Quantum geometry and entanglement from corner symmetry:

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APS-meeting

L. Ciambelli, W. Donnelly, E. Gesteau, F. Girelli, P. Jai-akson,
M. Geiller, F. Moosavian, R. Oliveri, D. Pranzetti, A. Raclariu,
S. Speziale, A. Speranza, N. Teh.

Based on 2302.12799 with M. Geiller and W. Wieland
A chapter in ``Handbook of Quantum Gravity''

Local Holography

- We want to understand the quantization of QFT and quantum gravity in finite regions
- This requires understanding the nature of quantum entanglement across sub-regions: In holography the emergence of classically connected spacetimes is related to the quantum entanglement of quantum gravity degrees of freedom
- In gravity the subregion entanglement is controlled by a symmetry group called the corner symmetry group, which follows from gauge invariance of the total space.
- At the quantum level finding the group representations amount to quantizing geometry and requires us to define the area as an operator

Corner symmetry = entanglement

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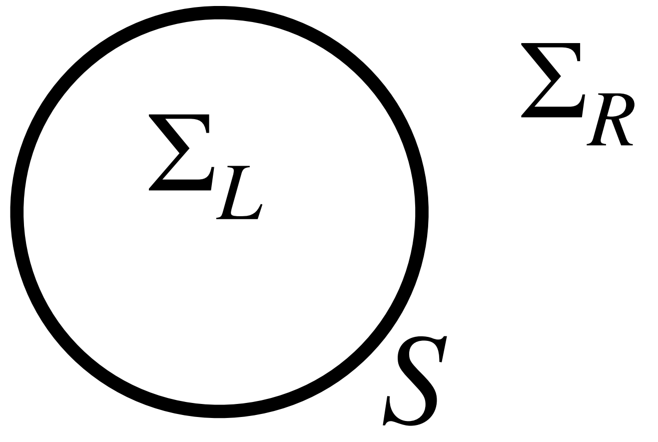
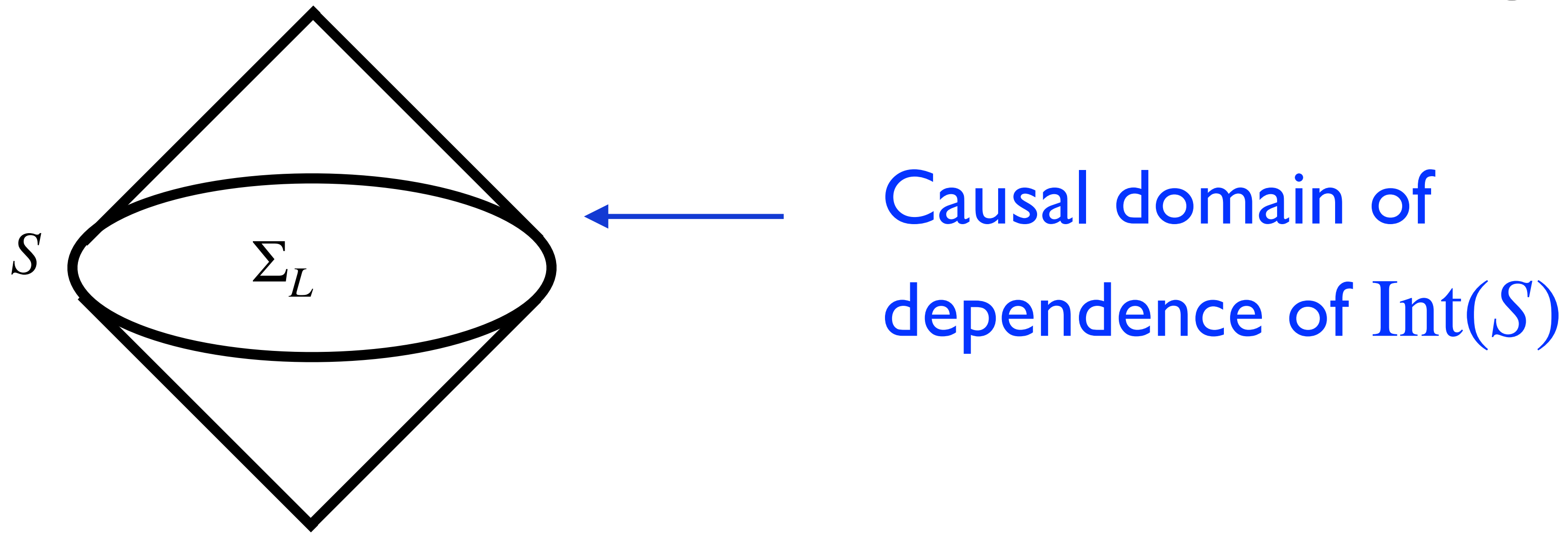
Corner symmetry= entanglement

- This allows us to show in the continuum and from quantization only that quantum geometry carries quanta of area
- Recent works have shown that this naturally connects with S-matrix quantization and soft theorems through celestial holography

Space entanglement

- Given Σ a Cauchy slice. We chose a 2d surface that divide the slice into 2 subregions $\Sigma = \Sigma_L \cup \Sigma_R$
- S is the entangling surface it defines the codimension 2 corner of the sustaining causal diamond





- We denote $\mathscr{A}_\Sigma$ the algebra of observable associated with the region Σ and $\mathscr{H}_\Sigma$ the corresponding Hilbert space obtained by acting with $\mathscr{A}_\Sigma$ on a vacuum state

- In Quantum mechanics we have double factorizability.
  $\mathscr{A}_\Sigma = \mathscr{A}_{\Sigma_L} \vee \mathscr{A}_{\Sigma_R}$ and $\mathscr{H}_\Sigma = \mathscr{H}_{\Sigma_L} \otimes \mathscr{H}_{\Sigma_R}$

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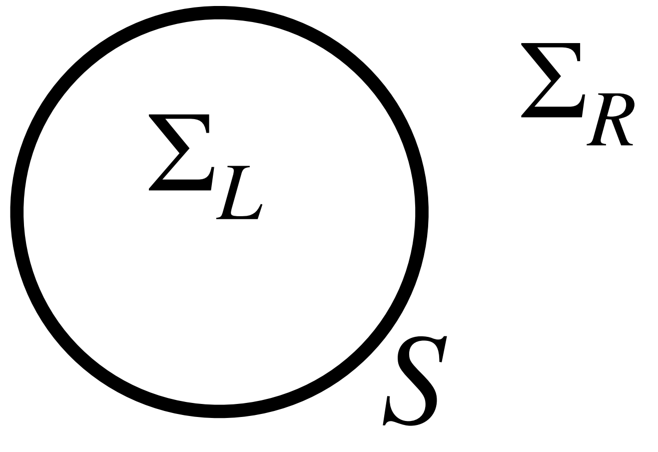

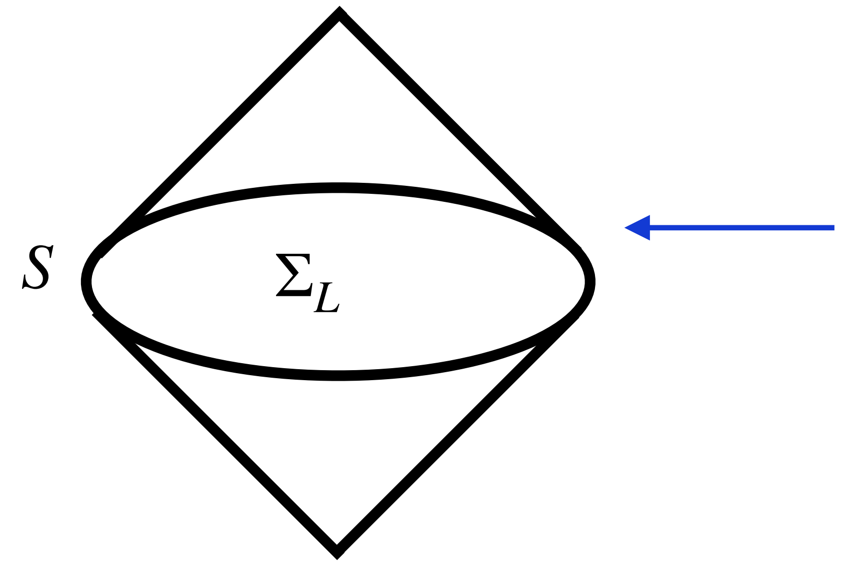

Causal domain of dependence of $\mathrm{Int}(S)$

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- In Relativistic QFT we loose factorizability of the Hilbert space
$\mathscr{A}_\Sigma = \mathscr{A}_{\Sigma_L} \vee \mathscr{A}_{\Sigma_R}$ and $\mathscr{H}_\Sigma \neq \mathscr{H}_{\Sigma_L} \otimes \mathscr{H}_{\Sigma_R}$ since $G(\mathscr{H}_{\Sigma_L}, \mathscr{H}_{\Sigma_R}) \neq 0$

Infinite vaccuum entanglement

Reeh-Schlieder theorem

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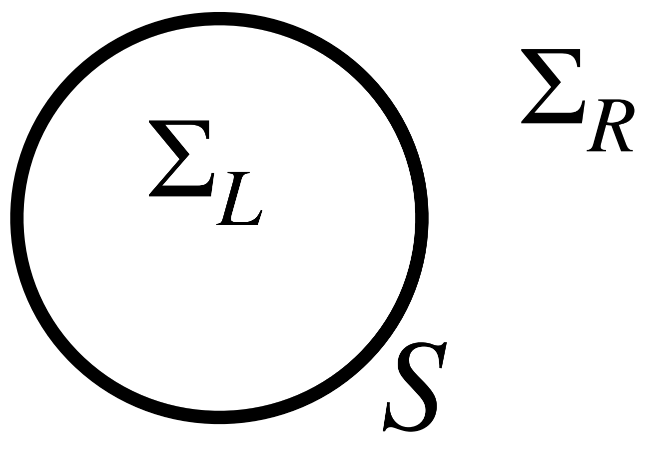

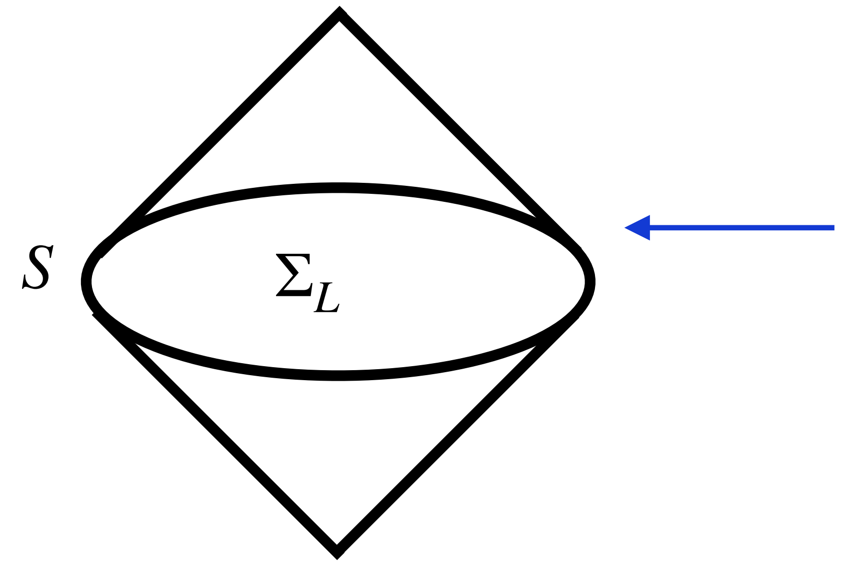

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- In Gravity and Gauge theory we also loose factorizability of observable algebra $\mathscr{A}_\Sigma \supsetneq \mathscr{A}_{\Sigma_L} \vee \mathscr{A}_{\Sigma_R}$ and $\mathscr{H}_\Sigma \neq \mathscr{H}_{\Sigma_L} \otimes \mathscr{H}_{\Sigma_R}$

Gauge invariant Observables are non local.

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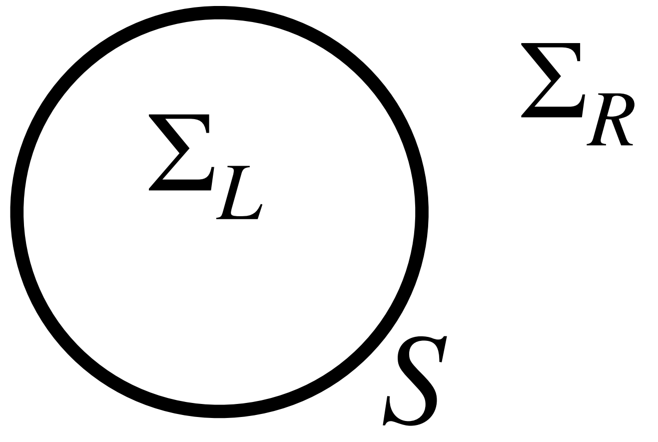

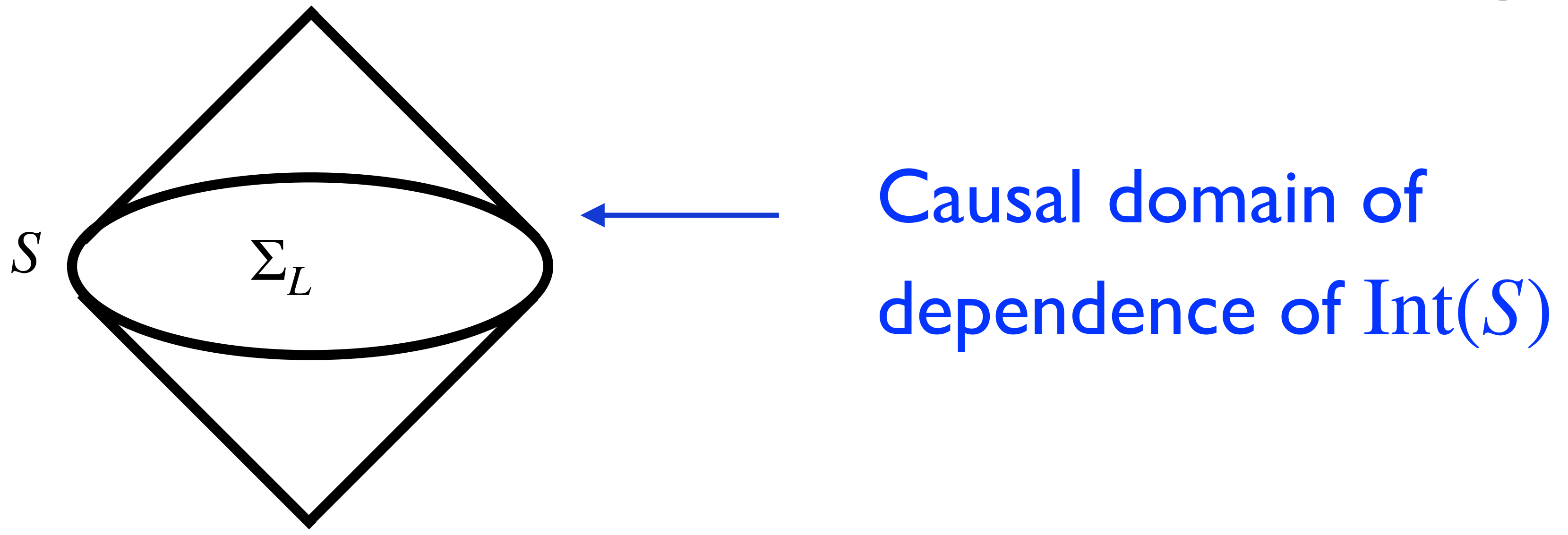

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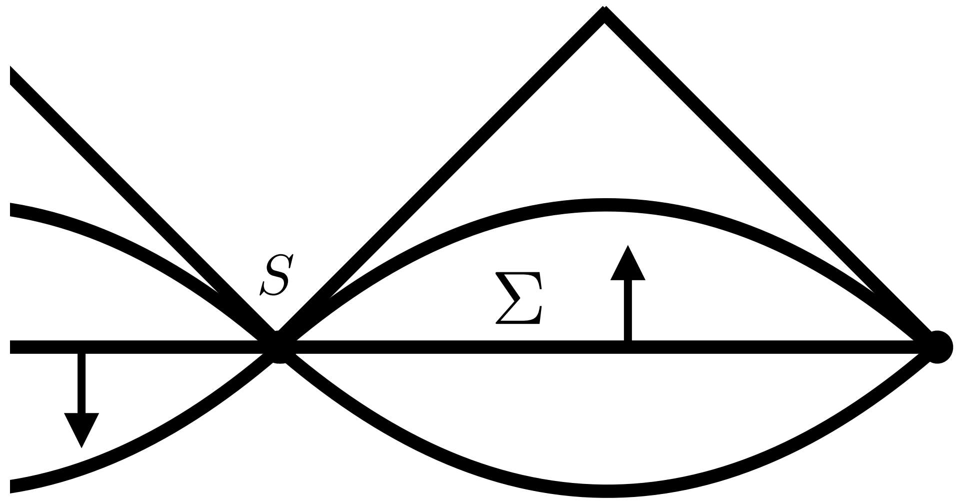
To recover quasi-locality of the gauge invariant observables we need to understand dressing $\longrightarrow$ symmetry

Gauge symmetry resolves entanglement



- In gauge theory and gravity, when no boundary exists the time evolution generator is a constraints $C_\xi \hat{=} 0$.
- The situation changes in the presence of a spacetime boundary or a spacetime corner
- In the presence of a spacetime boundary or a spacetime corner the time evolution operator is entirely supported on codimension 2 corners.

E. Noether 1918

$$Q_\xi(\Sigma_L) \hat{=} \int_{\partial\Sigma_L} q_\xi$$

- Corners unlike boundaries do not need the specification of boundary conditions

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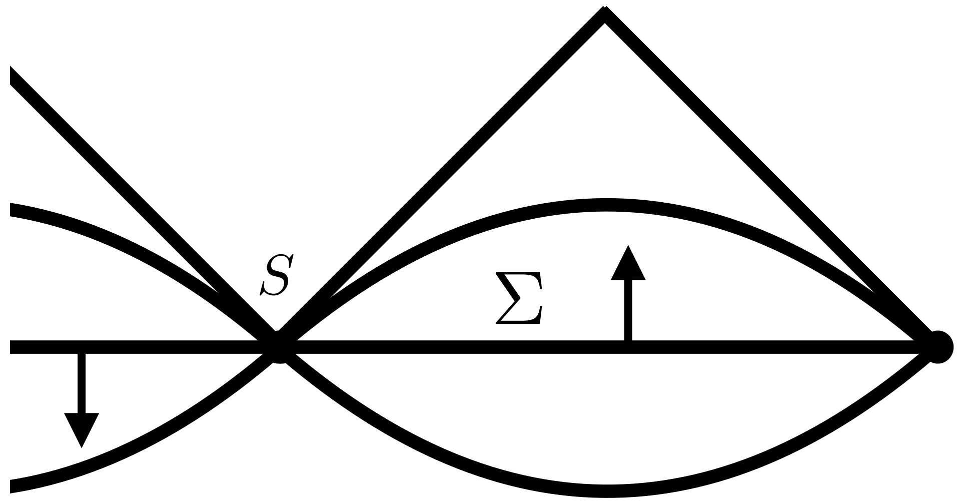

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- This property is the fundamental expression of gravitational holography D. Marolf 2008

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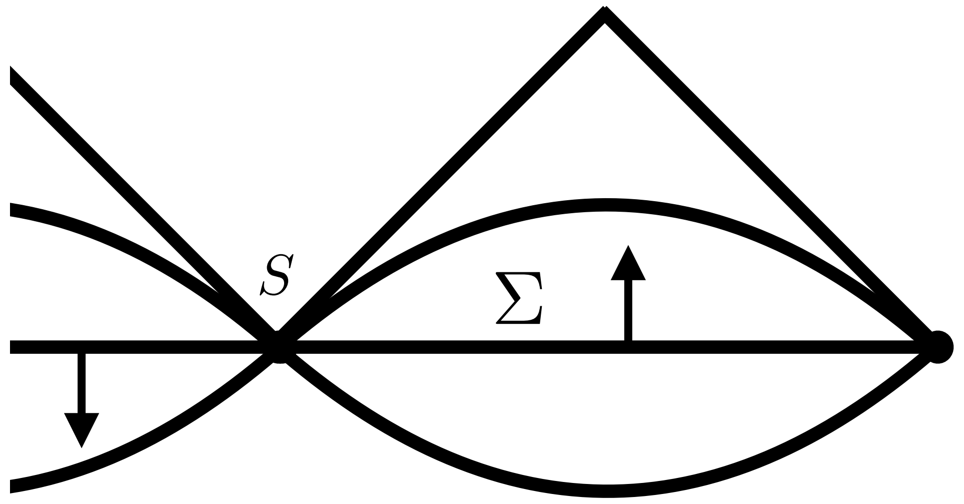

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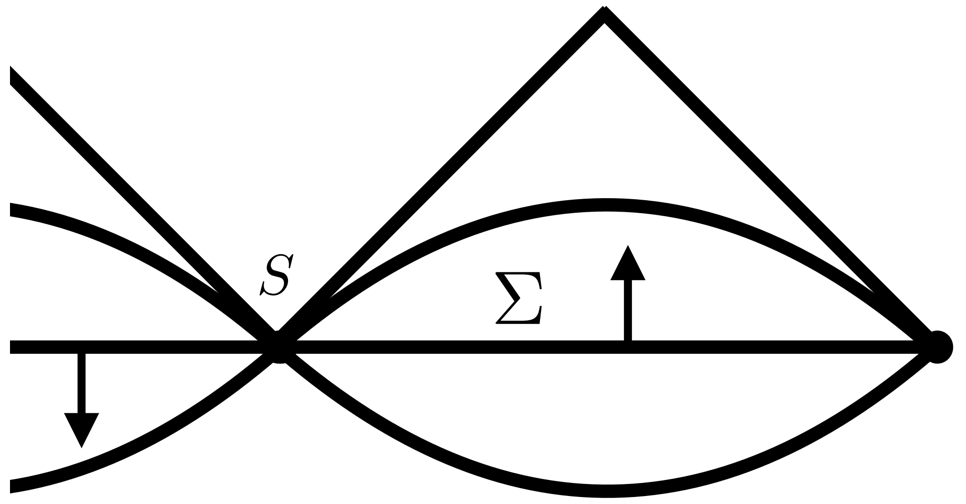

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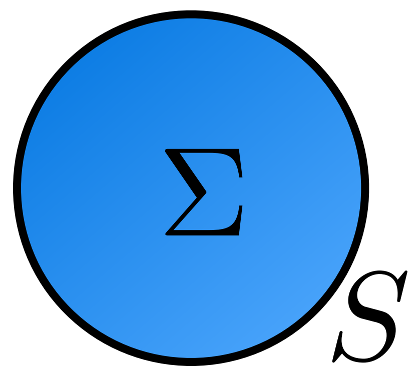
- Entangling corners carries the representation of a fundamental group of symmetry the corner symmetry group G_S. The modular group which contains boost hinging along S is a distinguished subgroup of G_S
- Noether theorem tells us that the charges represents elements of the spacetime geometry ⟶ Non-commutativity of the corner metric components

$$[Q_\xi, Q_\chi] = iQ_{[\xi,\chi]}$$

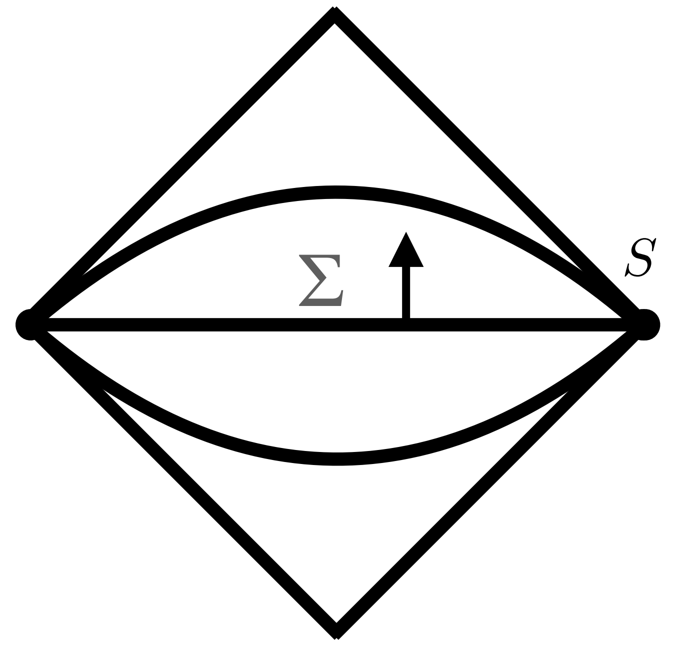
- Finding the quantum representation of G_S is equivalent to quantizing geometry

  ⟶ Understanding the quantum causal diamond

Symmetries and Gravity



- Given a region R with slice Σ the symmetry charges are supported on codimension 2 corners S= entangling sphere
- The extended corner symmetry group G_S is the subgroup of Diff(M) which and possesses non zero Noether charges in the presence of S, its with kinematical subgroup $G_S \subset E_S$ preserves the region R.



- In metric gravity

$$E_S = (\mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S) \ltimes \mathbb{R}^{2S}$$

Group = Kinematical + dynamical

W. Donnelly, L.F 2016
L.F, Leigh, Ciambelli' 21

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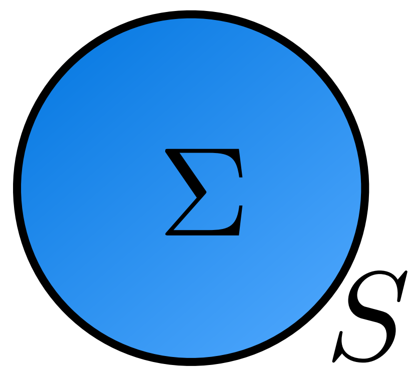

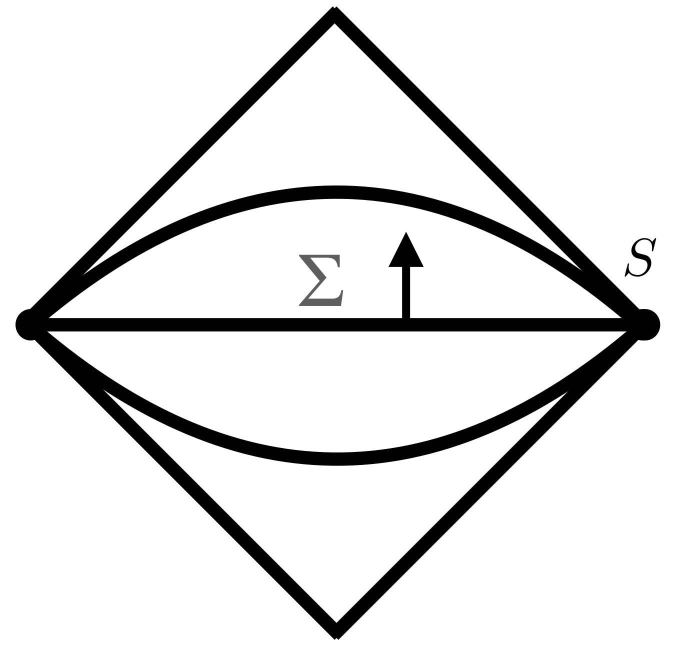

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W. Donnelly, L.F 2016
L.F, Leigh, Ciambelli' 21

- Double Universality of E_S, for metric gravity!
  -Same group for infinitesimal diamond or very large ones
  -Same group for Einstein gravity or any other higher derivative formulation of gravity no matter how many extra derivative

Wald, Speranza'17

- What changes is either the choice of representation or the canonical representation of the symmetry generators

Symmetry on null surfaces

- Local gravitational symmetries are attached to codimension 2 corner: In metric gravity this group is the extended corner symmetry group (Universal)

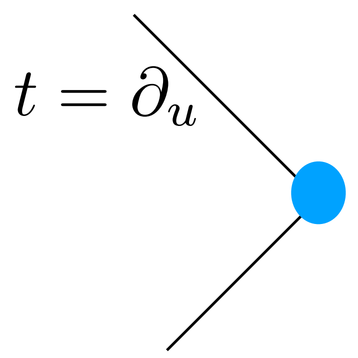
$$E_S = (\mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S) \ltimes \mathbb{R}^{2S}$$

- When we study Horizon, asymptotic infinity or the nature of quantum radiation one focuses our attention onto a specific null surface. In that case the subgroup preserving the preserving the null structure (Thermal Carrollian structure) is

$$\mathrm{BMSW} = (\mathrm{Diff}(S) \ltimes \mathrm{Weyl}) \ltimes \mathbb{R}^S$$

Barnich-Trossaert'10,
Chandrasekar, Flanagan, Prabhu'18
LF, Oliveri, Pranzetti Speziale '21

$$\xi = T\partial_u + Y^A\partial_A + W(u\partial_u - r\partial_r)$$



- At infinity, same group, conservation law are associated with GBMS

$$W = \frac{1}{2} D_A Y^A$$

Barnich Troesseart '11 Campiglia, Ladha '16
Compere, Fiorucci, Ruzziconi'18

Quantum Corner symmetry

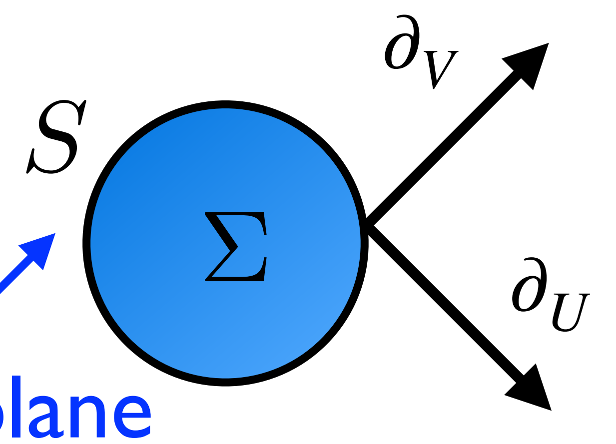
$$G_S = \mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S$$

Donnelly, Moosavian, Speranza, LF'

- What are the reps? what are the Casimirs?
- The little group is the group that preserves
- The subgroup generated by the $SL(2,\mathbb{R})$ Casimir

$$C_{\mathrm{SL}(2,\mathbb{R})_\perp} = \det(q) > 0$$

$\frac{\sqrt{q}}{4G}$ is the local affine boost group



Boost along the null plane

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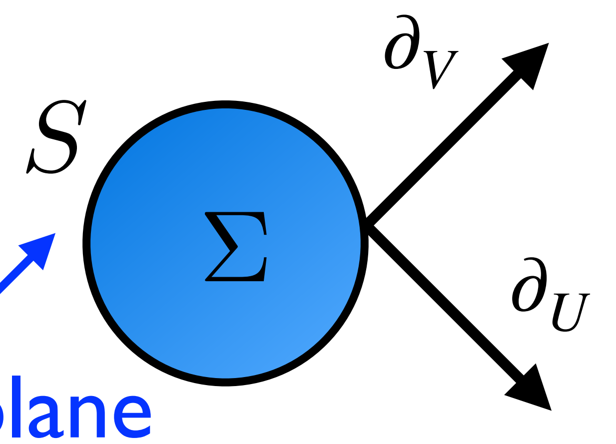

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- Denoting P_A the diff generator and N^a the $SL(2,\mathbb{R})$ one

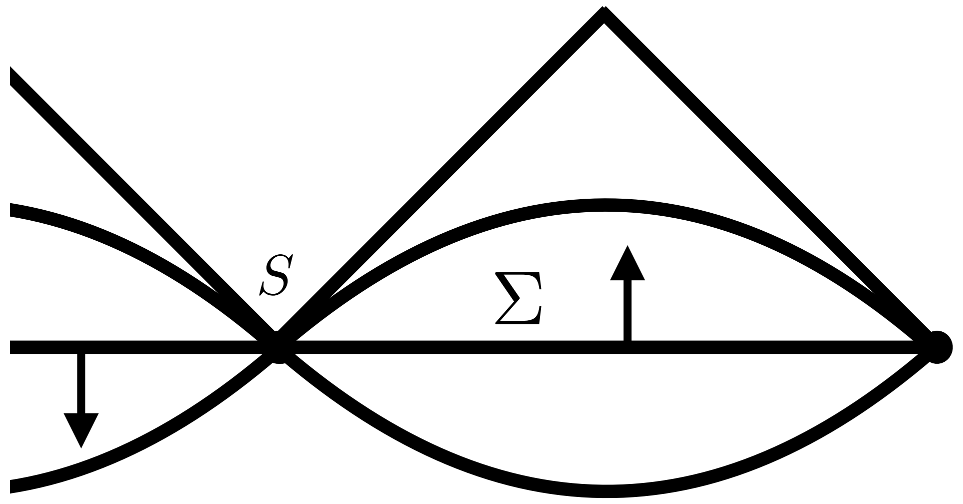
$$\Omega = \epsilon^{AB}\left[\partial_A P_B - \frac{1}{2}\epsilon_{abc}\partial_A N^a \partial_B N^b N^c\right]$$

- The Casimirs are then given by

$$C_n = \int_S \sqrt{q}\,\Omega^n$$

$\longrightarrow$ Complete classification of rep where $\sqrt{q} > 0$

Edge modes and Quantum space-time



- In order to define Gravity in finite region we need a field that tells us where the corner is situated $X : S \to \Sigma$
- This field, called the embedding field or edge mode field is part of the gravitational phase space: $[U(\sigma), V(\sigma), X^A(\sigma)] \to [\hat{U}(\sigma), \hat{V}(\sigma), \hat{X}^A(\sigma)]$

W. Donnelly, LF 2016
Speranza 2017
Leigh, Ciambelli 2021
LF 2021

Transverse null coordinates

Longitudinal coordinates

The location of the quantum corner is determined by the quantum state

- Edge modes allows the possibility to define gauge invariant observables through the dressing of observables. They render super-translation Hamiltonian
- This defines an extended algebra of observables $\mathscr{A}^{\text{ext}}_{\Sigma_L} = \mathscr{A}^{\text{ext}}_{\Sigma_L} \vee \mathscr{A}_S$
- The extended algebra of observable is a cross product algebra

Connes 1973
Venkatesa, Witten et al 2023

- One of the corner symmetry group Casimir is the modular hamiltonian!

Quantum geometry

Entanglement

$$\frac{\hat{A}}{4G} = \frac{1}{4G}\int_S \sqrt{g} \qquad = \qquad \hat{K} = -\frac{\hbar}{2\pi}\log \rho_\Sigma$$

Speranza et al. 2023
Leigh, 2023
LF, E Gesteau 2023 TA

- This simple fact implies a reduction of UV divergences Type $III \to$ Type II

Quantum fluid

G_S is isomorphic to the symmetry groups of 2d hydrodynamics

W. Donnelly, A. Speranza, F.M Moosavian, L.F 2020

- Analogy: the area density $\sqrt{q}$ plays the role of the fluid density ρ
  The outer curvature Ω plays the role of the fluid vorticity w
- The quantum representations are classified by a choice of area and vorticity densities (ρ, w) on S.
- (ρ, w) can be related to labels of the coadjoint orbits (hence representation) of the `fluid group' H_S

Arnold'66; Marsden, Ratiu'95 Khesin'17

- Classical fluid corresponds to a choice of density density measure $\rho > 0$ which is absolutely continuous with respect to the Lebesgue measure
- Quantum fluid corresponds to a choice where both ρ and w are counting measures.

  This gives a constituent picture to the fluid

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M. Geiller, D. Pranzetti, L.F 2021
Ciambelli, Leigh, L.F TA

- This provides a **constituent picture** where

  Fluid atomization = Area constituent
  Vortex quantization = momenta quantization

$$\rho = \sum_i \rho_i \delta^{(2)}(\sigma, \sigma_i)$$

- Each constituent carries a density, weight and spin (ρ_i, Δ_i, s_i)

$$P_A = \sum_i \delta^{(2)}(\sigma, \sigma_i) D_A + (\Delta_i \delta_A^B + s_i \epsilon_A{}^B)\partial_B \delta^{(2)}(\sigma, \sigma_i)$$

Geiller, Wieland, L.F 2022

- Area constituent in the continuum from quantization!

Discretization is derived not postulated

- Einstein Cartan gravity with an Immirzi parameter implies that $\rho_i = \gamma\sqrt{j_i(j_i+1)}$.
  Area gap in the continuum!

Wieland '19

Quantum fluid

W. Donnelly, A. Speranza, F.M Moosavian, L.F 2020

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Geiller, Wieland, L.F 2022

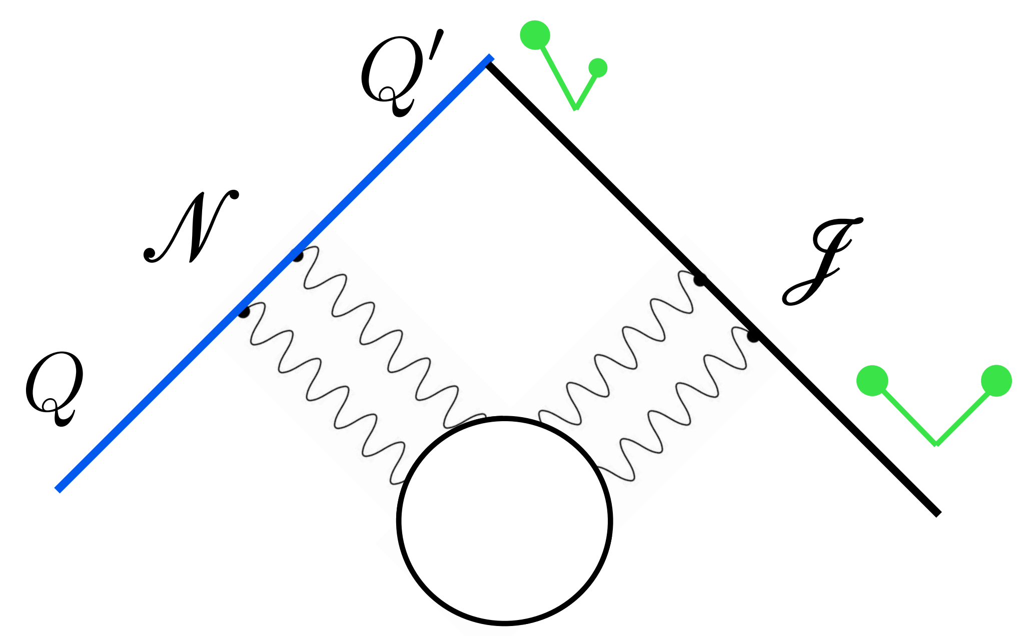
- Area constituent in the continuum from quantization!

- The area preserving diffeomorphisms arises as the large N limit of SU(N)

  ⟶ Matrix model deformation of Gravity and its symmetry.

W. Donnelly, A. Speranza, F.M Moosavian, L.F 2022

Dynamics along null surfaces



- The gravitational evolution along null surfaces can be entirely formulated as conservation laws for the corner charges ⟶ Quantization of the Einstein equation

Three main results for dynamics along causal Horizons:
Carollian structure (ℓ^a, q_{ab}) such that $\ell^a q_{ab} = 0$

- The Gravitational dynamics projected on $\mathcal{N}$ can be recast as a set of Null conservation Laws $D_b T_a{}^b = 0$ $\quad T_a{}^b = \tau_a \ell^b + \tau_a{}^b$

Carollian connection

Carollian energy-momentum tensor

Donnay, Marteau '19
LF, Hopfmueller,'19; Sheikh-Jabbari'20
Speranza, Flanagan, Chandrasekaran 21

- This dynamics can be understood as the conservation of charges for a universal null surface symmetry group BMSW
- The dynamics can be understood in terms of a canonical structure associated with

$$\Theta^{\text{can}} = \frac{1}{8\pi G}\int_{\mathcal{N}} \left(\frac{1}{2}\tau^{ab}\delta q_{ab} - \tau_a \delta \ell^a\right)\epsilon_{\mathcal{N}}$$

⟶ Quantum Ray-Chauduri $\quad$ Ciambelli, Leigh, F to appear

Summary:

- The profound consequences of Noether theorem for gravitational theories leads to a new picture of quantum geometry as a state of representation of the corner symmetry group which capture the essence of subregions entanglement.
- It encodes the non-commutativity of geometrical observables associated with subregions representing the quantization of geometry.
- It leads discretization of space from the representation of continuous non-commutative infinite dimensional algebras represented as quantum fluid.
- This discretization is two-fold: It allows the possibility of corner constituents through atomizisation of the density and the usual area gap from the presence of the immirzi parameter
- Edge modes allows the possibility to define quasi-localised gauge invariant observables and finite density matrix attached to subregions
- It gives a fundamental reason behind the type III $\rightarrow$ type II cross product reduction

- Dynamics along null surfaces is encoded into Carrolian conservation laws for the symmetry charges and activated at the quantum level by the representation of the dynamical charges. This dynamics is now open to quantization.