Unitarity and quantum resolution of gravitational singularities

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13 July 2023

based on joint work with Lucía Menéndez-Pidal:
`arXiv:2005.05357` (Class. Quant. Grav. **37** (2020) 205018)
`arXiv:2109.02660` (Class. Quant. Grav. **39** (2022) 075011)
`arXiv:2205.15387` (5th Prize GRF Essay Competition)

THE ROYAL SOCIETY Supported by a Royal Society University Research Fellowship.

Unitarity and general covariance

Unitarity of time evolution is a basic principle of quantum mechanics, closely tied in with the probability interpretation of the theory.

When we think of quantum gravity, would like unitarity to emerge somewhere.

But for which type of "time evolution"? t is just a label, but **relational** clocks (such as matter fields) are physical. Should time evolution with respect to a well-behaved ("physically reasonable") matter clock be unitary?

Clash with general covariance of classical relativity. In relativity, can always find clocks with unusual properties; unitarity with respect to these can lead to strange "predictions". Some higher principle seems needed, not clear what it is.

By choosing the right clock, it seems we can **always** resolve singularities.

A simple model

Consider a homogeneous, isotropic, spatially flat universe with metric

$$\mathrm{d}s^2 = -N(\tau)^2\mathrm{d}\tau^2 + a(\tau)^2 h_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

where h is a flat metric, $a(\tau)$ is the scale factor and $N(\tau)$ is the lapse function.

Matter: a free massless scalar $\phi(\tau)$ and perfect fluid with energy density $\rho(\tau)$ and equation of state parameter $w < 1$.

Hamiltonian can be written in the form

$$\mathcal{H} = \tilde{N}\left[-\pi_v^2 + \frac{\pi_\varphi^2}{v^2} + \lambda\right], \quad \{v, \pi_v\} = \{\varphi, \pi_\varphi\} = \{t, \lambda\} = 1$$

where $v \propto a^{\frac{3(1-w)}{2}}$, $\tilde{N} = Na^{-3w}$, λ is conserved momentum $\propto \rho\, a^{3(w+1)}$.
Gauge $\tilde{N} = 1$ leads to **simplest** dynamics, $\mathrm{d}t/\mathrm{d}\tau = 1$. **In this gauge** t becomes "time". Unimodular time for $w = -1$, conformal time for $w = \frac{1}{3}$, . . .

Unimodular gravity relation

For $w = -1$, this preferred standard of time is the one coming from the Henneaux–Teitelboim version of unimodular gravity

$$S[g,\Lambda,T] = \int \mathrm{d}^4x \left[\sqrt{-g}\left(\frac{R}{2}-\Lambda\right) + \Lambda\,\partial_\mu T^\mu\right] .$$

This action gives the Einstein equations with "dynamical" Λ plus

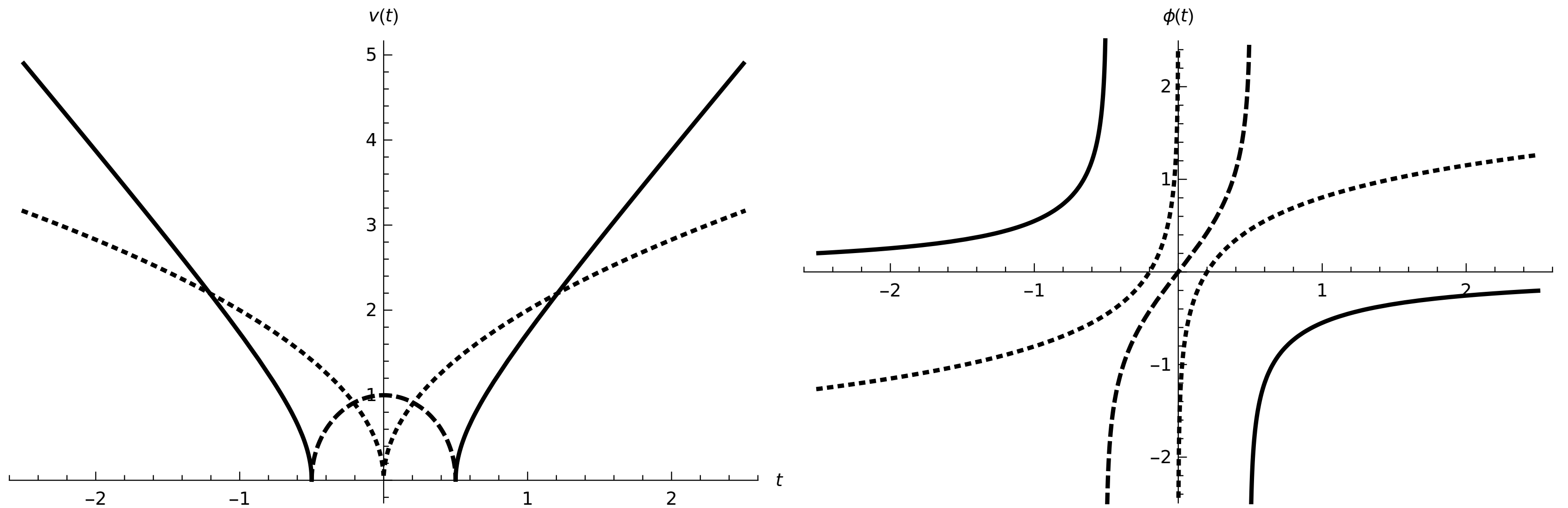
$$\sqrt{-g} = \partial_\mu T^\mu \,, \quad \partial_\mu \Lambda = 0\,.$$

"Preferred" global notion of time: four-volume between two boundary hypersurfaces is

$$\int_M \mathrm{d}^4x\sqrt{-g} = \int_{\partial M} \mathrm{d}^3x\, T^0\,.$$

Solutions in t time

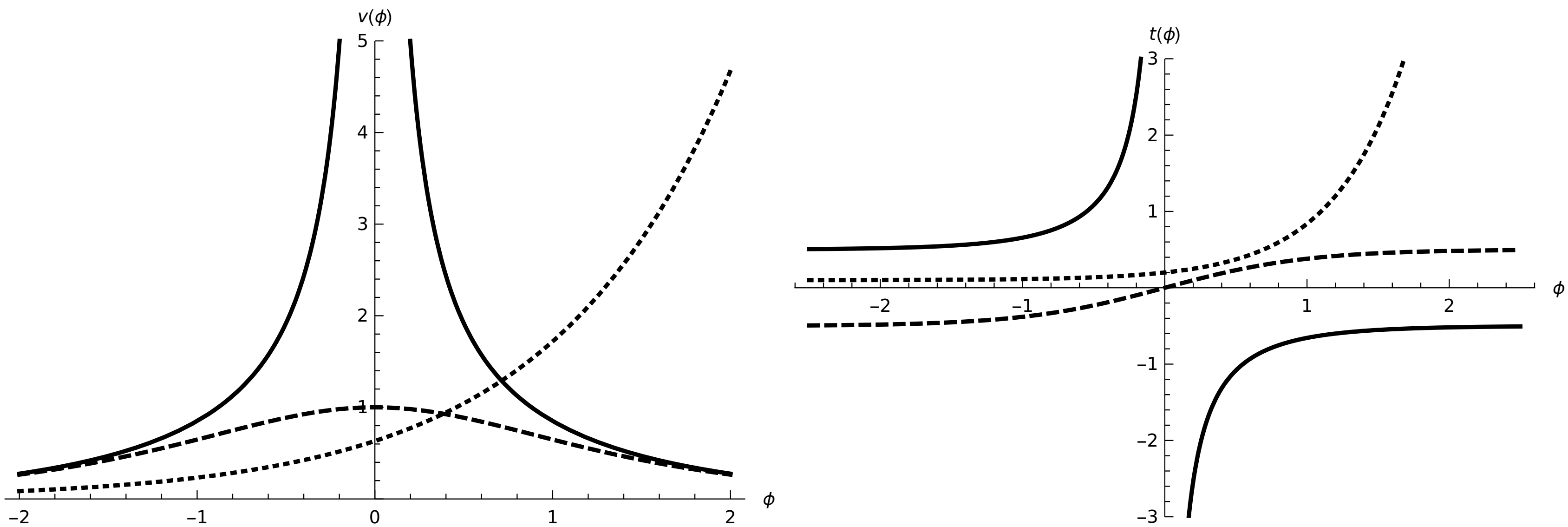
Classically, the variables t and φ evolve monotonically (if we exclude $\pi_\varphi = 0$) so are always good relational clocks.



Classical solutions $v(t)$ and $\varphi(t)$ as functions of the clock t, with $\pi_\varphi = 1$ and $\lambda = 1$ (solid), $\lambda = -1$ (dashed) and $\lambda = 0$ (dotted).

All solutions have a (Big Bang/Big Crunch) singularity with $v \to 0$ and $\varphi \to \infty$.

Solutions in φ time



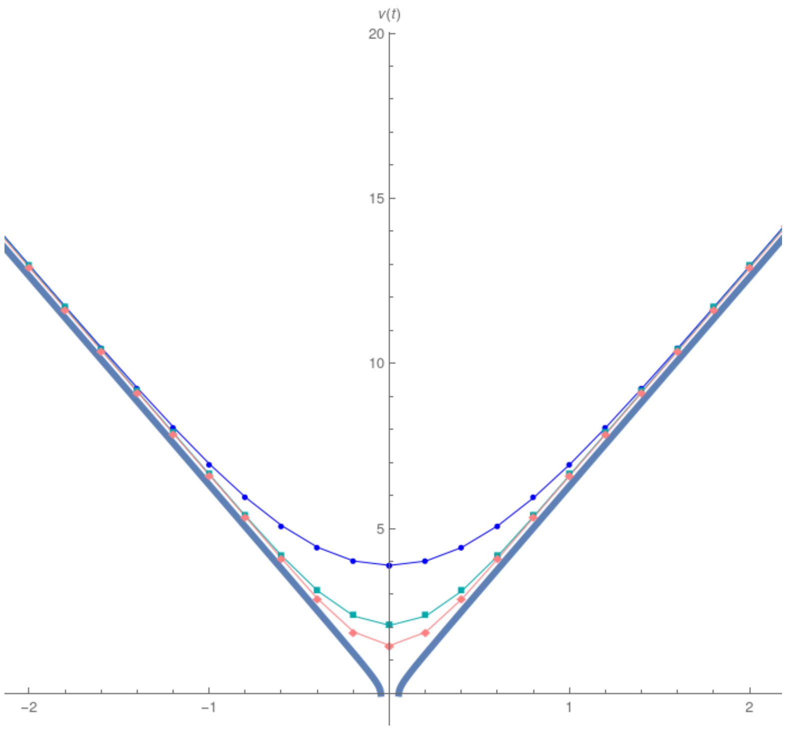
Parameters: $\pi_\varphi = 1$, $\lambda = 1$ (solid), $\lambda = -1$ (dashed) and $\lambda = 0$ (dotted).

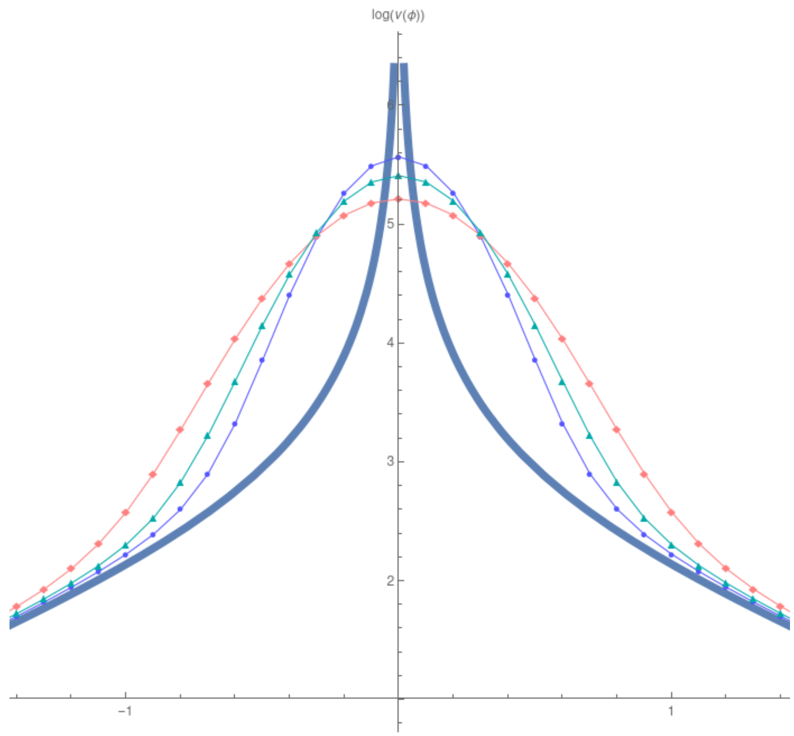
When φ is used as a clock, the Big Bang/Big Crunch singularity is pushed to $\varphi \to \pm\infty$. For $\lambda > 0$ there is a finite value of φ where v and t diverge.

The explicit form of cosmological solutions highly depends on the clock.

Singularity resolution and recollapse

Quantising these theories in the standard Wheeler–DeWitt fashion and requiring unitarity with respect to t or φ gives reflecting boundary conditions, leading to either singularity resolution or recollapse of the Universe:





Colours represent different values of the standard deviation in Gaussian states.

The role of unitarity

Classical solutions, when expressed in terms of one of the "natural" clock variables, can terminate at a finite time as measured by the clock.

In t time this reflects the Big Bang/Big Crunch singularity of classical GR.

In φ time and with $\lambda > 0$ it reflects the fact that $\varphi \to \varphi_0$ as the Universe expands and φ becomes an "infinitely slow" clock asymptotically.

Classically, clocks are not defined beyond the point where the solution terminates. But what happens quantum mechanically? If we require quantum theory to be **unitary** any state must have a globally well-defined time evolutuion.
$\Rightarrow$ Evolution must extend beyond points where classical solution terminates!

Conjecture [Gotay & Demaret 1983]: *unitary slow-time quantum dynamics is always nonsingular, while unitary fast-time quantum dynamics inevitably leads to collapse*.

Perspective of group averaging/Dirac quantisation

Choosing a particular clock with respect to which unitarity is required might seem *ad hoc*. Results can be seen from the more systematic perspective of group averaging/Dirac quantisation, where one defines a physical inner product through

$$|\psi_{\rm ph}\rangle = \delta(\hat{\mathcal{C}})|\psi\rangle \quad \Rightarrow \quad \langle\phi_{\rm ph}|\psi_{\rm ph}\rangle := \langle\phi|\delta(\hat{\mathcal{C}})|\psi\rangle$$

where $\mathcal{C}$ is our Hamiltonian constraint. Usually $\hat{\mathcal{C}}$ is required to be self-adjoint with respect to a particular kinematical inner product.

However, $\hat{\mathcal{C}}$ can be multiplied from the left by a nontrivial lapse function (again corresponding to a particular choice of clock or time coordinate). Different constraint operators will require different types of self-adjoint extension.

Path integral similarly requires different boundary conditions for unitary theory [Menéndez-Pidal's talk].

Extension to black holes

Mostly studied simple cosmological models so far, but arguments more general. In particular, unimodular formulation of gravity always comes with a preferred clock with respect to which we might require unitarity.

Interior Schwarzschild–de Sitter spacetime

$$\mathrm{d}s^2 = -\frac{\mathrm{d}r^2}{\Lambda r^2 + \frac{2M}{r} - 1} + \left(\Lambda r^2 + \frac{2M}{r} - 1\right)\mathrm{d}t^2 + r^2\,\mathrm{d}\Omega^2$$

can be rewritten in unimodular time as

$$\mathrm{d}s^2 = -\frac{\mathrm{d}T^2}{9(\Lambda T^2 + 2MT - T^{4/3})} + \left(\Lambda T^{2/3} + \frac{2M}{T^{1/3}} - 1\right)\mathrm{d}t^2 + T^{2/3}\,\mathrm{d}\Omega^2$$

3-volume goes as $\sqrt{T}$ for small T; singularity encountered in finite time. Requiring unitarity will resolve this singularity.

Summary

- Quantum theories defined with respect to different clocks inequivalent if we require unitarity. Non-classical behaviour triggered when classical solutions terminate in finite "time", leading to reflecting boundary conditions.

- Non-classical behaviour can be triggered at arbitarily low energies, when semiclassical arguments should be valid.

- Should we see one choice of clock as more fundamental and only demand unitarity for that clock? (e.g., the clock measuring proper time $N = 1$)

- Implications for claims of singularity resolution or other quantum corrections to classical cosmology?

Thank you!