QUANTUM REFERENCE FRAMES: A RELATIONAL PERSPECTIVE ON NONCLASSICAL SPACETIME

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ETH Zürich

Image credits: J. Palomino

Quantum Gravity Conference 2023
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What replaces spacetime when gravity acquires quantum properties?

GENERAL RELATIVITY

GRAVITY

CLASSICAL MATTER

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

QUANTUM THEORY

$$T_{\mu\nu} \rightarrow \hat{T}_{\mu\nu}$$

QUANTUM MATTER

Image credits: ISQG

QUANTUM ASPECTS OF SPACETIME

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PLANCK-SCALE ARGUMENTS
Heisenberg microscope
Fundamental discreteness
… more examples!

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Image credits: ISQG

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BOTTOM-UP APPROACH:
Start from quantum, include gravity

Image credits: ISQG

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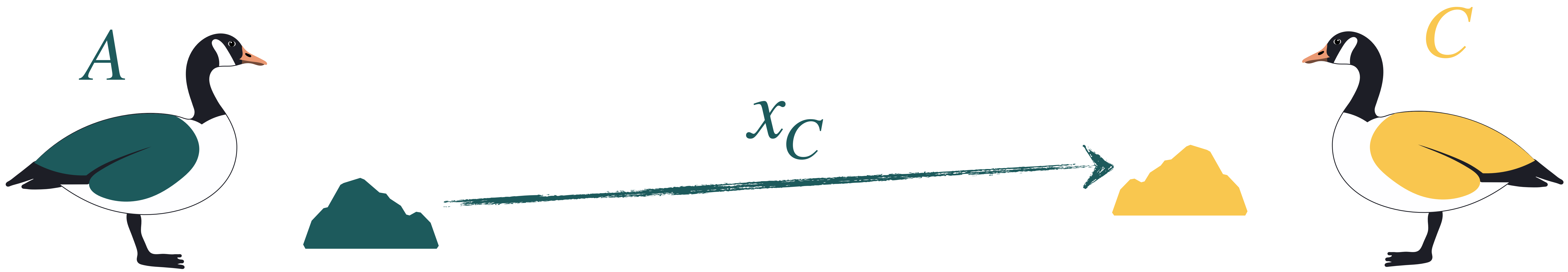

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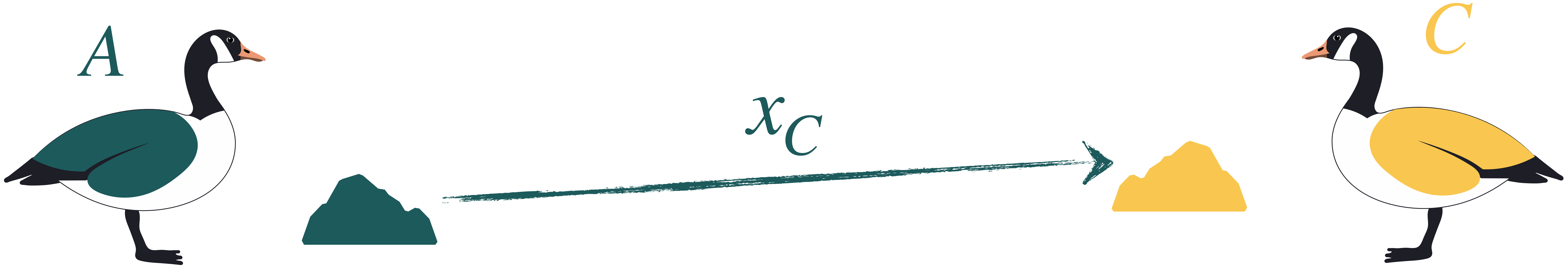

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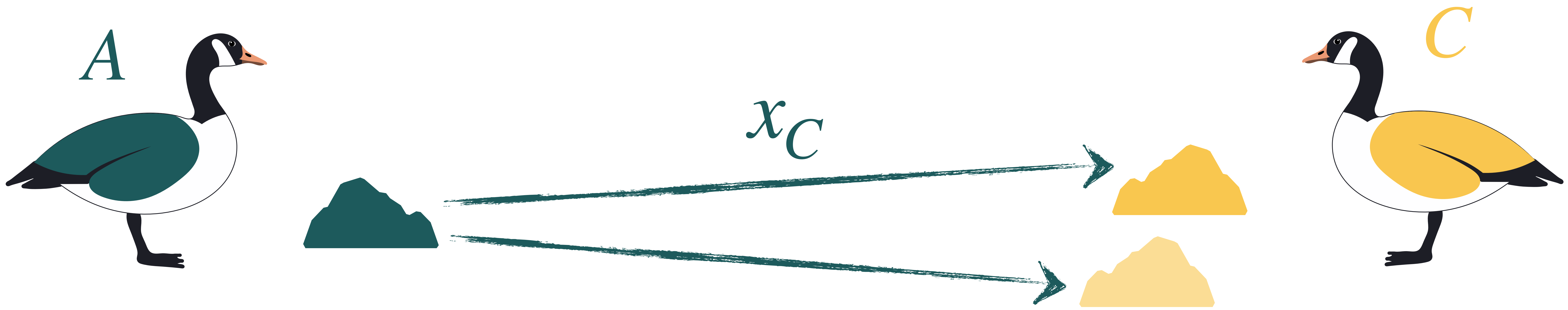

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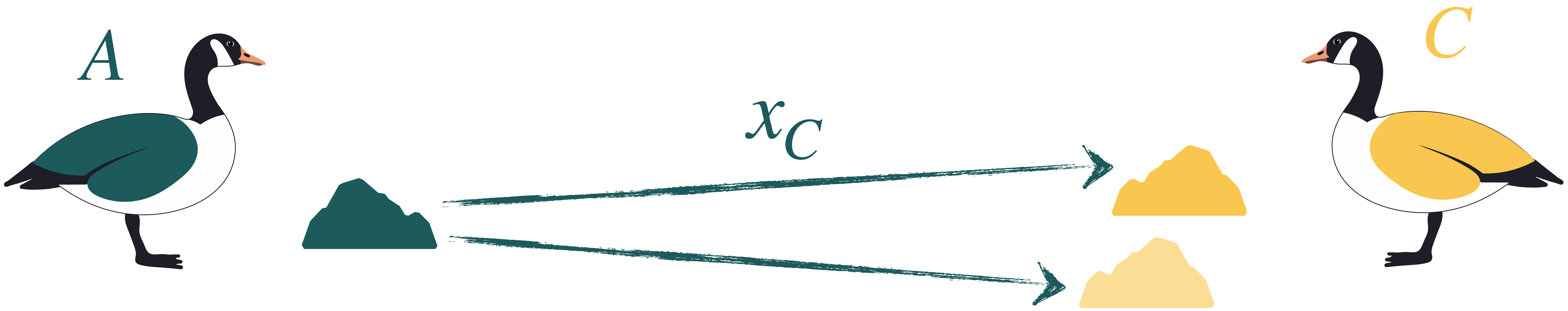

What is the state of A from the perspective of C?

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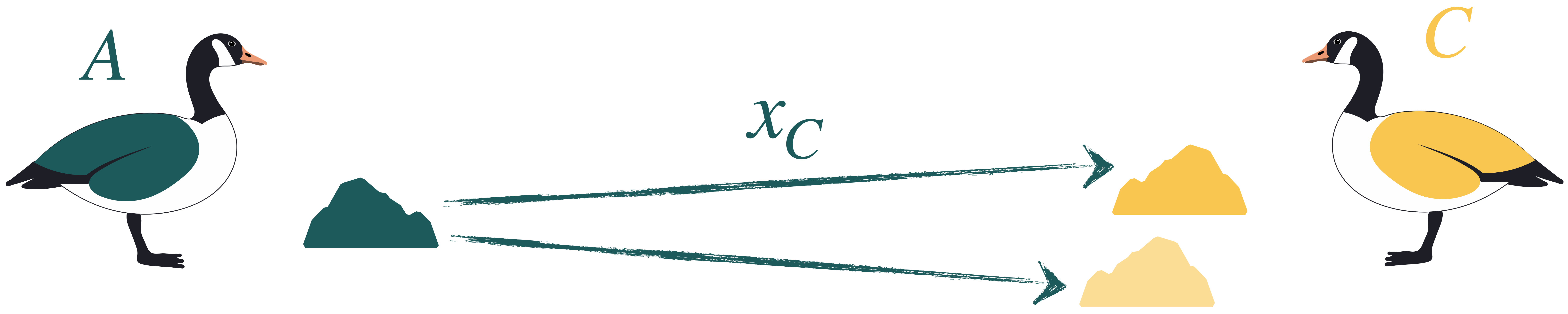
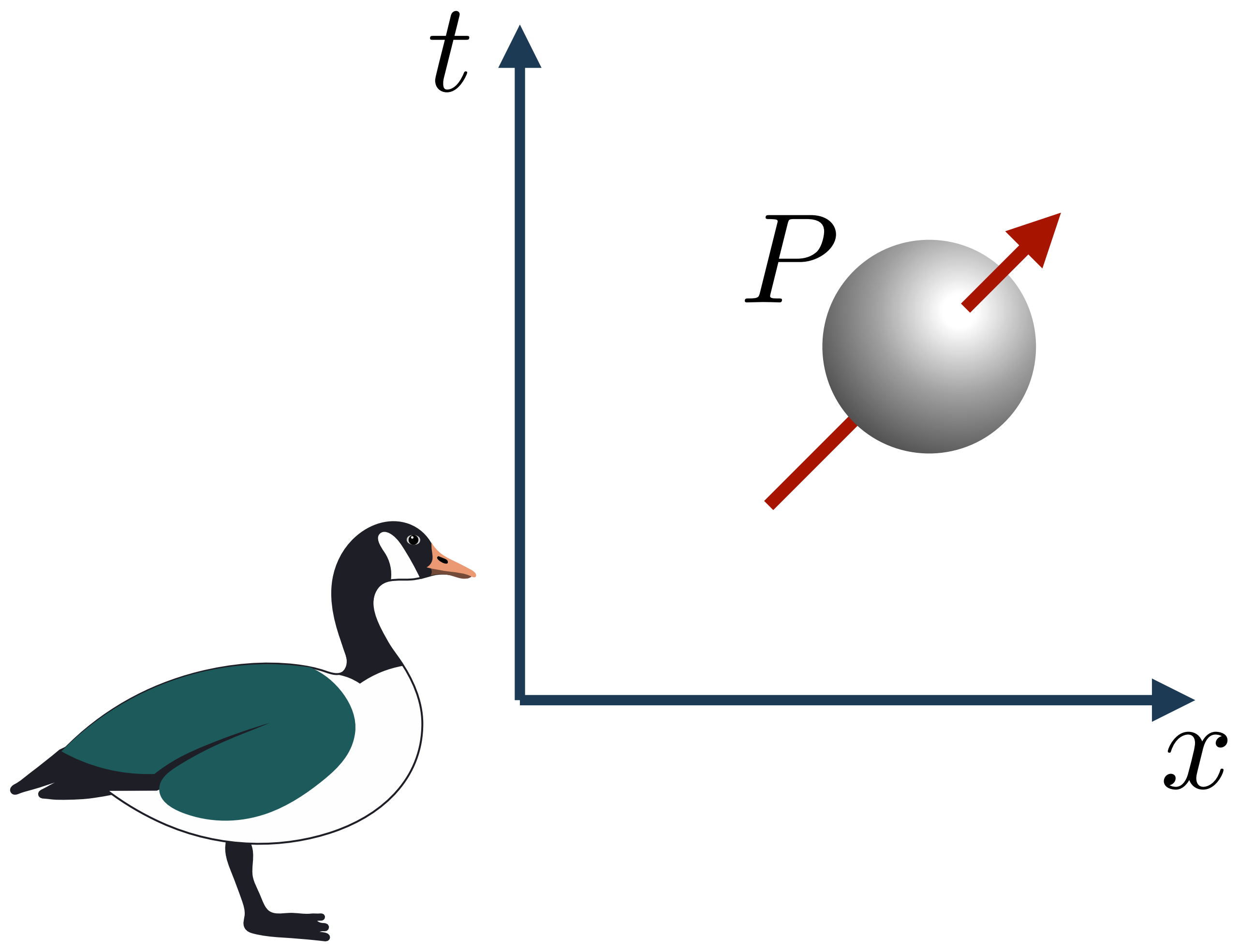


What is the state of A from the perspective of C?

Can we "attach" a reference frame to an object whose state is in a superposition of classical states (in some basis)?

COVARIANCE OF PHYSICAL LAWS IN QUANTUM MECHANICS



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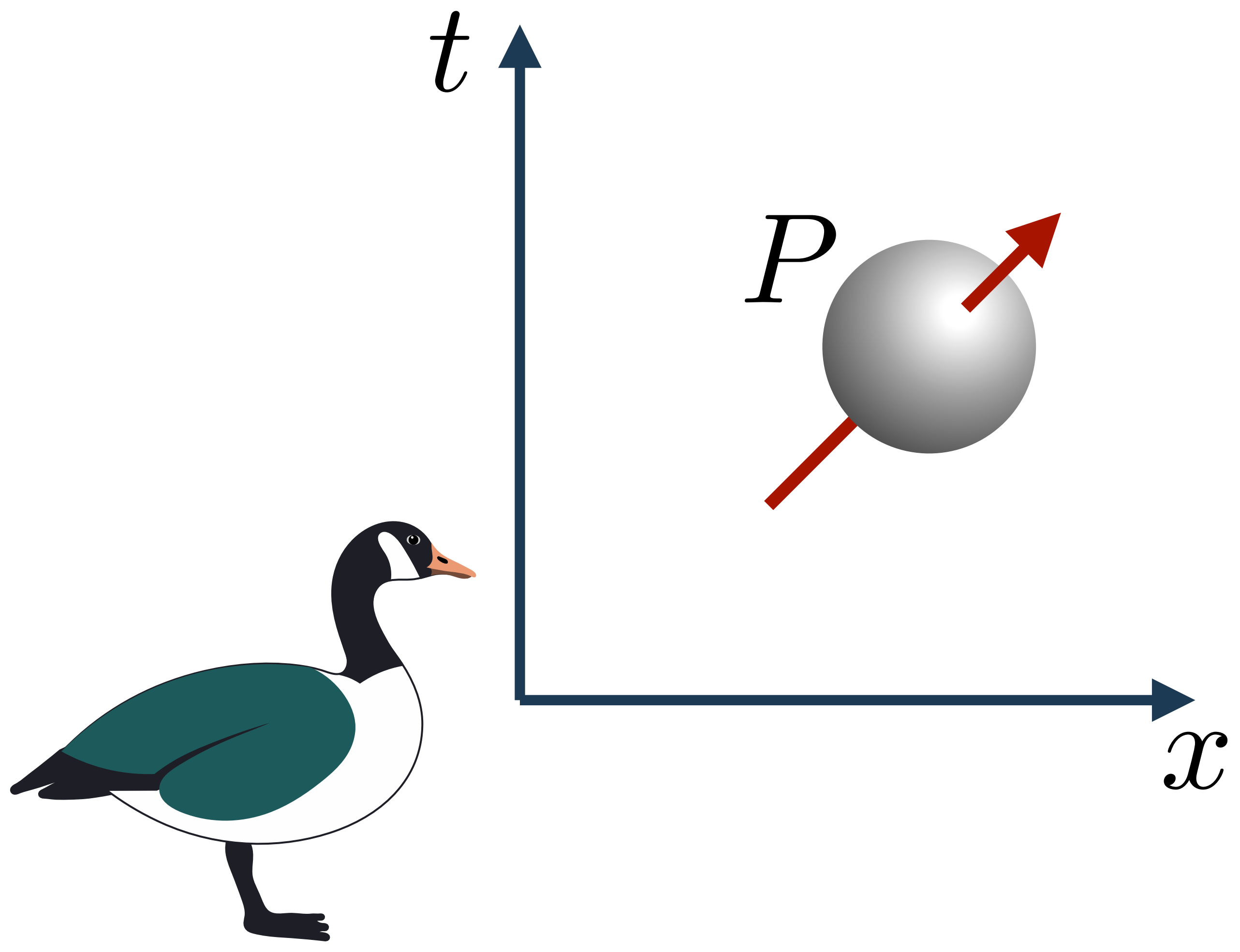

Translation $\hat{U}_T = e^{\frac{i}{\hbar}X_0\hat{p}}$

Galilean boost $\hat{U}_B = e^{\frac{i}{\hbar}v\hat{G}}$ $\hat{G} = \hat{p}t - m\hat{x}$

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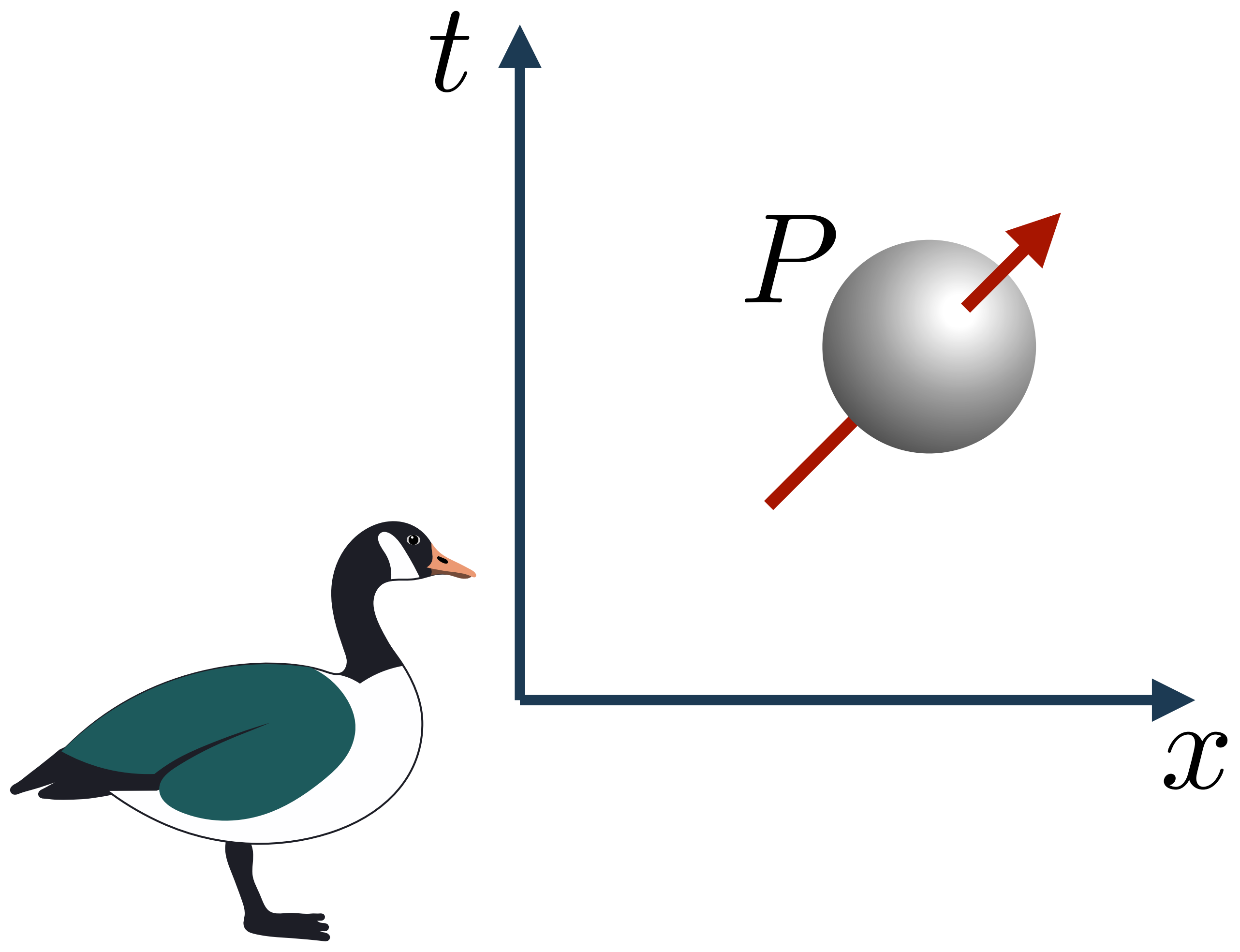
The reference frame enters the transformation as a parameter.



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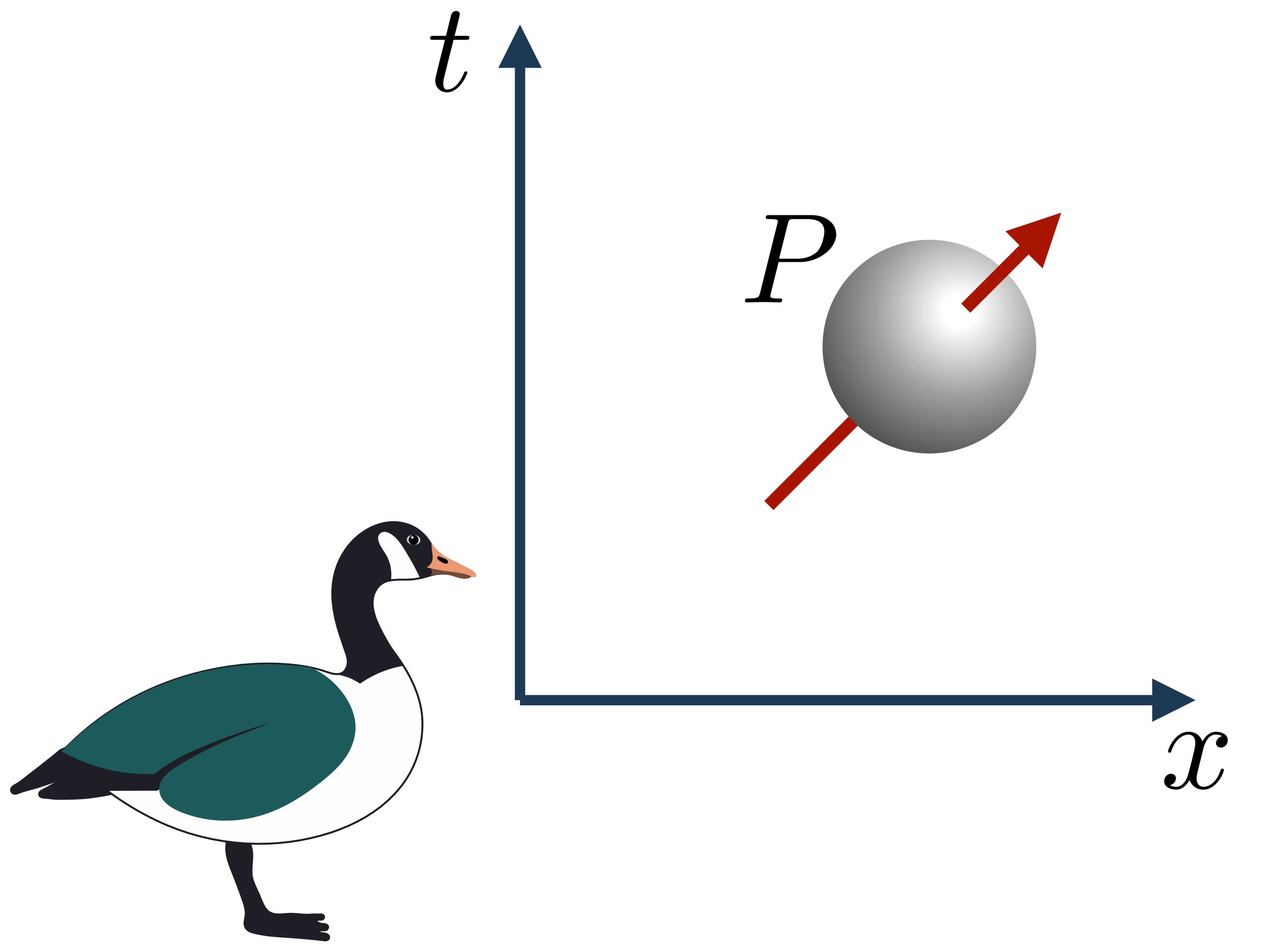

Covariance of physical laws

$$\hat{H}' = \hat{U}\hat{H}\hat{U}^\dagger + i\hbar \frac{d\hat{U}}{dt}\hat{U}^\dagger$$

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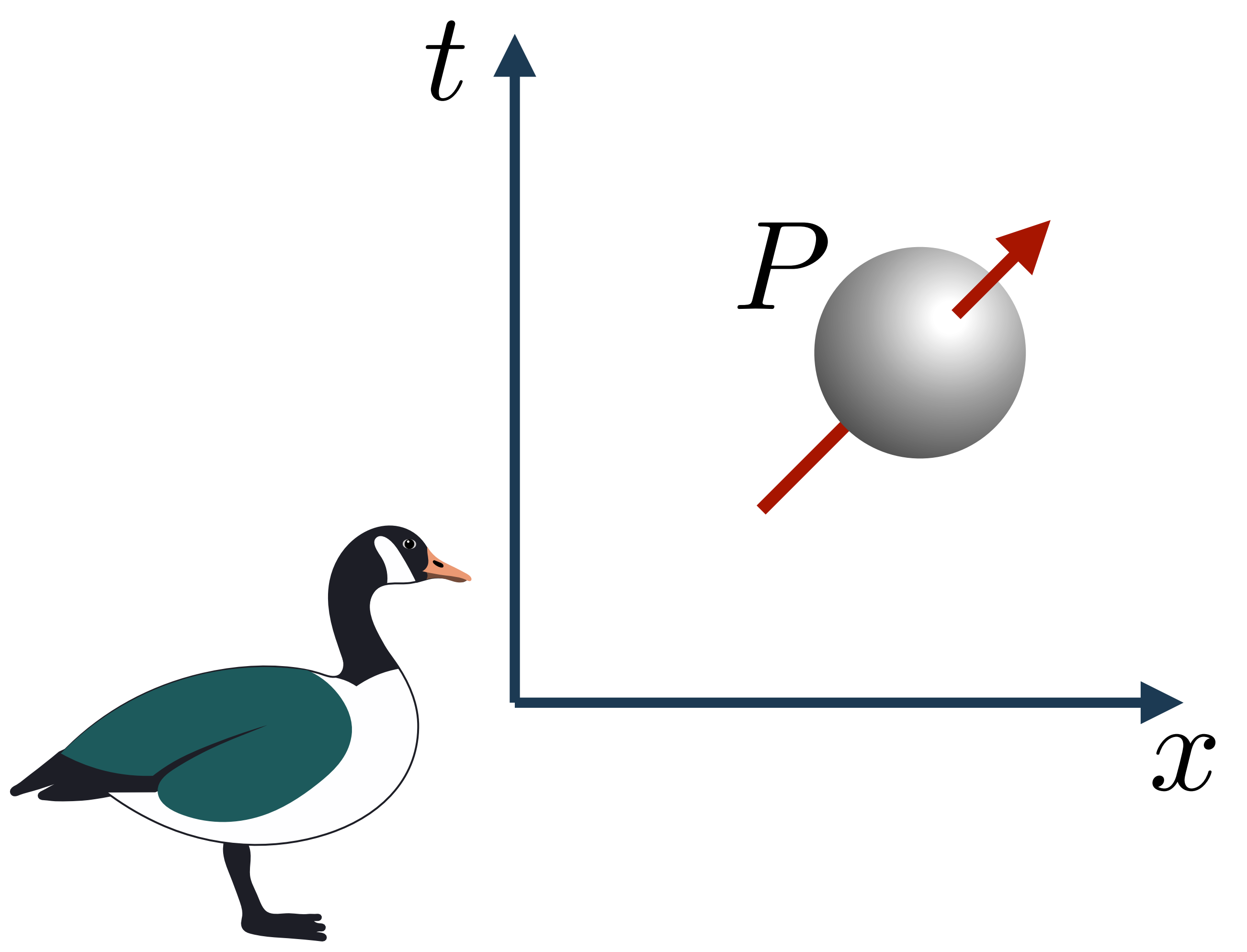

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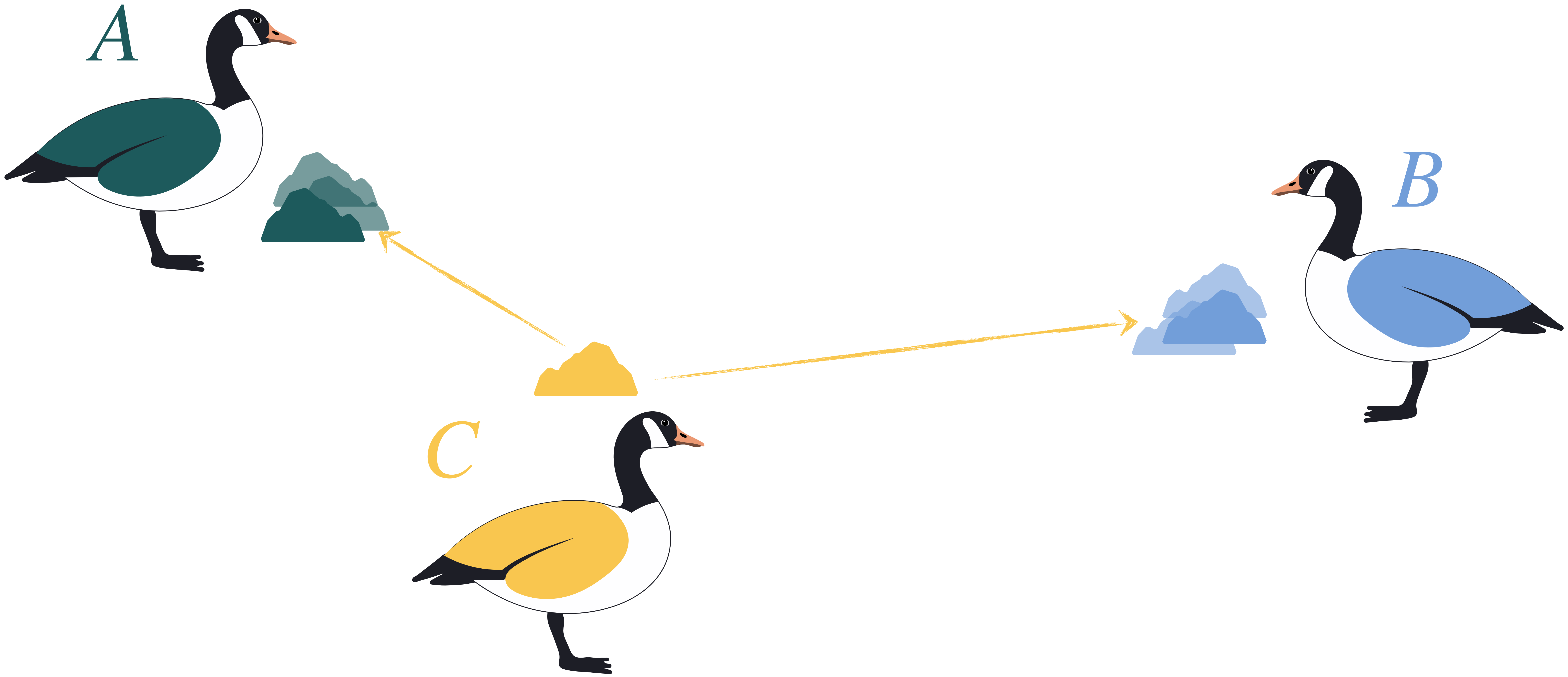
Symmetry

$$\hat{H}' = \hat{H}$$

QUANTUM REFERENCE FRAMES (QRFs)



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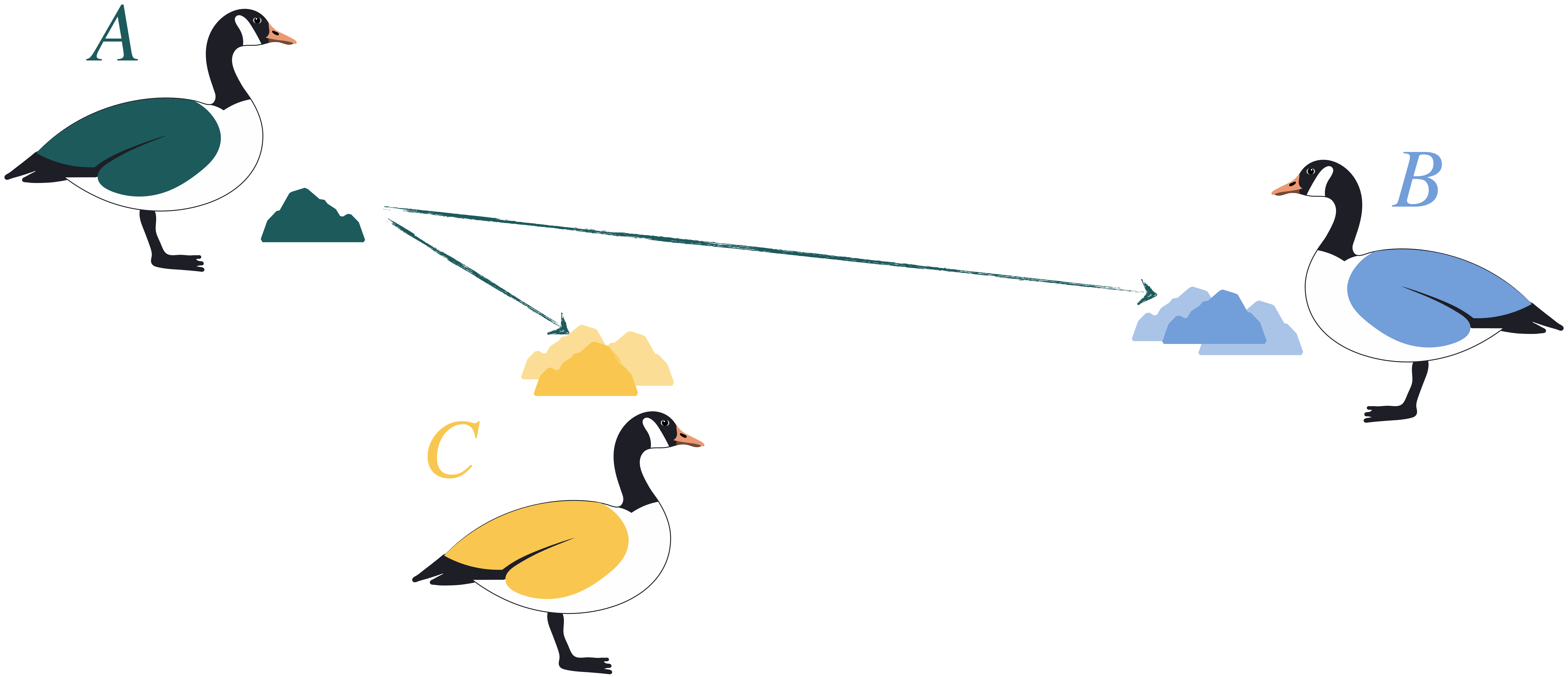
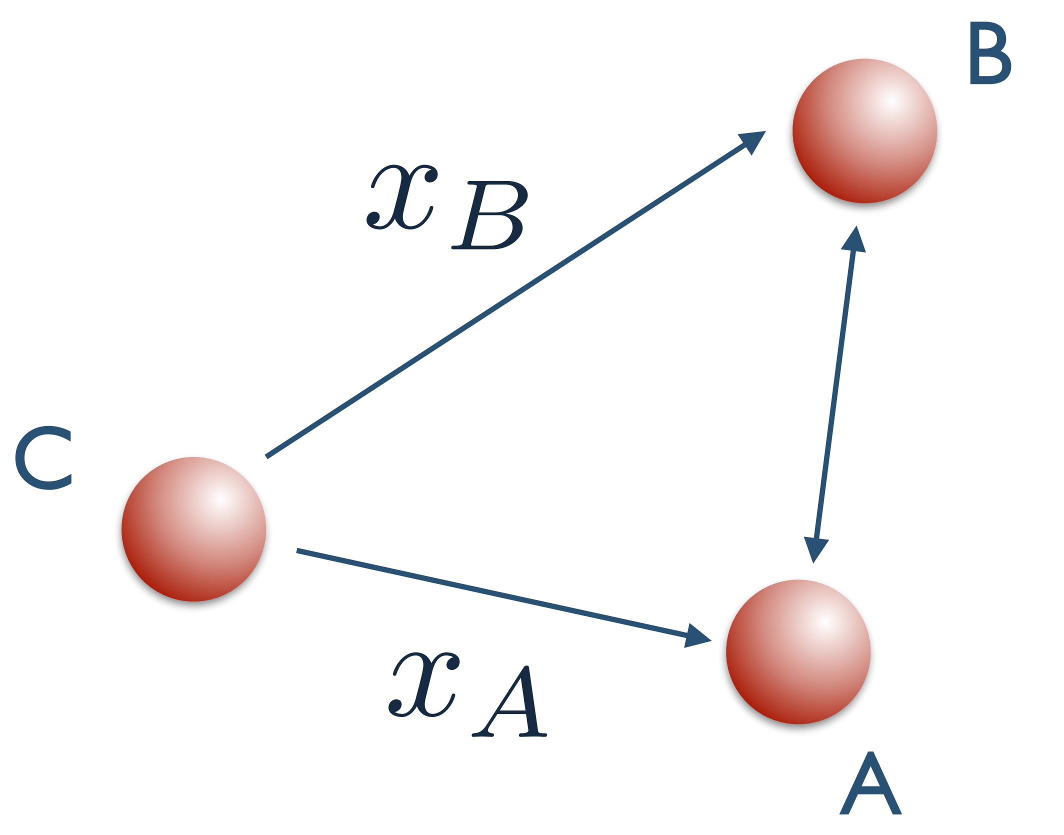


The simplest case: Transformation to relative coordinates

Giacomini, Castro-Ruiz, Brukner, Nat. Commun. (2019)

$$x_A \mapsto -q_C$$

$$x_B \mapsto q_B - q_C$$



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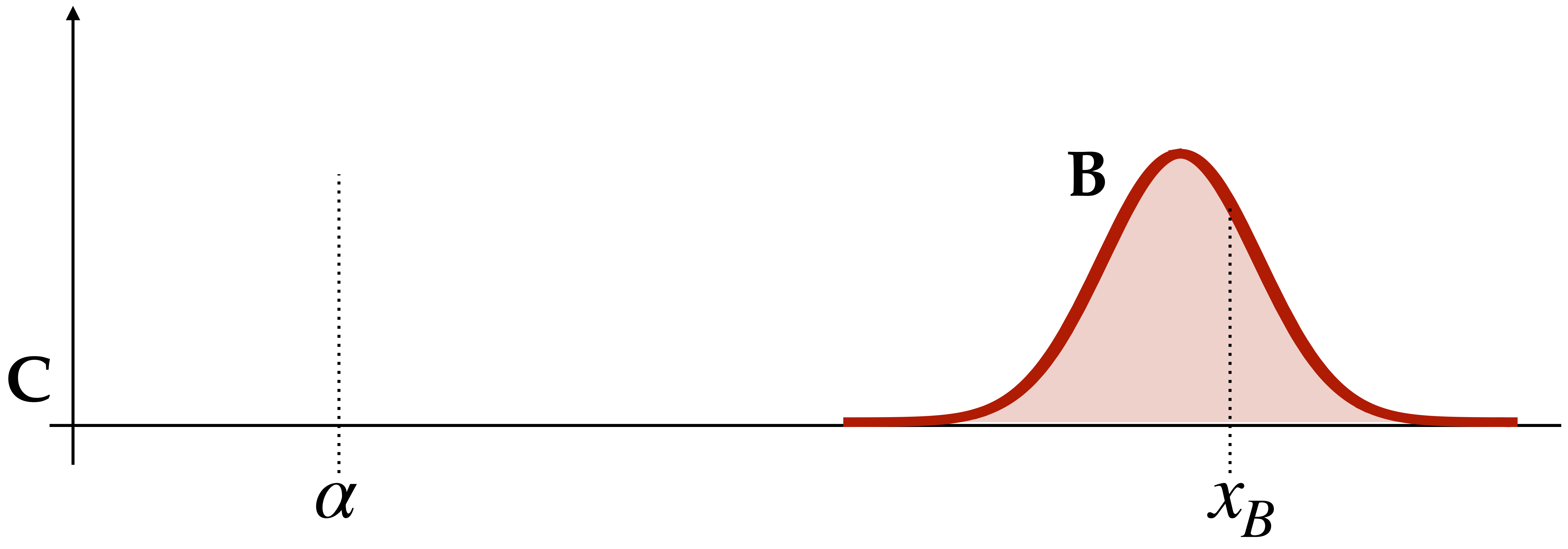

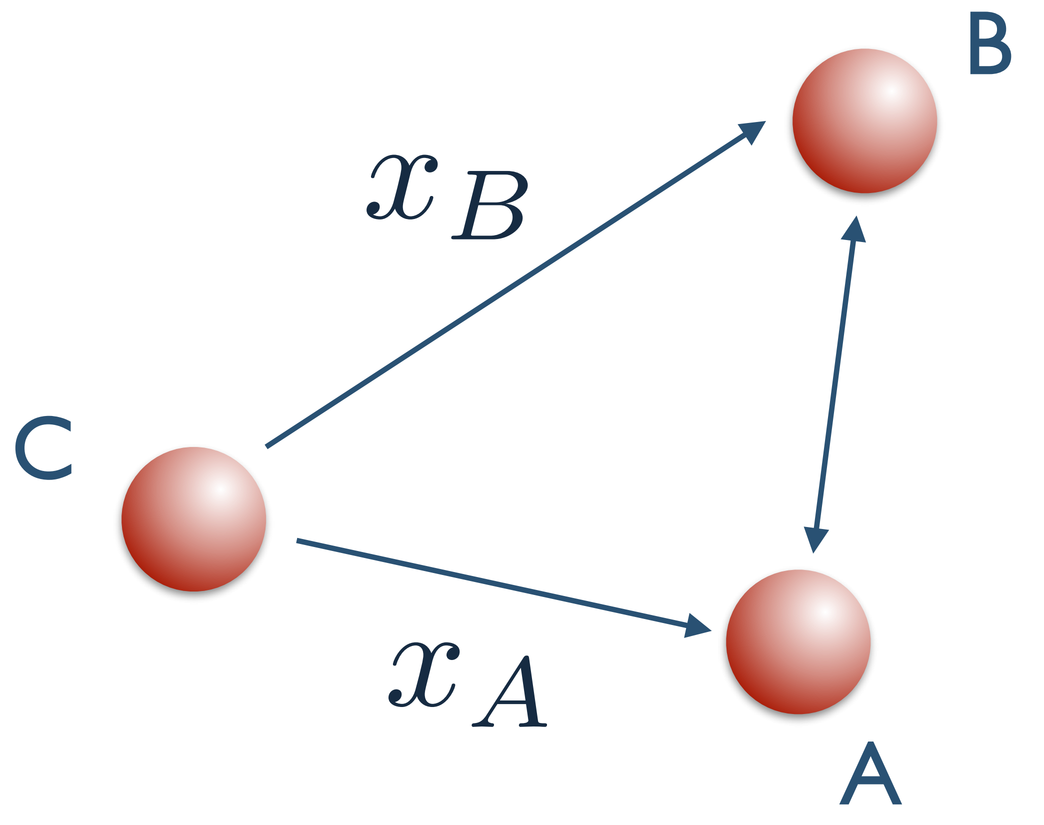

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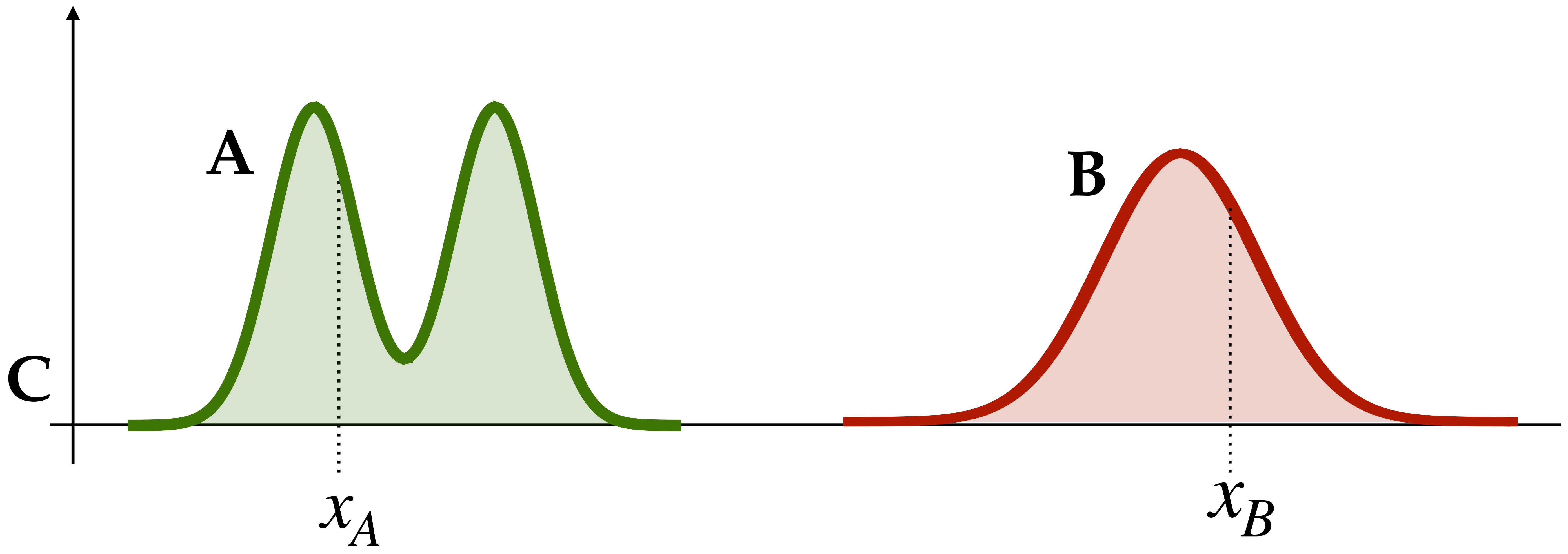

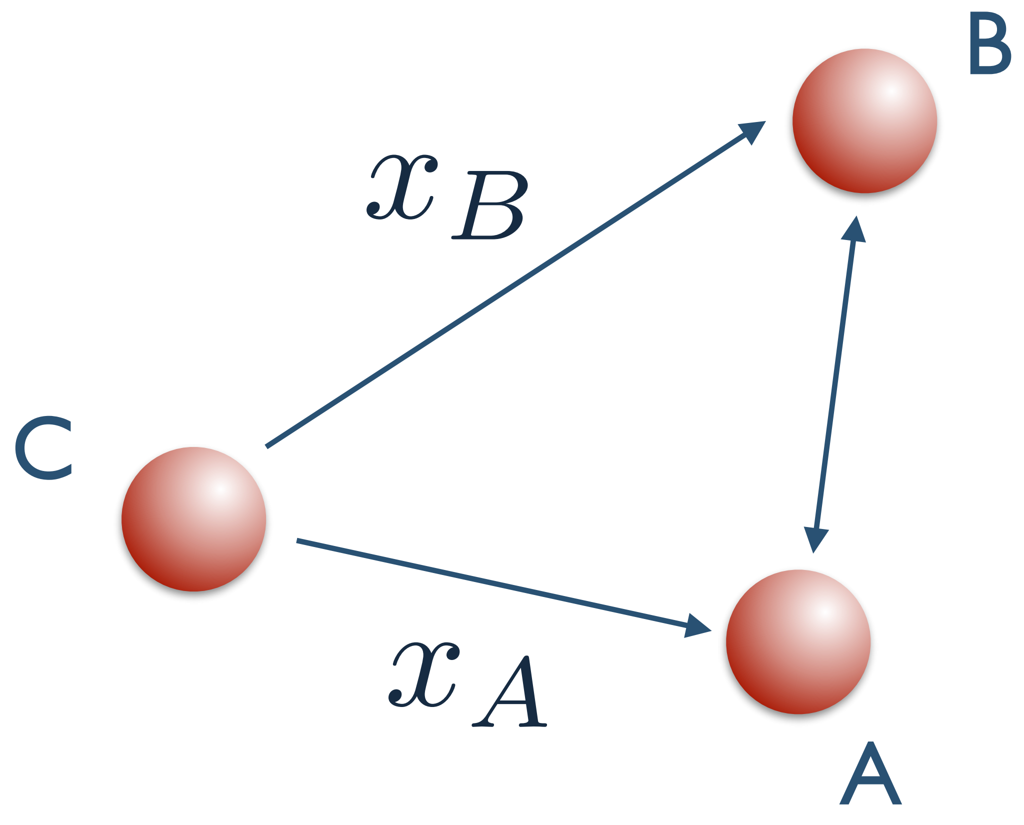

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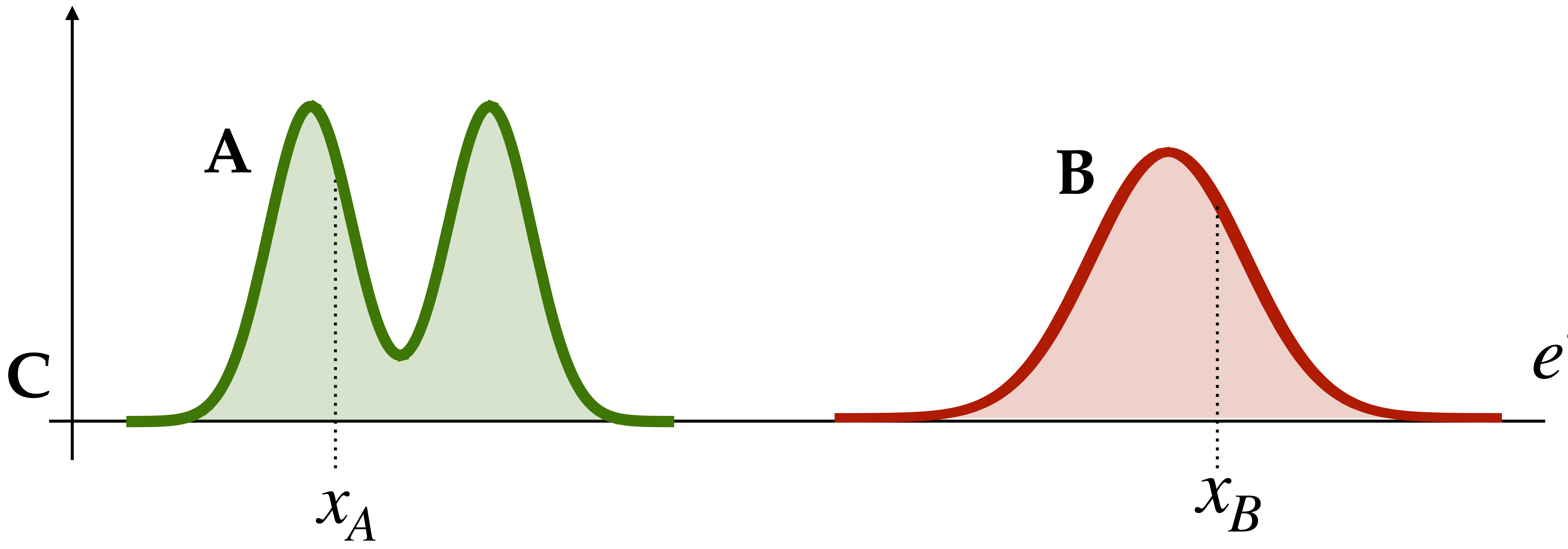
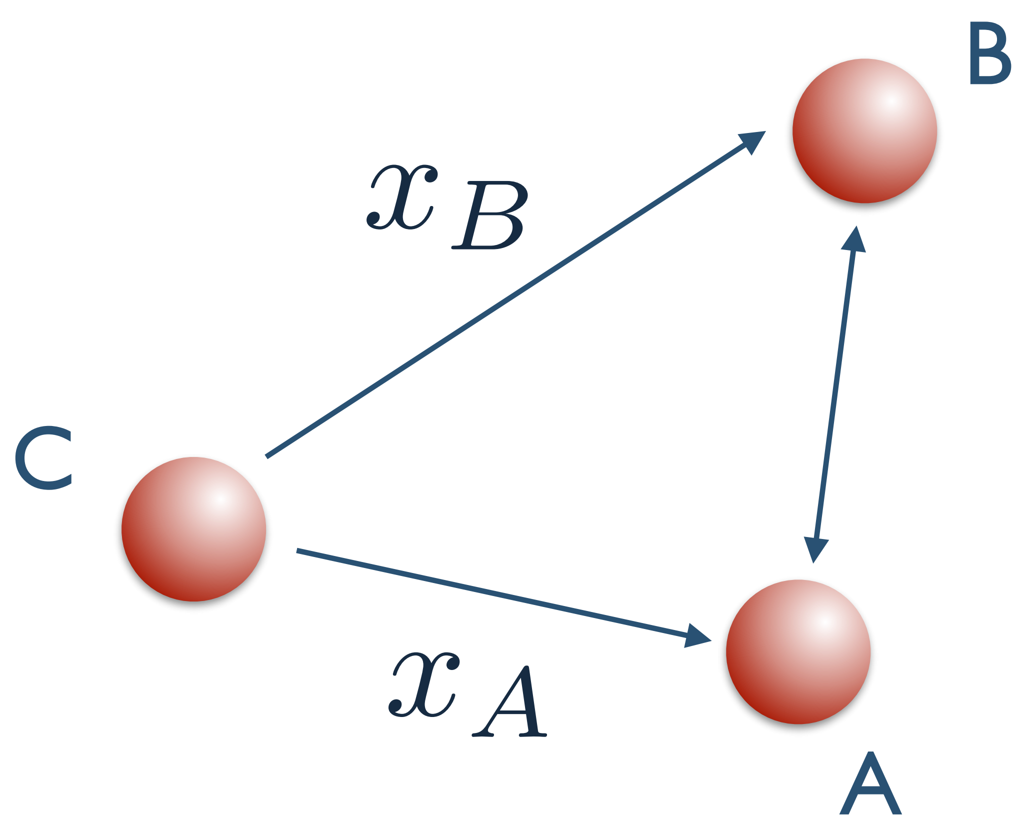


$$e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B} |\phi\rangle_A |\psi\rangle_B$$



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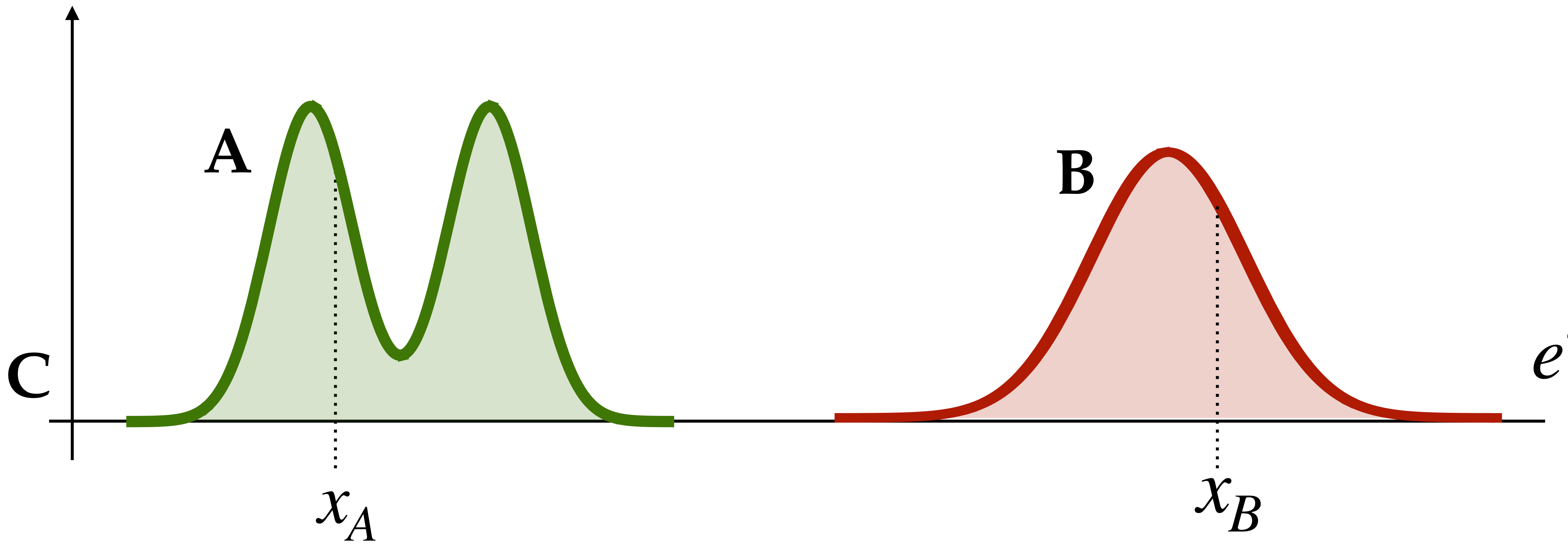

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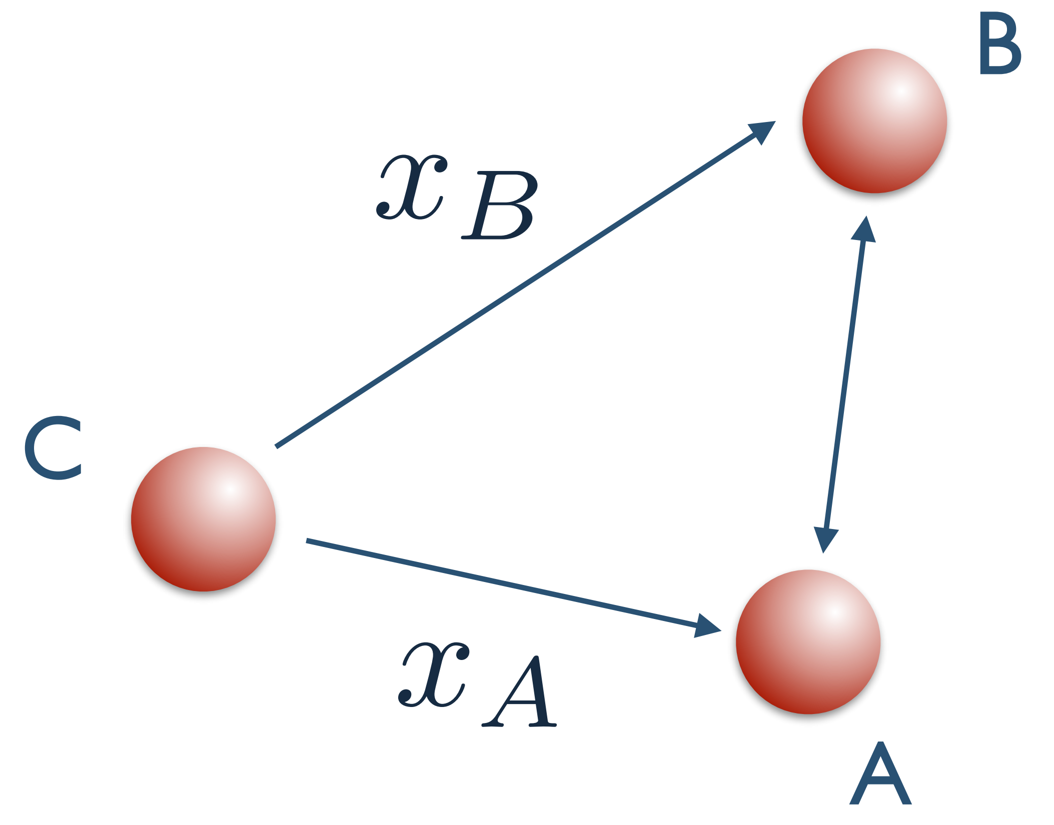

$$\hat{S}_x = \mathscr{P}_{AC} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}$$

$$\mathscr{P}_{AC} \hat{x}_A \mathscr{P}^{\dagger}_{AC} = -\hat{q}_C$$ **parity-swap operator**

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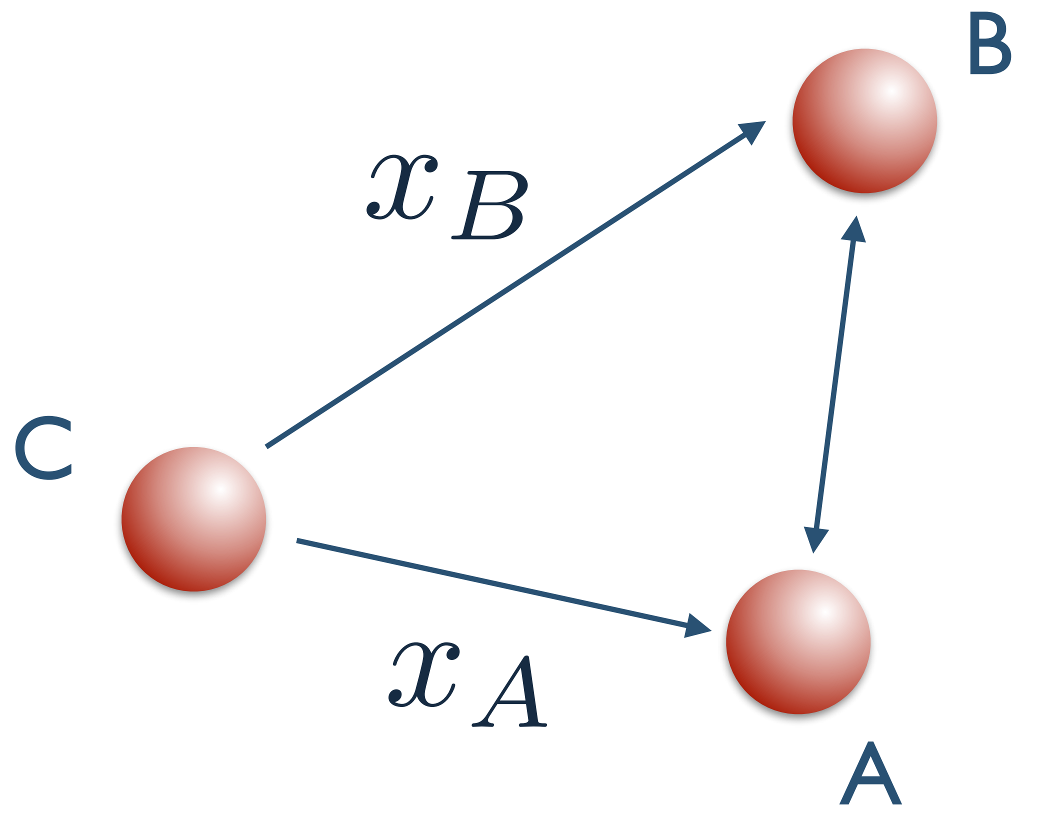

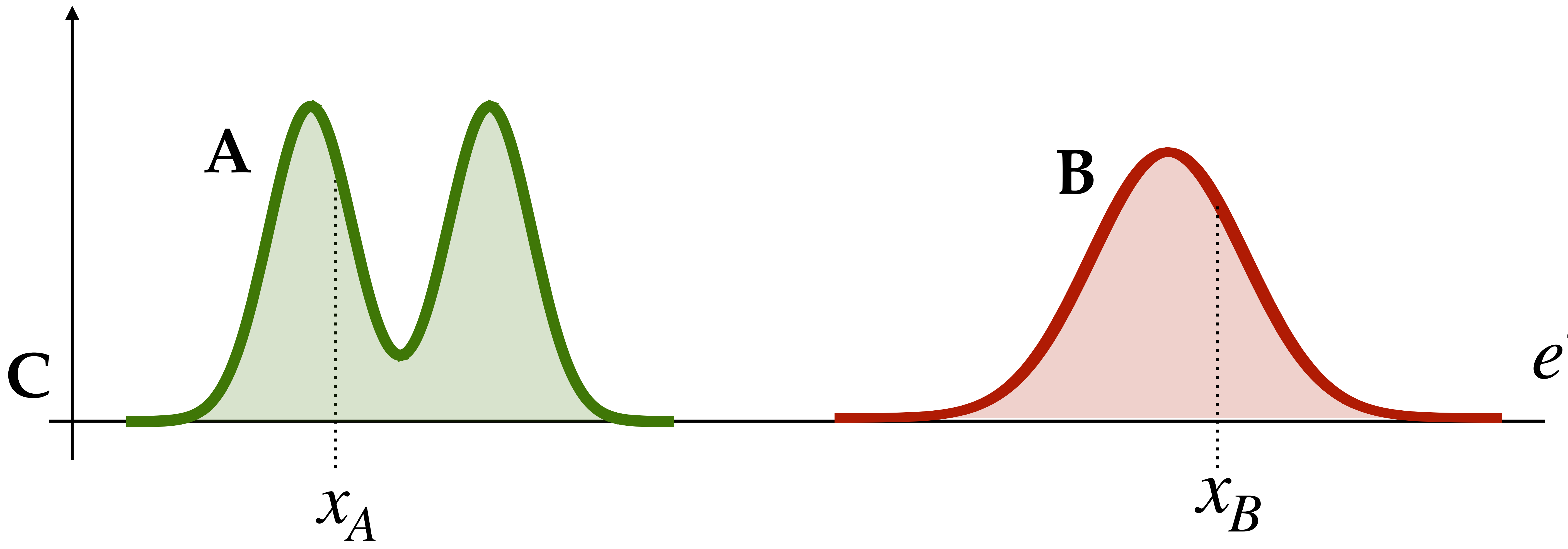

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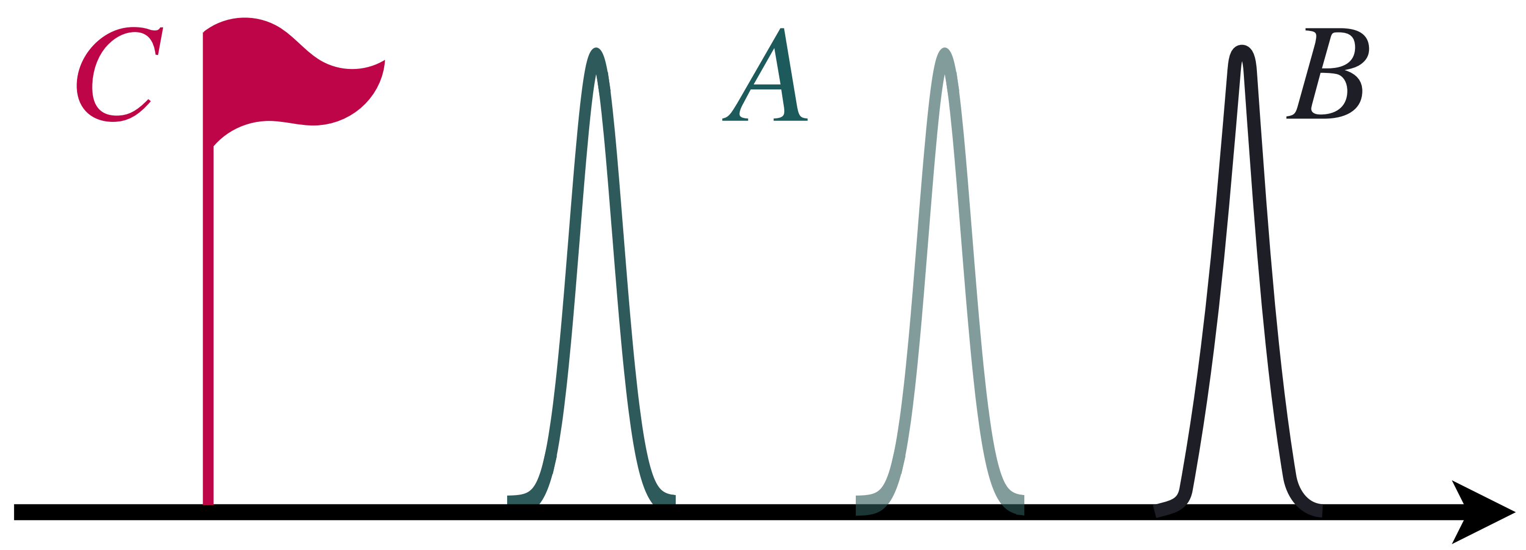
1. **Add Hilbert space of the QRF**
2. **Translate by a different amount for each position of A**

$$\mathscr{P}_{AC} \hat{x}_A \mathscr{P}^{\dagger}_{AC} = -\hat{q}_C$$ **parity-swap operator**

The simplest case: spatial translations in 1D

Giacomini, Castro-Ruiz, Brukner, Nat. Commun. (2019)

$$\rho_{BC}^{(A)} = \hat{S}_x \rho_{AB}^{(C)} \hat{S}_x^\dagger$$



$$\frac{1}{\sqrt{2}}(|x_1\rangle_A + |x_2\rangle_A)\,|x_0\rangle_B$$

Entanglement and superposition are QRF dependent

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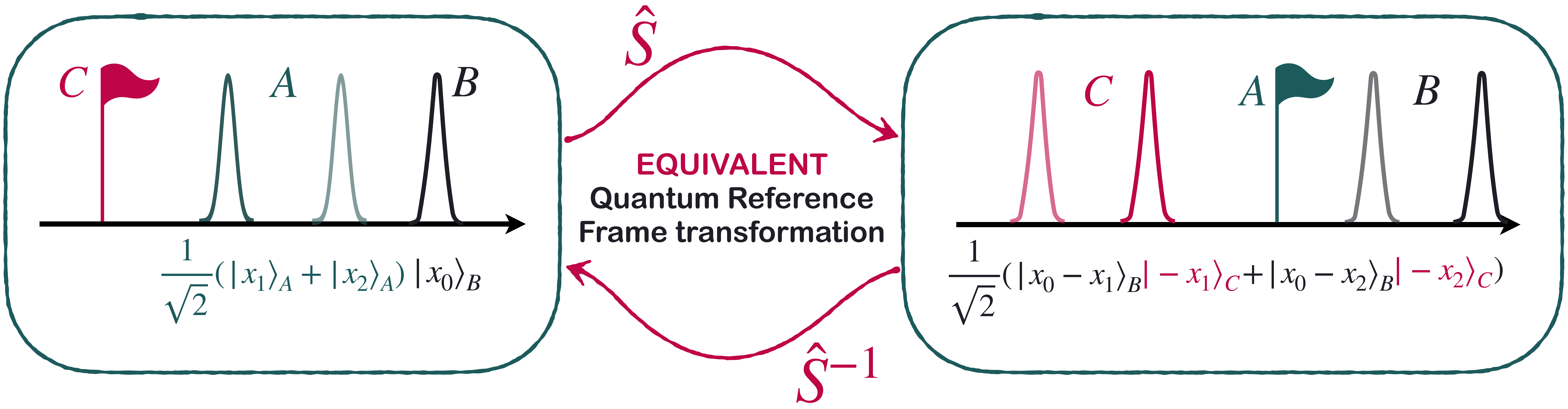

Entanglement and superposition are QRF dependent

EXTENDED COVARIANCE AND SYMMETRIES OF DYNAMICAL LAWS

A: new reference frame
B: quantum system
C: old reference frame

Giacomini, Castro-Ruiz, Brukner, Nat. Commun. (2019)

$$i\hbar \frac{d\hat{\rho}^{(C)}_{AB}}{dt} = [\hat{H}^{(C)}_{AB}, \hat{\rho}^{(C)}_{AB}]$$

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We define an extended symmetry transformation as:

$$\hat{S}\hat{H}^{(C)}_{AB}(\{m_i, \hat{x}_i, \hat{p}_i\}_{i=A,B})\hat{S}^\dagger + i\hbar \frac{d\hat{S}}{dt}\hat{S}^\dagger = \hat{H}^{(A)}_{BC}(\{m_i, \hat{q}_i, \hat{\pi}_i\}_{i=B,C})$$

Valid for:
- Superposition of translations
- Superposition of Galilean boosts

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Inertial QRF transformations form a group!

Ballesteros, Giacomini, Gubitosi, Quantum (2021)

1D model

$$\hat{H} = \sum_i \frac{\hat{p}_i^2}{2m_i} + \sum_{i<j} V(\hat{x}_i - \hat{x}_j) + \lambda \hat{P} \qquad \left(\hat{P} = \sum_i \hat{p}_i \approx 0 \right)$$

Vanrietvelde, Höhn, Giacomini, Castro Ruiz, Quantum (2020)
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RELATIONALISM IN QRF TRANSFORMATIONS

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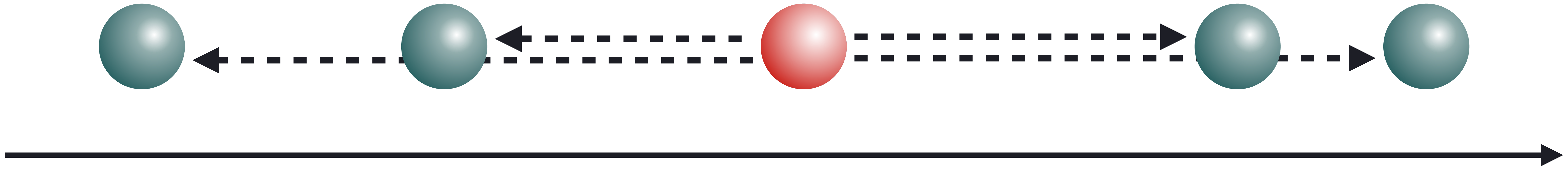

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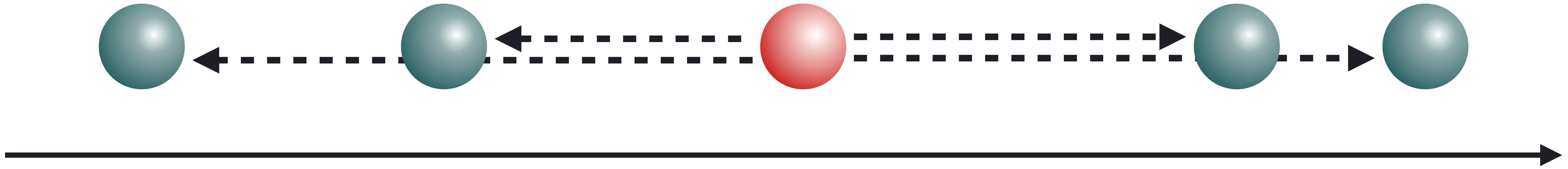

$$\hat{P} \, | \, \Psi \rangle_{ABC}^{ph} = 0$$

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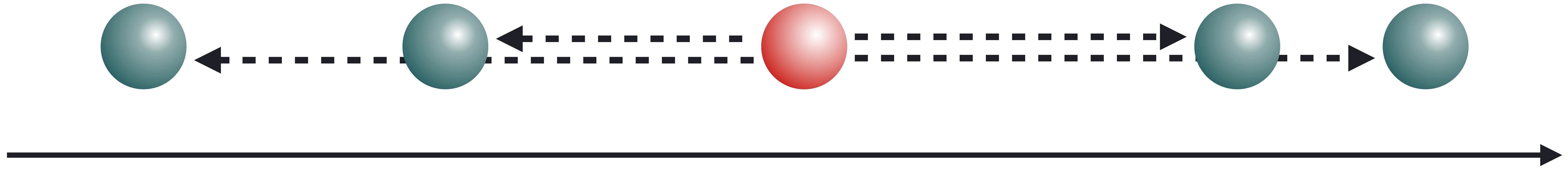

$$\hat{P} |\Psi\rangle^{ph}_{ABC} = 0 \longrightarrow |\Psi\rangle^{ph}_{ABC} = \frac{1}{2\pi} \int d\alpha e^{\frac{i}{\hbar}\alpha \hat{P}} |\phi\rangle_{ABC}$$

State with zero total momentum
= coherent group averaging

Vanrietvelde, Höhn, Giacomini, Castro Ruiz, Quantum (2020)
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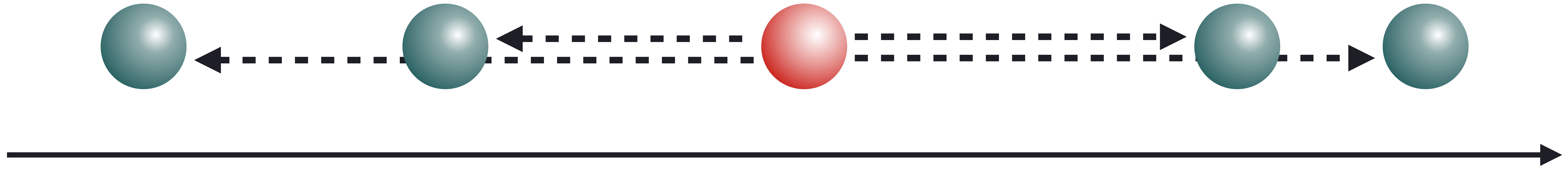

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State with zero total momentum = coherent group averaging

$$\mathcal{T}_{A|BC} = e^{\frac{i}{\hbar}\hat{x}_A(\hat{p}_B + \hat{p}_C)} \qquad \mathcal{T}_{A|BC}\,|\Psi\rangle^{ph}_{ABC} = |p_A = 0\rangle_A\,|\psi\rangle^{(A)}_{BC}$$

Map constraint on QRF Hilbert space

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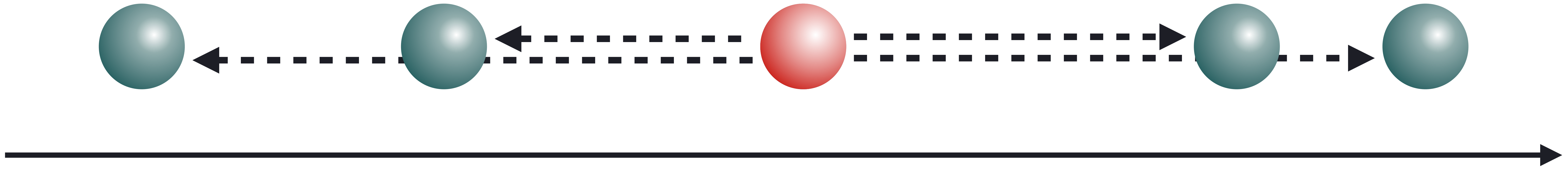
$$\hat{H} = \sum_i \frac{\hat{p}_i^2}{2m_i} + \sum_{i<j} V(\hat{x}_i - \hat{x}_j) + \lambda \hat{P} \qquad \left(\hat{P} = \sum_i \hat{p}_i \approx 0 \right)$$



$$\hat{P}\,|\Psi\rangle^{ph}_{ABC} = 0 \;\longrightarrow\; |\Psi\rangle^{ph}_{ABC} = \frac{1}{2\pi}\int d\alpha\, e^{\frac{i}{\hbar}\alpha\hat{P}}\,|\phi\rangle_{ABC}$$

State with zero total momentum = coherent group averaging

$$\mathcal{T}_{A|BC} = e^{\frac{i}{\hbar}\hat{x}_A(\hat{p}_B+\hat{p}_C)}$$

$$\mathcal{T}_{A|BC}\,|\Psi\rangle^{ph}_{ABC} = |p_A = 0\rangle_A\,|\psi\rangle^{(A)}_{BC}$$

Map constraint on QRF Hilbert space

$$|\psi\rangle^{(A)}_{BC} = \int dp'_A \langle p'|\,\mathcal{T}_{A|BC}\,|\Psi\rangle^{ph}_{ABC}$$

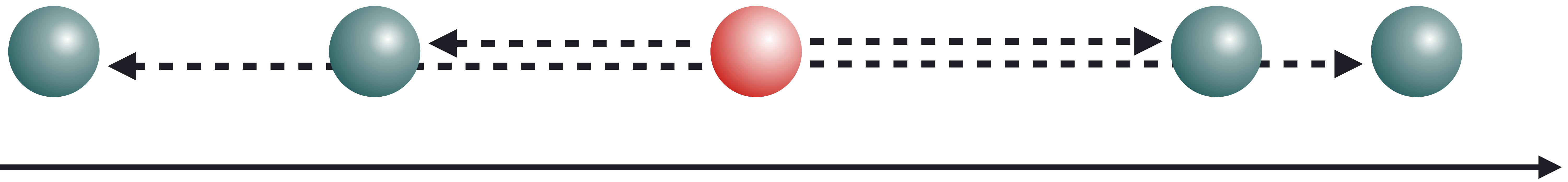
Reduced state in the relational variables

Vanrietvelde, Höhn, Giacomini, Castro Ruiz, Quantum (2020)
Vanrietvelde, Höhn, Giacomini, accepted in Quantum

RELATIONALISM IN QRF TRANSFORMATIONS

1D model

$$\hat{H} = \sum_i \frac{\hat{p}_i^2}{2m_i} + \sum_{i<j} V(\hat{x}_i - \hat{x}_j) + \lambda \hat{P} \qquad \left(\hat{P} = \sum_i \hat{p}_i \approx 0 \right)$$



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Reduced state in the relational variables

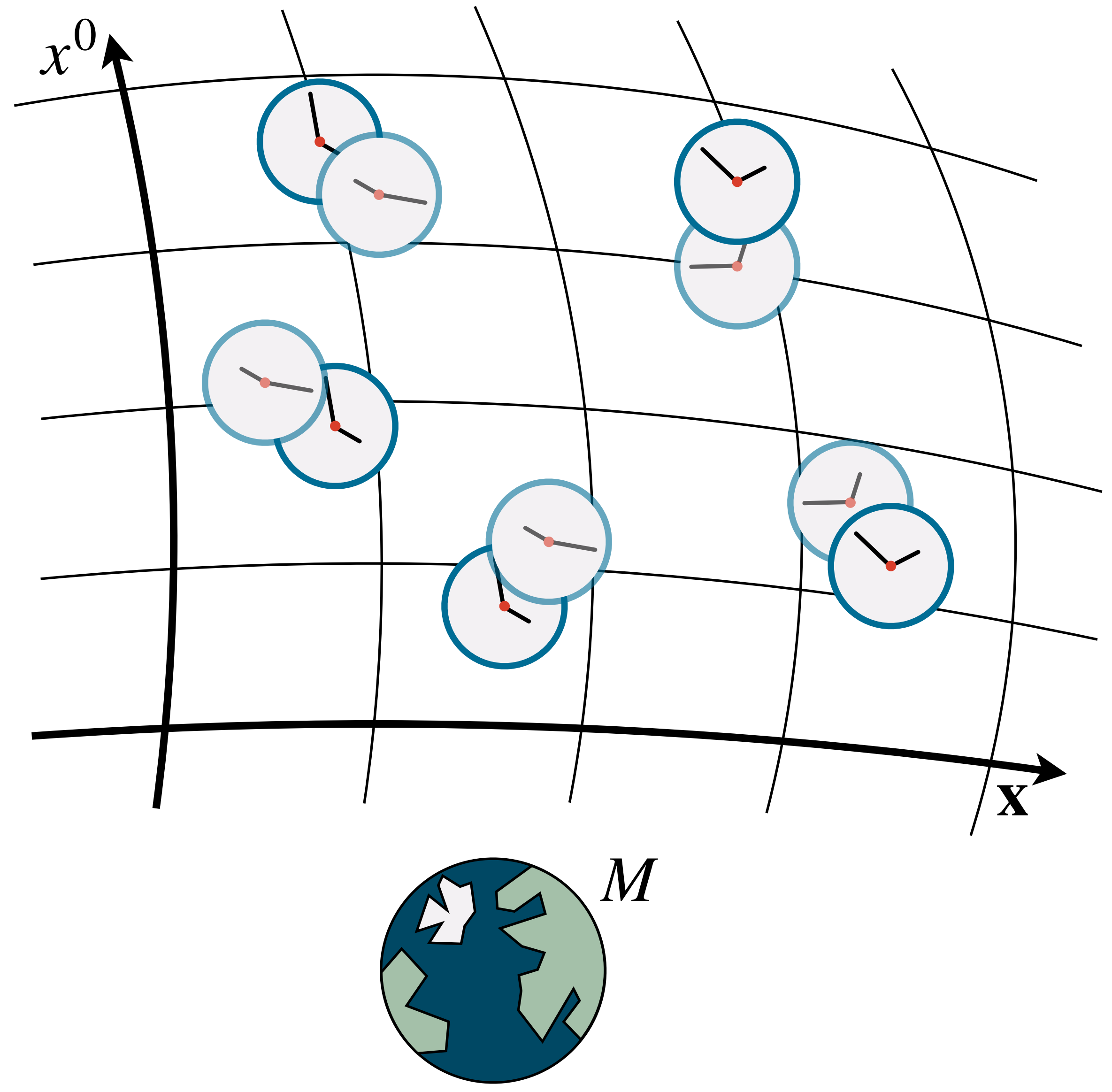
$$\hat{S}_x = \mathcal{P}_{AC} e^{\frac{i}{\hbar}\hat{x}_A\hat{p}_B}$$

Remember?

Vanrietvelde, Höhn, Giacomini, Castro Ruiz, Quantum (2020)
Vanrietvelde, Höhn, Giacomini, accepted in Quantum

QRF IN SPACETIME

Quantum clocks: external and internal d.o.f.

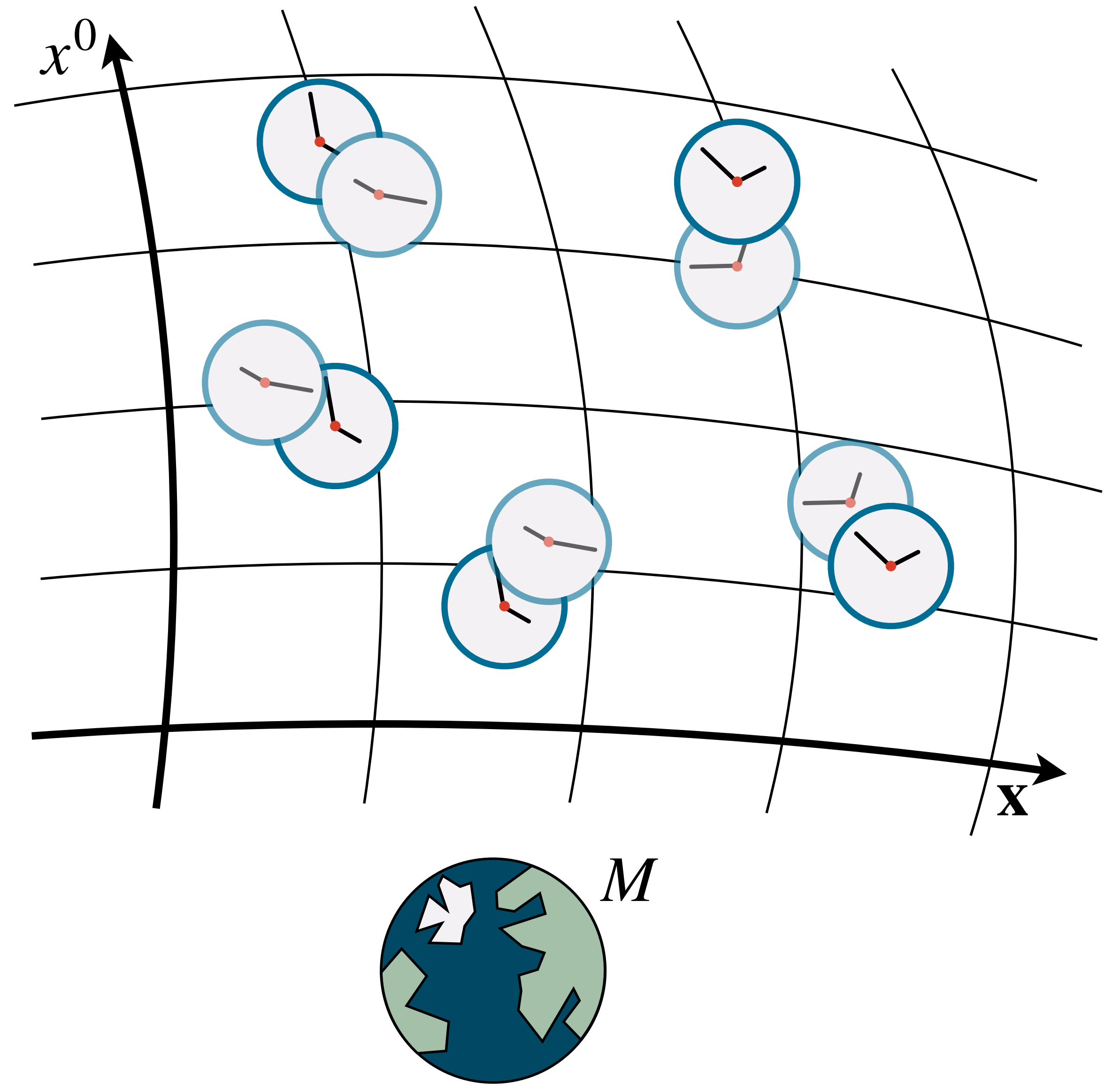


Giacomini, Quantum (2021)

Quantum clocks: external and internal d.o.f.

$$\hat{C}_i = \sqrt{g^{00}(\hat{x}_i - \hat{x}_M)}\hat{p}^i_0 - \frac{\hat{p}_i^2}{2m_i}$$

Dispersion relation of the single clock



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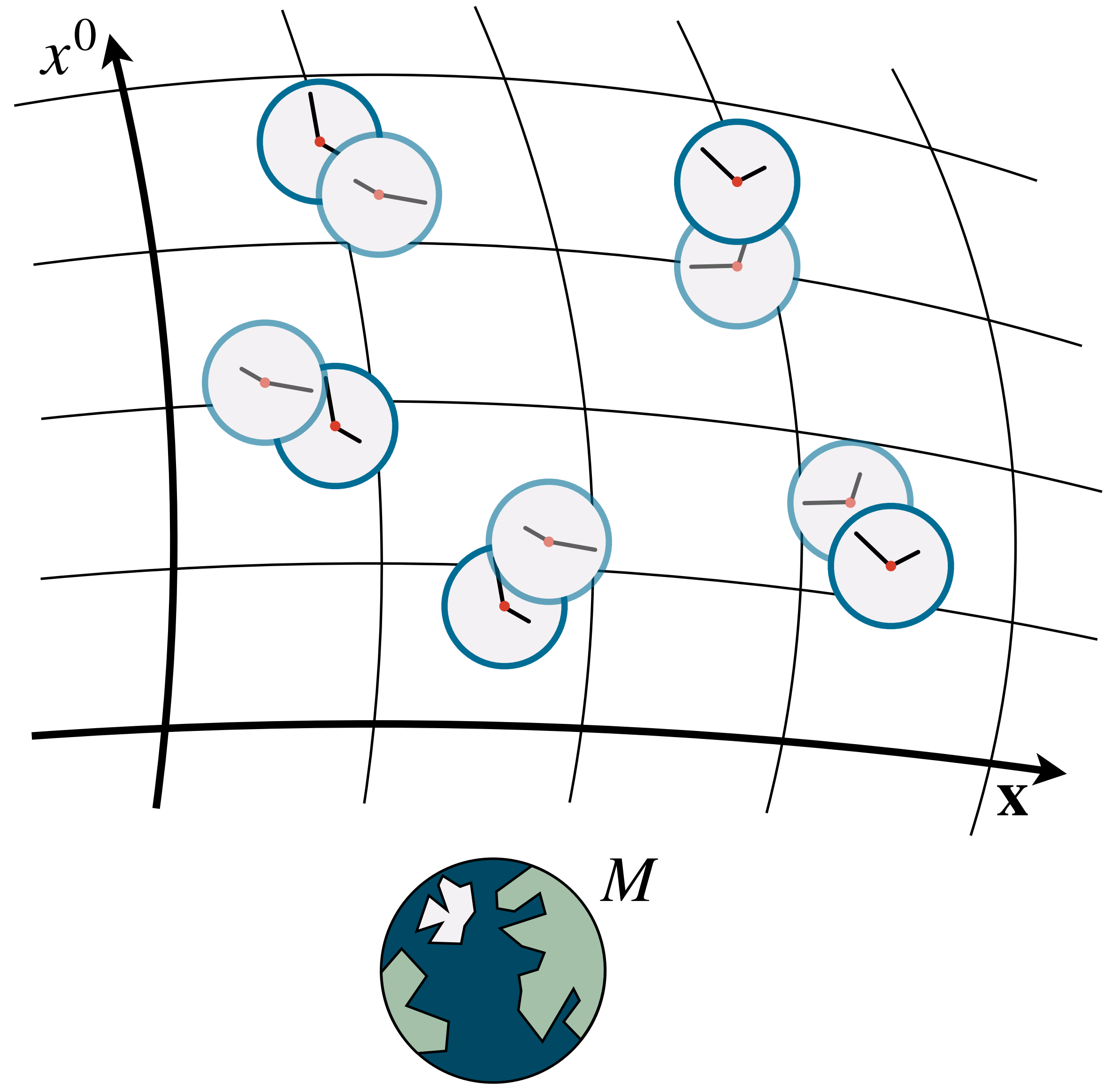

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$$\begin{cases} \hat{f}^0 = \sum_i \left(\hat{p}_0^i + \sqrt{g_{00}(\hat{x}_i - \hat{x}_M)} \frac{\hat{H}_i}{c} \right) + \hat{p}_0^M \\ \hat{f} = \sum_i \hat{p}^i + \hat{p}^M \end{cases}$$

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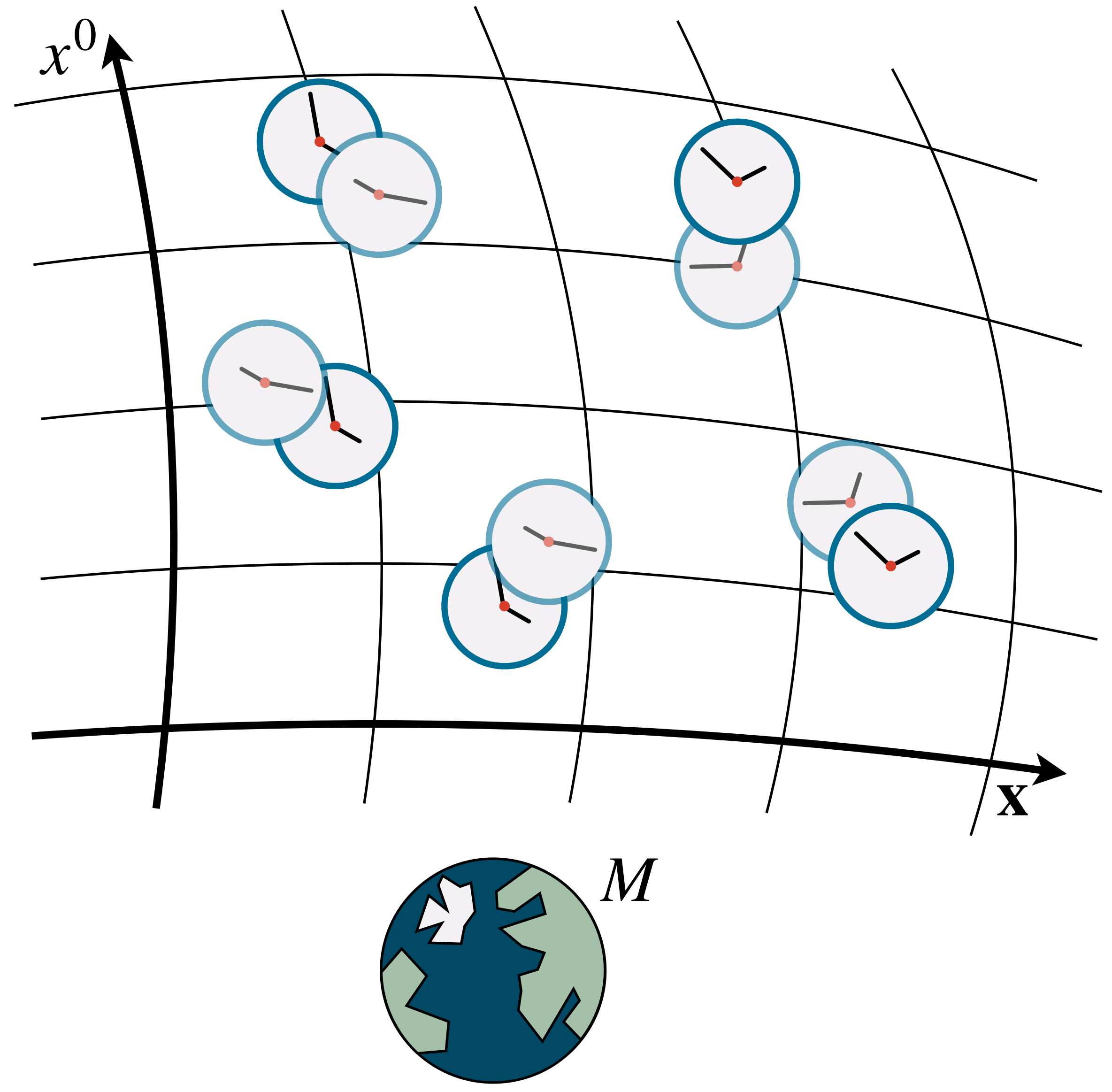
→ **See Page-Wootters**



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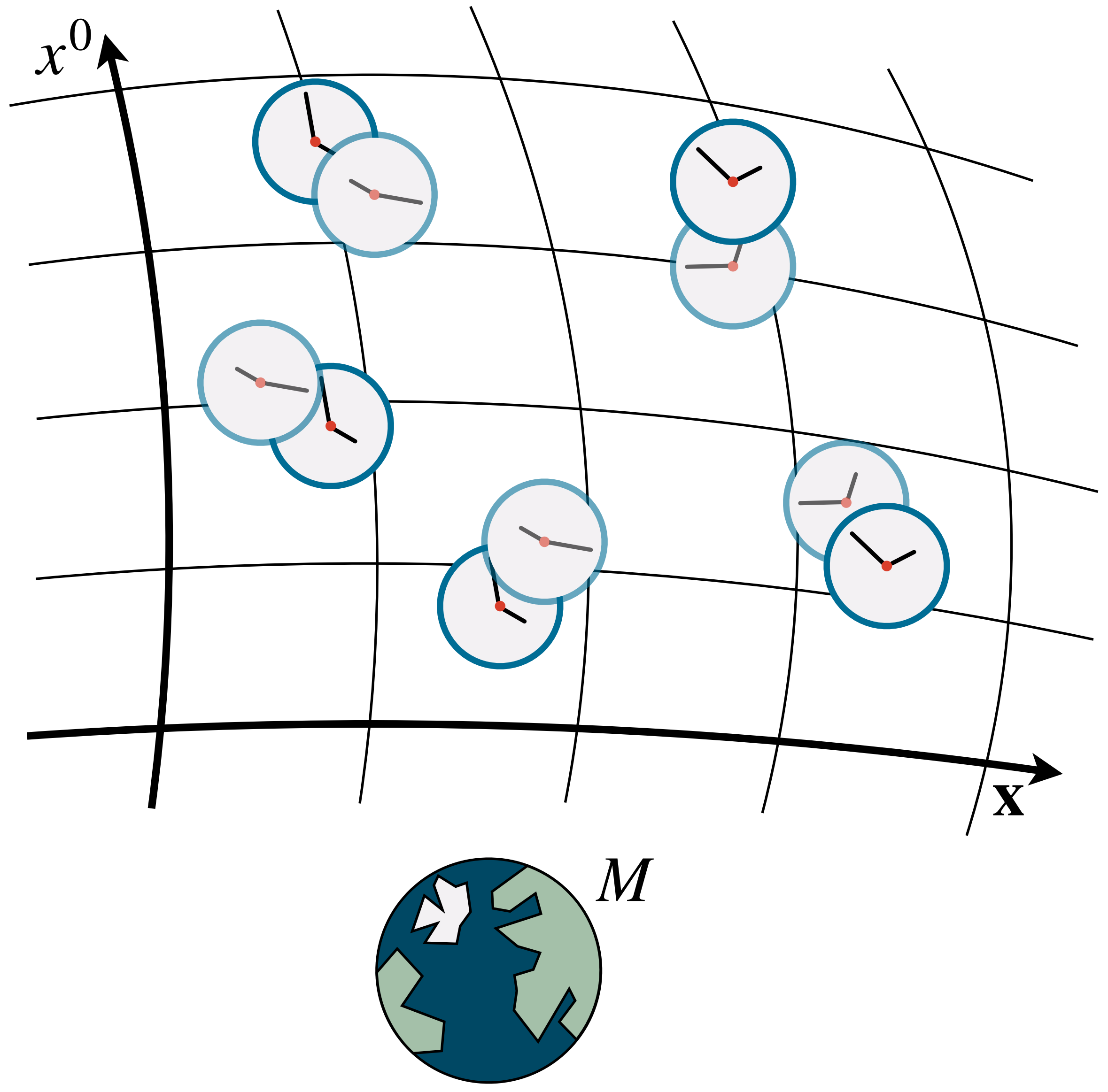
→ **See relational construction of**

Vanrietvelde, Höhn, Giacomini, Castro Ruiz, Quantum (2020)



Giacomini, Quantum (2021)

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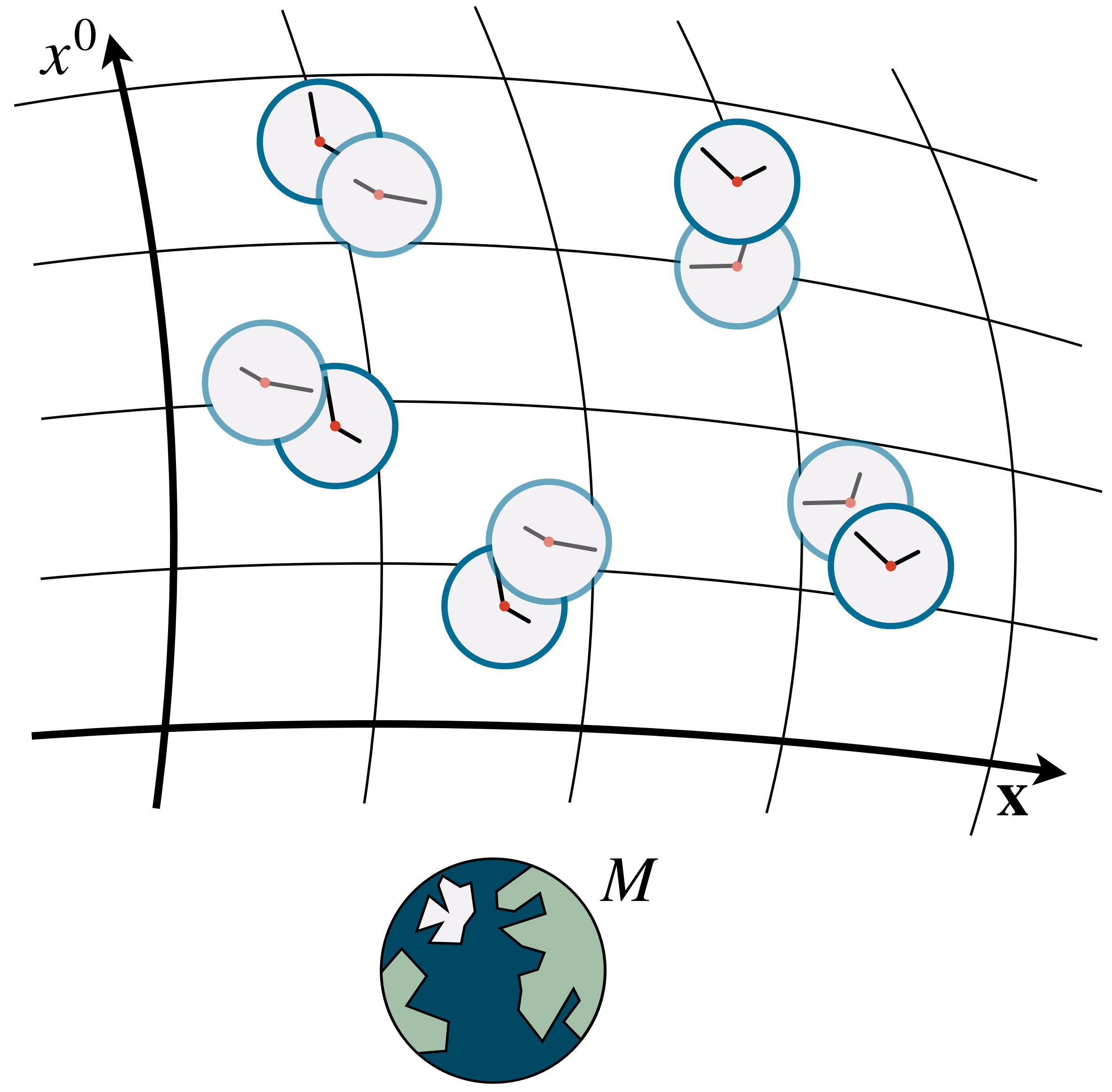

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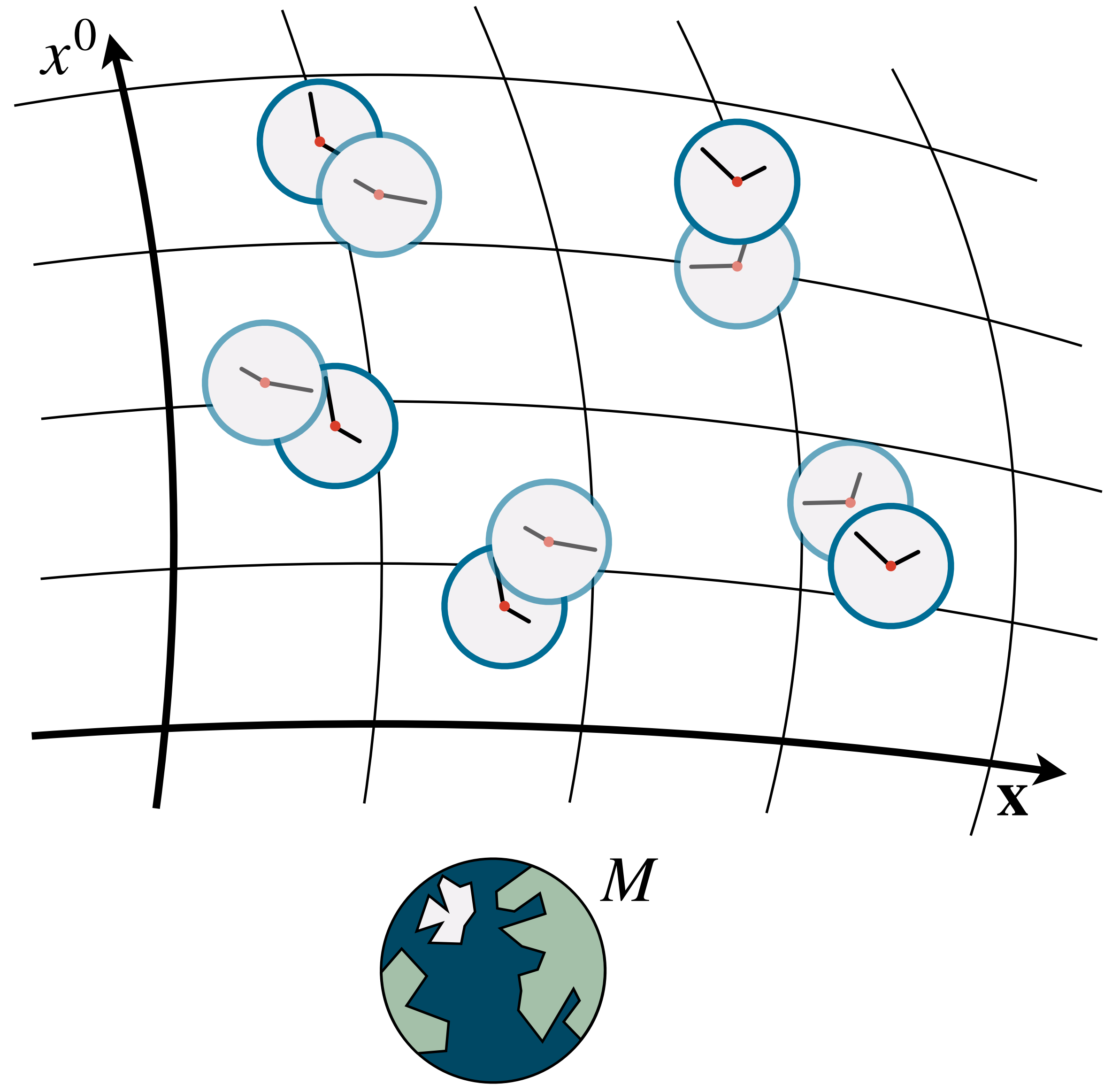
→ **See relational construction of**

Vanrietvelde, Höhn, Giacomini, Castro Ruiz, Quantum (2020)

Conservation of the 4-momentum

Giacomini, Quantum (2021)

Quantum clocks: external and internal d.o.f.



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$\hat{f}^0$ → See Page-Wootters

$\hat{f}$ → See relational construction of Vanrietvelde, Höhn, Giacomini, Castro Ruiz, Quantum (2020)

Conservation of the 4-momentum

$$i\hbar \frac{d|\psi(\tau_i)\rangle^{(i)}}{d\tau_i} = \sum_{j \neq i} \left[\hat{H}_j^{ext} + \Delta(\hat{x}_j, \hat{x}_M)\hat{H}_j \right] |\psi(\tau_i)\rangle^{(i)}$$

Giacomini, Quantum (2021)

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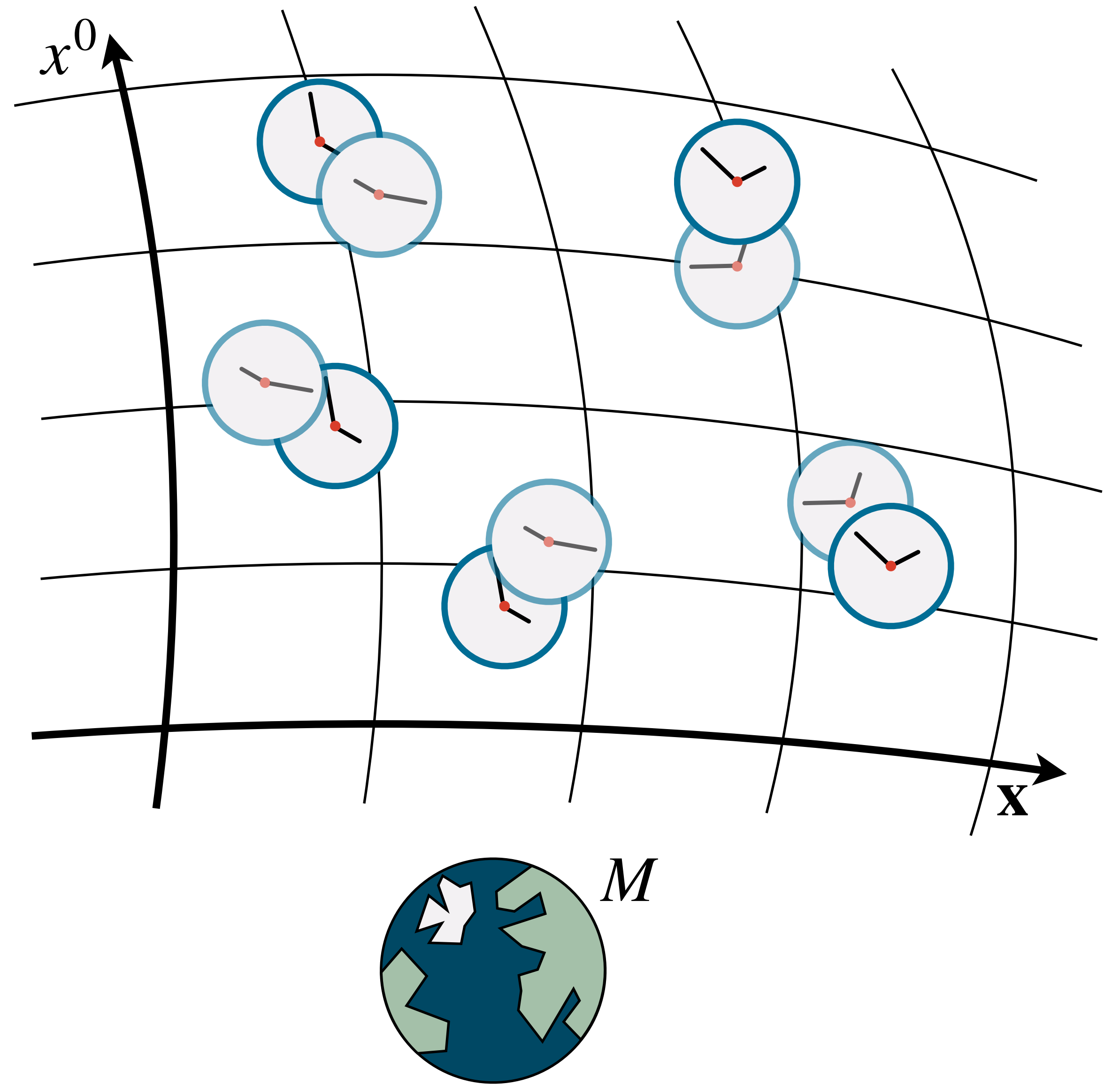

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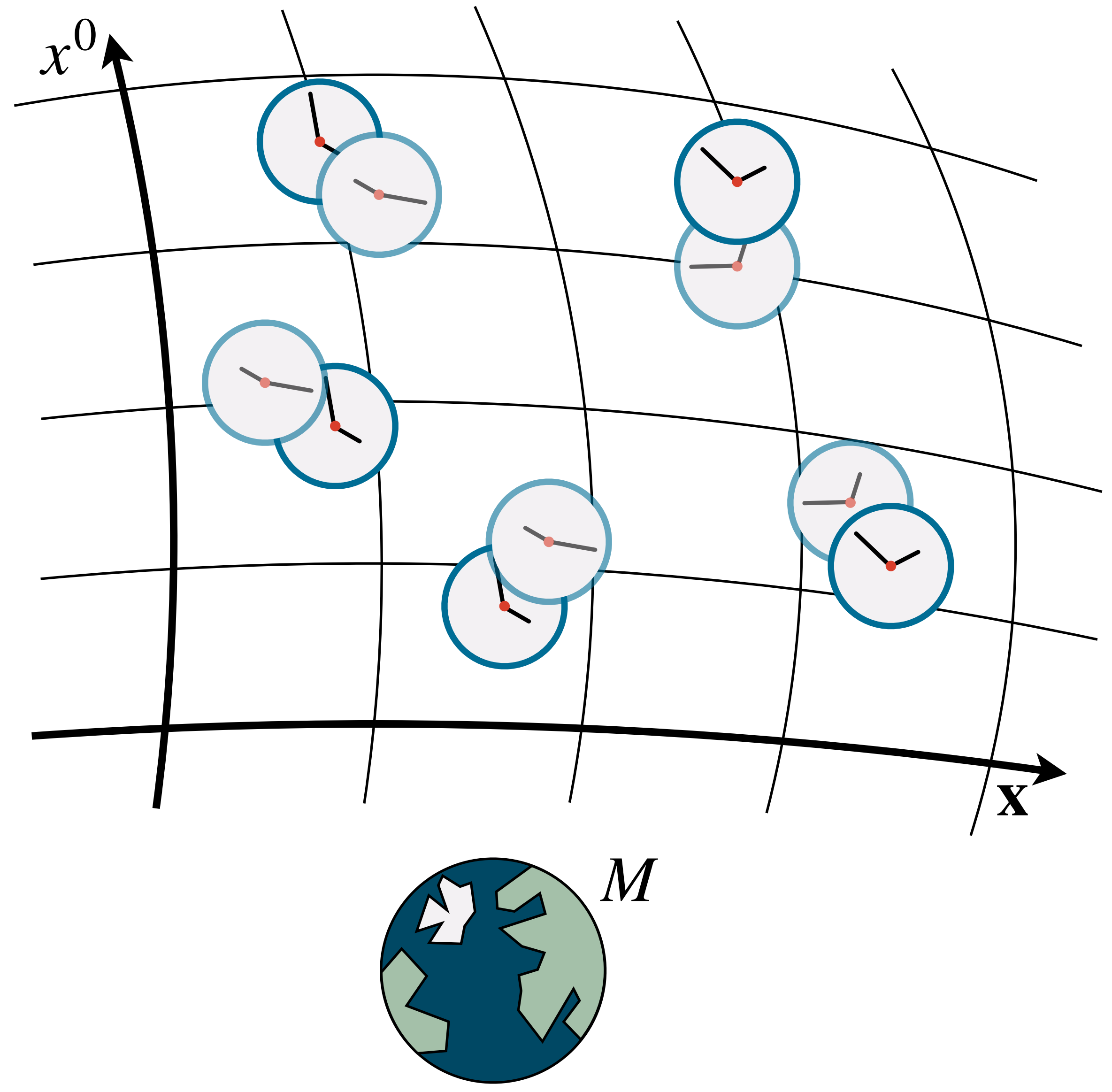
Proper time of the i-th clock

Giacomini, Quantum (2021)

QRF IN SPACETIME

Quantum clocks: external and internal d.o.f.



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Proper time of the i-th clock

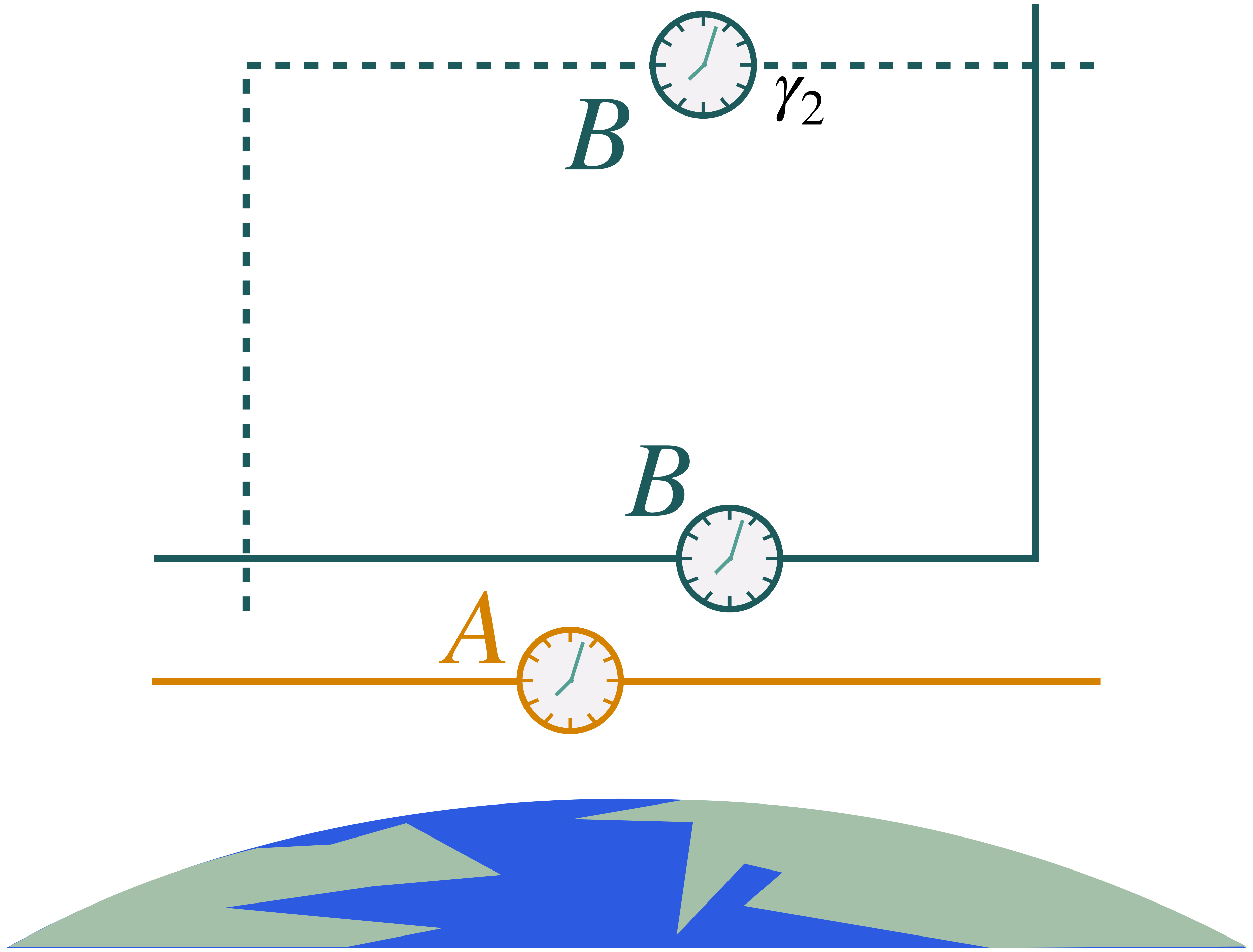
Gravitational time dilation $\Delta(\hat{x}_j, \hat{x}_M) = 1 + \frac{\Phi(\hat{x}_j - \hat{x}_M) - \Phi(\hat{x}_M)}{c^2}$

Giacomini, Quantum (2021)

QRF IN TIME - RELATIVE LOCALISATION OF EVENTS

Giacomini, Quantum (2021)
Cepollaro, Giacomini, 2112.03303 (2021)

QRF IN TIME - RELATIVE LOCALISATION OF EVENTS



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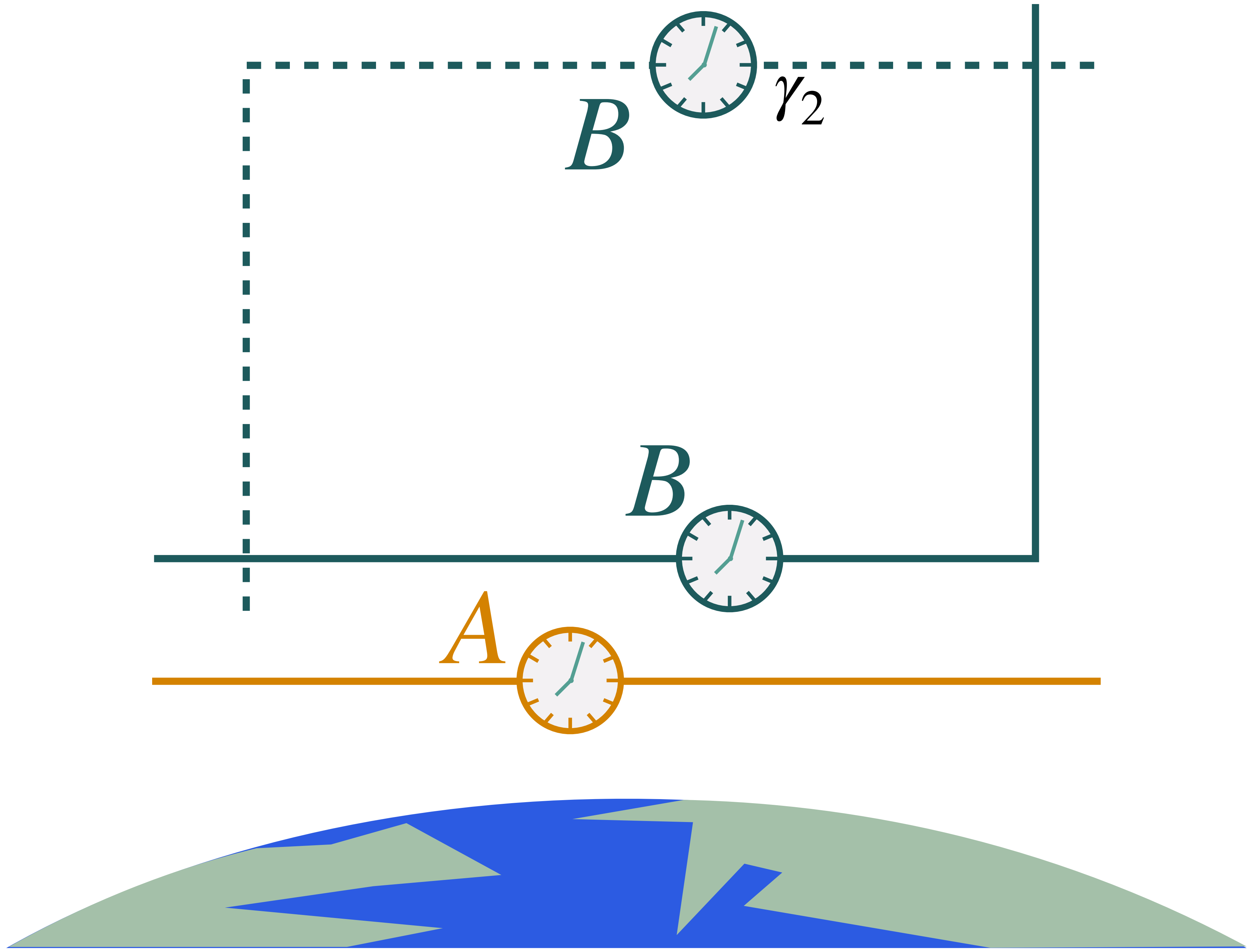

B in a superposition from A
A in a superposition from B

Giacomini, Quantum (2021)
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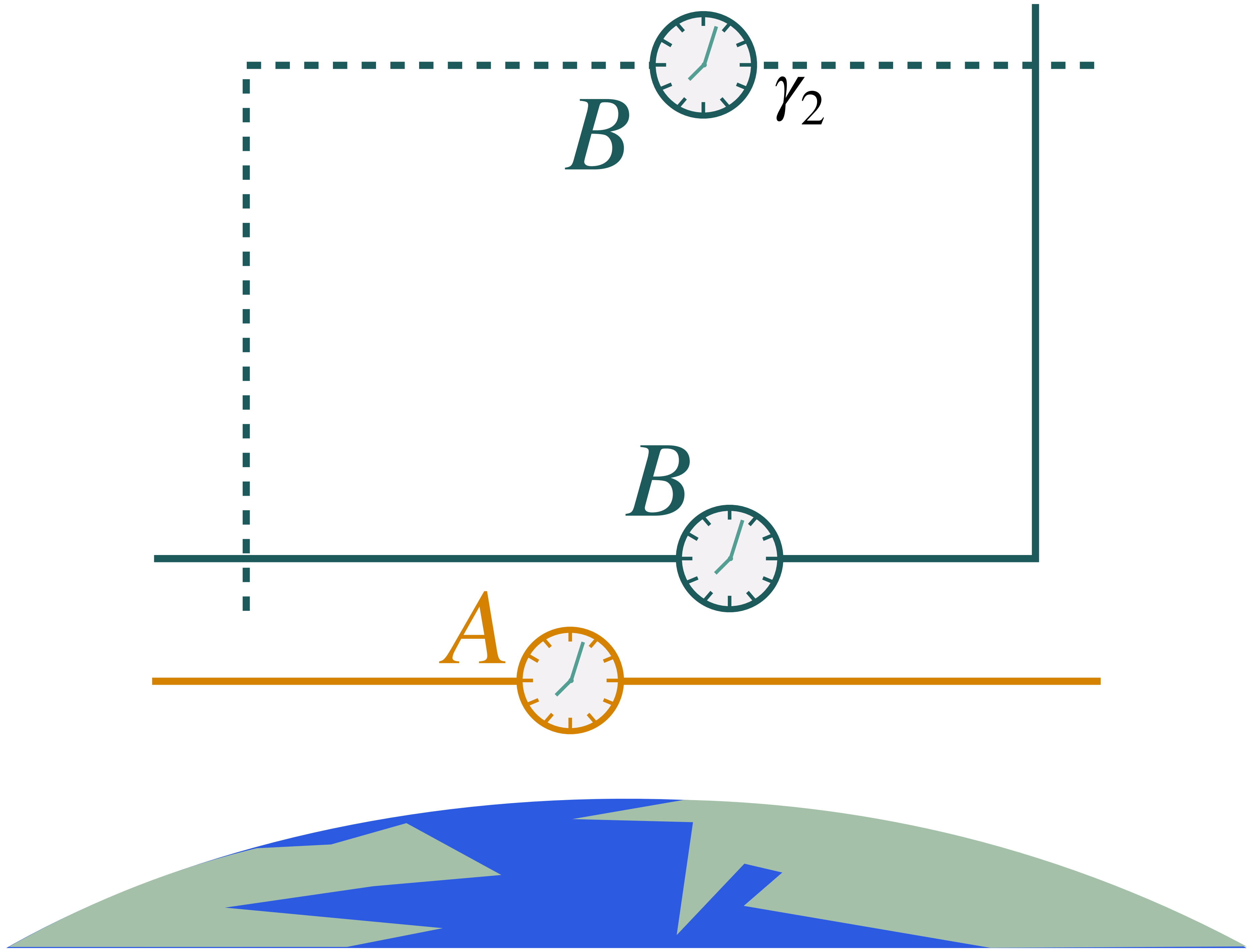
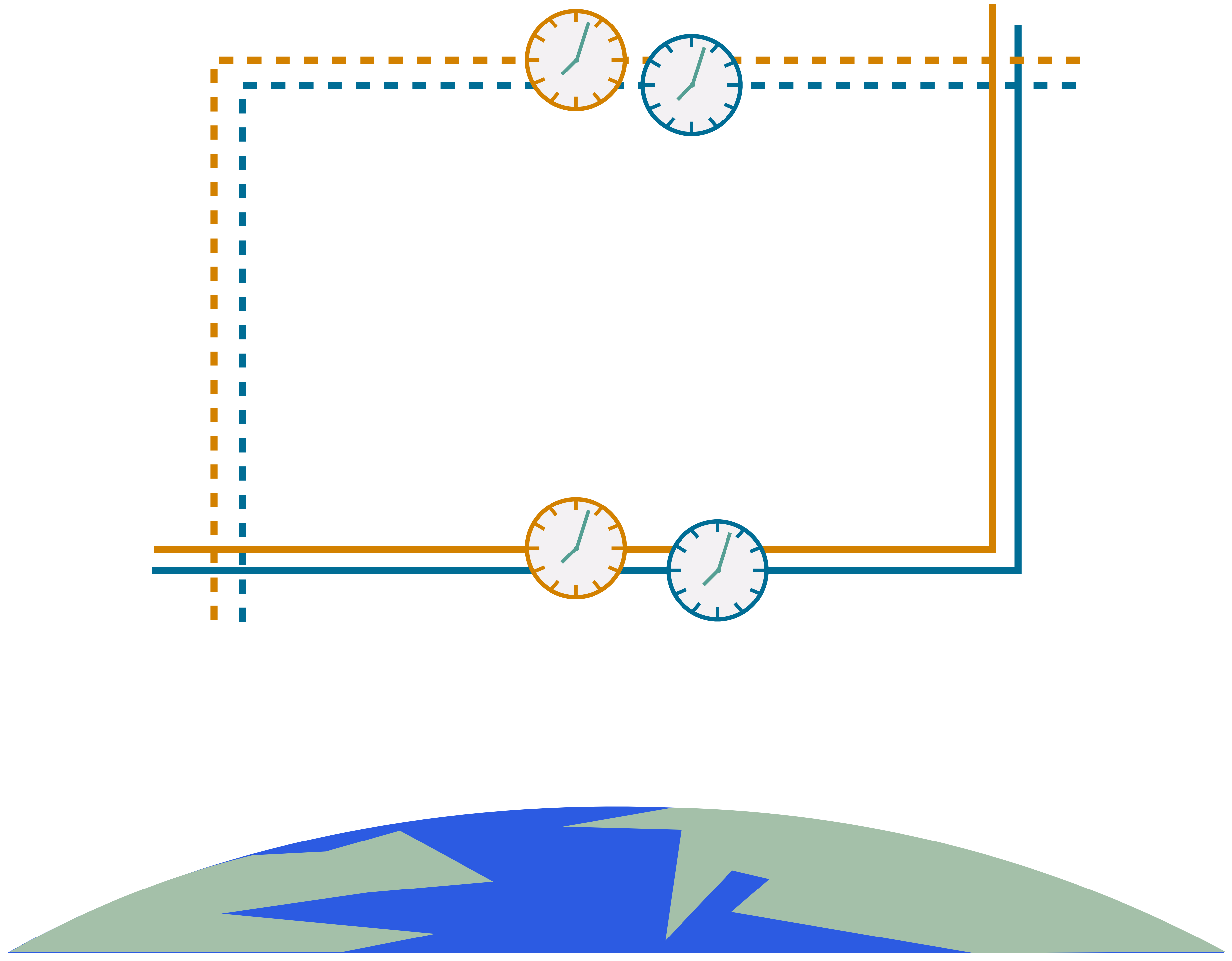
QRF IN TIME - RELATIVE LOCALISATION OF EVENTS





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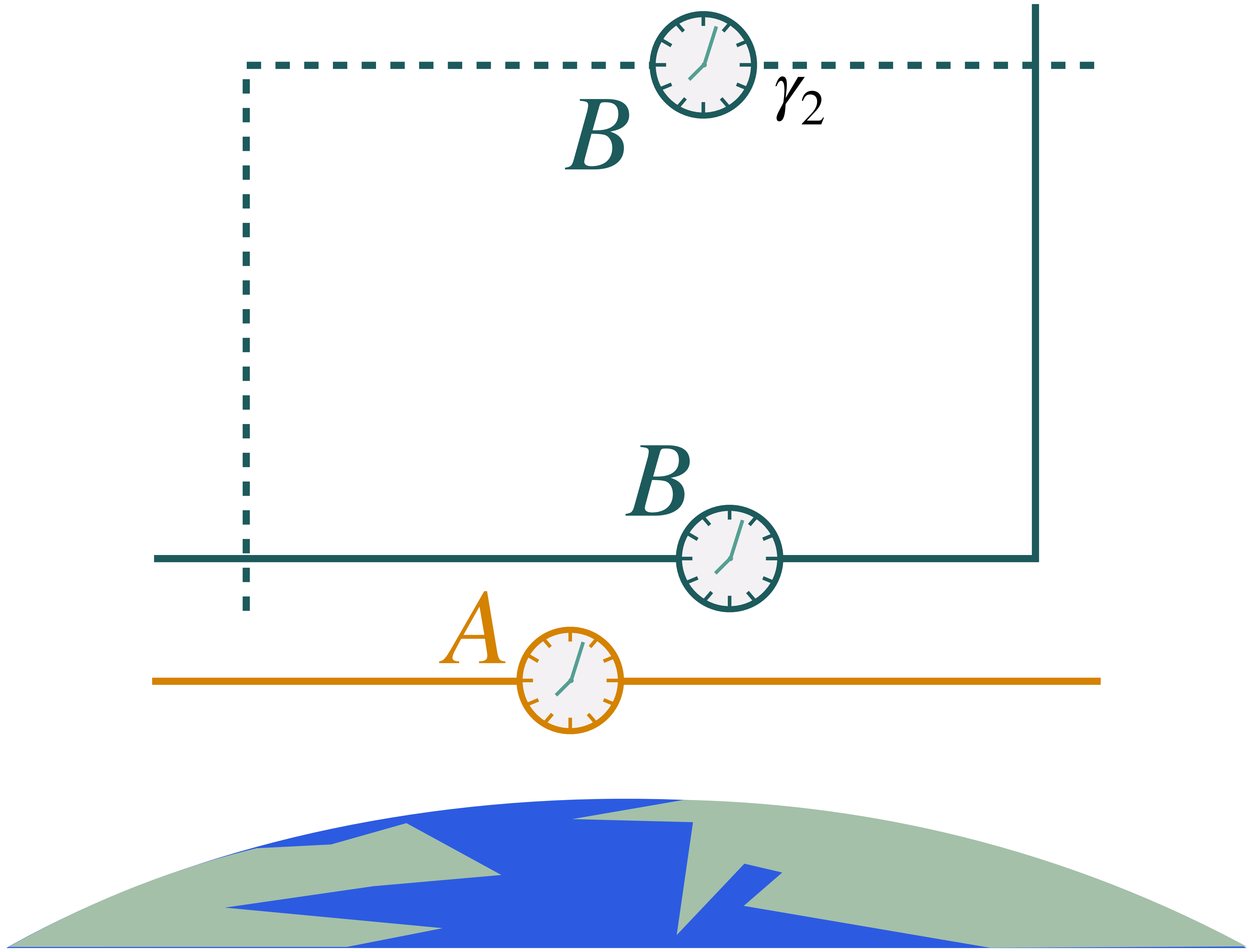

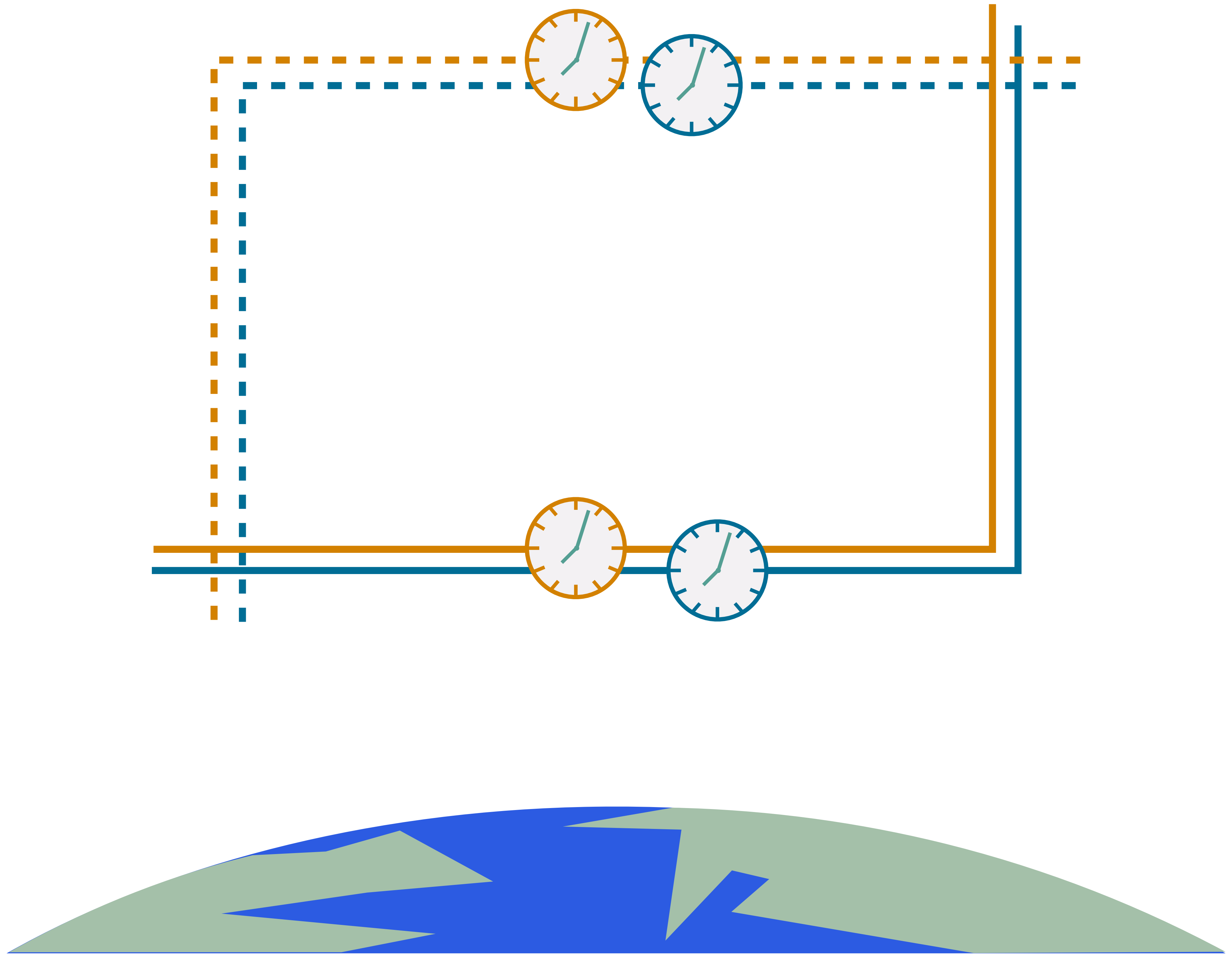

B in a superposition from A
A in a superposition from B

A and B tell the same time;
Sharp from each other's perspective

Giacomini, Quantum (2021)
Cepollaro, Giacomini, 2112.03303 (2021)

CONCLUSIONS

Operational and relational formalism for quantum reference frames:
associate a reference frame to a quantum system.

Conceptually, space and time are quantum when quantum systems serve as reference frames.

Where do we go now?
Need to modify/extend the framework to include gravity explicitly

Relation to NONCLASSICAL SPACETIME is still unknown
Structure of transformations?
Properties of nonclassical spacetime?
Observable features?
Relation to experiments?

THANK YOU!