Isometric boundary-boundary maps in QG entanglement graphs

based on ArXiv:2207.07625

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From black holes, AdS/CFT, celestial holography, corner symmetries $\Longrightarrow$
Holographic correspondences encode parts of gravity

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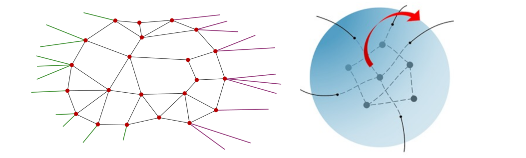

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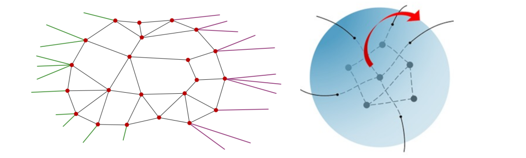

Study restricted class of quantum gravity states using random tensor network techniques:
Fix graph structure and intertwiner data, randomise the rest.

Concrete example: Spin networks as entanglement graphs

Describe a network Γ of *loosely glued polyhedra* (on vertices), with geometric data, glued along faces (as links).

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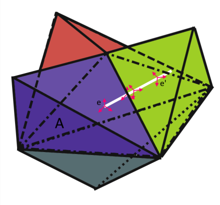
Typical data:

$$\text{Parallel transports of frames} \qquad \text{Areas, volumes} \tag{1}$$

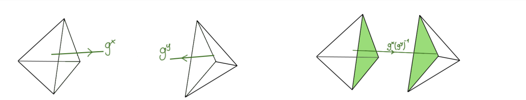
$$g \in G = SO(3), SU(2), SL(2;\mathbb{C}) \quad \leftrightarrow \quad \mathbf{j} \in \frac{\mathbb{N}}{2}, \iota \in \mathrm{Inv}_{SU(2)}(V_{\mathbf{j}}) \tag{2}$$



Simple states with combinatorial structure play role in QG approaches like LQG,SFM,GFT.

Spin networks as tensor networks

Glueing $\equiv$ Entanglement



Individual vertex states, apply projection onto maximally entangled link state

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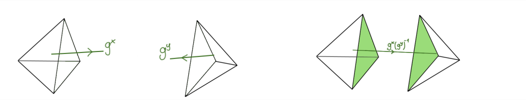

Individual vertex states, apply projection onto maximally entangled link state

$\implies$ Dependence on single group element on glued edge.

$$\Psi(g_1, g_2) = \int dh\, \psi_1(g_1 h)\psi_2(g_2 h) = \Psi(g_1 (g_2)^{-1}, e) \tag{3}$$

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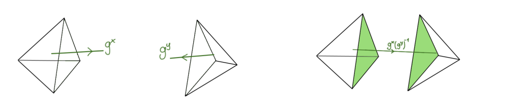

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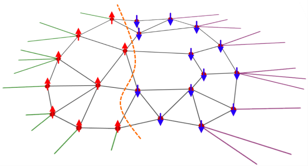
Spin network basis states are naturally tensor network states, with individual tensors: $\Psi_x \in L^2(G^V/G_{Diag})$

Context and previous work

Strategy: Use *random tensor network techniques* for typicality statements.

(Han, Hung '17)[1610.02134]: Similar method, with focus on boundary entropy.

(Colafranceschi, Oriti, Chirco '21,'22)[2105.06454 , 2012.12622 , 2110.15166]: Isometry degree of maps from intertwiners to boundary edges, with *fixed edge spins.* $\longrightarrow$ **Goffredo's talk tomorrow**



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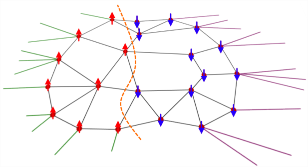

Study entropy and isometry questions through Ising model on graph.

$\implies$ Holographic maps only with inhomogeneous local areas of faces.

$\implies$ Ryu-Takayanagi formula for bulk entropy.

Methods

Extend this to **superpositions** of different link data. Same method - work in tensor network (PEPS) class of spin network superpositions and randomise over vertex wavefunctions. (Harlow, Yang, Hayden, Qi, ...) [1601.01694 , 1503.06237 , 1510.03784, JHEP08(2017)060]

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Main technical step: Calculate entropies of states

$$|\phi\rangle = \langle\zeta|_{\mathcal{I}} \bigotimes_{e\in E} \langle e| \bigotimes_{x\in V} |\Psi_x\rangle \text{, where } |\Psi_x\rangle = U_x |\Psi_0\rangle \text{ random} \tag{4}$$

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- **Renyi 2-Entropy** of reduced state ρ_A maximal $\leftrightarrow$ isometry.
- Replica trick, introduction of $\mathbb{Z}_2$ spin to account for swaps

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$\implies$ Equal to partition sum(s) of **random/weighted Ising model**.

$$\langle e^{-S_2}\rangle \hat{=} Z = \sum_{J,K} P(J,K) Z^{J,K} \tag{5}$$

Notions of holographic transport

Would like **bulk$\rightarrow$ boundary operator maps** as holography.
Issue: *No natural bulk or boundary Hilbert spaces!*

$$\mathbb{H}_\Gamma = \bigoplus_{j_\partial} \mathcal{I}_{j_\partial} \otimes V_{j_\partial} \tag{6}$$

Resolution: Define holography as *isometry of an operator map*

$$\mathcal{T}_\rho : \mathcal{A}_I \rightarrow \mathcal{A}_O \tag{7}$$

in Hilbert-Schmidt norm. Use the *Choi map* of state ρ of entanglement graph

$$\mathcal{T}_\rho(X) = K \, \mathrm{Tr}[(X \otimes \mathbb{I}_O)\rho^{t_I}] \tag{8}$$

Choice of algebras (and centers $Z = \mathcal{A}_I \cap \mathcal{A}_O$) matter!

Holographic mappings in the presence of centers

Can define 'proper' notion of holography via operator map.
If ρ is pure, ρ_I max entropy $\equiv$ $\mathcal{T}_\rho$ isometric.

Condition for isometry:

- Restrictions on input/output dimensions per-sector
- weights related to product of areas $p_n \sim \frac{D_{I_n}}{D_I}$

Results

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For high spins, $P(m,n) = p_m p_n$. Then needed:

$$p_n \approx \frac{\dim(\mathbb{H}_{I,n})}{\dim(\mathbb{H}_I)} = \frac{\prod_{e\in I} d_{j_{n,e}}}{\sum_m \prod_{e\in I} d_{j_{m,e}}} \tag{9}$$

Results

A quantum geometry application: Area operator (from canonical quantisation of GR)

$$\hat{A} = \sum_j j\mathbb{I}_j \tag{10}$$

Calculate EV of area of a boundary region: In a holographic state, given by simple combination of individual values. For many similar geometries:

$$\langle\langle A_C\rangle_\rho\rangle_U \approx \frac{4}{3}\overline{A_C} \qquad \langle\langle(\Delta A_C)^2\rangle_\rho\rangle_U \approx \frac{2}{9}\overline{A_C}^2 \tag{11}$$

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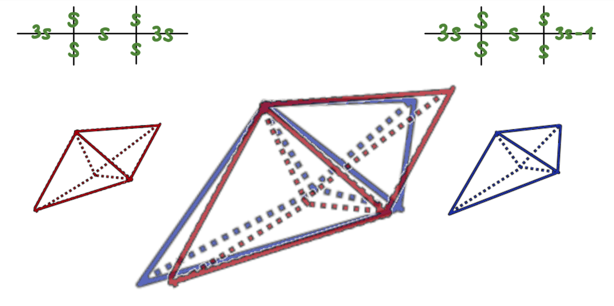
$$\frac{(\Delta A_C)^2}{(A_C)^2} \approx \frac{1}{6} \implies \text{nonvanishing relative uncertainty} \tag{12}$$



Summary

- Bounded regions of quantum space as entanglement graphs.
- Holography as operator mapping, characterised by entropic properties via random tensor networks.
- Typical large-spin superpositions of spin networks feature boundary/boundary isometry under simple conditions.

Outlook

Work in progress

- Give explicit characterisation of *local* conditions on labels for isometry
- Relate isometry condition to geometric scaling laws
- Numerical Monte-Carlo studies?

Possible applications

- Spin network states as channels between boundary models of punctures
- With Dynamics: Causal characterisation of entanglement shadow
- AdS/CFT or semiclassical superpositions with spin networks?

Thank you for your attention!