COMMON CHALLENGES IN QUANTUM GRAVITY

Focus: Problem of Time

Claus Kiefer

Institut für Theoretische Physik
Universität zu Köln

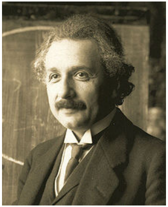
Contents

Albert Einstein 1953:

Es hat schweren Ringens bedurft, um zu dem für die theoretische Entwicklung unentbehrlichen Begriff des selbständigen und absoluten Raums [und der Zeit, C.K.] zu gelangen. Und es hat nicht geringerer Anstrengung bedurft, um diesen Begriff nachträglich wieder zu überwinden – ein Prozeß , der wahrscheinlich noch keineswegs beendet ist.[1]

[1] It took a lot of struggle to arrive at the concept of independent and absolute space [and time, C.K.], which is indispensable for the theoretical development. And it has taken no less struggle to overcome this concept afterwards – a process that is probably by no means finished.

General theory of relativity



$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}}$$

(November 25, 1915)

- left: geometry (gravitational field)
- right: matter fields (Standard Model of particle physics)

Problem of time I: Background independence

Albert Einstein:
Es widerstrebt dem wissenschaftlichen Verstande, ein Ding zu setzen, das zwar wirkt, aber auf das nicht gewirkt werden kann.[2]

There are no absolute fields in general relativity, that is, there is no background structure.

[2] It is contrary to the scientific mode of understanding to postulate a thing that acts, but which cannot be acted upon.

General relativity in canonical form

Einstein's equations can be written as a dynamical system (for the three-metric h_{ab} and its canonical momentum π^{ab} on a spacelike hypersurface Σ) of evolution equations together with constraints:

$$\begin{aligned} \mathcal{H}_\perp &= 2\kappa\, G_{ab\,cd}\pi^{ab}\pi^{cd} - (2\kappa)^{-1}\sqrt{h}({}^{(3)}R - 2\Lambda) + \sqrt{h}\rho \approx 0 \\ \mathcal{H}^a &= -2\nabla_b \pi^{ab} + \sqrt{h}j^a \approx 0\,, \end{aligned}$$

with the DeWitt metric

$$G_{ab\,cd} = \frac{1}{2\sqrt{h}}(h_{ac}h_{bd} + h_{ad}h_{bc} - h_{ab}h_{cd})$$

and

$$\kappa = 8\pi G/c^4$$

$H \approx 0$ is called "Hamiltonian constraint", $\mathcal{H}^a \approx 0$ are called "momentum (diffeomorphism) constraints".

Constraints and evolution

I

Constraints are preserved in time $\Longleftrightarrow$ energy–momentum tensor of matter has vanishing covariant divergence

compare with electrodynamics: Gauss constraint preserved in time $\Longleftrightarrow$ charge conservation

II

Einstein's equations are the unique propagation law consistent with the constraints

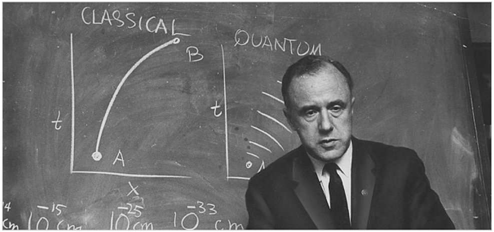
compare with electrodynamics: Maxwell's equations are the unique propagation law consistent with the Gauss constraint

Restriction to *compact* three-spaces Σ:

- The total Hamiltonian is a combination of pure constraints; all of the evolution will be generated by constraints;
- no external time parameter exists
- all physical time parameters are to be constructed from within our system, that is, as functional of the canonical variables; a priori there is no preferred choice of such an intrinsic time parameter

The absence of an extrinsic time and the non-preference of an intrinsic one is known as the problem of time in (classical) canonical gravity. Still, spacetime exists.

Quantum geometrodynamics



(a) John Archibald Wheeler

(b) Bryce DeWitt

Application of Schrödinger's procedure to general relativity leads to

$$\hat{\mathcal{H}}_\perp \Psi \equiv \left(-16\pi G \hbar^2 G_{abcd} \frac{\delta^2}{\delta h_{ab} \delta h_{cd}} - (16\pi G)^{-1} \sqrt{h} \left({}^{(3)}R - 2\Lambda \right) + \sqrt{h} \hat{\rho} \right) \Psi = 0$$

Wheeler–DeWitt equation

$$\hat{\mathcal{H}}^a \Psi \equiv -2 \nabla_b \frac{\hbar}{\mathrm{i}} \frac{\delta \Psi}{\delta h_{ab}} + \sqrt{h} \hat{j}^a \Psi = 0$$

quantum diffeomorphism (momentum) constraint

Comparison between geometrodynamics and particle dynamics		
underlying notion	geometrodynamics	particle dynamics
dynamical object	space	particle
configuration	h_{ij} 3-geometry	x, t events
classical description	$h_{ij}(t)$ 4-geometry	$x(t)$ trajectory
dynamical arena	superspace	spacetime
wave function(al)	$\Psi[h_{ij}]$	$\psi(x, t)$

From: C. Kiefer and P. Peter, *Universe* **8**, 36 (2022).

Problem of time II

- Spacetime has vanished from the formalism; only space remains
- This holds also for loop quantum gravity and probably for string theory;
- it holds for *any* theory that has no absolute time at the classical level.
- The Wheeler–DeWitt equation has the structure of a wave equation any may therefore allow the introduction of an intrinsic time.
- The Hilbert-space structure in quantum mechanics is connected with the probability interpretation, in particular with probability conservation *in time* t; what happens with this structure in a timeless situation?

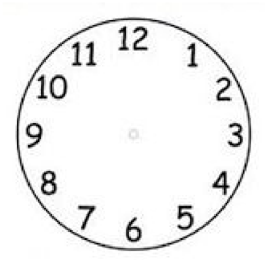


Figure: Absence of time in full quantum gravity

Figure from: C.K, B. Nikolić, *J.Phys.Conf.Ser.* **880**, 012002 (2017).

Quantum Cosmology

Closed Friedmann–Lemaître universe with scale factor a, containing a homogeneous massive scalar field ϕ (two-dimensional *minisuperspace*)

$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\mathrm{d}\Omega_3^2$$

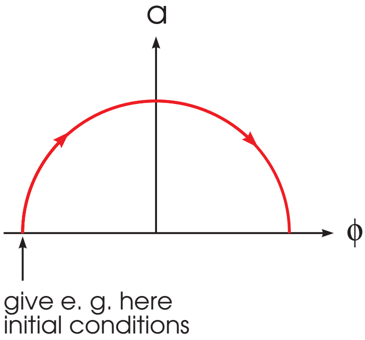
The Wheeler–DeWitt equation reads (with units $2G/3\pi = 1$)

$$\frac{1}{2}\left(\frac{\hbar^2}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) - \frac{\hbar^2}{a^3}\frac{\partial^2}{\partial \phi^2} - a + \frac{\Lambda a^3}{3} + m^2 a^3 \phi^2\right)\psi(a,\phi) = 0$$

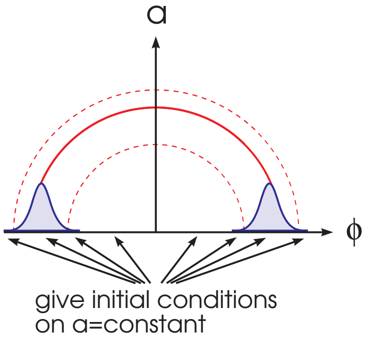
Factor ordering chosen in order to achieve covariance in minisuperspace

Determinism in classical and quantum theory

Classical theory



Quantum theory



Recollapsing part is deterministic successor of expanding part

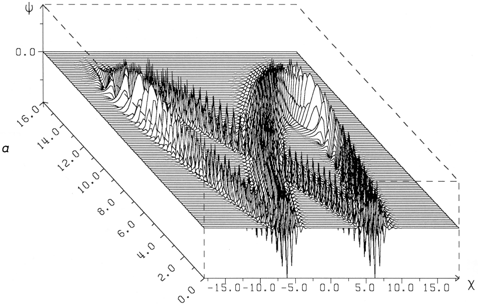
‘Recollapsing’ wave packet must be present ‘initially’

No intrinsic difference between ‘big bang’ and ‘big crunch’!

Example

Indefinite Oscillator (C.K. 1990)

$$\hat{H}\psi(a,\chi) \equiv (-H_a + H_\chi)\psi \equiv \left(\frac{\partial^2}{\partial a^2} - \frac{\partial^2}{\partial \chi^2} - a^2 + \chi^2\right)\psi = 0$$



Remark on path integrals: Such solutions cannot be found from the corresponding (e.g. no-boundary) path integral; there, the solutions either diverge at infinity or along the lightcone in minisuperspace.

Recovery of semiclassical time

An expansion of the Wheeler–DeWitt equation with respect to the Planck mass leads to the functional Schrödinger equation for non-gravitational fields (with Hamiltonian $\hat{H}^{\mathrm{m}}$) in a spacetime that is a solution of Einstein's equations. In this way, a semiclassical (WKB) time emerges as an approximate concept.
(Born–Oppenheimer type of approximation)

In this limit, one recovers the usual Hilbert space structure and the associated probability interpretation.

Lapchinsky and Rubakov 1979, *Banks* 1985, *Halliwell and Hawking* 1985, *Hartle* 1986, *C.K.* 1987, ...

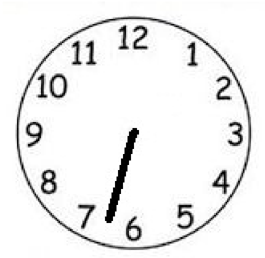


Figure: Emergence of semiclassical time

Figure from: C.K, B. Nikolić, *J.Phys.Conf.Ser.* **880**, 012002 (2017).

Quantum-gravitational corrections

The next order in the Born–Oppenheimer approximation gives

$$\hat{H}^{\mathrm{m}} \to \hat{H}^{\mathrm{m}} + \frac{1}{m_{\mathrm{P}}^2} \times (\text{various terms})$$

(C.K. and T.P. Singh (1991); A. O. Barvinsky and C.K. (1998)), ...

Corrections to the CMB power spectrum?

Observations

Does the anisotropy spectrum of the cosmic background radiation contain information about quantum gravity?

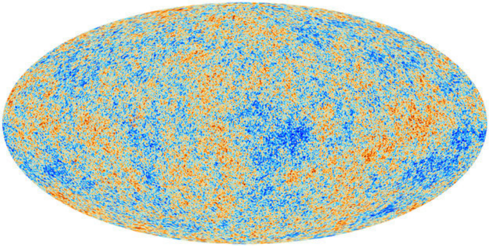


Figure credit: ESA/PLANCK Collaboration

See our recent review: L. Chataignier, C. Kiefer, P. Moniz, arXiv:2306.14948 [gr-qc], and references therein.

Decoherence in quantum cosmology

- 'System': global degrees of freedom (scale factor, inflaton field, ...)
- 'Environment': small density fluctuations, gravitational waves, ...

(Zeh 1986, C.K. 1987)

Example: scale factor a of a de Sitter universe ($a \propto \mathrm{e}^{H_{\mathrm{I}}t}$) ('system') experiences decoherence by gravitons ('environment') according to

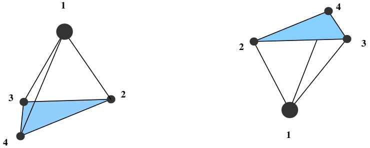
$$\rho_0(a,a') \to \rho_0(a,a') \exp\left(-CH_{\mathrm{I}}^3 a(a-a')^2\right) \ , \ C > 0$$

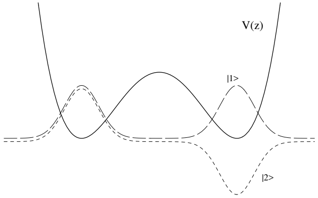
The Universe assumes classical properties at the beginning of inflation

(Barvinsky, Kamenshchik, C.K. 1999)

Time from symmetry breaking

Analogy from molecular physics: emergence of chirality





dynamical origin: decoherence through scattering by light or air molecules

Quantum cosmology: decoherence between $\exp(\mathrm{i}S_0/G\hbar)$- and $\exp(-\mathrm{i}S_0/G\hbar)$-components of the wave function through interaction with e.g. weak gravitational waves

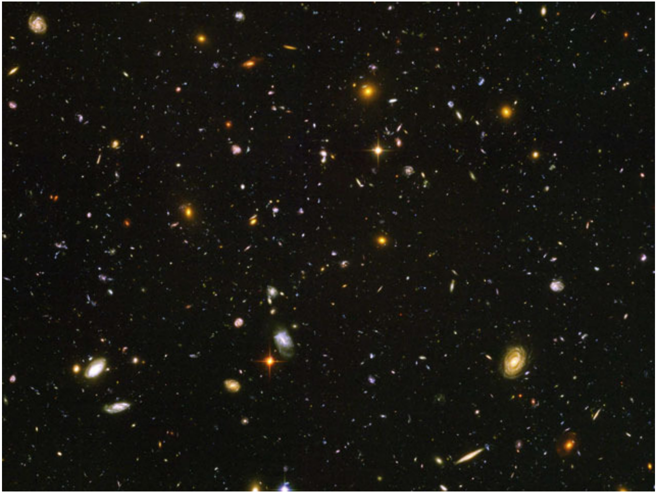
Example for decoherence factor: $\exp\left(-\frac{\pi m H_0^2 a^3}{128\hbar}\right) \sim \exp\left(-10^{43}\right)$ (C.K. 1992)

Decoherence of primordial fluctuations

During the inflationary phase (ca. 10^{-34} after the Big Bang) there is a quantum-to-classical transition for the ubiquitous fluctuations of the inflaton and the metric.
The process of decoherence is crucial in understanding this transition

The fluctuations then behave like classical stochastic quantities and yield the seeds for the structures in the Universe. Quantum gravity is needed to understand the power spectrum.

(C.K., Polarski, Starobinsky 1998, . . .)



Arrow of time from cosmology

Where does the Sun come from?

$\Downarrow$

Gravitational instability of dust clouds

$\Downarrow$

Cosmology

Ludwig Boltzmann (1898):

That in Nature the transition from a probable to an improbable state does not happen equally often as the opposite transition, should be sufficiently explained by the assumption of a very improbable initial state of the whole Universe surrounding us . . . .

Arrow of time from quantum cosmology

Fundamental asymmetry with respect to “intrinsic time”:

$$\hat{H}\Psi = \left(\frac{\partial^2}{\partial \alpha^2} + \sum_i \left[-\frac{\partial^2}{\partial x_i^2} + \underbrace{V_i(\alpha, x_i)}_{\to 0 \text{ for } \alpha \to -\infty} \right] \right) \Psi = 0$$

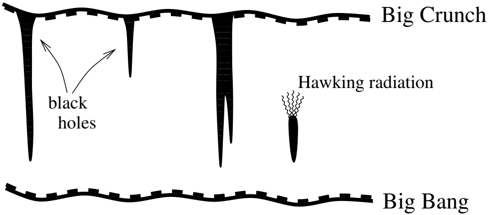
Is compatible with simple boundary condition:

$$\Psi \stackrel{\alpha \to -\infty}{\longrightarrow} \psi_0(\alpha) \prod_i \psi_i(x_i)$$

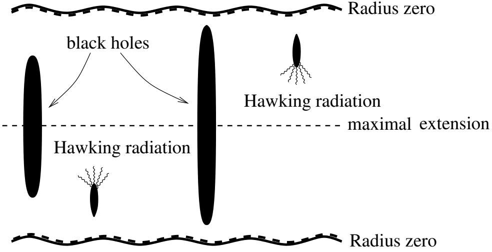
Entropy $-k_\mathrm{B} \mathrm{Tr} \rho \ln \rho$ increases with increasing α, since entanglement with other degrees of freedom increases; this defines the direction of time

Is the expansion of the Universe a tautology?

Arrow of time in a recollapsing quantum universe



(Penrose 1979)



(C.K. and Zeh 1995)