\sigma,\ \sigma = x, y, z, t$, taking values in a circle of circumference δ and winding around S^1 once in the direction x, y, z, and t, respectively.
- The continuous Euclidean action for a massless scalar field:

$$S[g,\varphi] = \frac{1}{2}\int \mathrm{d}^4x\,\sqrt{g(x)}\,\partial^\mu\varphi(x)\partial_\mu\varphi(x), \quad \varphi(x) \in S^1(\delta)$$

- The discrete counterpart of the matter action decomposes into the quantum and the classical parts:

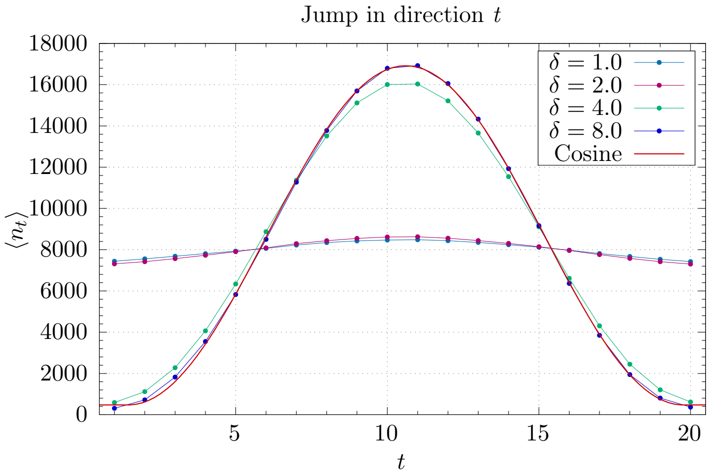
$$S[\mathcal{T},\varphi = \bar\varphi + \eta] = \sum_{i\leftrightarrow j}(\varphi_i - \varphi_j - \delta\mathbf{B}_{ij})^2 = \eta^\top\mathbf{L}\eta + \delta^2\tilde{S}^{\mathrm{clas}}[\mathcal{T}]$$

$$\tilde{S}^{\mathrm{clas}}[\mathcal{T}] = \tilde\varphi^\top\mathbf{L}\tilde\varphi - 2b^\top\tilde\varphi + \|\mathbf{B}\|^2, \quad L\tilde\varphi = b$$

L is the discrete Laplacian matrix and **B** is an antisymmetric *boundary* matrix.

Single field winding in time direction

Single dynamical scalar field winding once around a circle of circumference δ in the time direction. For large δ, the volume profile is described by a *pinched* cosine function.



Minisuperspace model with scalar field

The behavior $\langle n_t \rangle \propto \cos\left(\frac{2\pi}{\tau} t\right)$, can be explained by assuming spatial homogeneity and isotropy.

Minisuperspace action $(v = v(t) \propto a^3(t),\ \varphi = \varphi(t))$

$$S[v, \varphi] = \int \mathrm{d}^4x \sqrt{g} \left(\# R - \Lambda + (\partial\varphi)^2 \right) = \int_{-T/2}^{T/2} \mathrm{d}t \, \frac{\dot{v}^2}{v} + v\dot{\varphi}^2$$

Constraints

$$V = \int \mathrm{d}t \, v, \quad \delta = \int \mathrm{d}t \, \dot{\varphi} = \varphi\left(\frac{T}{2}\right) - \varphi\left(-\frac{T}{2}\right), \quad v(t) \geq \varepsilon$$

Constant solution for $\delta \leq 2\pi$

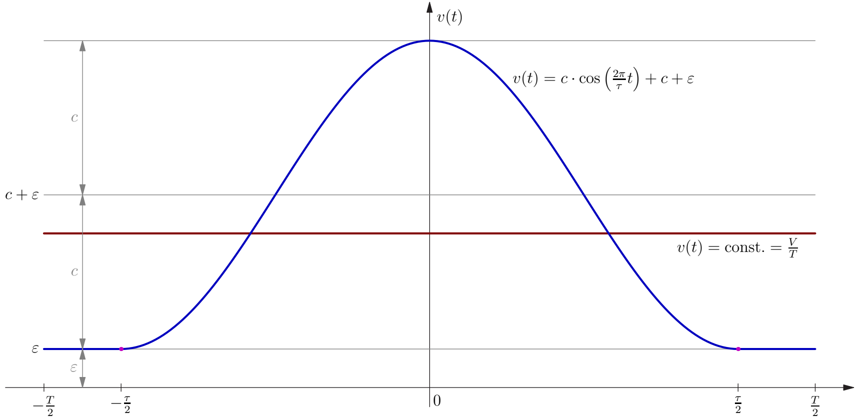
$$v(t) = \frac{V}{T}, \qquad \dot{\varphi}(t) = \frac{\delta}{T}, \qquad S = \frac{V}{T^2} \cdot \delta^2$$

Cosine solution for $\delta \geq 2\pi$

$$v(t) = \begin{cases} c \cdot \cos\left(\frac{2\pi}{\tau} t\right) + c + \varepsilon & |t| \leq \frac{\tau}{2} \\ \varepsilon & \frac{\tau}{2} \leq |t| \leq \frac{T}{2} \end{cases}, \quad \dot{\varphi}(t) = \frac{\beta}{v(t)}$$

Minisuperspace model with scalar field

Classical solutions



Phase transition at $\delta = \frac{2\pi}{\sqrt{G}}$

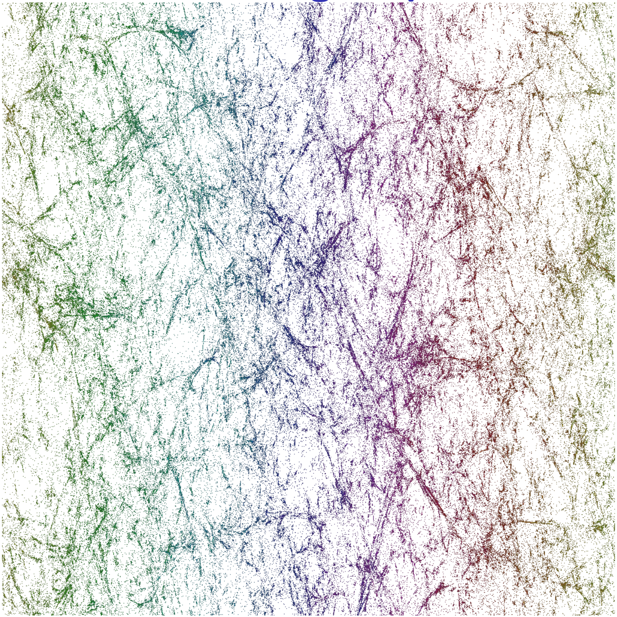
Density maps

- The matter action decomposes into quantum and classical parts:

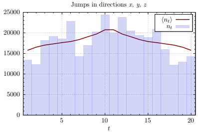
$$S[\mathcal{T}, \varphi = \bar{\varphi} + \eta] = \eta^\top \mathbf{L} \eta + \delta^2 \tilde{S}^{\mathrm{clas}}[\mathcal{T}]$$

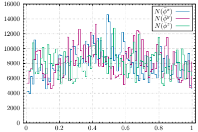
- Quantum fluctuations can be integrated out and are negligible.
- The *nontrivial* classical solution contributes to the classical action $\delta^2 \tilde{S}^{\mathrm{clas}}[\mathcal{T}]$ which depends in a crucial way on the triangulation $\mathcal{T}$.
- For small δ the geometric part of the action dominates.
- For large δ the matter action dominates and the total action is the lowest for *pinched* configurations.
- To visualize a triangulation and a field configuration, we reintroduce four classical scalar fields. Each four-simplex is assigned coordinates $(\varphi_x, \varphi_y, \varphi_z, \varphi_t)$ and is visible as a single pixel on the following images.
- Dense *fibers* correspond to *evolving* outgrowths.

Scalar fields winding in spatial directions



Dynamical fields $\varphi^x, \varphi^y, \varphi^z$. Projection on $\varphi^t - \varphi^x$. Circumference $\delta = 0.0$.





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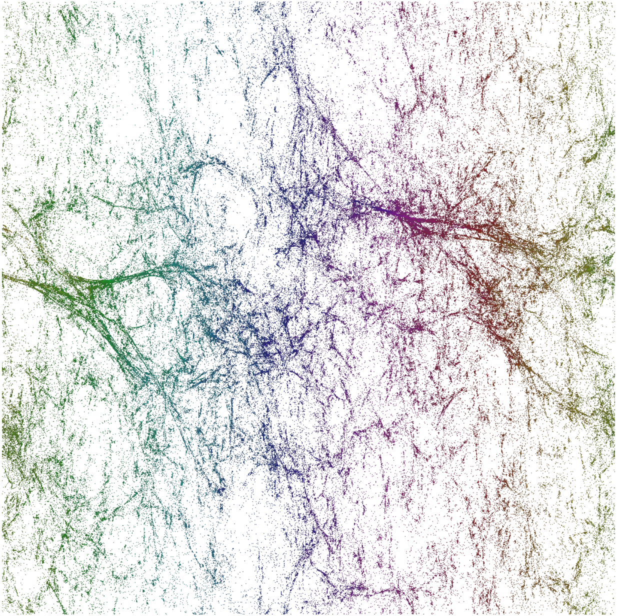
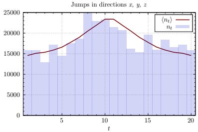
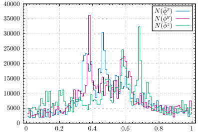
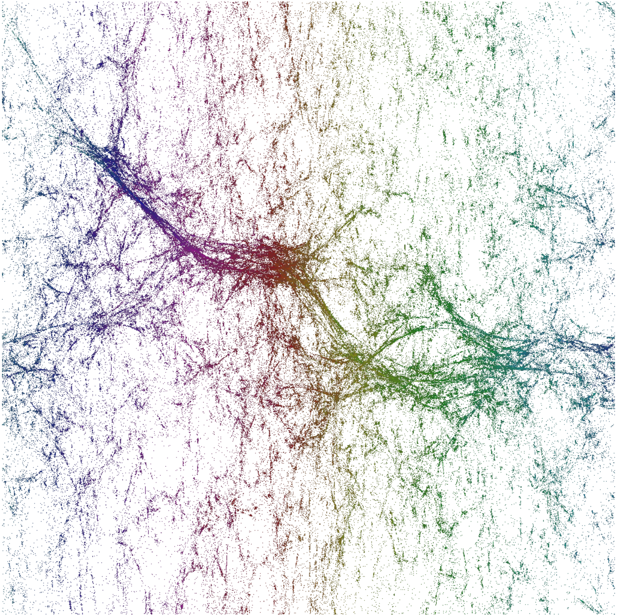


Dynamical fields $\varphi^x, \varphi^y, \varphi^z$. Projection on $\varphi^t - \varphi^x$. Circumference $\delta = 2.0$.

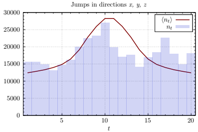


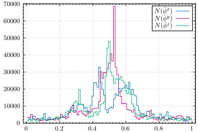


Scalar fields winding in spatial directions

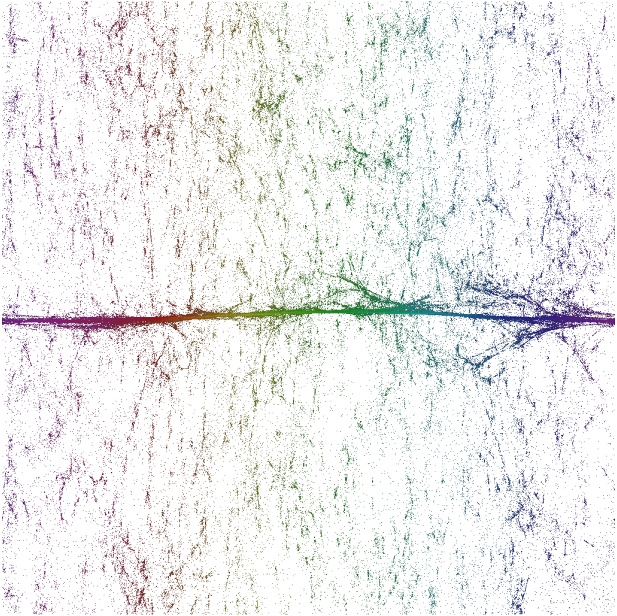


Dynamical fields $\varphi^x, \varphi^y, \varphi^z$. Projection on $\varphi^t - \varphi^x$. Circumference $\delta = 3.0$.

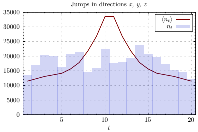


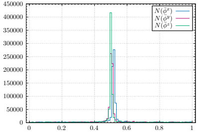


Scalar fields winding in spatial directions

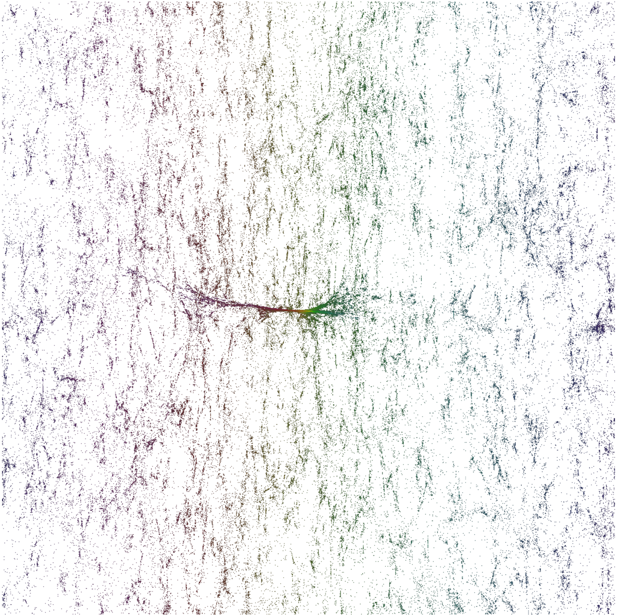


Dynamical fields $\varphi^x, \varphi^y, \varphi^z$. Projection on $\varphi^t - \varphi^x$. Circumference $\delta = 4.0$.

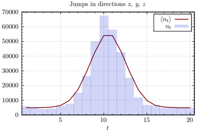


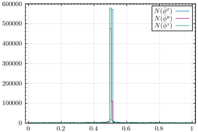


Scalar fields winding in spatial directions

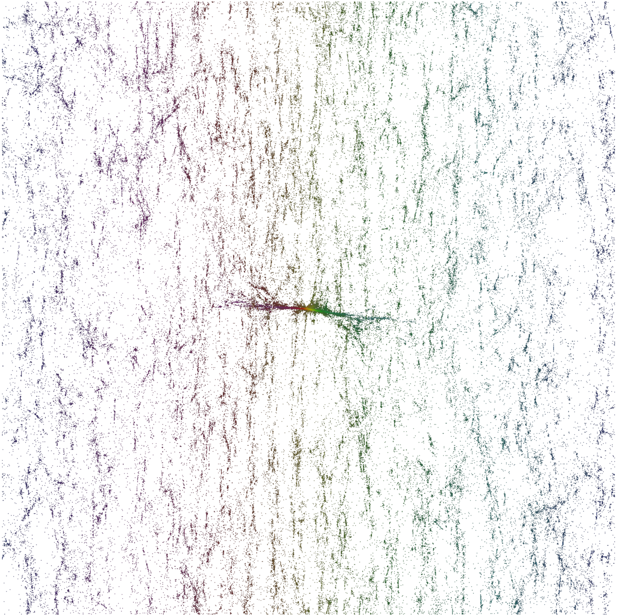


Dynamical fields $\varphi^x, \varphi^y, \varphi^z$. Projection on $\varphi^t - \varphi^x$. Circumference $\delta = 7.0$.

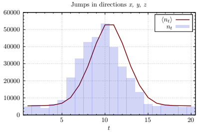


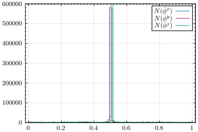


Scalar fields winding in spatial directions

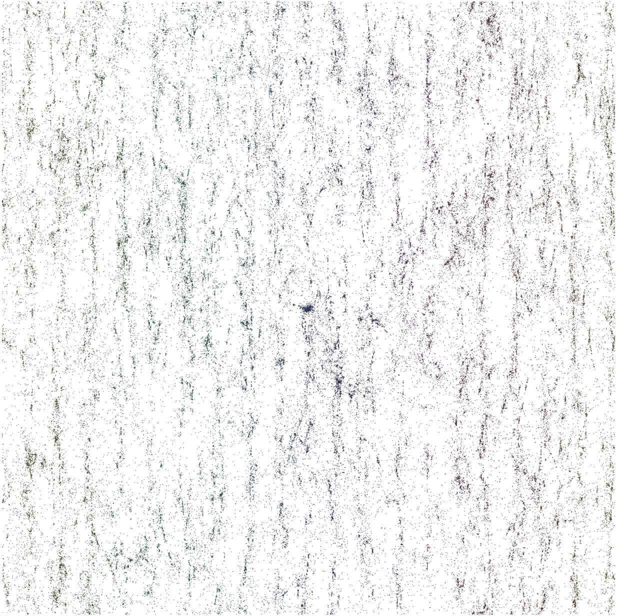


Dynamical fields $\varphi^x, \varphi^y, \varphi^z$. Projection on $\varphi^t - \varphi^x$. Circumference $\delta = 8.0$.

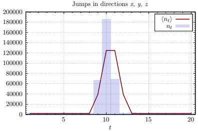


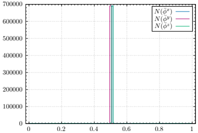


Scalar fields winding in spatial directions

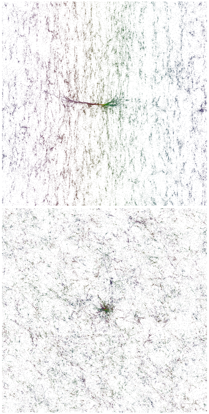


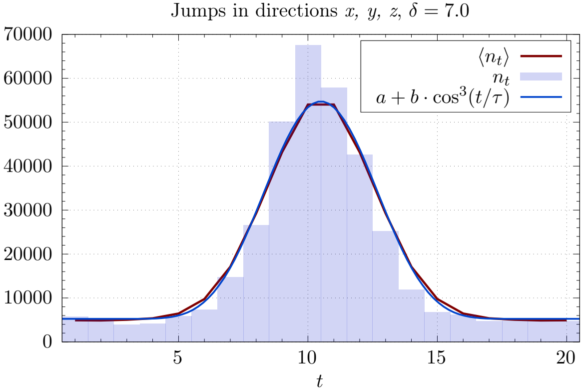
Dynamical fields $\varphi^x, \varphi^y, \varphi^z$. Projection on $\varphi^t - \varphi^x$. Circumference $\delta = 10.0$.





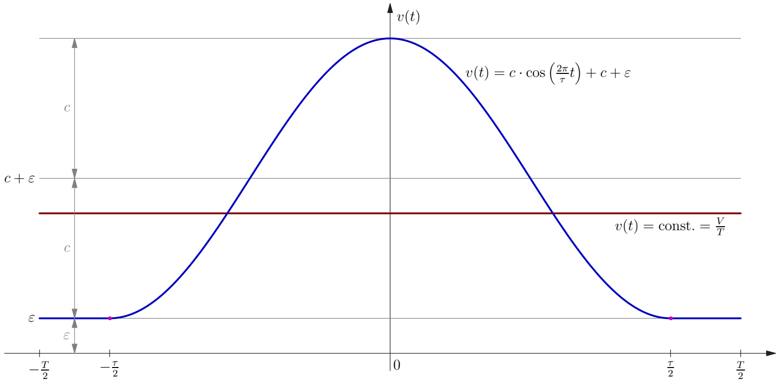
Volume profile for circumference $\delta = 7.0$





Topology change

For large enough circumference δ, a scalar field winding in a spatial direction introduces a *pinching*, which results in an *effective* **change of spatial topology**.



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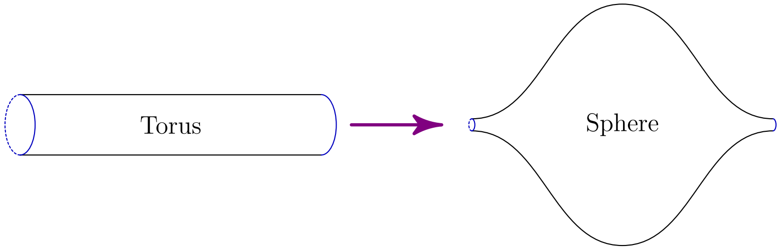

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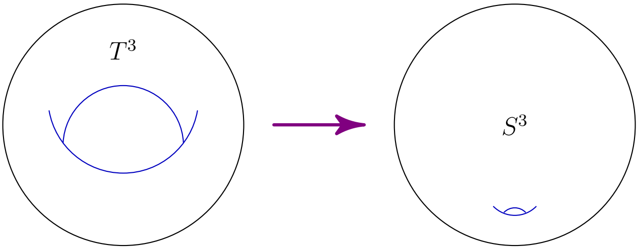

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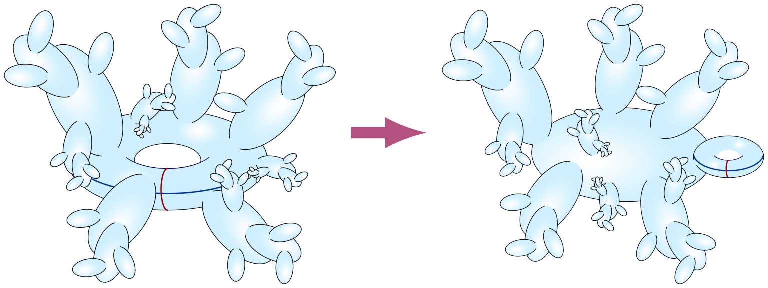
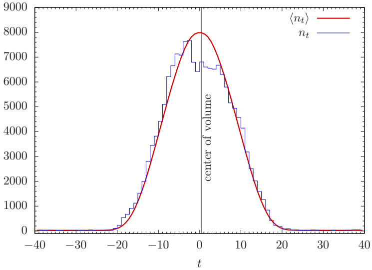


Spherical vs toroidal spatial topology

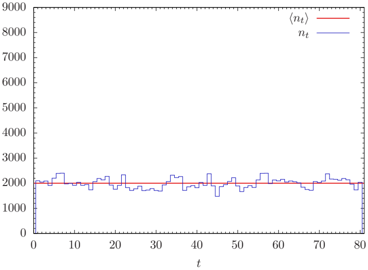
Volume profile

Spherical ($S^3 \times S^1$)



$$L[v] = \frac{1}{\Gamma}\frac{\dot{v}^2}{v} + \mu v^{1/3} - \lambda v$$
$$v(t) = a + b \cdot \cos^3(t/\tau)$$

Toroidal ($T^3 \times S^1$)



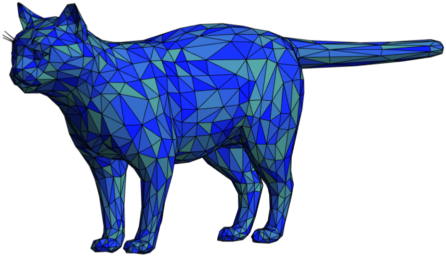
$$L[v] = \frac{1}{\Gamma}\frac{\dot{v}^2}{v} + \mu v^{-3/2} - \lambda v$$
$$v(t) = \mathrm{const.}$$

The difference between **spherical** and **toroidal** **spatial topology** is visible in the volume profile - as predicted by the minisuperspace model.

Summary

- *Causal dynamical triangulations* is a model of generic **geometry fluctuations at the Planck scale**.
- Introduction of **dynamical scalar fields** with matching topological boundary conditions has a **dramatic effect on the geometries** that dominate the CDT path integral.
- Scalar fields induce a new type of phase transition, where the **effective spacetime topology** changes from **toroidal** to a **simply connected** one.
- This new kind of coupling between the topology of the matter fields and the topology of spacetime is likely to result in a **phase transition** for sufficiently strong coupling.
- The simple minisuperspace model predicts a *phase transition* when changing the circumference δ.
- The classical limit agrees with *the minisuperspace model*.

Thank You!



In collaboration with

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Jakub Gizbert-Studnicki Dániel Németh