A Zoo of Axionic Wormholes

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MAX PLANCK INSTITUTE FOR GRAVITATIONAL PHYSICS (Albert Einstein Institute)

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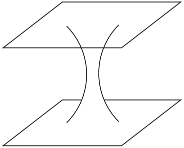
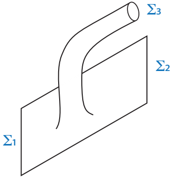
Based on 2306.11129

[CJ, Jean-Luc Lehners and George Lavrelashvili]

Euclidean wormholes

Are **topology changes** allowed in quantum gravity?





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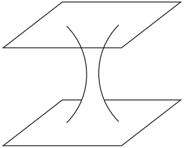

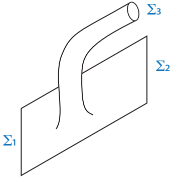

→ explicit wormhole solution in axion-gravity theory [Giddings, Strominger 1983]

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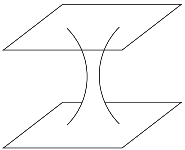

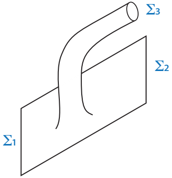

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Puzzles:

- apparent non-unitarity, random values of coupling constant [Coleman 1988]
- AdS/CFT non-locality puzzle [Maldacena, Maoz 2004]
- negative modes and linear stability [Hertog et al. 2019, Loges et al. 2022]
- ... → see [Hebecker, Mikhail, Soler 2018]

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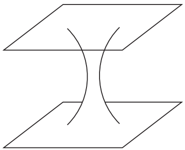

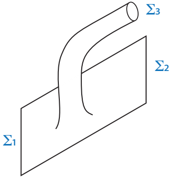

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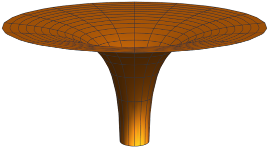
This talk: study of wormhole solutions in axion-dilaton-gravity theory with a massive dilaton

Giddings-Strominger wormholes [Giddings, Strominger 1983]

$$S_{\mathrm{E}} = \int \mathrm{d}^4x \sqrt{g} \Big[-\frac{1}{2\kappa} R + \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi + V(\phi) + \frac{1}{12 f^2} e^{-\beta\phi\sqrt{\kappa}} H_{\mu\nu\rho} H^{\mu\nu\rho} \Big]$$



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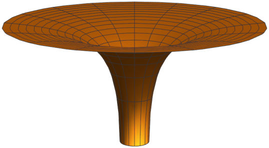

Spherically symmetric & homogeneous ansatz:

$$\begin{cases} \mathrm{d}s^2 = h^2(\tau)\mathrm{d}\tau^2 + a(\tau)^2\mathrm{d}\Omega_3^2\,, \\ \phi = \phi(\tau)\,, \\ H_{0ij} = 0\,,\ H_{ijk} = q\varepsilon_{ijk}\,. \end{cases}$$

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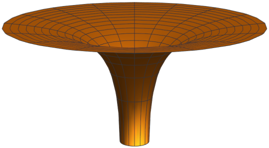

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GS solution for $V = 0$: exist $\forall\ \beta < \beta_c = 2\sqrt{2/3} \simeq 1.63$

Generalised GS solutions to non-zero potential

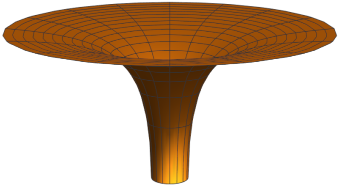
Initial conditions: $\dot{a}(0) = 0$, $\dot{\phi}(0) = 0$

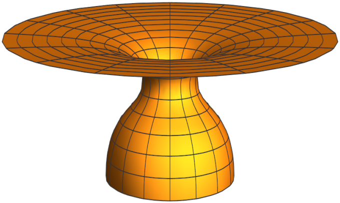
Asymptotic future conditions: $\dot{a}(\tau_f) = 1$, $\phi(\tau_f) = 0$

$\Rightarrow$ Necessary condition: $\kappa^3 N^2 e^{-\beta\phi_0\sqrt{\kappa}} V(\phi_0)^2 < 4$ $(N^2 = \frac{q^2}{2f^2})$

$\rightarrow$ two real solutions for a_0:

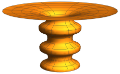
- one leading to **contracting baby universe ($\ddot{a}(0) > 0$)**
- the other leading to **expanding baby universe ($\ddot{a}(0) < 0$)**



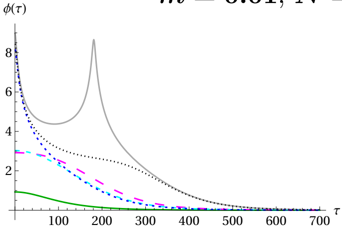


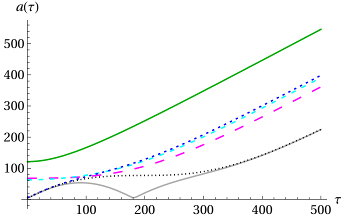
Massive dilaton potential $V(\phi) = m^2\phi^2/2$

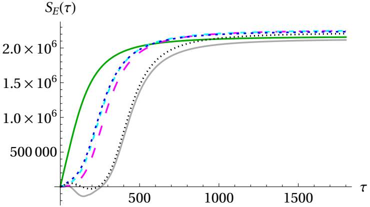
- studied already by [Andriolo, Shiu, Van Riet 2022]→ solutions exist above β_c
- we find new branches of solutions + solutions with several minima:



$$m = 0.01,\ N = 47089 \text{ and } \beta = 1.58$$







color	$\phi(0)$	$S_{\mathrm{E}}(\tau_f)$
green	0.9267658893	$2.17037895 \cdot 10^6$
pink	2.9202136114	$2.26069056 \cdot 10^6$
cyan	3.0261054894	$2.26141352 \cdot 10^6$
blue	8.1578681214	$2.24856190 \cdot 10^6$
black	8.4314038628	$2.22003030 \cdot 10^6$
grey	8.9744628254	$2.12898137 \cdot 10^6$

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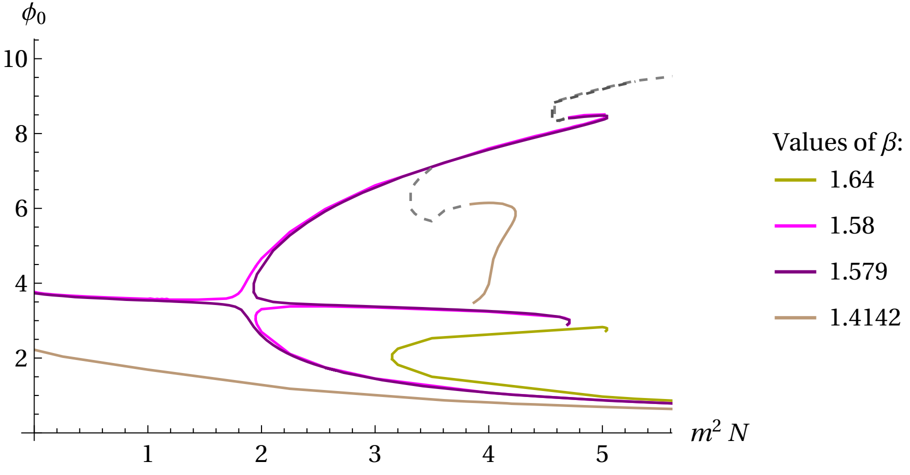

Branching structure of the GS-like wormhole solutions in the massive dilaton case

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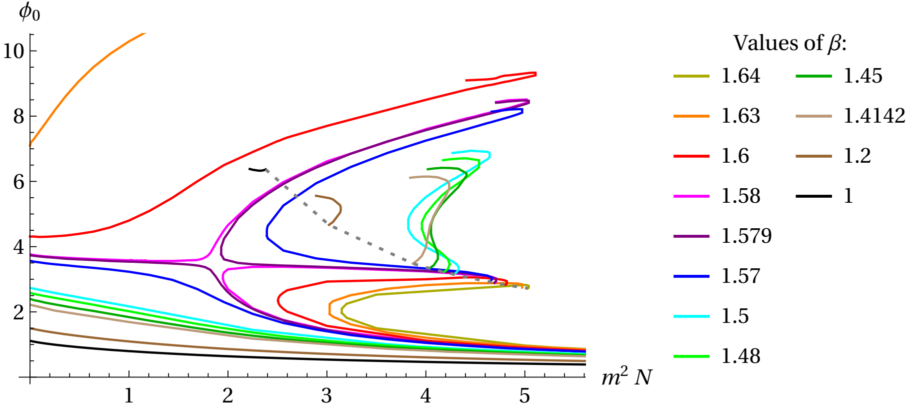
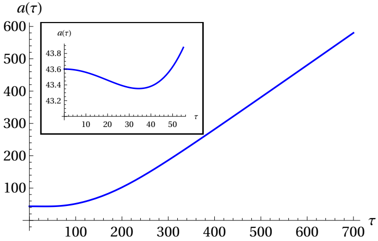
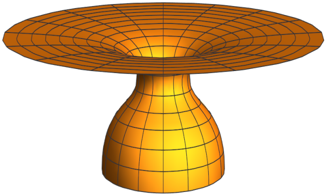


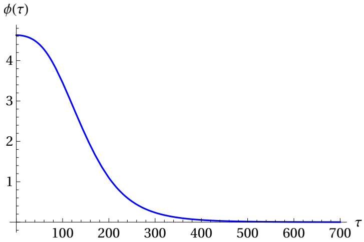
All GS-like solutions for the massive dilaton case

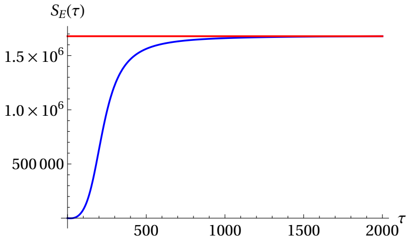
Expanding baby-universe solutions

One example - $\beta = 1.2$, $N = 30000$, $m = 0.01$









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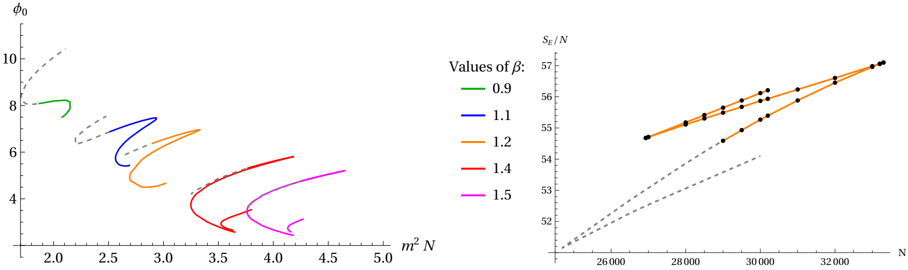

- bifurcating behavior
- also oscillating wormholes with lower Euclidean action

Summary and outlook

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- if we replace the dilaton by a **scalar field with double well potential**, we also find expanding baby universe wormholes. Their action is better behaved but some have negative Euclidean action!

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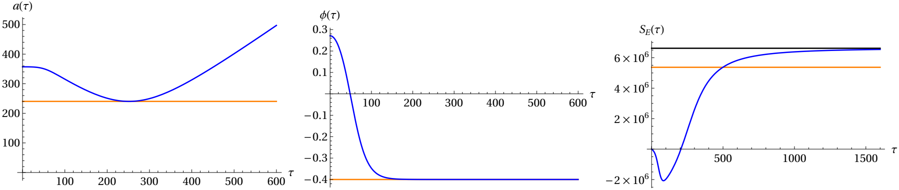
- solutions in asymptotically dS [Hertog et al. 2023] and AdS?

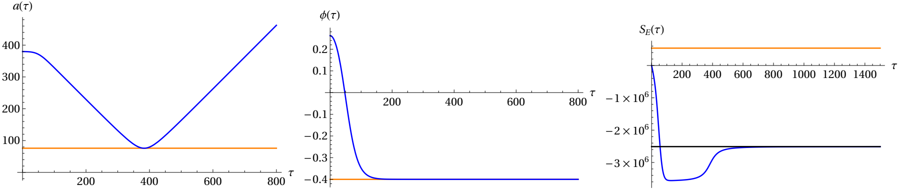
Double well scalar potential $V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2$

- Solutions found before in asymmetric double well potential [Lavrelashvili, Rubakov, Tinyakov 1987]
- Potential barrier necessary to the existence of wormhole solutions

Expanding solution with $\lambda = 0.01$, $v = 0.4$ **and** $N = 10^5$



Expanding solution with $\lambda = 0.01$, $v = 0.4$ **and** $N = 10^3$



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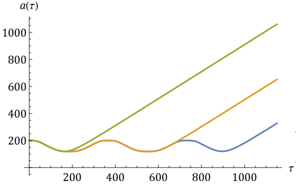
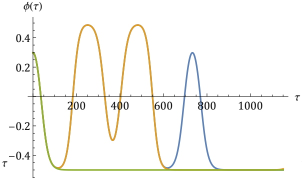
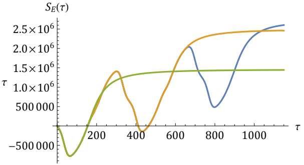
Oscillating solutions:







Summary of results: