Trace anomaly and the stability of stars

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In progress hep-th 2307…

The Planck scale

Classical gravity with matter,

$$T_{\mu\nu} = (8\pi G)^{-1} \left(R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} \right)$$

Now introduce quantum corrections for matter

$$\langle : \hat{T}_{\mu\nu} : \rangle \sim \hbar R^2 g_{\mu\nu} + \ldots$$

Comparing components,

$$T_{\mu\nu} \approx \langle : \hat{T}_{\mu\nu} : \rangle \quad \Rightarrow \qquad R^{-1} \sim G\hbar \quad \leftrightarrow \quad \rho \sim \rho_P = M_P^4$$

We can neglect quantum effects of curvature below Planck scale.

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Derivation:

Trace Anomaly

Trace anomaly

QFT curved spacetime:

$$T_{\mu\nu} = T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(A)}$$

break conformal symmetry classically

not explicit dependence

$g_{\mu\nu}$

anomaly

explicitly dependent

Weyl anomaly:

$c, a > 0$

$$T^{(A)} = cW^2 - aE$$

$$\int E = \chi$$

Assumptions

Much below the Planck density $\rho_P = M_p^4$:

(A1) Semi-classical approximation is valid

$$G_{\mu\nu} = 8\pi G \left(T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(A)} \right)$$

(A2) Matter equation of state is conformal at leading order

$$T^{(m)} = -\rho + 3p \underset{m^4 \ll \rho \ll \rho_P}{\approx} -m^{4(1-\alpha)} \rho^{\alpha} \qquad \alpha < 1$$

(A3) Individual components

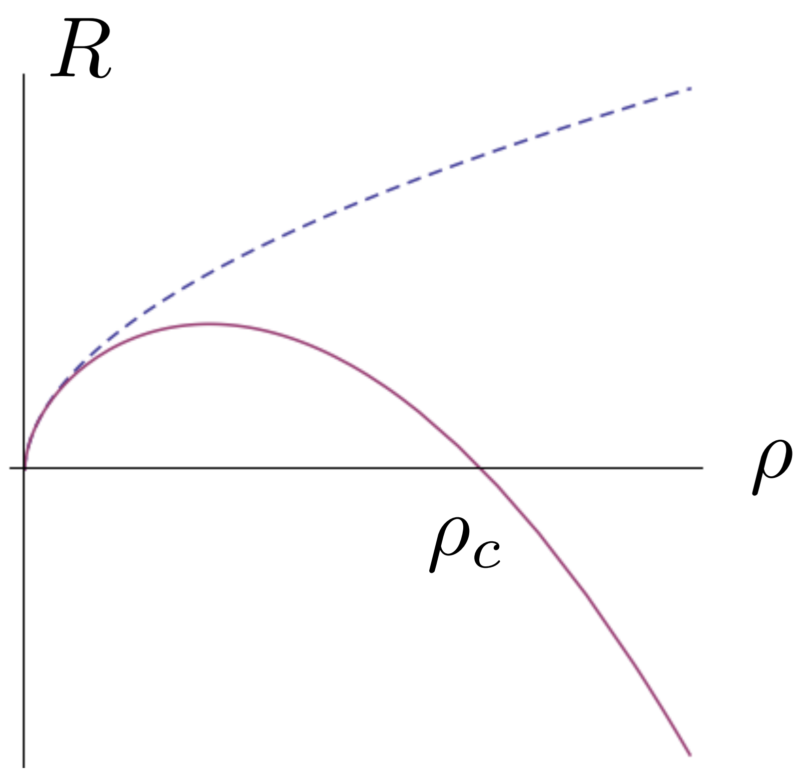
$$|T_{\mu\nu}^{(A)}| \ll |T_{\mu\nu}^{(m)}|$$

Anomalous field equations

Trace of semi-classical equations:

$$R = -8\pi G \left(T^{(m)} + T^{(A)} \right)$$

Using assumptions (A1)-(A3) one easily finds



$$R \approx M_p^{-2} \left(m^{4(1-\alpha)} \rho^{\alpha} - M_p^{-4} \rho^2 \right)$$

$$\rho_c \sim (m/M_p)^{\frac{4(1-\alpha)}{2-\alpha}} \rho_p$$

Cold Fermi gas:

$$\alpha = 1/2 \qquad m = m_e \qquad \rho_c \sim 10^{-30} \rho_P$$

Application:

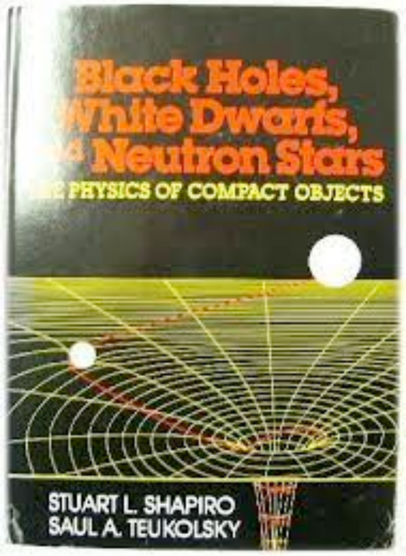
Compact stars

Relativistic Stars

Two main results:

Stability: maximum mass (1939)

Theory of degenerate stars uses *exclusion principle* for fermions.



Existence: maximum compactness (1959)

Buchdahl's theorem, independent of equation of state.

Buchdahl's Theorem [1959]

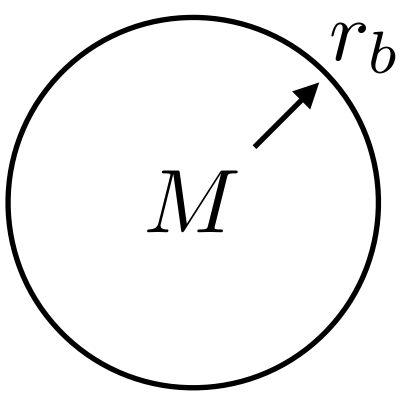
Static, spherically symmetric isotropic perfect fluid

$$T^\mu_\nu = \mathrm{diag}(-\rho, p, p, p)$$

$$ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2 d\Omega^2$$

Then, assuming nothing about the EOS but only

$$\rho > 0 \quad , \quad \partial_r \rho \leq 0 \quad , \quad G_{\mu\nu} = 8\pi G T_{\mu\nu}$$



$$f(0) \geq 0 \qquad \Rightarrow \qquad \frac{r_b}{GM} \geq \frac{9}{4} > 2$$

Buchdahl limit: $\quad f(0) \to 0^+$

Stability in Buchdahl limit

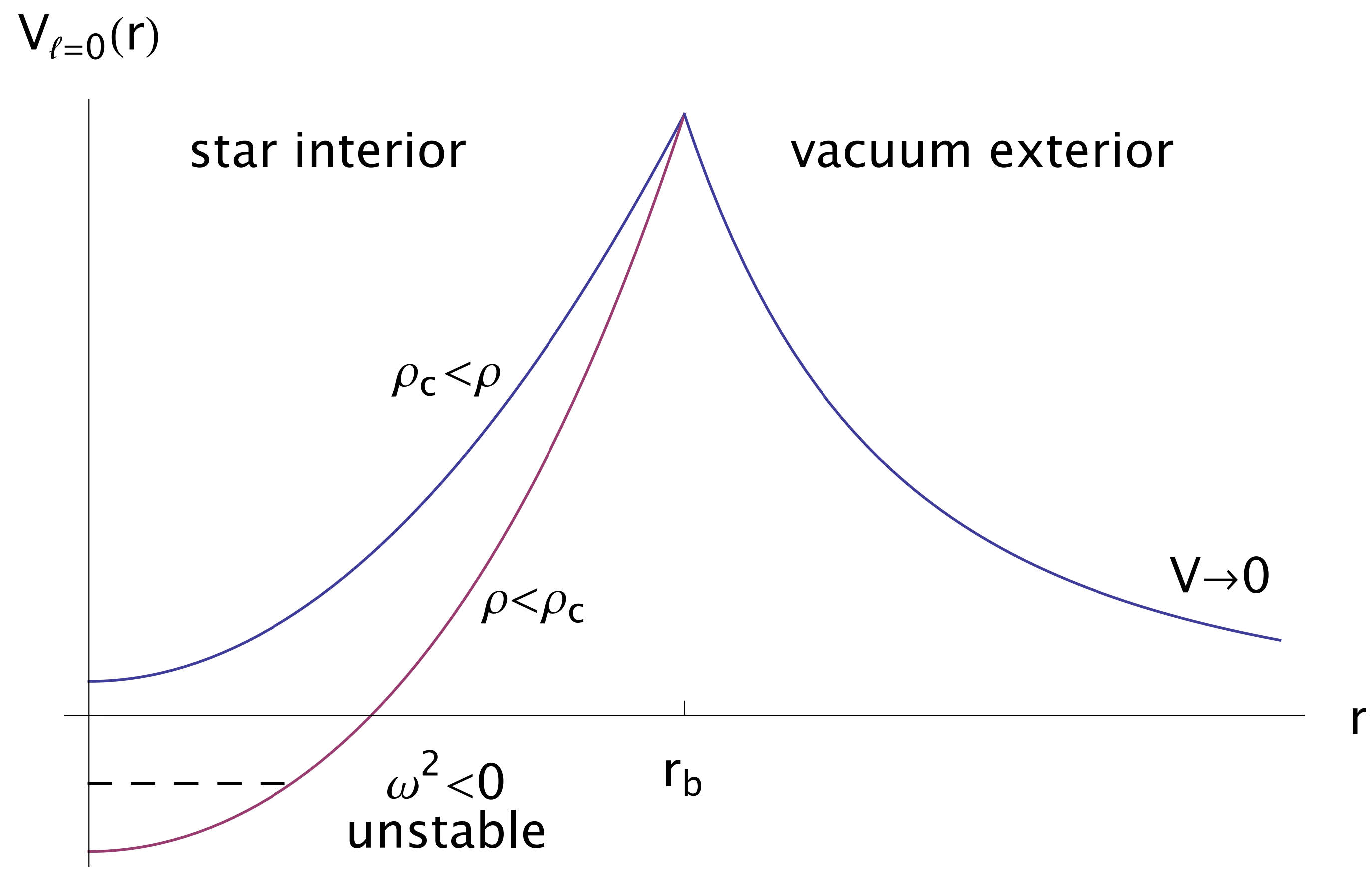
Massless scalar proxy $\quad \Box\Phi = 0 \quad\quad \Phi = \frac{u(r)}{r} Y^{\ell}_{m}(\theta, \varphi) e^{-i\omega t}$

$$\frac{dr_*}{dr} = \sqrt{\frac{h(r)}{f(r)}} \qquad -\partial^2_{r_*} u + V(r_*) u = \omega^2 u$$

$$V_{\ell=0}(0) = -\frac{1}{6} f(0) R(0)$$

$\rho < \rho_c :$ unstable

$\rho_c < \rho :$ stable (anomaly)



Conclusions

The effects of the trace anomaly become *macroscopic* at

$$\rho_c \sim (m/M_P)^{\frac{4(1-\alpha)}{2-\alpha}} \rho_P \ll \rho_P \qquad \alpha < 1$$

For scalar perturbations of stars close to Buchdahl limit,

$$\Box\Phi = 0 \qquad \rho < \rho_c : \longrightarrow \rho_c < \rho :$$

unstable stable

Outlook

- Spin perturbations
- Gravitational collapse