Quantum Gravity in de Sitter Space

QG 2022, Nijmegen

Thomas Hertog
Institute for Theoretical Physics
KU Leuven

hep-th, 2211.05907; 2305.15440

The KS-criterion constrains inflation in the no-boundary state

Thomas Hertog[♠], Oliver Janssen[♣] and Joel Karlsson[♠]

[♠]*Institute for Theoretical Physics, KU Leuven, Celestijnenlaan 200D, 3001 Leuven, Belgium*
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We show that the Kontsevich–Segal (KS) criterion, applied to the complex saddles that specify the semiclassical no-boundary wave function, acts as a selection mechanism on inflationary scalar field potentials. In this context the KS-criterion effectively bounds the tensor-to-scalar ratio of cosmic microwave background fluctuations to be less than 0.08, in line with current observations. We trace the failure of complex saddles to meet the KS-criterion to the development of a tachyon in their spectrum of perturbations.

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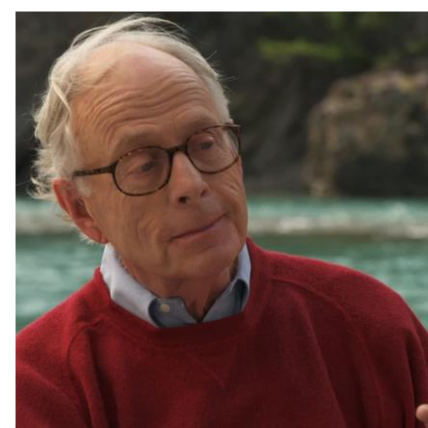
Dedicated to the memory of Jim Hartle, whose innate quantum outlook on cosmology will be a source of inspiration for many years to come.



PHYSICAL REVIEW D VOLUME 28, NUMBER 12 15 DECEMBER 1983

Wave function of the Universe

J. B. Hartle
Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637
and Institute for Theoretical Physics, University of California, Santa Barbara, California 93106

S. W. Hawking
Department of Applied Mathematics and Theoretical Physics, Silver Street, Cambridge, England
and Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
(Received 29 July 1983)

The quantum state of a spatially closed universe can be described by a wave function which is a functional on the geometries of compact three-manifolds and on the values of the matter fields on these manifolds. The wave function obeys the Wheeler-DeWitt second-order functional differential equation. We put forward a proposal for the wave function of the "ground state" or state of minimum excitation: the ground-state amplitude for a three-geometry is given by a path integral over all compact positive-definite four-geometries which have the three-geometry as a boundary. The requirement that the Hamiltonian be Hermitian then defines the boundary conditions for the Wheeler-DeWitt equation and the spectrum of possible excited states. To illustrate the above, we calculate the ground and excited states in a simple minisuperspace model in which the scale factor is the only gravitational degree of freedom, a conformally invariant scalar field is the only matter degree of freedom and $\Lambda > 0$. The ground state corresponds to de Sitter space in the classical limit. There are excited states which represent universes which expand from zero volume, reach a maximum size, and then recollapse but which have a finite (though very small) probability of tunneling through a potential barrier to a de Sitter-type state of continual expansion. The path-integral approach allows us to handle situations in which the topology of the three-manifold changes. We estimate the probability that the ground state in our minisuperspace model contains more than one connected component of the spacelike surface.

A Quantum Universe

If the universe is a quantum mechanical system it has a quantum state.
What is it?

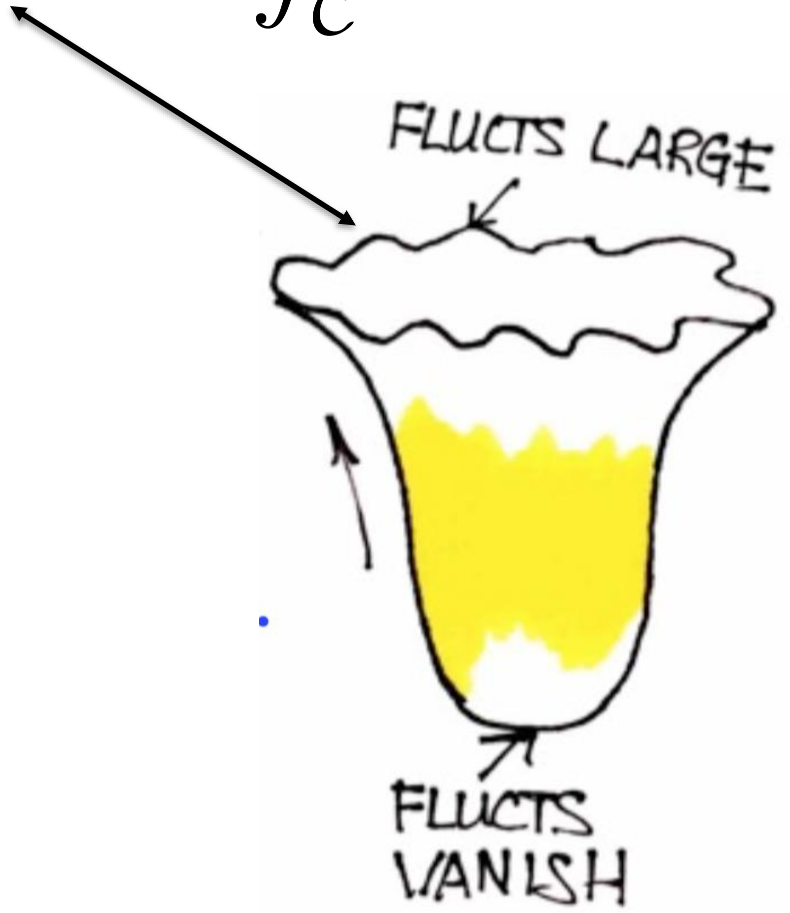
Ψ

That is the problem of Quantum Cosmology.

A theory of the
quantum state
of the universe
is as much a part of a
final theory
as a theory of dynamics.

No-Boundary Proposal

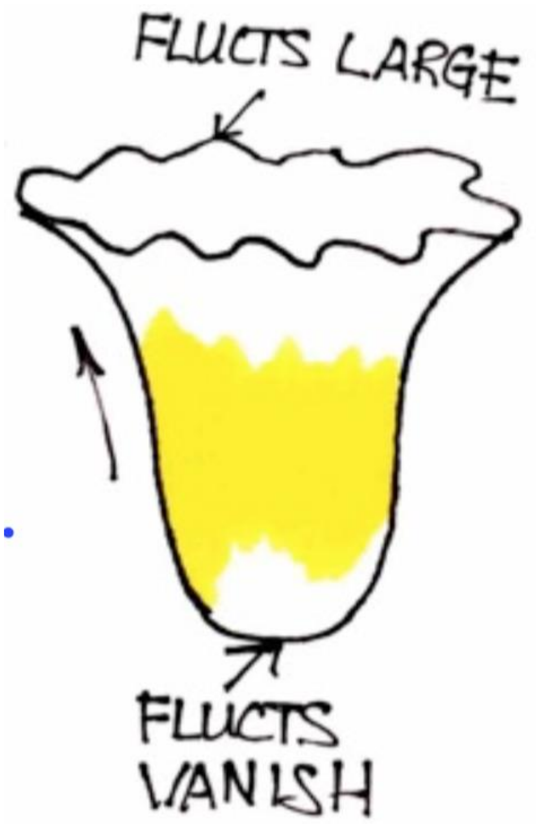
$$\Psi[{}^3g, \phi_f] = \int_{\mathcal{C}} \delta g \delta\phi \exp\left(-I[g,\phi]/\hbar\right)$$



No-Boundary Proposal

$$\Psi[{}^3g, \phi_f] = \int_{\mathcal{C}} \delta g \delta \phi \exp\left(-I[g, \phi]/\hbar\right)$$

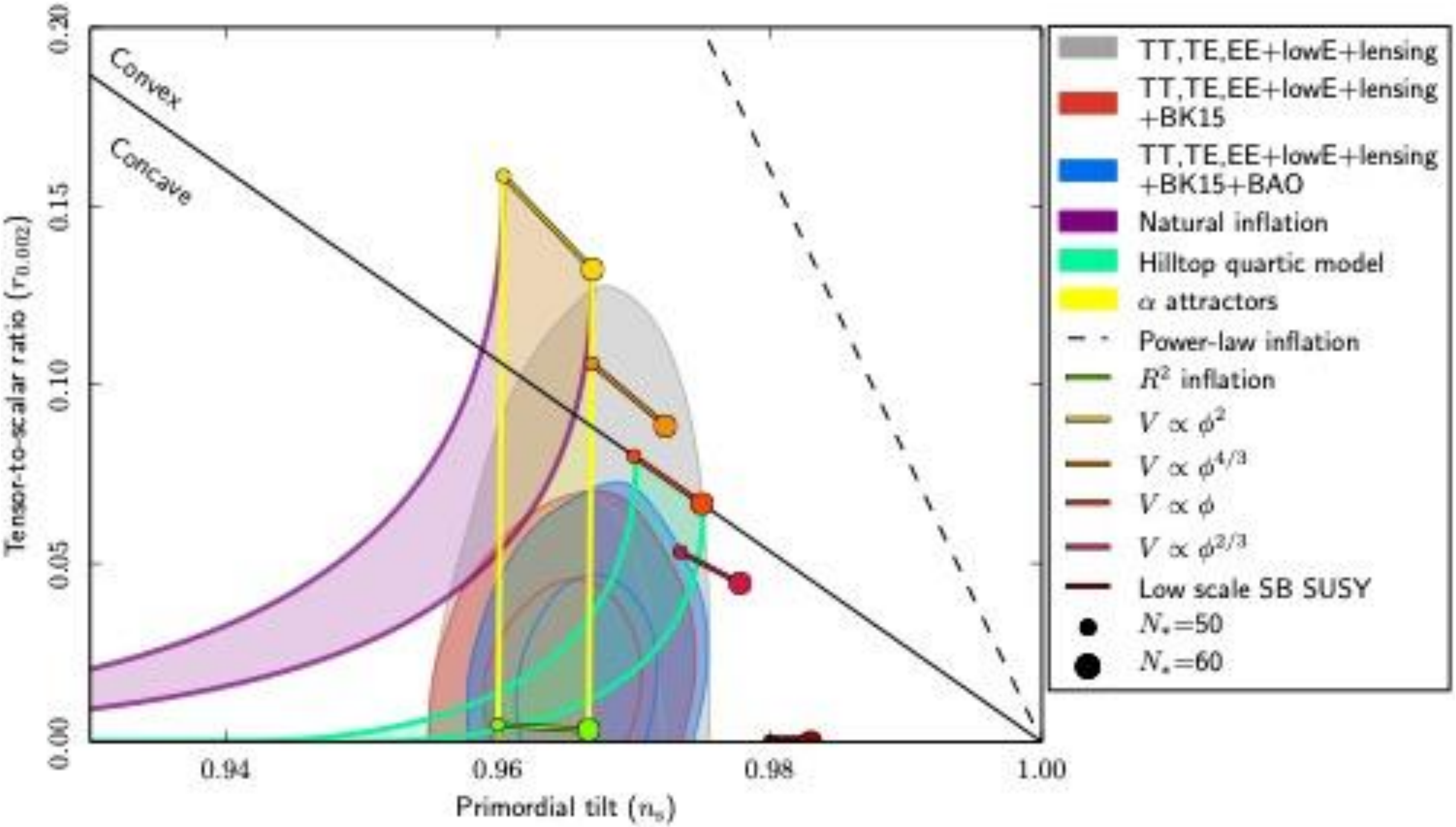
- Classical cosmological evolution emerges through an early `de Sitter-like' phase of inflation.



- Fluctuations are initially small, giving rise to a physical arrow of time.

[Hartle, Hawking, TH, 2008]

Which de Sitter like phase?



[Planck 2018 results: Constraints on inflation, 1807.06211]

No-Boundary Prior

[Submitted on 27 May 2013 (v1), last revised 10 Feb 2014 (this version, v3)]

Predicting a Prior for Planck

Thomas Hertog

The quantum state of the universe combined with the structure of the landscape potential implies a prior that specifies predictions for observations. We compute the prior for CMB related observables given by the no-boundary wave function (NBWF) in a landscape model that includes a range of inflationary patches representative of relatively simple single-field models. In this landscape the NBWF predicts our classical cosmological background emerges from a region of eternal inflation associated with a plateau-like potential. The spectra of primordial fluctuations on observable scales are characteristic of concave potentials, in excellent agreement with the Planck data. By contrast, alternative theories of initial conditions that strongly favor inflation at high values of the potential are disfavored by observations in this landscape.

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- Predictions for observations involve conditional probabilities P(O|D,Ψ) [Hartle, Hawking, TH, 2009 – 2011]
- **The nature of the microscopic degrees of freedom behind the wave function/path integral remains obscure.**
- Can the no-boundary theory be refined -> its predictions strengthened?

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Saddle selection

Kontsevich- Segal: consider only those (complex) saddle backgrounds on which an arbitrary QFT can be defined [2105.10161]

$$\mathrm{Re}\left(\sqrt{g}\, g^{\mu_1\nu_1}\cdots g^{\mu_p\nu_p} F_{\mu_1\cdots\mu_p} F_{\nu_1\cdots\nu_p}\right) > 0$$

Witten: elevate this criterion to a selection principle of saddle geometries in gravitational path integrals [2111.06514]

Some evidence: the criterion eliminates pathological wormholes but it allows for the complexified Kerr solution

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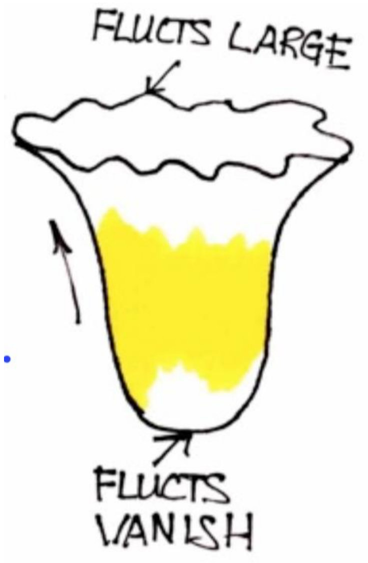
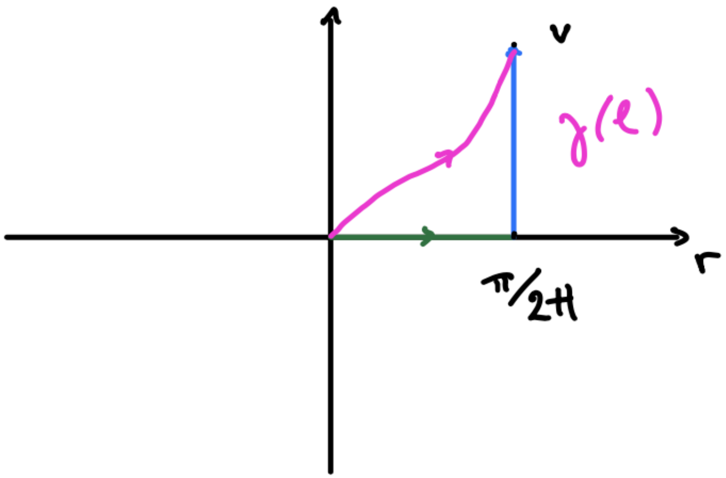
Do all no-boundary saddles satisfy the KSW criterion?

Saddle selection

What can go wrong?





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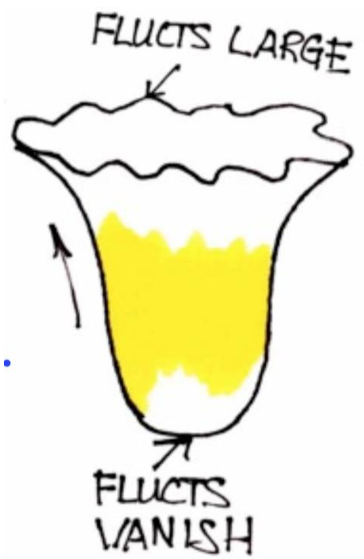
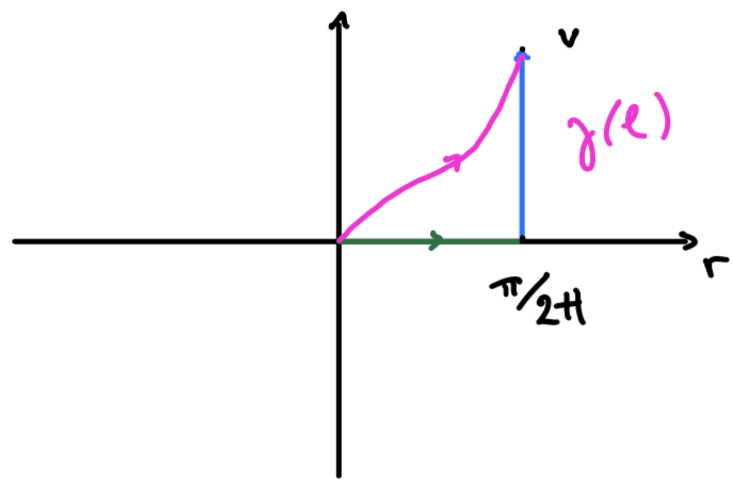
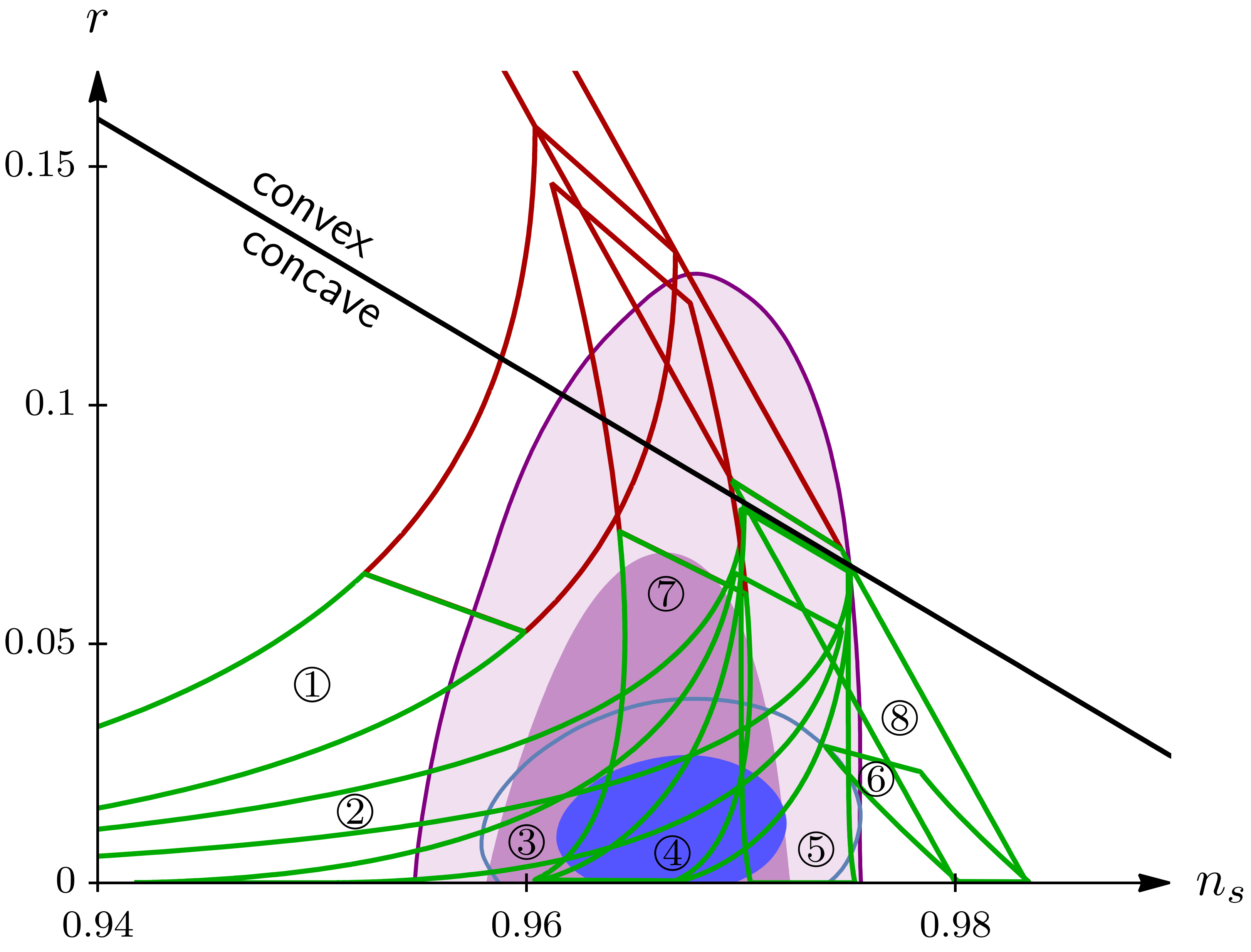




→ The KSW-criterion selects those no-boundary saddles in which the universe emerges on a concave patch of the scalar slow-roll potential.

Observational implications



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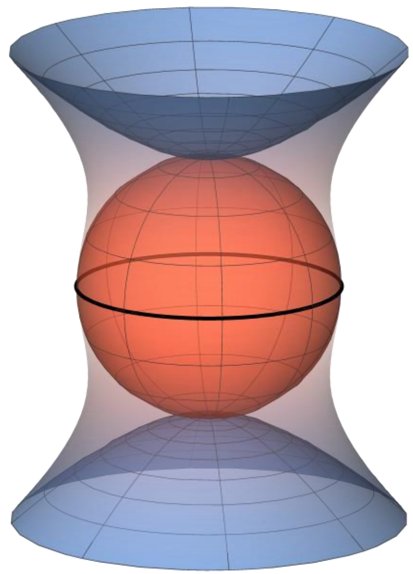
Microscopics of de Sitter entropy from precision holography



2211.05907

with Nikolay Bobev, Junho Hong, Joel Karlsson, Valentin Reys

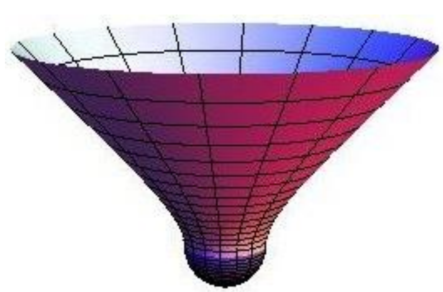
de Sitter saddle

$$ds^2 = \left[-dt^2 + L^2 \cosh^2(t/L)\, d\Omega_3^2\right]$$

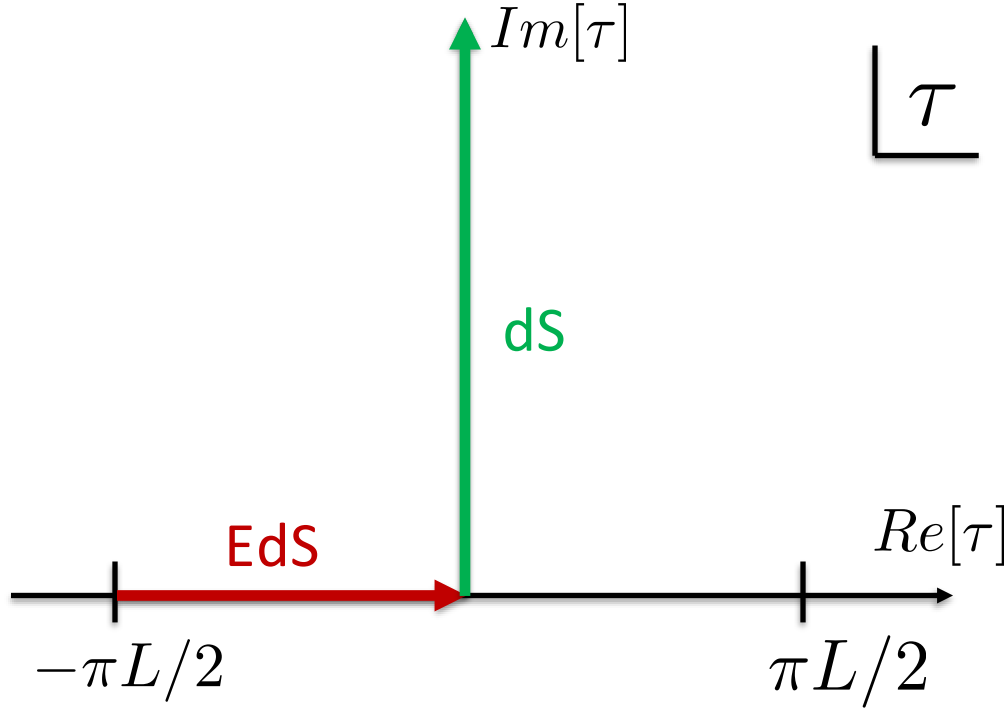
$$t \longrightarrow i\tau$$

$$ds^2 = \left[d\tau^2 + L^2 \cos^2(\tau/L)\, d\Omega_3^2\right]$$

[Hartle Hawking '83]



$$\Psi \sim e^{-I_{EdS}/2} e^{iI_{dS}}$$



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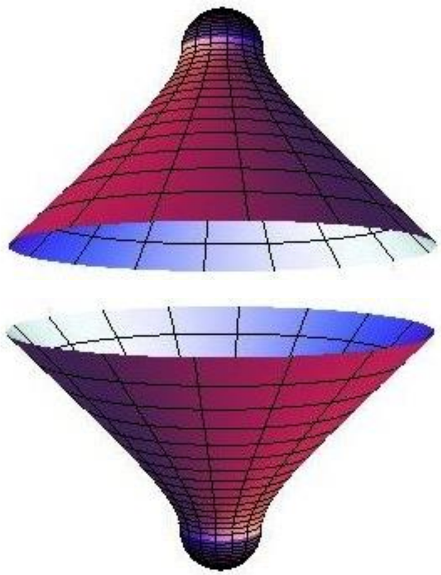
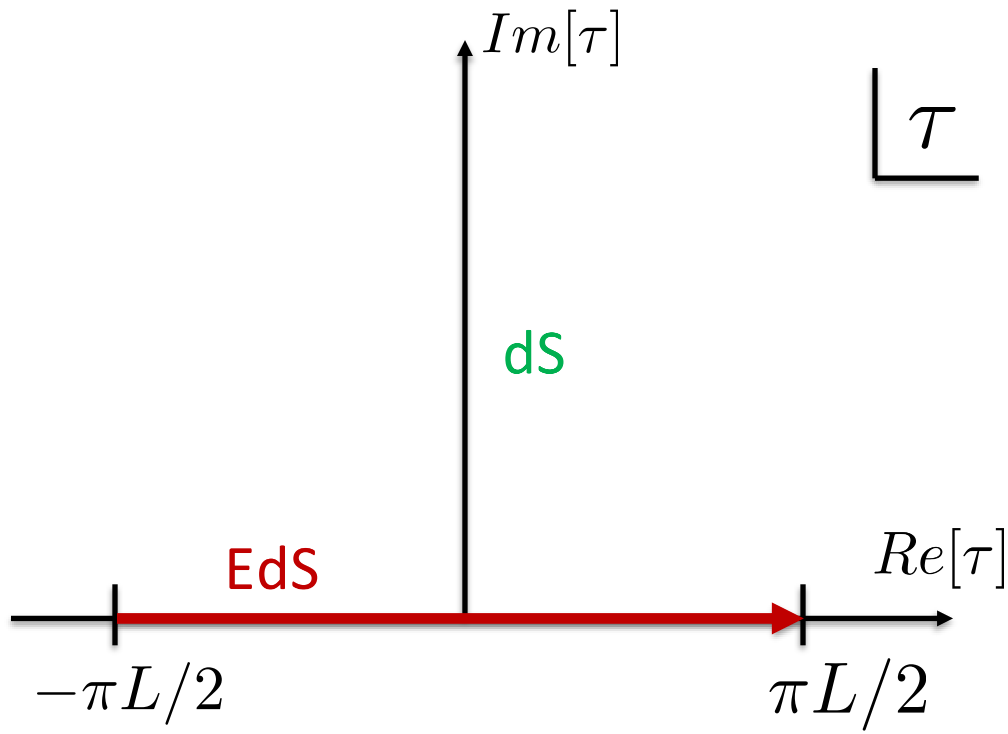


$$\Psi^*\Psi \sim e^{-I_{EdS}}$$



de Sitter entropy

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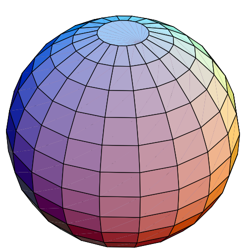

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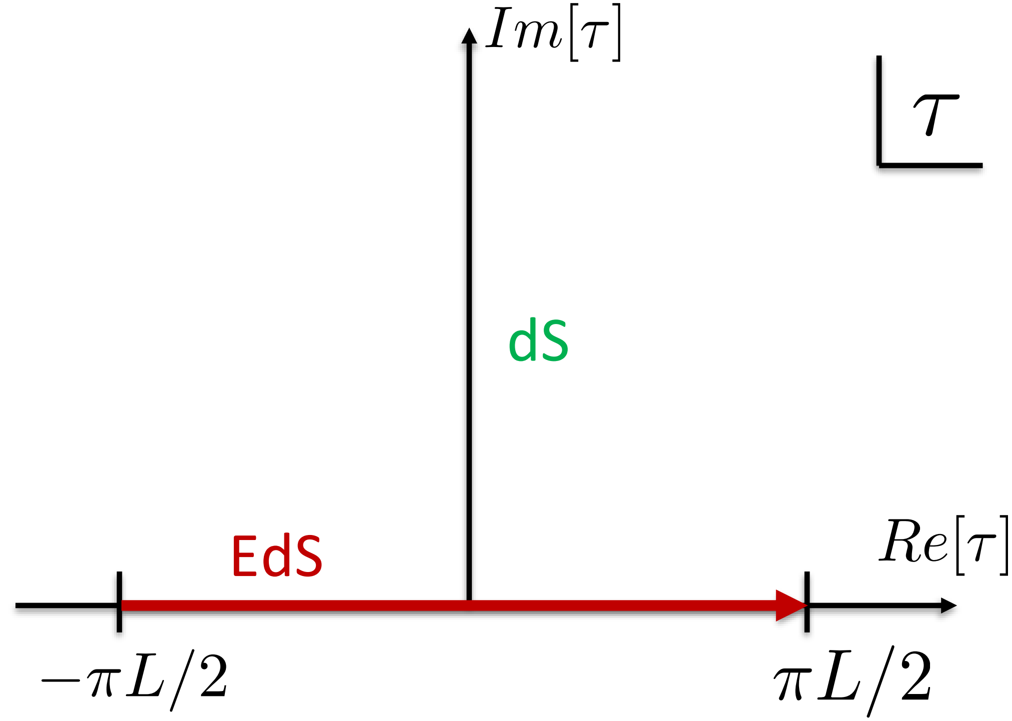

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[Hartle, TH '11; Harlow, Stanford '11]

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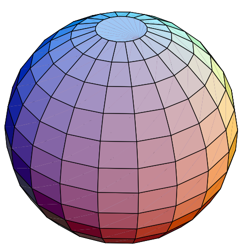

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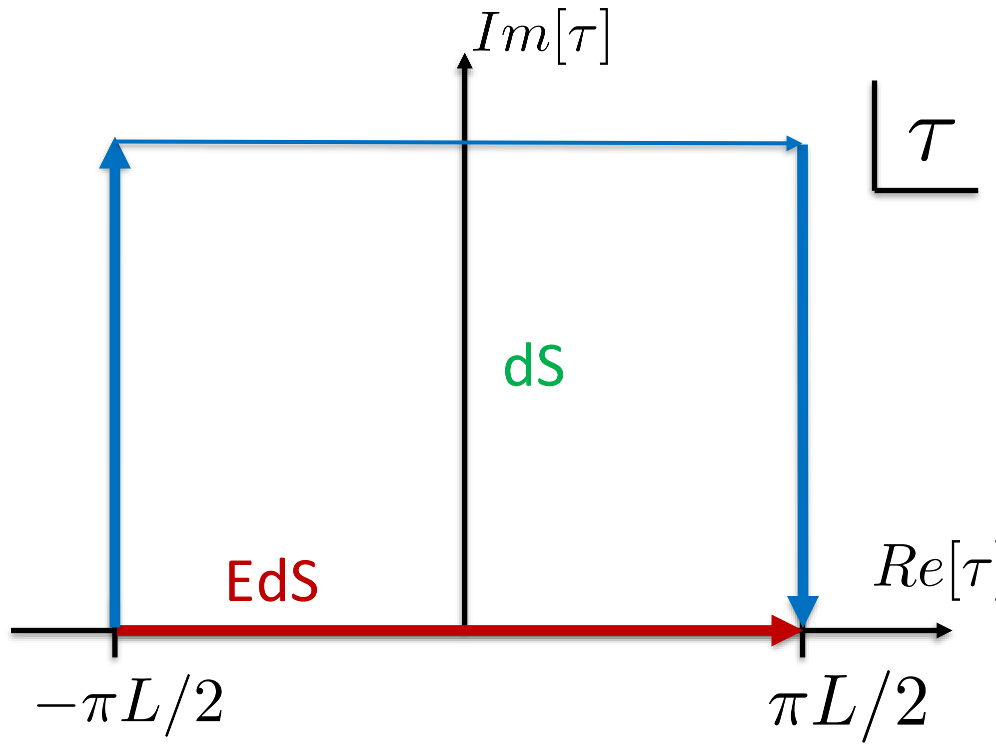

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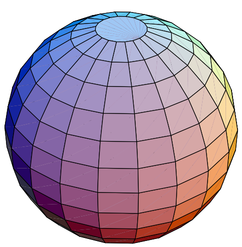
$$ds^2 = \left[d\tau^2 + L^2 \cos^2(\tau/L)\, d\Omega_3^2 \right]$$

$$\tau = -\pi L/2 + ir$$

$$ds^2 = \left[-dr^2 - L^2 \sinh^2(r/L)\, d\Omega_3^2 \right]$$



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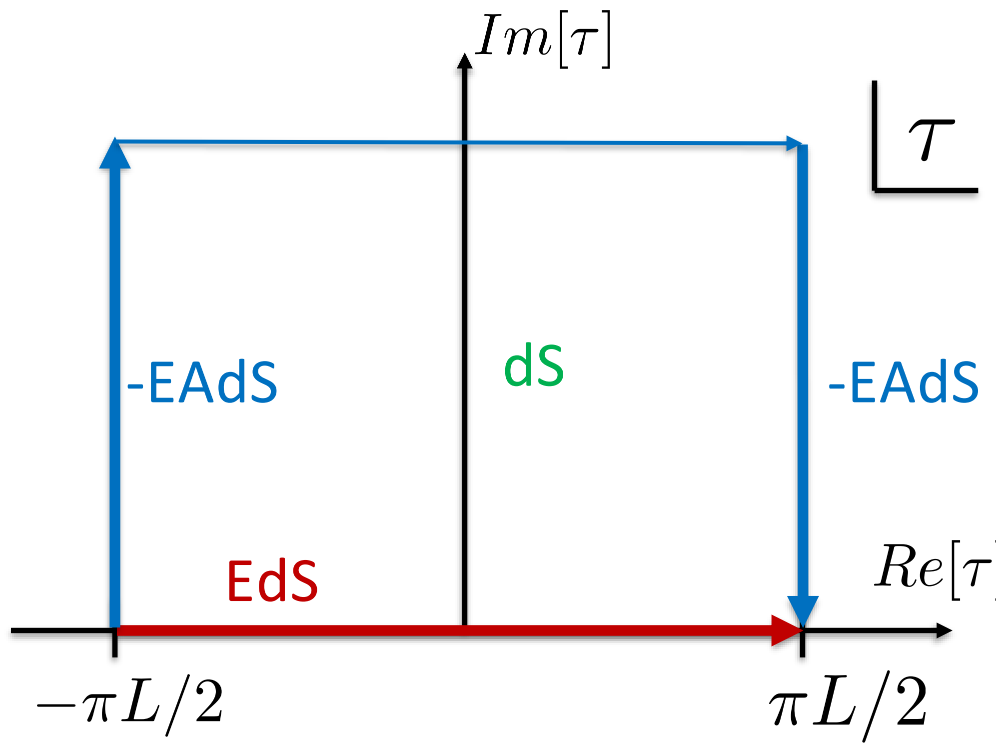


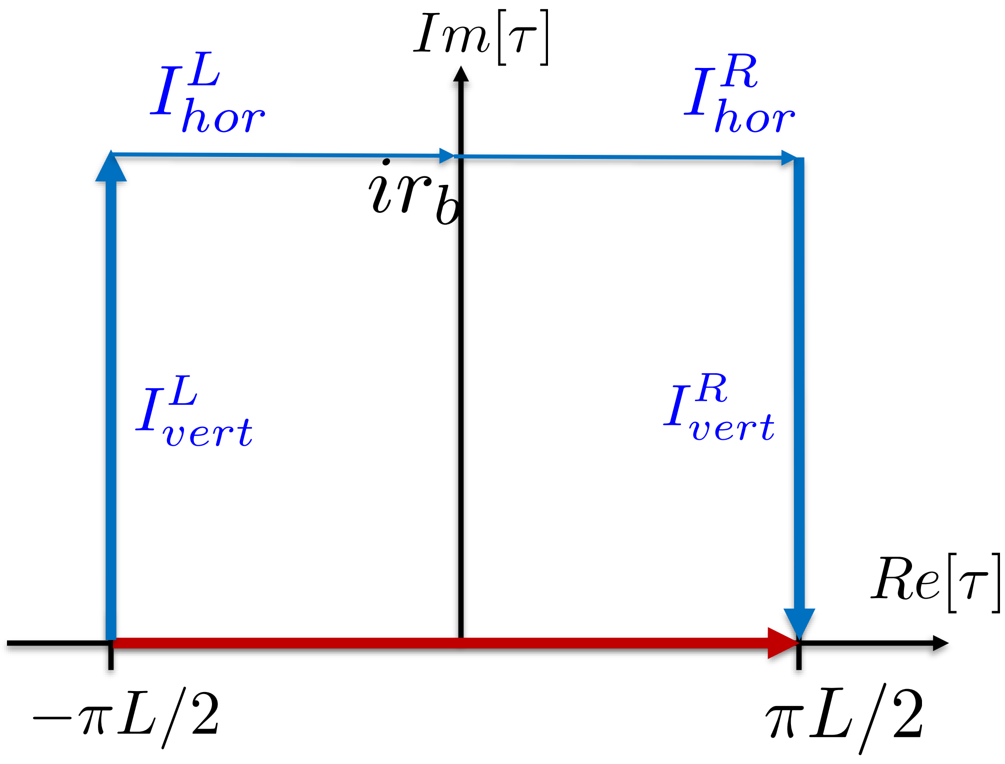
de Sitter entropy

$$I_{vert}^{L} = -I_{\text{EAdS}}^{\text{reg}} - I_{ct} + \mathcal{O}(e^{-r_b/L})$$

$$I_{hor}^{L} = +I_{ct} - iI_{ct} + \mathcal{O}(e^{-r_b/L})$$

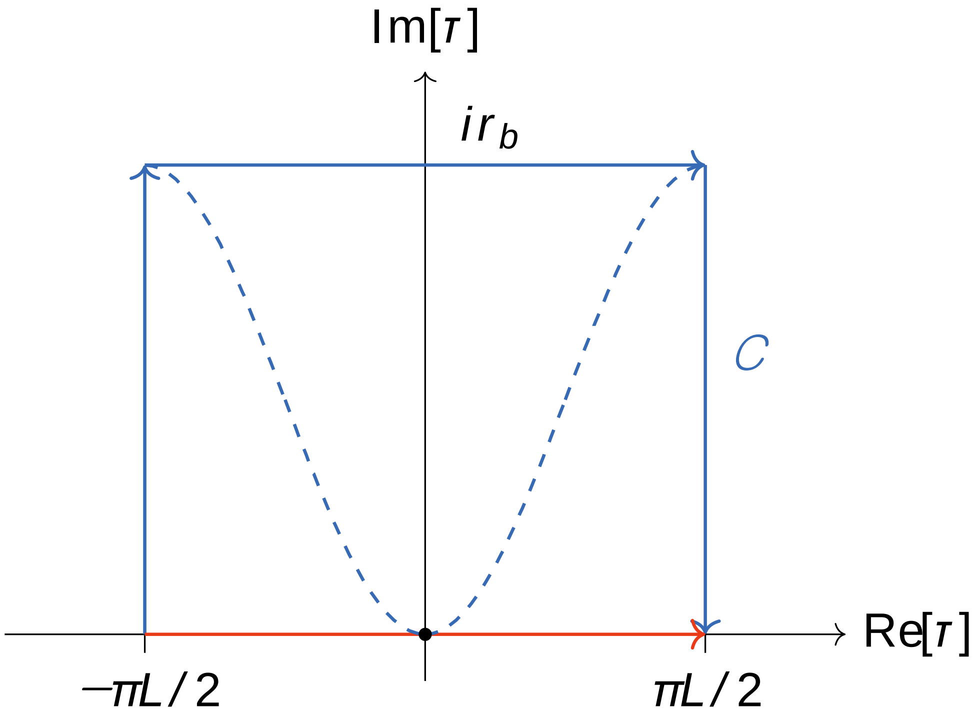
$$I^{R} = (I^{L})^{*}$$

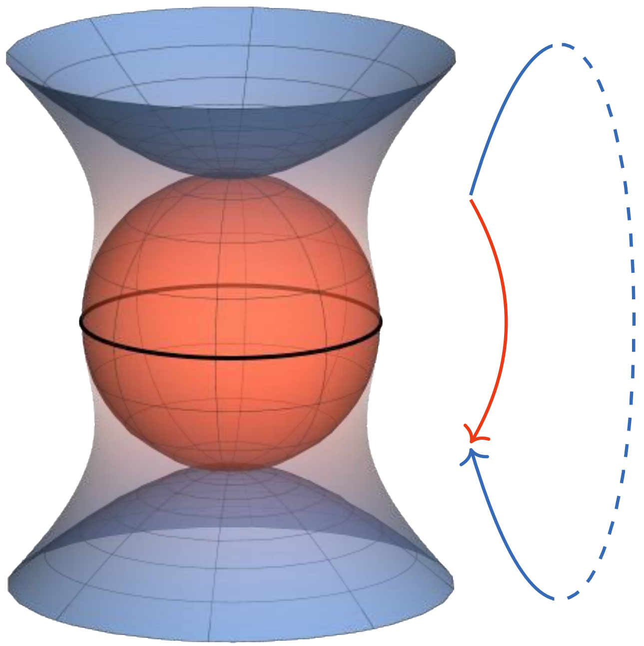
$$I_{EdS} = -2\, I_{\text{EAdS}}^{\text{reg}}$$



de Sitter entropy

$$S_{dS} = -I_{EdS} = 2I_{\rm EAdS}^{\rm reg}$$





de Sitter entropy: microscopics

$$S_{dS} = -I_{EdS} = 2I^{\mathrm{reg}}_{\mathrm{EAdS}}$$

- AdS/CFT $EAdS_4 \times S^7$: $-I^{\mathrm{reg}}_{\mathrm{EAdS}} + \cdots = \log Z^{\mathrm{CFT}}_{S^3}$
- Conjecture: $\boxed{S_{dS} = -2\log Z^{\mathrm{CFT}}_{S^3}}$

de Sitter entropy: microscopics

- AdS/CFT $EAdS_4 \times S^7$:

$$S_{dS} = -I_{EdS} = 2I^{\mathrm{reg}}_{\mathrm{EAdS}} = -2\log Z^{\mathrm{ABJM}}_{S^3}$$

- Leading term matches !

- **Q:** What about quantum corrections?

de Sitter entropy: microscopics

- AdS/CFT *EAdS$_4$ x S^7* :

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- **Q:** What about quantum corrections?

$$\mathcal{S}_{\mathrm{dS}} = \frac{2\pi\sqrt{2k}}{3}\,N^{3/2} - \frac{\pi(k^2+8)}{12\sqrt{2k}}\,N^{1/2} + \frac{1}{2}\log N + \mathcal{O}(N^0)$$

Quid bulk?

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Quid bulk?

- Compute one-loop determinants in 11d Euclidean SUGRA on $-S^4 \times S^7/\mathbb{Z}_k$

  [Bhattacharyya, Grassi, Marino, Sen ’12; Bobev, TH, Hong, Karlssopn, Reys ‘23]

- Leading correction and logarithmic correction match !

So.. what does S_{dS} count?

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- The microscopic de Sitter entropy does not quite count microstates, since there is no time, and thus no Hamiltonian.
- In effect, exp (S_{dS}) not an integer for low (N,k)
- But, being given by a (QFT) path integral, the entropy does represent some sort of measure of degrees of freedom.

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