The road towards non-perturbative gravitational scattering amplitudes in Asymptotic Safety

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A Perspective on Quantum Gravity

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lack of smoking gun
quantum gravity experiments

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many big ideas
and even bigger claims
on QG

recall Bianca's talk yesterday

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**why trust any
approach in particular?**

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  1. set up quantum theory of gravity and matter (at least SM)
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- tool of choice: **gravitational scattering amplitudes**

Outline

- Asymptotic Safety
- Gravity-mediated scattering amplitudes
- Summary

Asymptotic Safety

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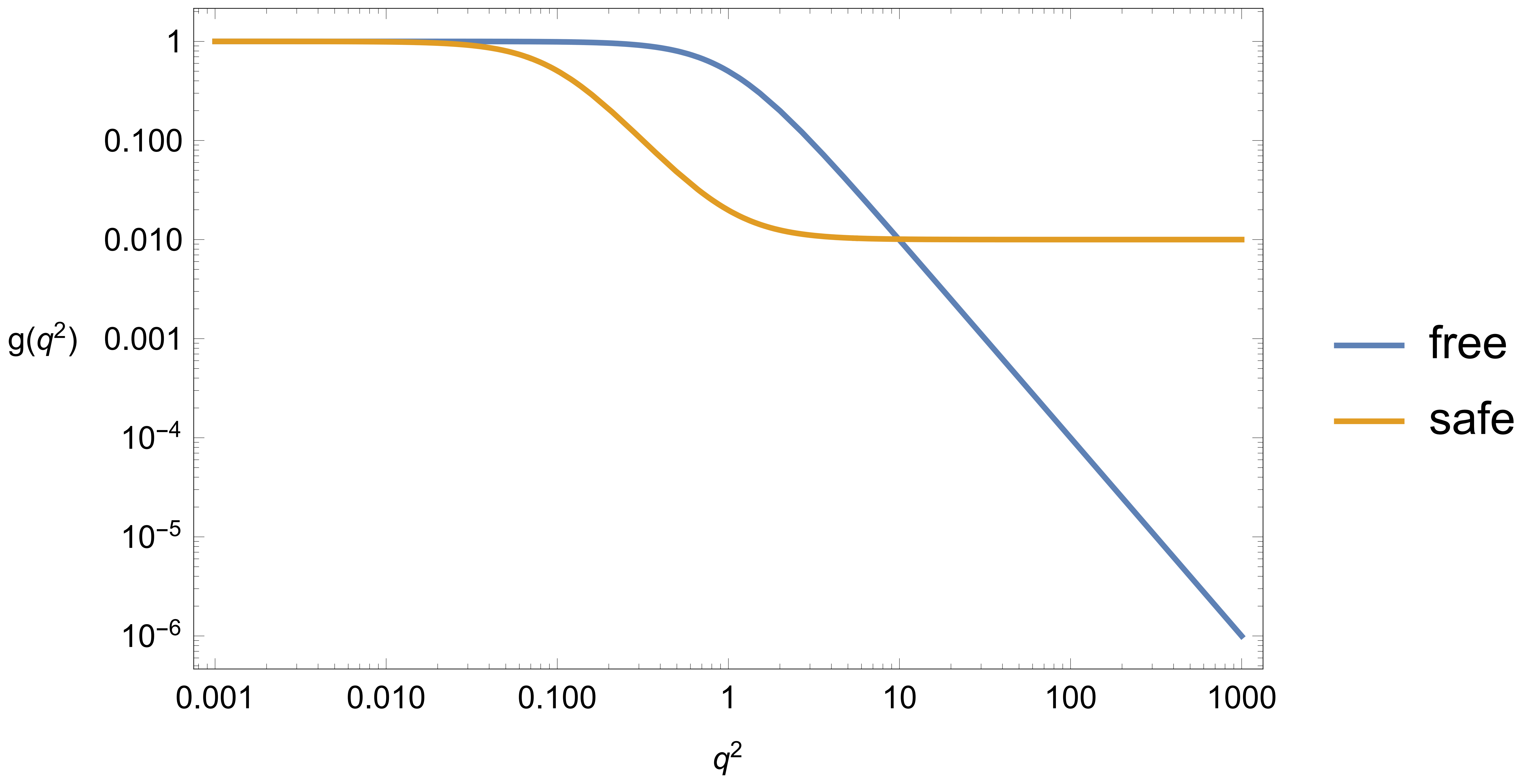

Asymptotic Safety

- hypothesis: metric gravity (+suitable matter) can be formulated as a QFT in a consistent, non-perturbative way

- conditions:

  - all dimensionless versions of essential couplings approach a finite value at high energies = fixed point

  - only finitely many relevant operators = finitely many measurements needed to uniquely fix theory

The stage for the rest of the talk

- perturbation theory for QG has problems, but let us not give up QFT entirely
- test Asymptotic Safety by connecting it to theoretical and experimental constraints via investigating gravitational scattering amplitudes
- starting point: quantum effective action

Gravity-mediated scattering amplitudes

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Graviton-mediated scattering amplitudes

- my mid-term goal: compute all $2 \to 2$ gravitational scattering amplitudes within Asymptotic Safety and confront them with experiment
- benefits:
  - probe quantum gravity effects
  - direct link to observables
  - independent of arbitrary choices
  - use effective action = tree-level diagrams encode “everything”

Graviton-mediated scattering amplitudes

- strategy for a given scattering amplitude:

  1. parameterise all possible terms in the effective action that contribute to the scattering event

  2. compute ingredients from RG flow

  3. confront with experimental data and theoretical constraints (finiteness, unitarity, causality, …)

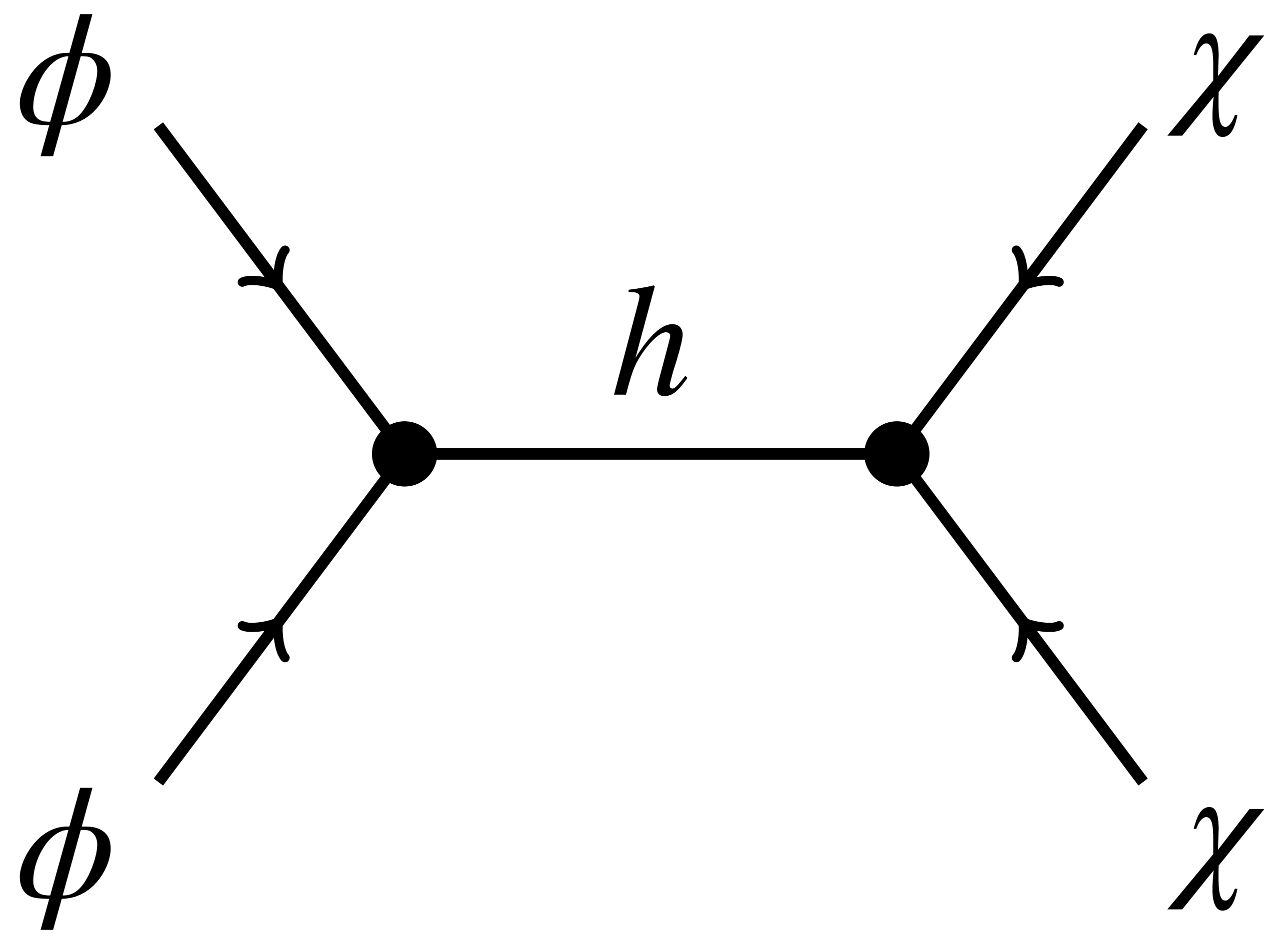
Graviton-mediated scattering amplitudes

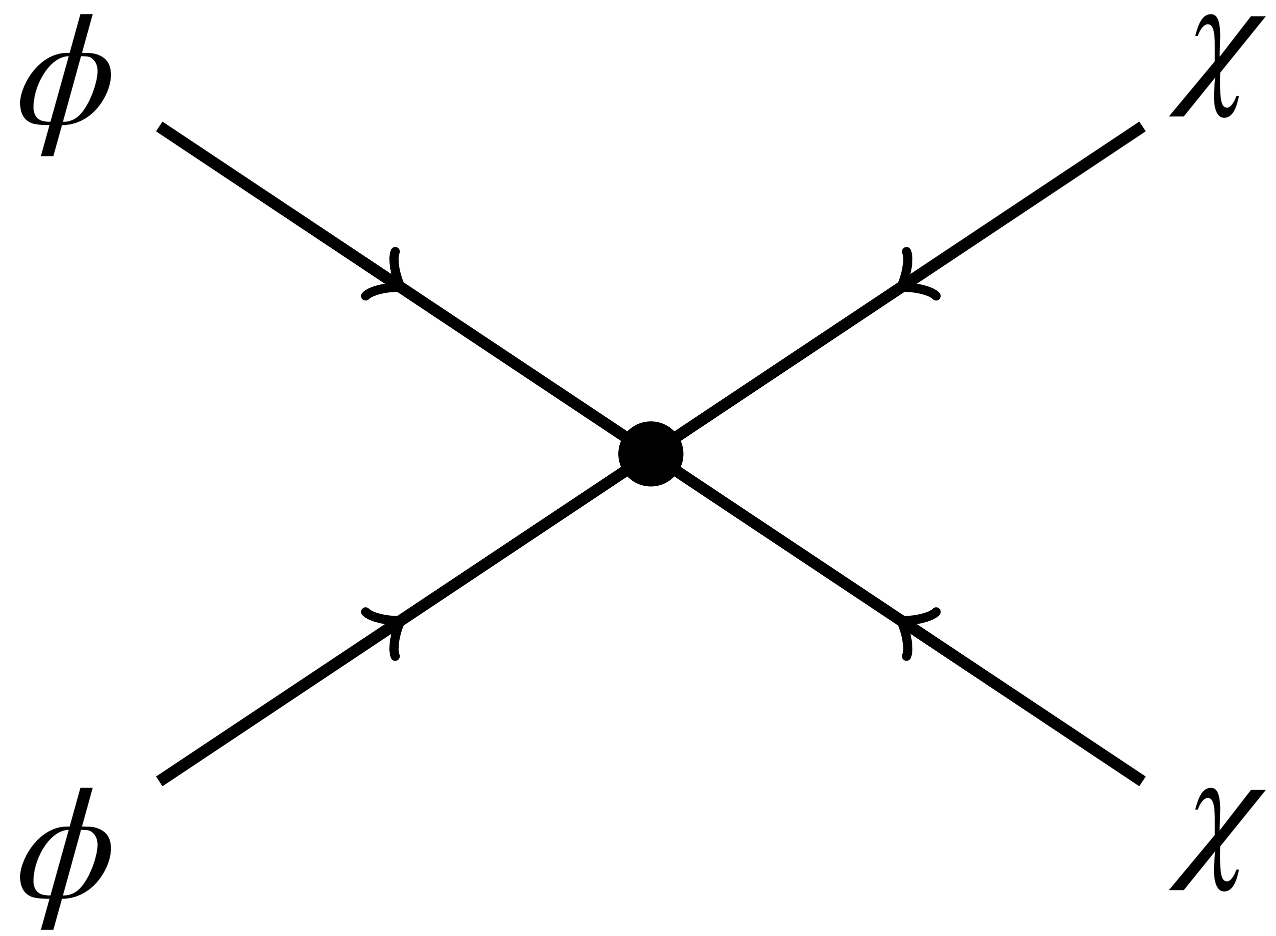
- disclaimers:
  - no external gravitons - difficult to define what an on-shell graviton is
  - flat spacetime and “standard” asymptotic states

1. Parameterising

Scalar-scalar scattering

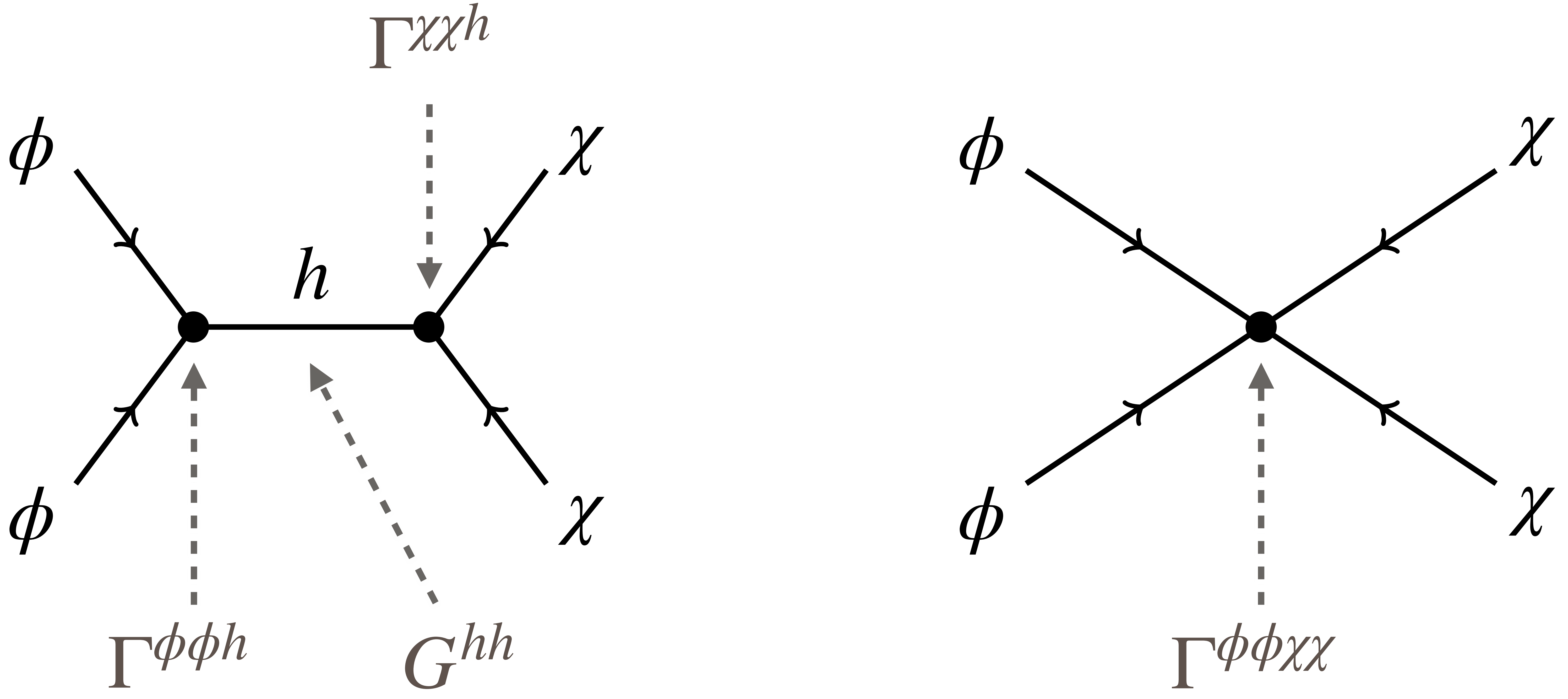
- gravity-mediated scalar scattering:





Scalar-scalar scattering

- gravity-mediated scalar scattering:



Scalar-scalar scattering

- necessary ingredients in the effective action:

G^{hh}

$$\Gamma \simeq \frac{1}{16\pi G_N}\int \mathrm{d}^4x\sqrt{-g}\left[-R - \frac{1}{6}R f_R(\Delta)\,R + \frac{1}{2}C^{\mu\nu\rho\sigma} f_C(\Delta)\,C_{\mu\nu\rho\sigma}\right]$$

$$+\int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}\phi f_\phi(\Delta)\,\phi + f_{R\phi\phi}(\Delta_1,\Delta_2,\Delta_3)\,R\,\phi\,\phi + f_{Ric\phi\phi}(\Delta_1,\Delta_2,\Delta_3)\,R^{\mu\nu}\left(D_\mu D_\nu\phi\right)\phi\right]$$

$\Gamma^{\phi\phi h}$

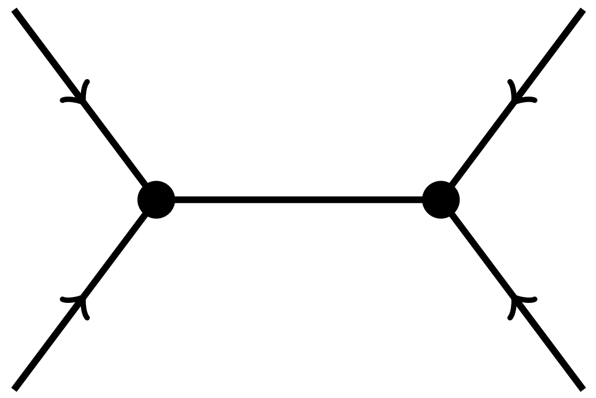
$$+(\phi\to\chi) + \frac{1}{(2!)^2}\int \mathrm{d}^4x\sqrt{-g}\,f_{\phi\chi}(\{-D_{ij}\})\phi\phi\chi\chi$$

$\Gamma^{\chi\chi h}$ $\Gamma^{\phi\phi\chi\chi}$

full momentum dependence is key

form factor toolbox:
BK, Ripken, Saueressig '19

Scalar-scalar scattering



$$\mathcal{A}^{\phi\chi}_{\mathfrak{s}} = \frac{4\pi}{3}\Bigg[-\left(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)\left(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_C(\mathfrak{s}) \left\{ \mathfrak{t}^2 - 4\mathfrak{t}\mathfrak{u} + \mathfrak{u}^2 + 2\left(m_\phi^2 - m_\chi^2\right)^2 \right\}$$

$$+\left((\mathfrak{s} + 2m_\phi^2)(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)) - 12\mathfrak{s} f_{R\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)$$

$$\times \left((\mathfrak{s} + 2m_\chi^2)(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)) - 12\mathfrak{s} f_{R\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_R(\mathfrak{s}) \Bigg]$$

$$G_X(z) = \frac{G_N}{z(1+f_X(z))}$$

$$p_1^2 = p_2^2 = m_\phi^2$$

$$p_3^2 = p_4^2 = m_\chi^2$$

$$\mathfrak{s} = (p_1 + p_2)^2$$

$$\mathfrak{t} = (p_1 + p_3)^2$$

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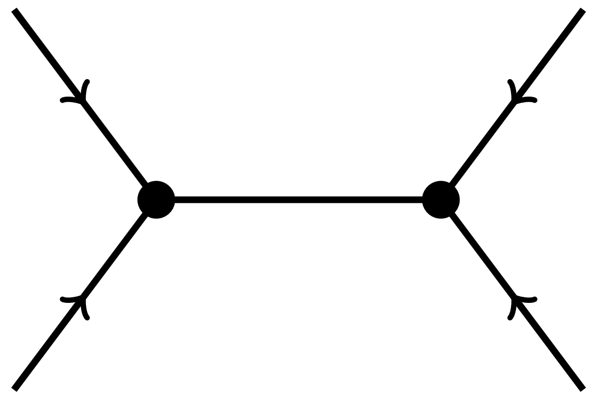
Draper, BK, Ripken, Saueressig '20

Scalar-scalar scattering



vertex factors — graviton propagator — contraction factor — **spin 2**

$$\mathscr{A}_{\mathfrak{s}}^{\phi\chi} = \frac{4\pi}{3}\Big[-\left(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)\left(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_C(\mathfrak{s}) \left\{ \mathfrak{t}^2 - 4\mathfrak{t}\mathfrak{u} + \mathfrak{u}^2 + 2\left(m_\phi^2 - m_\chi^2\right)^2 \right\}$$

$$+ \left((\mathfrak{s}+2m_\phi^2)(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)) - 12\mathfrak{s} f_{R\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)$$

$$\times \left((\mathfrak{s}+2m_\chi^2)(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)) - 12\mathfrak{s} f_{R\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_R(\mathfrak{s}) \Big]$$

spin 0

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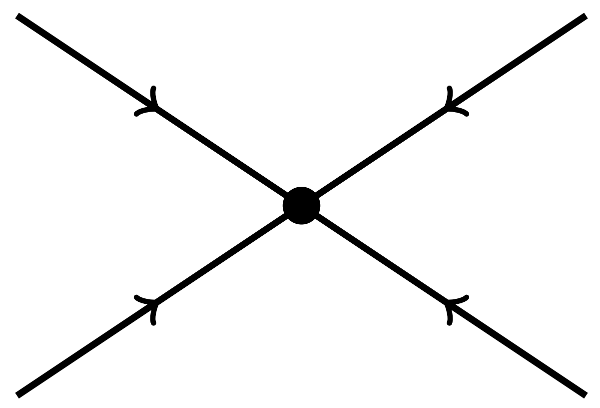

$$\mathscr{A}_4^{\phi\chi} = f_{\phi\chi}\left(\frac{\mathfrak{s} - 2m_\phi^2}{2}, \frac{\mathfrak{t} - m_\phi^2 - m_\chi^2}{2}, \frac{\mathfrak{u} - m_\phi^2 - m_\chi^2}{2}, \frac{\mathfrak{u} - m_\phi^2 - m_\chi^2}{2}, \frac{\mathfrak{t} - m_\phi^2 - m_\chi^2}{2}, \frac{\mathfrak{s} - 2m_\chi^2}{2}\right)$$

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Draper, BK, Ripken, Saueressig '20

Beyond scalar-scalar scattering

- similar computations can be done for any scattering event
  - scalar-photon, photon-photon

    BK, Pirlo, Ripken, Saueressig ’22
    book chapter: BK, Ripken, Saueressig ‘22
  - to do: fermions
- focussing on **essential** couplings/form factors will be helpful, reduces complexity severely

Baldazzi, Ben Alì Zinati, Falls ’21
Baldazzi, Falls ’21
BK ’22
BK, Ripken wip
Baldazzi, Falls, Kluth, BK wip
BK, Platania wip

2. Computing

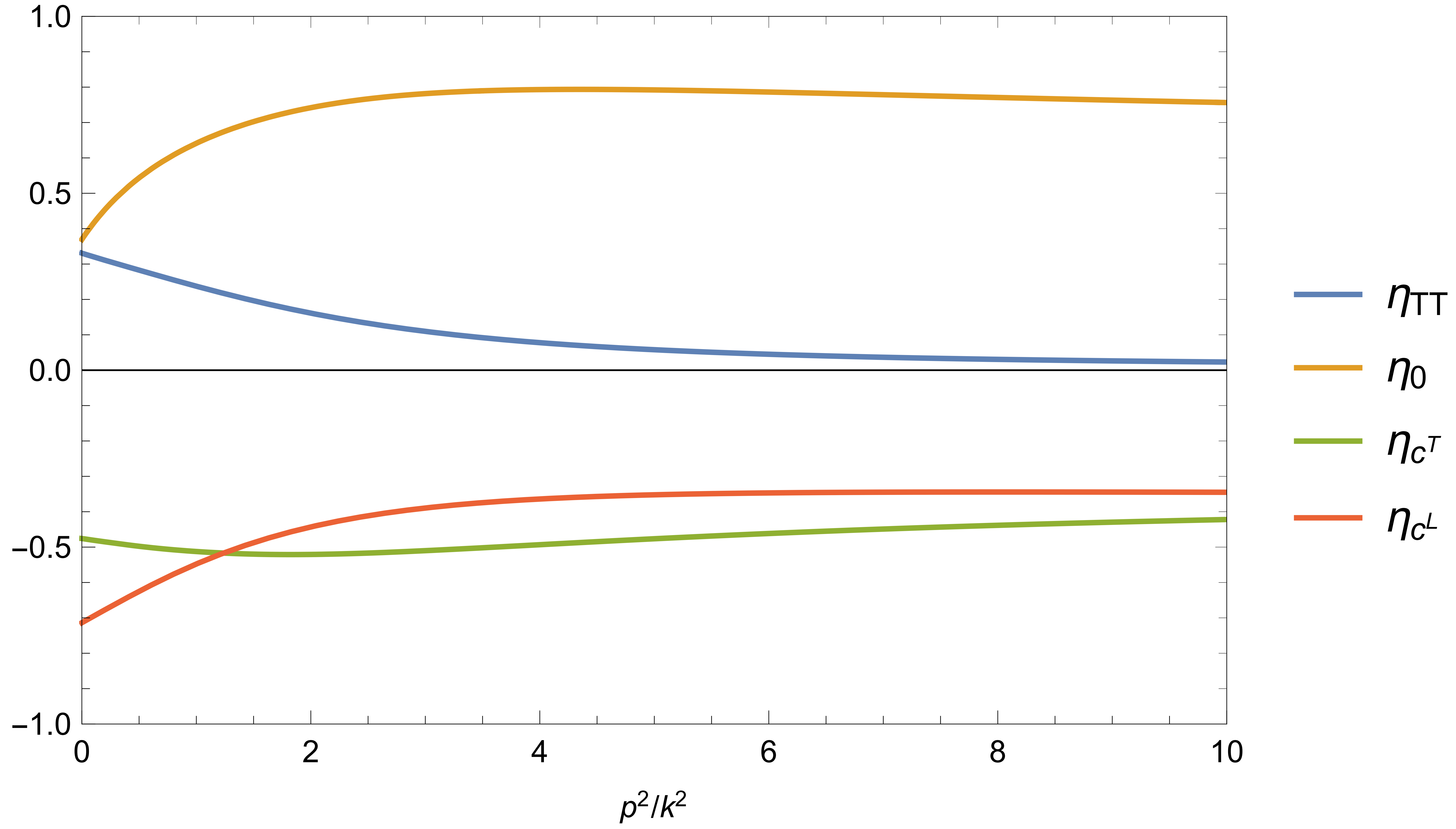
Momentum dependence in Asymptotic Safety

- a lot of work on graviton propagator:
  - no additional modes beyond the massless graviton
  - spin zero and spin two sector behave qualitatively differently
  - first efforts to rotate to/compute directly in Lorentzian signature
- beyond propagators: only limited information, concerted effort needed

cutting-edge
computer tensor algebra
and numerics necessary!

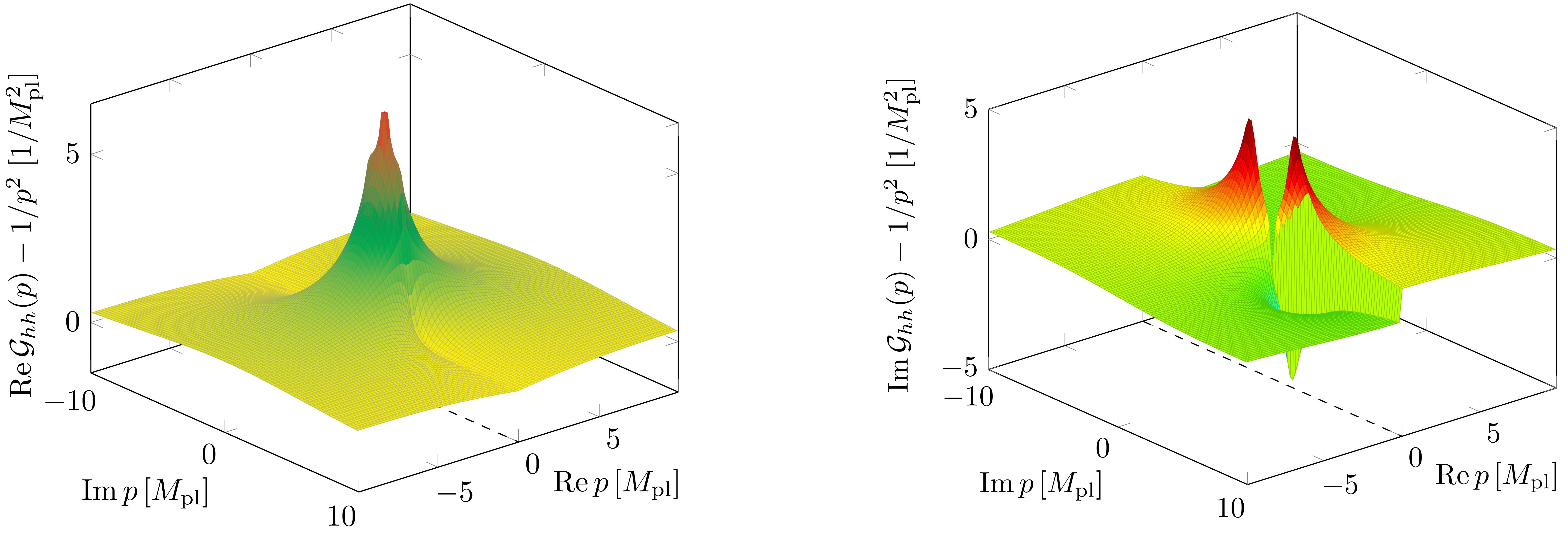
Christiansen, BK, Pawlowski, Rodigast '14
Christiansen, BK, Meibohm, Pawlowski, Reichert '15
Denz, Pawlowski, Reichert '16
Bonanno, Denz, Pawlowski, Reichert '21
BK, Schiffer '21
Fehre, Litim, Pawlowski, Reichert '21
…

Momentum dependence in Asymptotic Safety



BK, Schiffer '21

Momentum dependence in Asymptotic Safety



recall Manuel's talk yesterday

Fehre, Litim, Pawlowski, Reichert '21

3. a) Confronting with experimental data

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- not yet enough computational input to make statements about scattering amplitudes

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- however: first insights directly from RG flow
  - prediction of Higgs mass *Shaposhnikov, Wetterich '09*
  - postdiction of top mass *Eichhorn, Held '17*
  - …

3. b) Confronting with theoretical constraints

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Confronting with theoretical constraints

- not yet enough computational input to make statements about scattering amplitudes
- however: proof of principle that Asymptotic Safety can coexist with unitarity, causality, …
- surprising finding: theoretical constraints need specific relations between a priori unrelated couplings

Draper, BK, Ripken, Saueressig '20

Summary

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- falsifiability is at the heart of science, and it should also be at the heart of quantum gravity research
- scattering amplitudes are promising tool to probe quantum gravity
- ingredients can be computed ab initio, no need to guess
- encouraging progress:
  - AS fixed point is very stable result
  - we have the technology to compute correlation functions - **we are getting there!**

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  - we have the technology to compute correlation functions - **we are getting there! - but there is no free lunch either…**