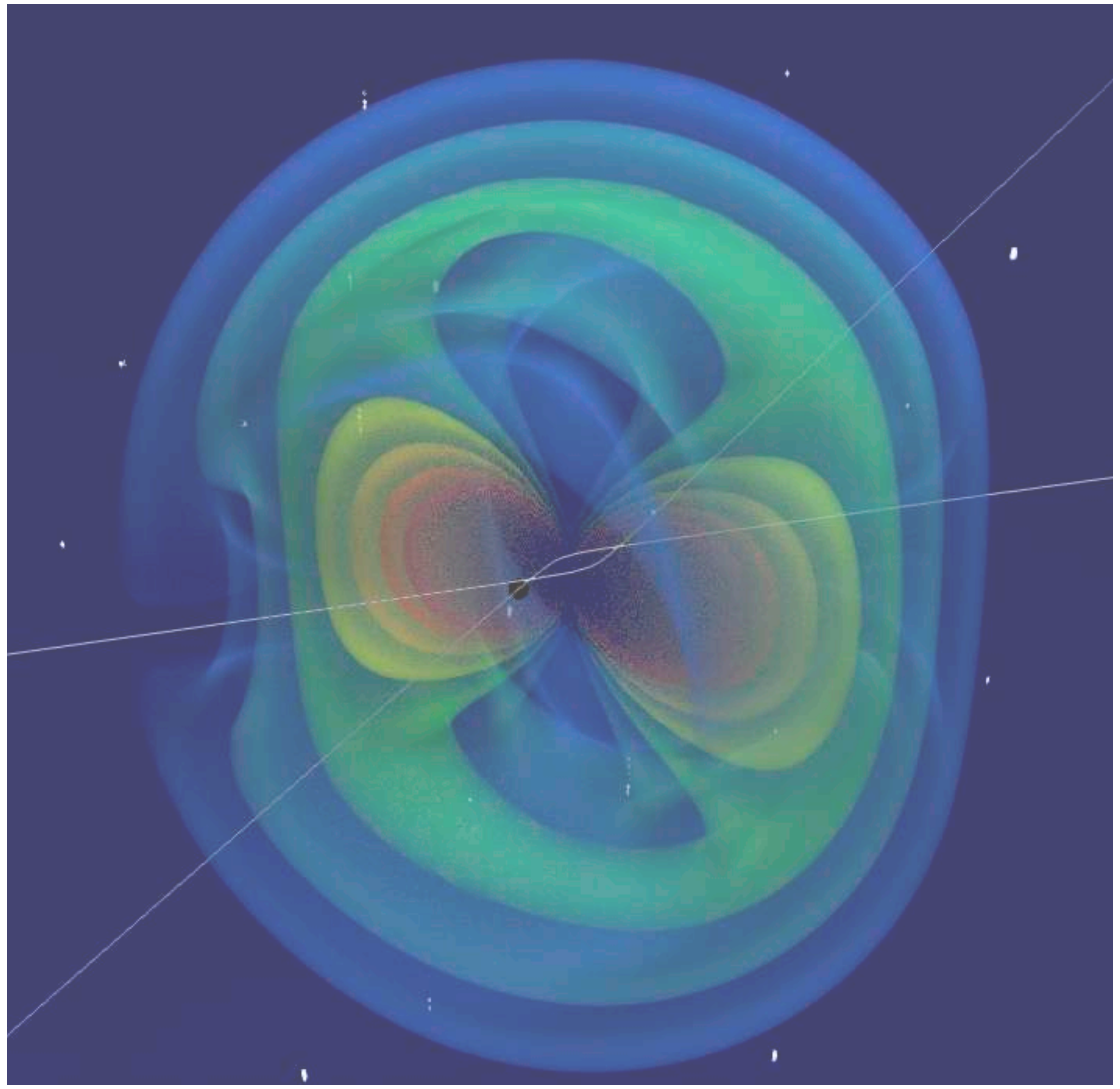
Classical ↓

HIGH PRECISION GRAVITATIONAL WAVE PHYSICS FROM A WORLDLINE QUANTUM FIELD THEORY



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Based on joint work with

Gustav Uhre Jakobsen, Gustav Mogull, Benjamin Sauer,
Jan Steinhoff (AEI)

2010:02865, *JHEP* 02 (2021) 048
2101.12688, ***PRL*** 126 (2021) 20
2106.10256, ***PRL*** 128 (2022) 1
2109.04465, *JHEP 01 (2022) 027*
2201.07778, ***PRL*** 128 *(2022) 14*
2207.00569, *JHEP* 10 (2022) 128
2306.01714,

RTG 2575:
Rethinking
Quantum Field Theory

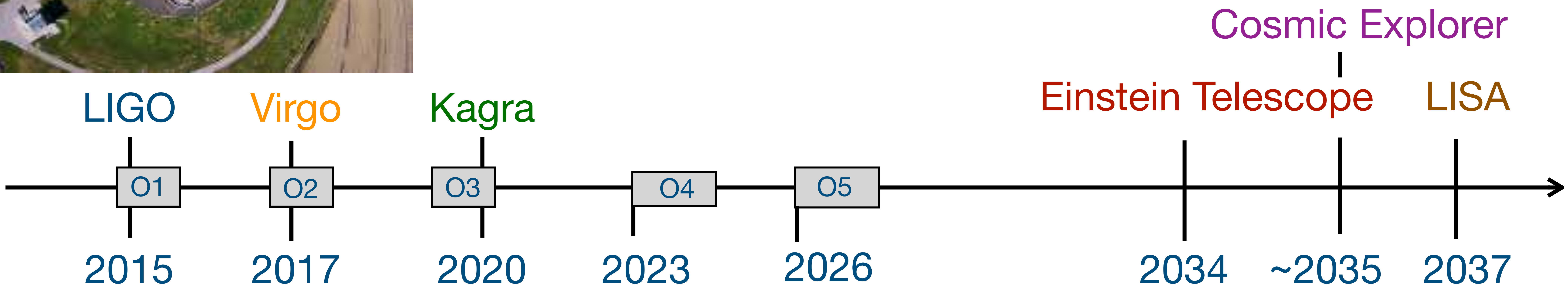

$\gamma = 1.1, \ \frac{m_1}{m_2} = 1, \ \frac{b}{m_1} = 50$
Time: -5.840000

ERA OF GRAVITATIONAL WAVE PHYSICS: NEED FOR HIGH-PRECISION PREDICTIONS

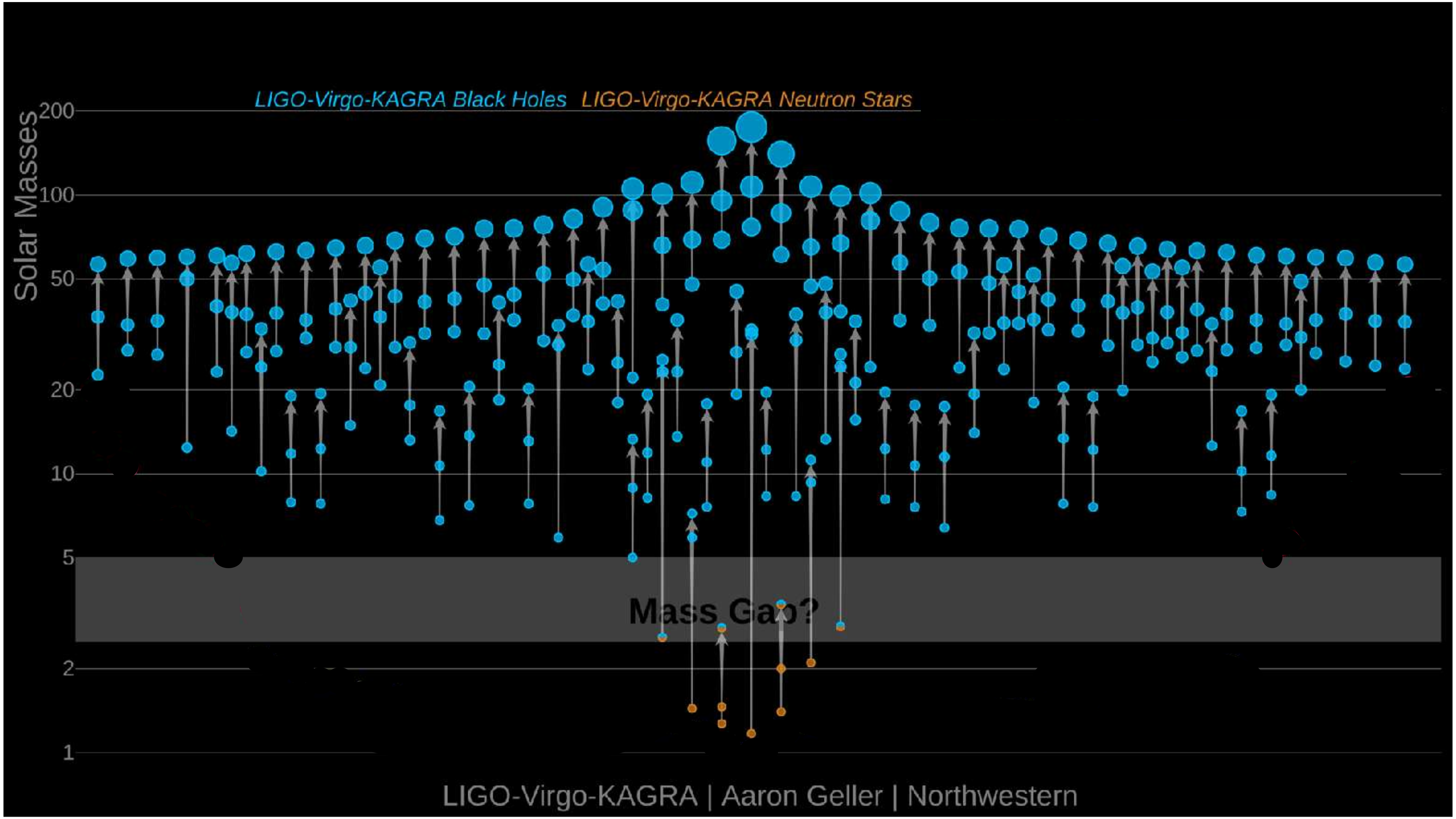
- Upcoming 3rd generation of gravitational wave observatories with 10^2 sensitivity increase
- Need for accurate waveform predictions well beyond state-of-the art

High-precision predictions necessary basis to study fundamental questions in physics:

- Is Einstein’s theory correct?
- Black hole formation & population?
- Neutron star properties?
- Physics beyond the standard model?



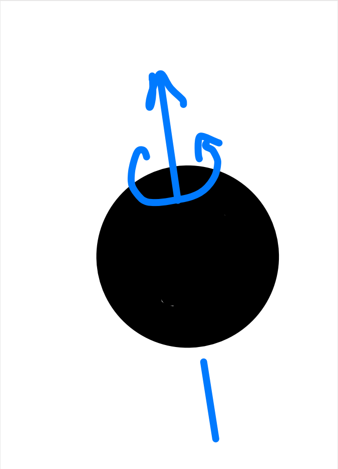
GRAVITATIONAL WAVES: A NEW OBSERVATIONAL ERA



Following GW150914: To date 90 binary mergers detected by LIGO-Virgo-Karga Collaboration

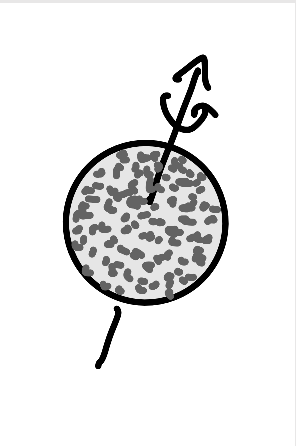
GRAVITATIONAL TWO-BODY PROBLEM

Black Hole



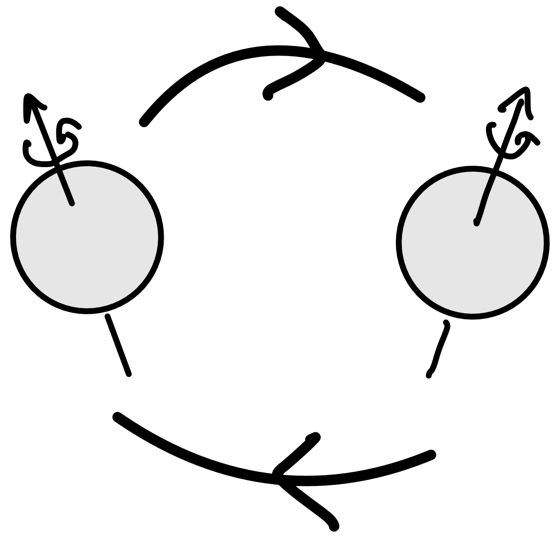
mass, spin

Neutron Star

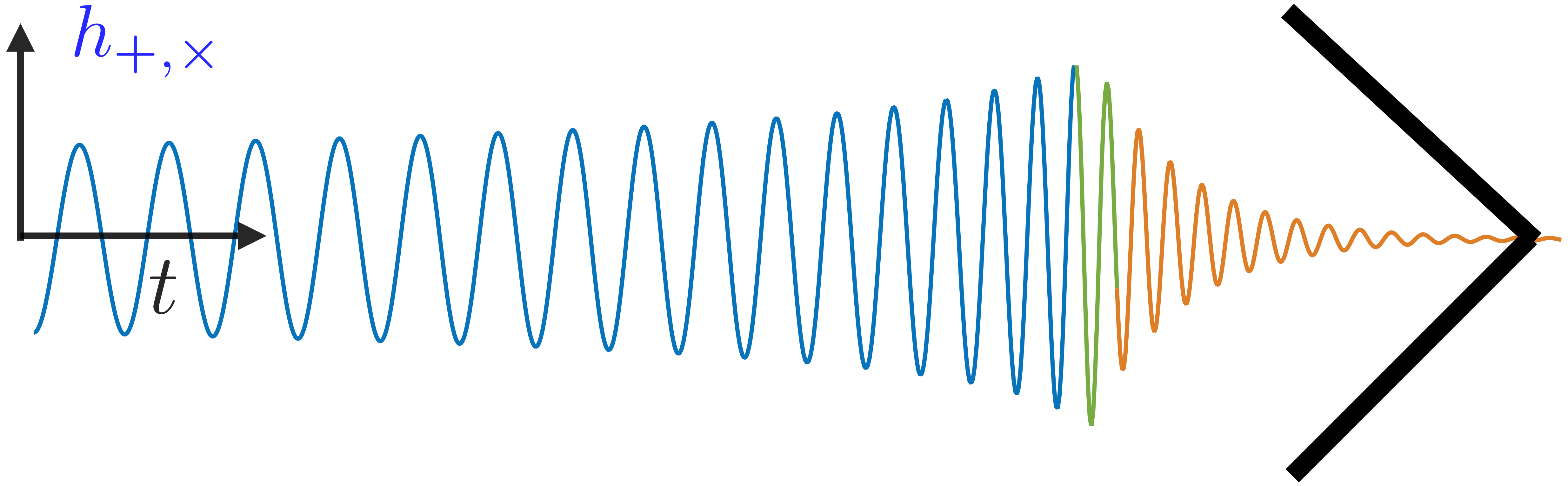


mass, spin, radius, tidal deformability

Black Hole/Neutron Star Binaries:



Bound state



inspiral

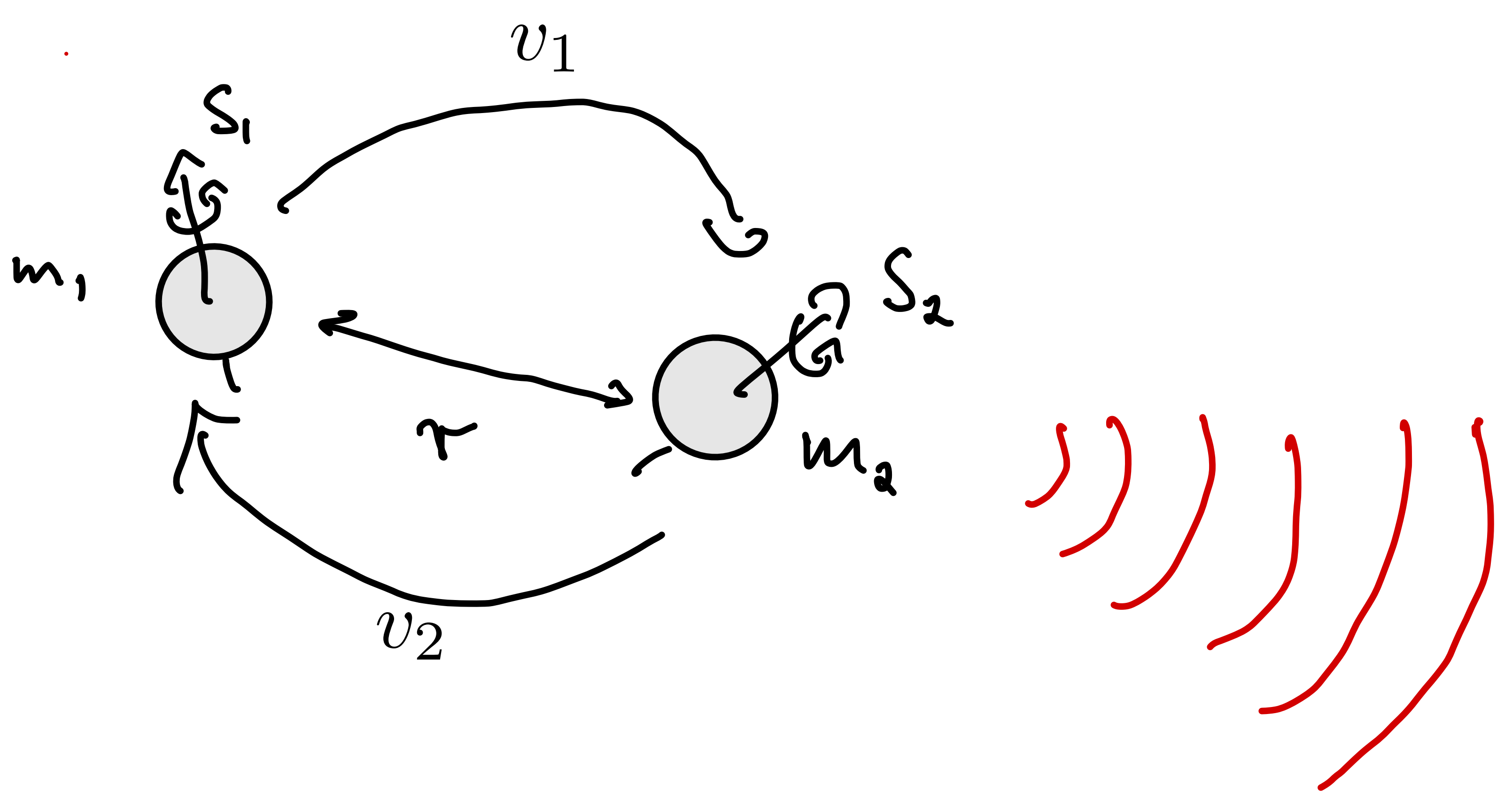
merger

- During **inspiral**: weak gravitational fields $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\, h_{\mu\nu}$
- **Quantum** field theory formalism for **classical** two-body problem:

WORLDLINE QUANTUM FIELD THEORY

THE GENERAL RELATIVISTIC 2-BODY PROBLEM

As in Newtonian case has either **bound** or **unbound** orbits.



Inspiral of 2 black holes or neutron stars:

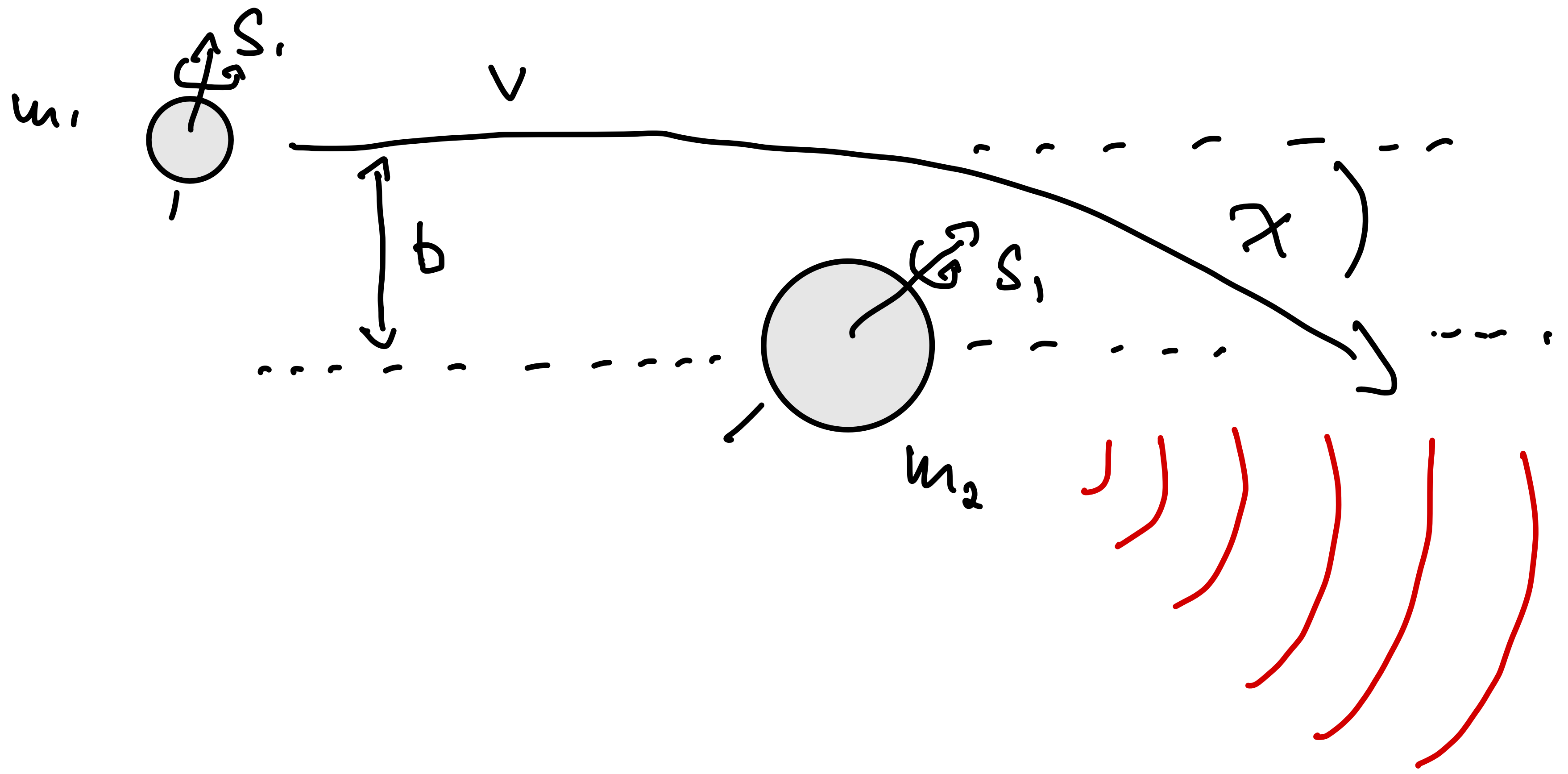
Virial-theorem: $\frac{GM}{r} \sim v^2$ $(c = 1)$

post-Newtonian (PN) expansion in G & v^2

Weak field expansion: $g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}$

$\kappa = \sqrt{32\pi G}$

Newton's constant



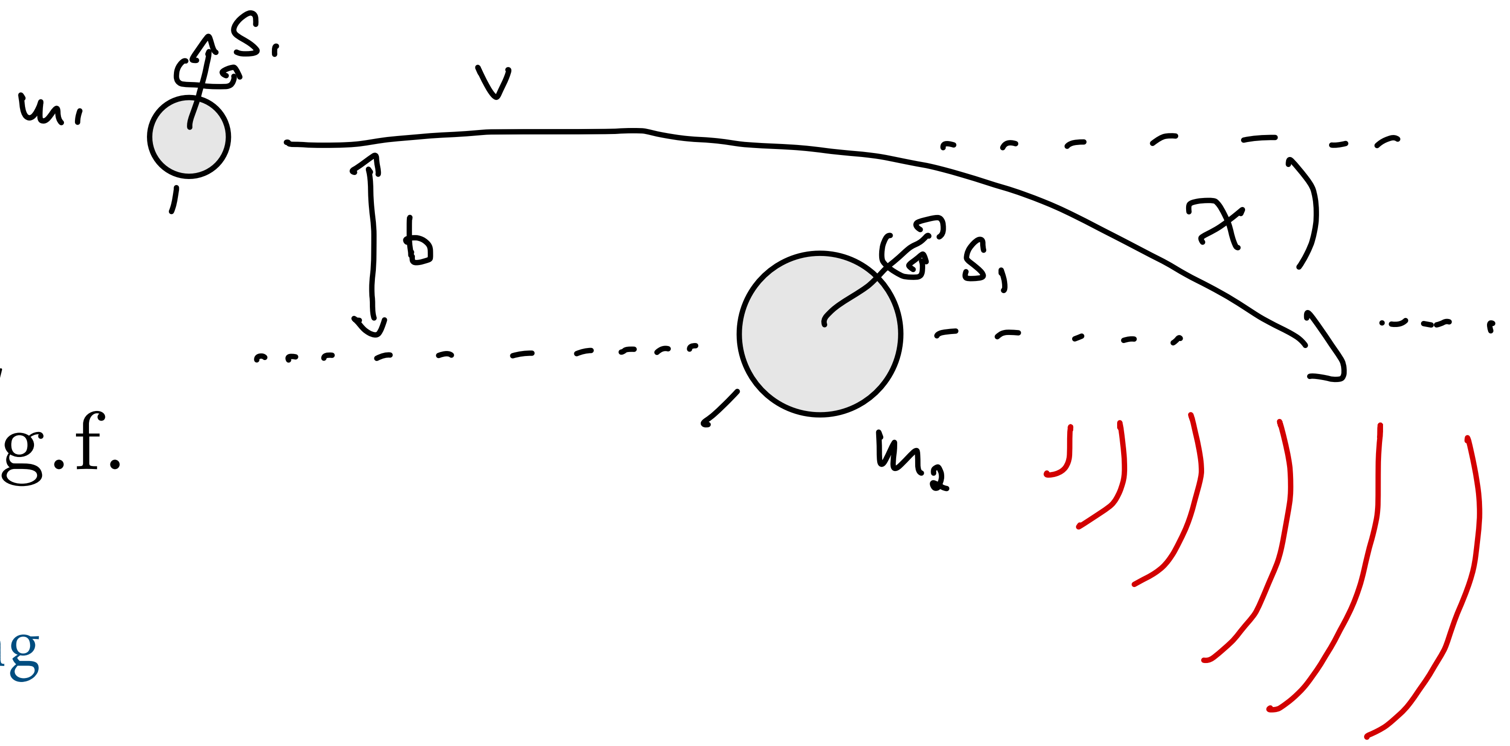
Scattering of 2 black holes or neutron stars:

Weak field (G), but exact in v^2

post-Minkowskian (PM) expansion

RELATIVISTIC TWO BODY PROBLEM IN PM: TRADITIONAL APPROACH

Point-particle approximation for BHs (or NSs)



$$S = -\sum_{i=1}^{2}\int d\tau_i\sqrt{g_{\mu\nu}\dot{x}_i^{\mu}(\tau_i)\dot{x}_i^{\nu}(\tau_i)} + \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + S_{\text{g.f.}}$$

Point particle approximation

Bulk gravity & gauge fixing

1) Equations of motion:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} = \frac{\kappa^2}{8}T_{\mu\nu}$$

Einstein's eqs.

$$\ddot{x}_i^{\mu} + \Gamma^{\mu}{}_{\nu\rho}\dot{x}_i^{\nu}\dot{x}_i^{\rho} = 0$$

Geodesic eqs.

2) Solve iteratively in G

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\sum_{n=0}^{\infty}G^n h_{\mu\nu}^{(n)}(x)$$

emitted radiation

$$x_i^{\mu}(\tau) = b_i^{\mu} + v_i^{\mu}\,\tau + \sum_{n=1}^{\infty}G^n z_i^{(n)\,\mu}(\tau)$$

straight line: „in“ state

deflections

3) Construct observables

Far field waveform:

$$\lim_{r\to\infty} h_{\mu\nu} = \frac{f_{\mu\nu}(t-r,\theta,\varphi)}{r} + \mathcal{O}(\frac{1}{r^2})$$

„Impulse“ (change in momentum):

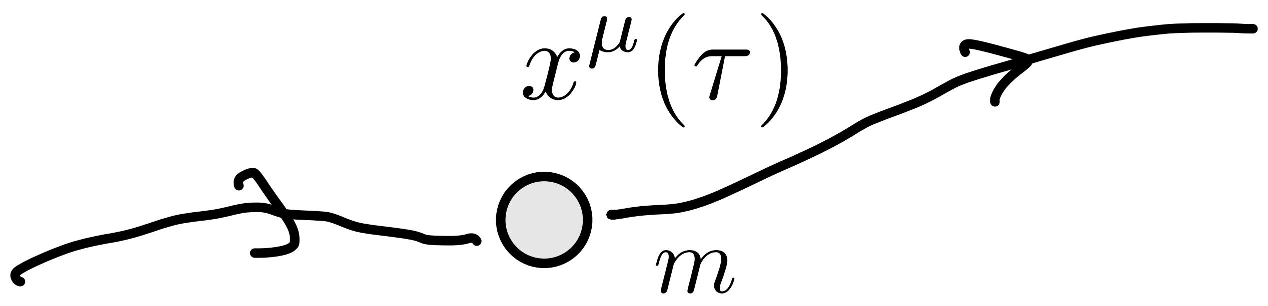
$$\Delta p_i^{\mu} = m_i\dot{x}_i^{\mu}\Big|_{\tau=-\infty}^{\tau=+\infty} = m_i\int d\tau\ddot{x}_i^{\mu}(\tau)$$

WORLDLINE QUANTUM FIELD THEORY

Mogull, **JP**, Steinhoff *JHEP* 02 (2021) 048

- Model Black Holes/Neutron Stars as a point particles

$$S_{\mathrm{BH/NS}} = -\frac{m}{2}\int d\tau g_{\mu\nu}\,\dot{x}^{\mu}(\tau)\,\dot{x}^{\nu}(\tau) \quad + \text{[spin \& tidal effects]}$$



They interact through Einstein's gravity:

$$S = S_{\mathrm{BH/NS}} + \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R(g)$$

- Scattering scenario: $x_i^{\mu}(\tau) = b_i^{\mu} + v_i^{\mu}\tau + z^{\mu}(\tau)$ $\qquad g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\,h_{\mu\nu}$

- Path integral quantisation perturbative in Newton's constant G but exact in velocity

$$\langle\mathcal{O}\rangle_{\mathrm{WQFT}} = \int D[h,z]\mathcal{O}\,e^{-\frac{i}{\hbar}S[z,h]} \xrightarrow{\hbar\to 0}$$

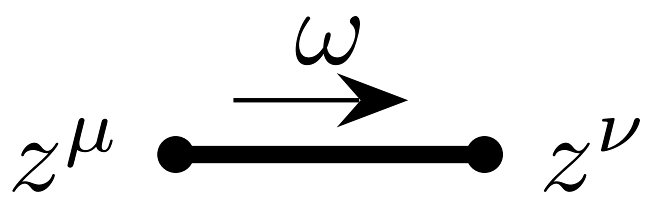
Tree-level one-point functions $\langle h_{\mu\nu}\rangle$ and $\langle z^{\mu}\rangle$ solve classical equations of motion

$\Rightarrow$ Advanced quantum field theory technology for classical gravitational wave physics

WORLDLINE QUANTUM FIELD THEORY: PROPAGATORS

$$S_{\mathrm{WQFT}} = -\frac{m}{2}\int d\tau\, g_{\mu\nu}\,\dot{x}^{\mu}(\tau)\,\dot{x}^{\nu}(\tau) + \frac{1}{16\pi\, G}\int d^4x\sqrt{-g}\,R(g)$$

- Scattering scenario: $x_i^{\mu}(\tau) = b_i^{\mu} + v_i^{\mu}\tau + z^{\mu}(\tau)$ $\qquad g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\,h_{\mu\nu}$
- Worldline propagators:



$$\langle z^{\mu}(\omega) z^{\nu}(-\omega)\rangle = -\frac{i}{m}\frac{\eta_{\mu\nu}}{(\omega+i0)^2}$$

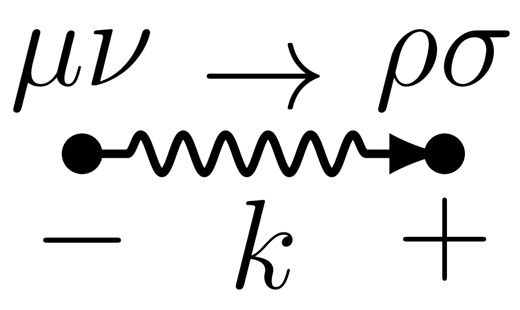
- Perturbative (quantum) gravity:

$$\sqrt{-g}\,R(g) \;\Longrightarrow\; \boxed{-\frac{1}{2}h_{\mu\nu}(P^{-1})^{\mu\nu;\rho\sigma}\Box h_{\rho\sigma} + \sqrt{G}[\partial^2 h^3] + \sqrt{G}^2[\partial^2 h^4] + \sqrt{G}^3[\partial^2 h^5] + \ldots}$$

$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\,h_{\mu\nu}$

$$P_{\mu\nu;\rho\sigma} = \eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{1}{2}\eta_{\mu\nu}\eta_{\rho\sigma}$$

⇒ graviton propagator:

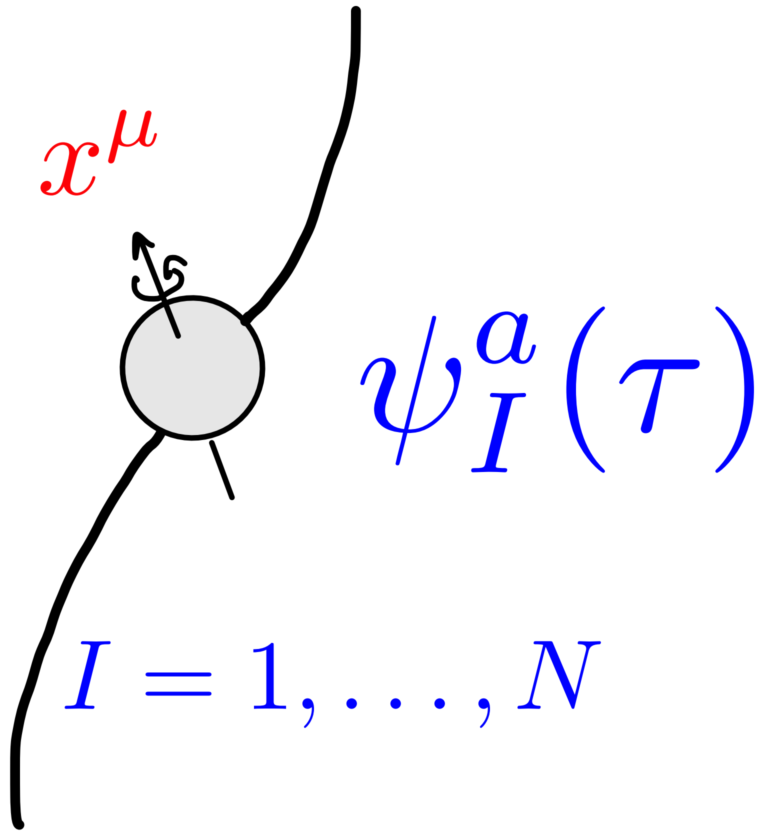


$$= i\frac{P_{\mu\nu;\rho\sigma}}{(k^0+i0)^2 - \mathbf{k}^2}$$

N.B. need to take retarded propagator (in-in formalism)

PUTTING SPIN ON THE WORLD-LINE

[Jakobsen,Mogull,JP,Steinhoff]



- Hidden **supersymmetry** of spinning-black holes!

  Add anti-commuting fields ψ^a: Captures (Spin)^N interactions

- Spin-orbit & spin-spin interactions via $N=2$ superparticle action

$$S_{\mathrm{BH/NS}} = -m\int d\tau\left[\tfrac{1}{2}g_{\mu\nu}\dot{x}^\mu\,\dot{x}^\nu + i\bar{\psi}D_\tau\psi + \tfrac{1}{2}R_{abcd}\bar{\psi}^a\psi^b\bar{\psi}^c\psi^d + C_E R_{a\mu b\nu}\dot{x}^\mu\dot{x}^\nu\bar{\psi}^a\psi^b\,\bar{\psi}\cdot\psi\right]$$

spin degrees of freedom

neutron star term

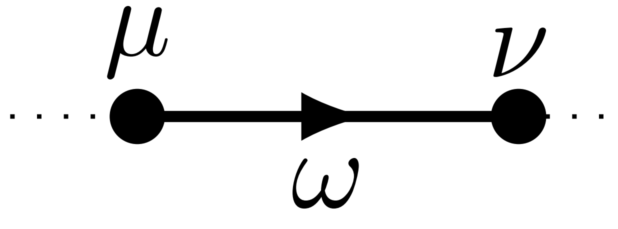
Scattering scenario:

$$x_i^\mu(\tau) = b_i^\mu + v_i^\mu\,\tau + z_i^\mu(\tau)$$
$$\psi_i^a(\tau) = \Psi_i^a + \psi_i^{\prime a}(\tau)$$

Spin tensor of BHs/NSs

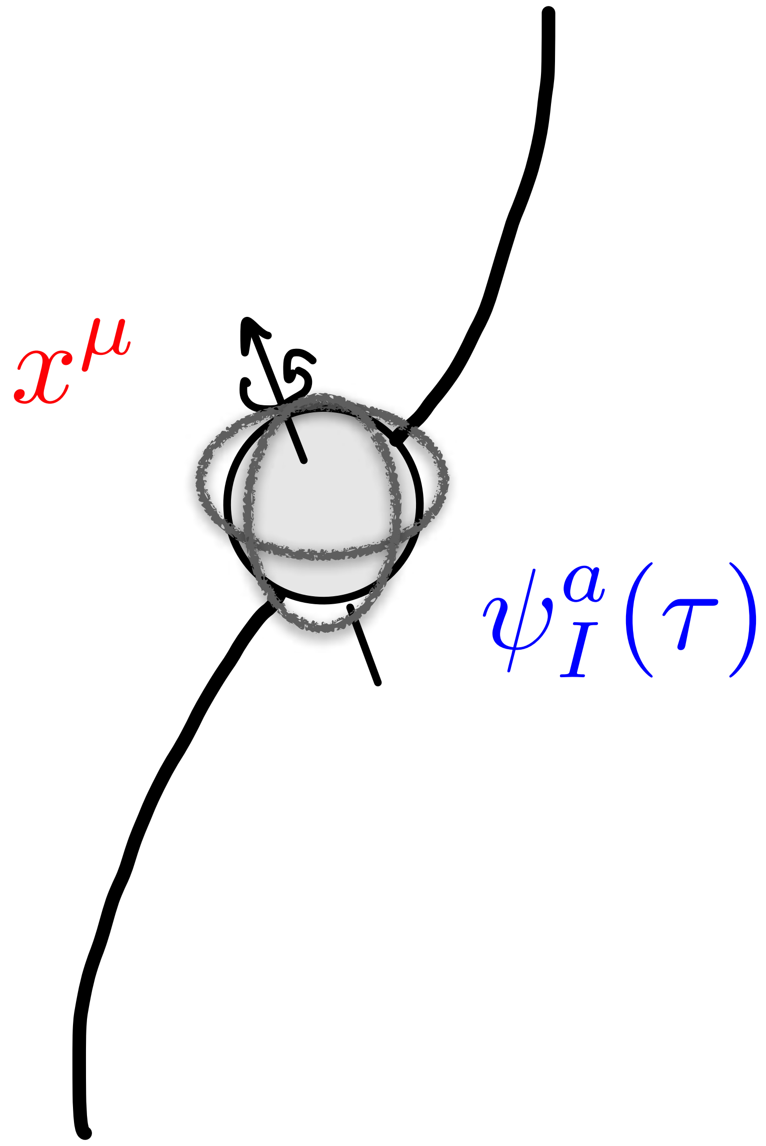
$$S_i^{ab} = -2im\bar{\psi}_i^{[a}\psi_i^{b]}$$

Quantize $z_i^\mu, \psi_i^{\prime a}, \bar{\psi}_i^{\prime a}$



$$\langle\psi^a(\omega)\bar{\psi}^b(-\omega)\rangle = \frac{-i\eta^{ab}}{m(\omega+i0)}$$

TIDAL INTERACTIONS



- First layer of tidal & finite size effects:

$$S_{\text{tidal}} = m \int d\tau \left[c_{E^2} E_{\mu\nu} E^{\mu\nu} + c_{B^2} B_{\mu\nu} B^{\mu\nu} \right]$$

Electric and magnetic curvature:

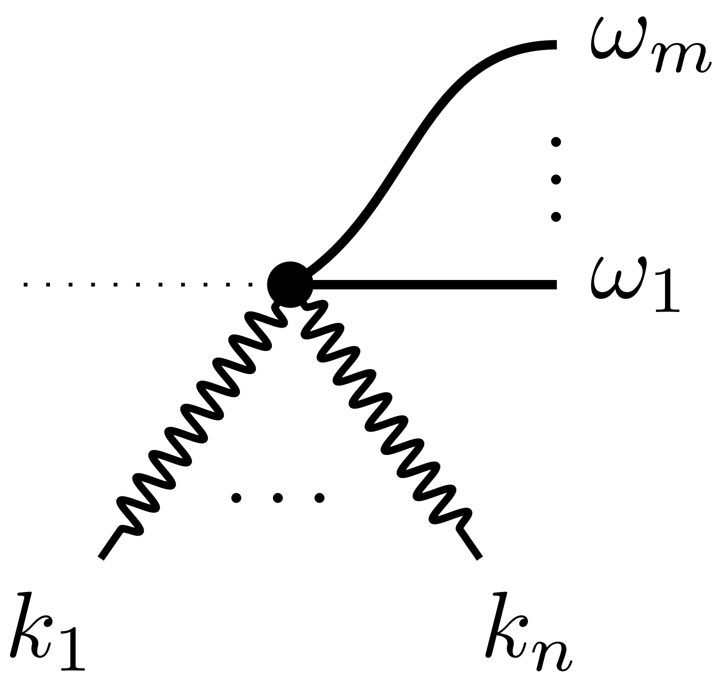
$$E_{\mu\nu} := R_{\mu\alpha\nu\beta} \dot{x}^\alpha \dot{x}^\beta \qquad B_{\mu\nu} := R^*_{\mu\alpha\nu\beta} \dot{x}^\alpha \dot{x}^\beta$$

Wilson coefficients (or „Love numbers“): c_{E^2} & c_{B^2} (vanish for black holes)

WORLDLINE QUANTUM FIELD THEORY: VERTICES

- Worldline vertices: n-gravitons & m world-line fluctuations

$V_{n|m} =$

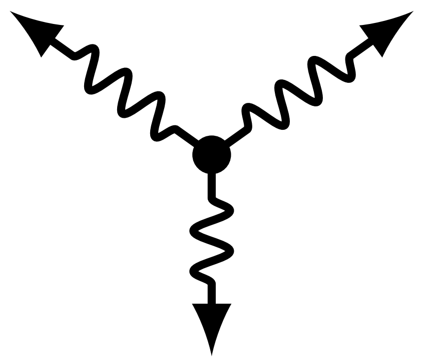


$$= m\sqrt{G}^n\, e^{ib\cdot\sum_j k_j}\,\delta\Big(v\cdot\sum_{j=1}^{n} k_j + \sum_{i=1}^{m}\omega_i\Big) \times \begin{pmatrix}\text{polynomial in } \omega_i, k_j \\ \text{of degree } 2n+m \\ \text{depending on } v^\mu,\ S^{\mu\nu}\end{pmatrix}$$

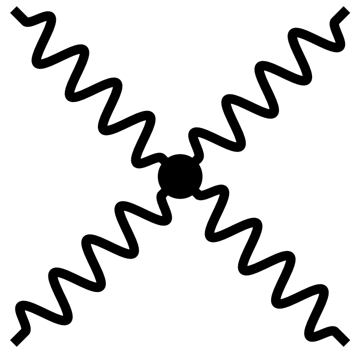
+ C_E, c_{E^2}, c_{B^2} for neutron stars

Energy conservation on worldline

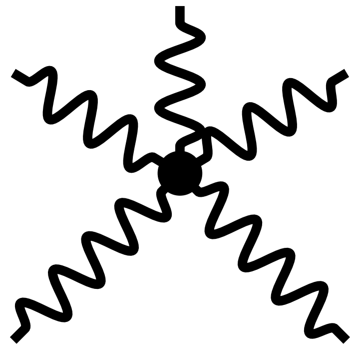
- „Bulk“ graviton vertices:



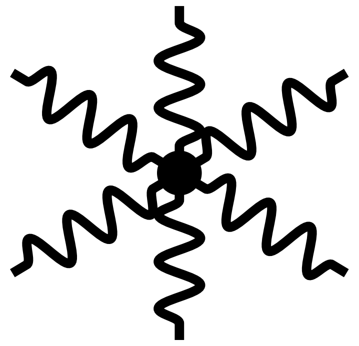
$\sim \sqrt{G}\, k^2$



$\sim \sqrt{G}^2 k^2$,



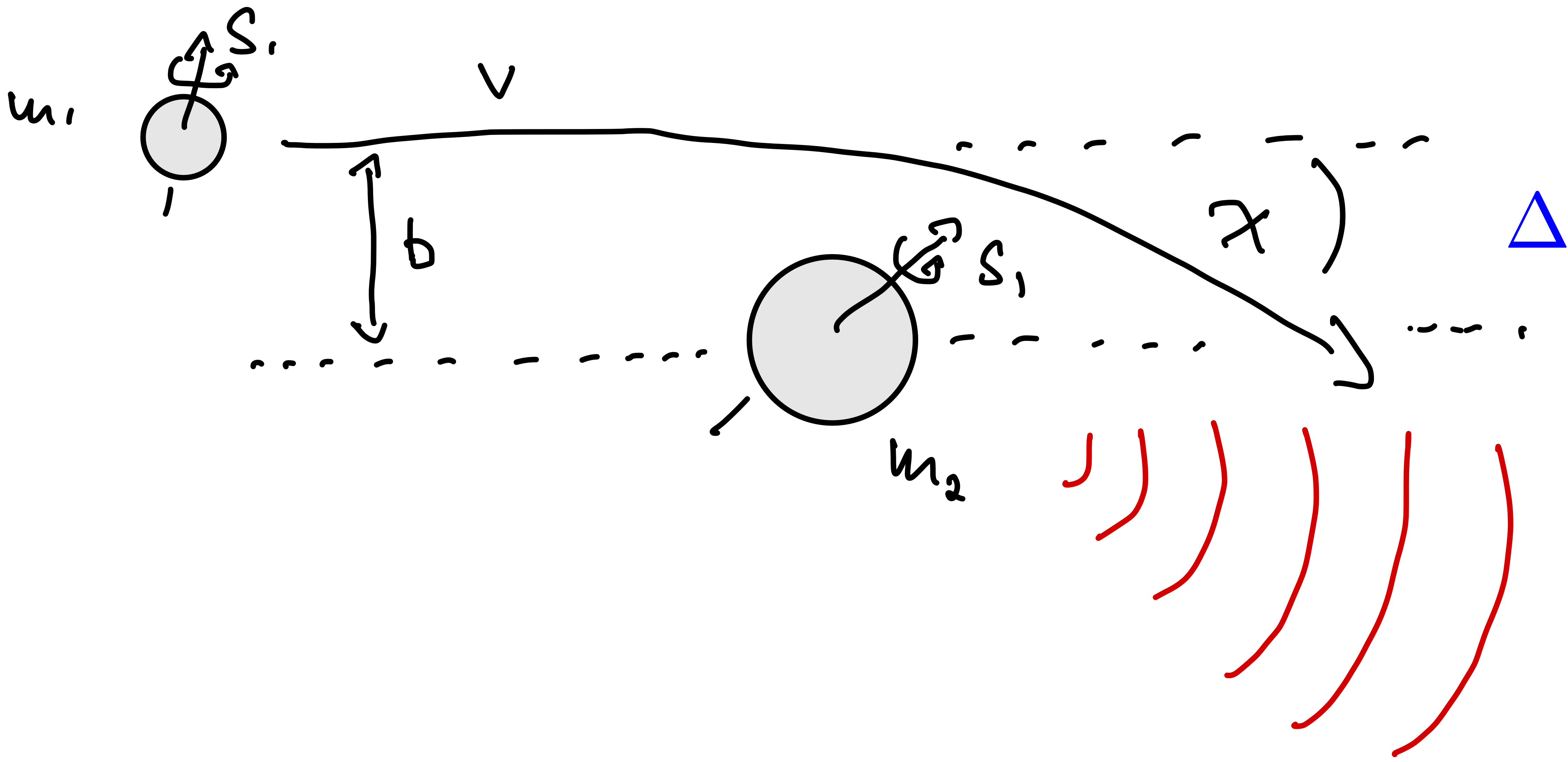
$\sim \sqrt{G}^3 k^2$



$\sim \sqrt{G}^4 k^2, \dots$

Four-momentum conservation in bulk $\delta^4(\sum p)$

WQFT OBSERVABLES

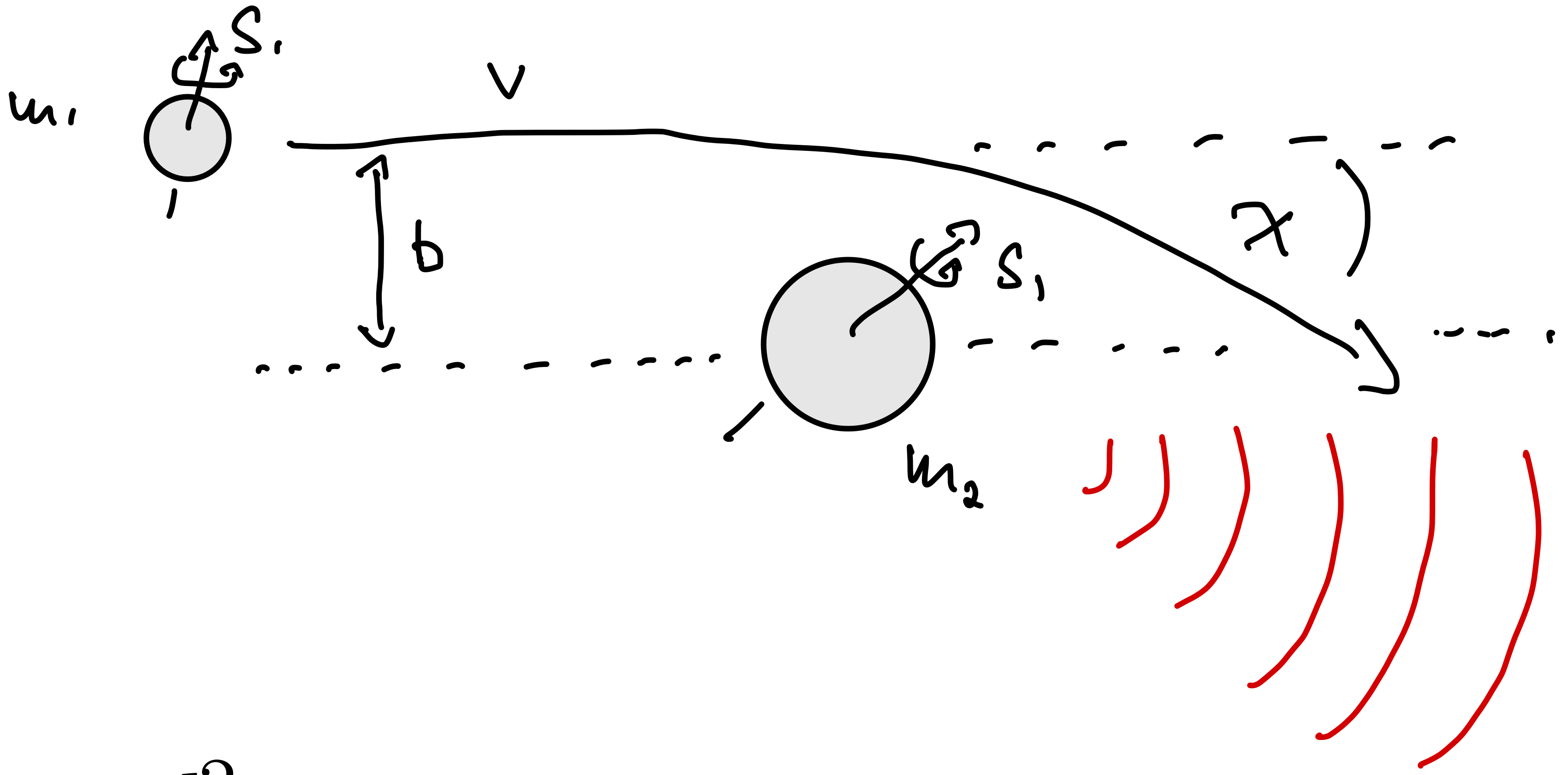


$$\Delta p_1^\mu = \sum_{n=1}^{\infty} G^n \, \Delta p_1^{(n)\,\mu}$$

$$f_{\mu\nu} = \sum_{n=1}^{\infty} G^n \, f_{\mu\nu}^{(n)}$$

OBSERVABLES OF WQFT: ONE POINT FUNCTIONS

Spin-less BH/NS scattering:



1) Impulse (change of momentum)

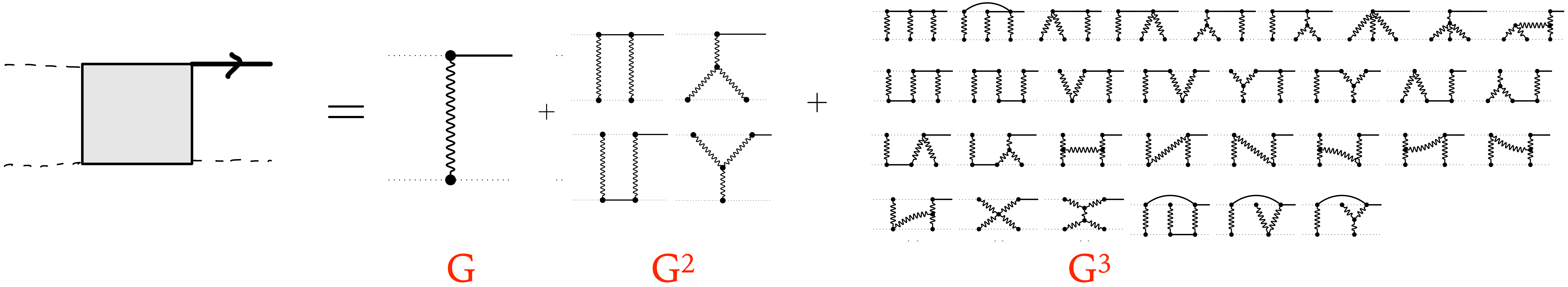
$$\Delta p_i^\mu = m_i \langle \dot{x}_i^\mu \rangle \Big|_{\tau=-\infty}^{\tau=+\infty} = m_i \int d\tau \langle \ddot{x}_i^\mu(\tau) \rangle = m_i \int d\tau \frac{d^2}{d\tau^2} \langle z_i^\mu(\tau) \rangle = -m_i \omega^2 \langle z_i^\mu(\omega) \rangle \Big|_{\omega \to 0}$$

Fourier trans.

Needs sum of all graphs with outgoing z-line:

Jakobsen, Mogull, ***PRL*** 128 *(2022) 14;* Jakobsen, Mogull, **JP**, Sauer, *JHEP* 10 (2022) 128

$$\langle \Delta p^\mu \rangle =$$

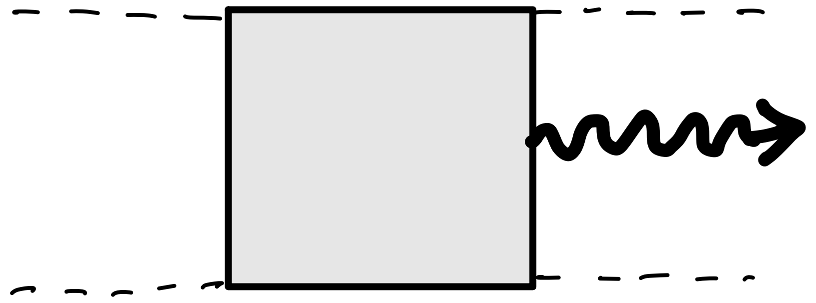


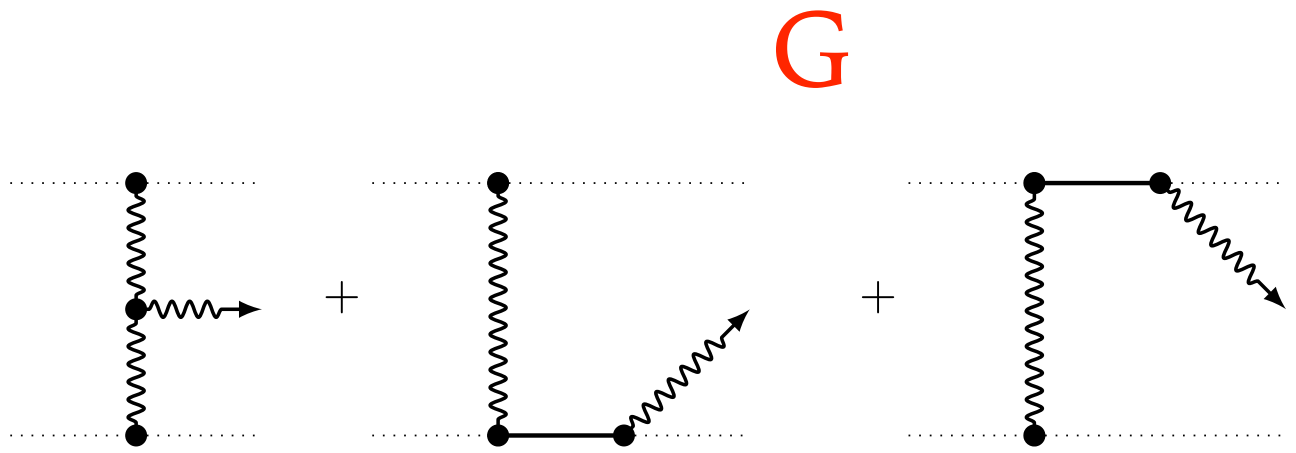
OBSERVABLES OF WQFT: ONE POINT FUNCTIONS

2) **Emitted Waveform** (Gravitational Bremsstrahlung)

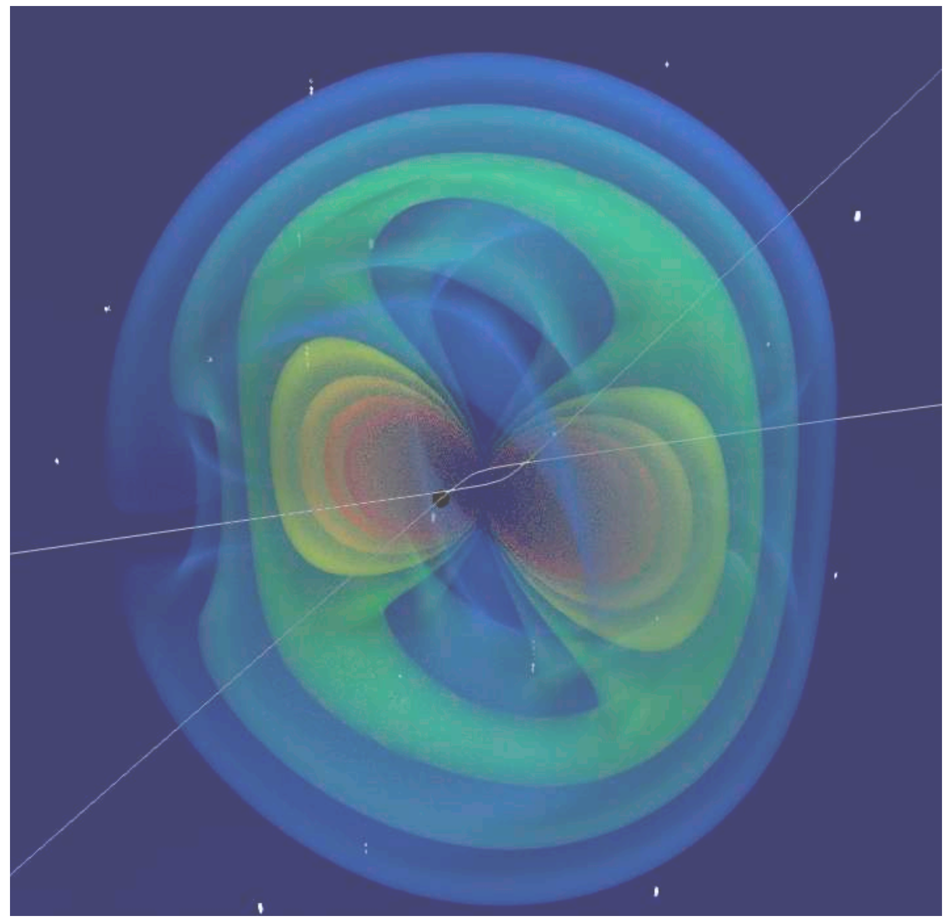
Jakobsen, Mogull, **JP**, Steinhoff, ***PRL*** 126 (2021) 20, ***PRL*** 128 (2022) 1

$$\langle h_{\mu\nu} \rangle =$$



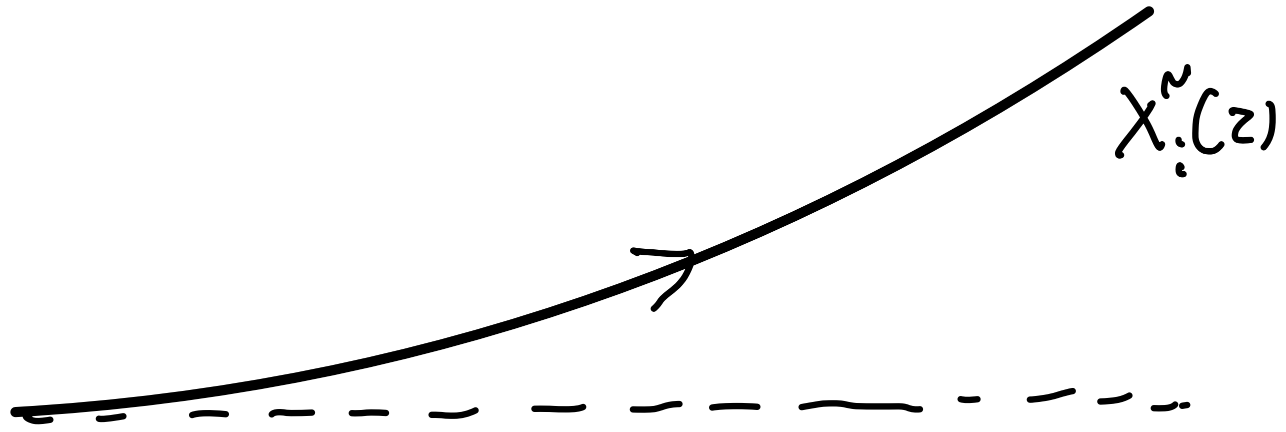


$$+ \ldots$$



3) **Trajectory!**

$$x^\mu(\tau) = b^\mu + v^\mu\,\tau + \int_\omega e^{i\omega\tau}\,\langle z^\mu(\omega)\rangle$$

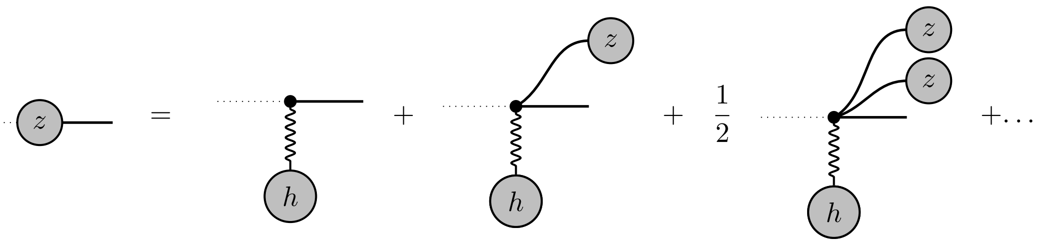


WORKFLOW WITH RETARDED INTEGRALS

[Jakobsen,Mogull,JP,Sauer]

Generate graphs

Berends-Giele recursion
(tree-level!)



↓

Insert Feynman rules

Form script
(4PM: 5g vertex)

↓

Tensor reduction to
scalar integrals

$$\int \prod_i d^D l_i \frac{l_{i_1}^{\mu_1} \dots l_{i_n}^{\mu_n}}{\prod_i D(l_i)}$$

↓ $i\epsilon$

IBPs: Basis of Master
Integral

Fire/Kira
Reduced symmetries
due to retarded prop.

$l \to -l \quad \epsilon \to -\epsilon$

→

Solve masters by Diff.
Eq. technique

Single scale
integrals

$x = \gamma - \sqrt{\gamma^2 - 1}$

↑

Fix boundary conditions
in static limit (PN) $\gamma \to 1$

Method of regions
(potential & radiation
gravitons)

↗ $i\epsilon$

$\Delta p_i^\mu \quad f_{\mu\nu}(u, \hat{x})$

ΔS_i^μ

Find same set of Master integrals for spin and tidal effects

Tensor reduction to scalar integrals

STRUCTURE OF WQFT INTEGRALS: IMPULSE & SPIN KICK

[Jakobsen,Mogull,JP,Sauer]

- Order **N-PM** : Single scale **(N-1)-loop** integral

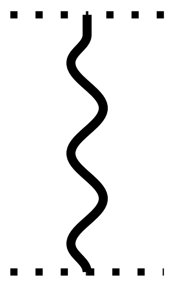
$$I_{\mathrm{nPM}} = \int_q e^{-q\cdot b}\delta(q\cdot v_1)\delta(q\cdot v_2)\int_{l_1,l_2\ldots l_{n-1}} \frac{\mathrm{num}[l_i]}{D_1\ldots D_j}\delta(l_1\cdot v_*)\delta(l_2\cdot v_*)\ldots\delta(l_{n-1}\cdot v_*)$$

$$v_* \in \{v_1, v_2\}$$

Retarded propagators $D_i(l_i, q, v_*)$ are linear $(l_i \cdot v_*) \pm i0$ or quadratic $(l_i + q)^2$

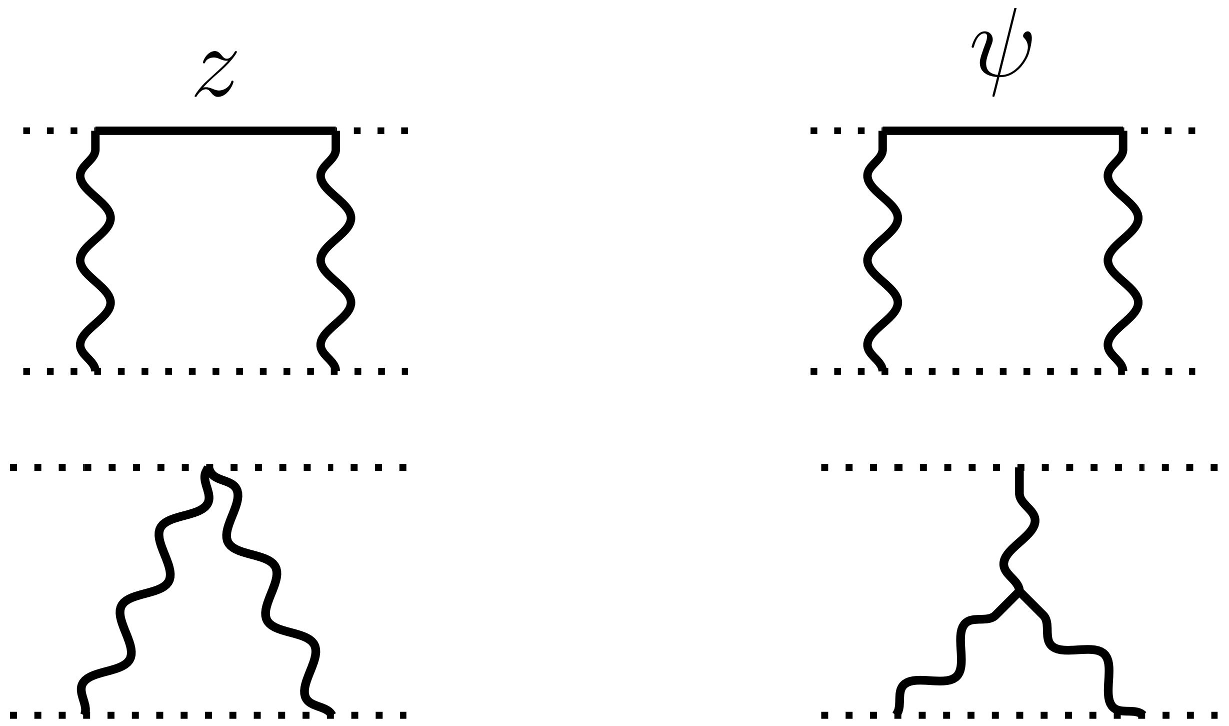
Depend on single dimensionful scale q —> left with **single parameter** integral $\gamma = v_1 \cdot v_2$

- **1PM: Trivial - pure Fourier transform**



$$\sim Gm_1m_2$$

- **2PM: 1-loop**



$$\sim G^2m_1m_2^2$$

$$D_1 = l\cdot v_1 \pm i\epsilon\ ,$$
$$D_2 = l^2\ ,$$
$$D_3 = (l+q)^2\ .$$

$$\int_l \frac{\mathrm{num}[l]}{D_1D_2D_3}\delta(l\cdot v_*)$$

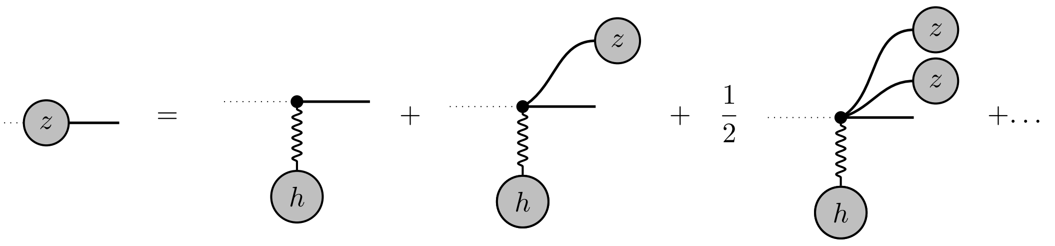
WORKFLOW WITH RETARDED INTEGRALS

[Jakobsen,Mogull,JP,Sauer]

Generate graphs

Berends-Giele recursion

(tree-level!)



Insert Feynman rules

Form script

(4PM: 5g vertex)

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$$\int \prod_i d^D l_i \frac{l_{i_1}^{\mu_1} \dots l_{i_n}^{\mu_n}}{\prod_i D(l_i)}$$

$i\epsilon$

IBPs: Basis of Master Integral

Fire/Kira

Reduced symmetries due to retarded prop.

$l \to -l \quad \epsilon \to -\epsilon$

Solve masters by Diff. Eq. technique

Single scale integrals

$$x = \gamma - \sqrt{\gamma^2 - 1}$$

Fix boundary conditions in static limit (PN) $\gamma \to 1$

Method of regions (potential & radiation gravitons)

$i\epsilon$

$\Delta p_i^\mu \quad f_{\mu\nu}(u, \hat{x})$

ΔS_i^μ

Find same set of Master integrals for spin and tidal effects

RESULT IMPULSE @ 3PM PRECISION:

[Jakobsen,Mogull,JP,Sauer]

$$\boxed{\Delta p_1^\mu = p_\infty \sin\theta \frac{b^\mu}{|b|} + (\cos\theta - 1)\frac{m_1 m_2}{E^2}\left[(\gamma m_1 + m_2)v_1^\mu - (\gamma m_2 + m_1)v_2^\mu\right] - v_2 \cdot P_{\rm rad}\, w_2^\mu}$$

$$w_1^\mu = \frac{\gamma v_2^\mu - v_1^\mu}{\gamma^2 - 1}$$

$$\gamma = v_1 \cdot v_2$$

- Scattering angle:

$$\frac{\theta}{\Gamma} = \underbrace{\frac{GM}{|b|}\frac{2(2\gamma^2-1)}{\gamma^2-1}}_{\text{1PM}} + \underbrace{\left(\frac{GM}{|b|}\right)^2 \frac{3\pi(5\gamma^2-1)}{4(\gamma^2-1)}}_{\text{2PM}} + \underbrace{\left(\frac{GM}{|b|}\right)^3 \left(2\frac{64\gamma^6 - 120\gamma^4 + 60\gamma^2 - 5}{3(\gamma^2-1)^3}\Gamma^2 - \frac{8\nu\gamma(14\gamma^2+25)}{3(\gamma^2-1)} - 8\nu\frac{(4\gamma^4 - 12\gamma^2 - 3)}{(\gamma^2-1)}\frac{\text{arccosh}\gamma}{\sqrt{\gamma^2-1}}\right)}_{\text{3PM conservative}}$$

$$+ \underbrace{\left(\frac{GM}{|b|}\right)^3 \frac{4\nu(2\gamma^2-1)^2}{(\gamma^2-1)^{3/2}}\left(-\frac{8}{3} + \frac{1}{v^2} + \frac{(3v^2-1)}{v^3}\text{arccosh}\gamma\right)}_{\text{3PM radiation-reaction}}$$

$$\Gamma = E/M = \sqrt{1 + 2\nu(\gamma - 1)}$$

$$\nu = \frac{m_1 m_2}{M^2}$$

- Radiated 4-momentum: $P^\mu_{\rm rad} = -\Delta p_1^\mu - \Delta p_2^\mu$ Dissipation! Need for retarded propagators

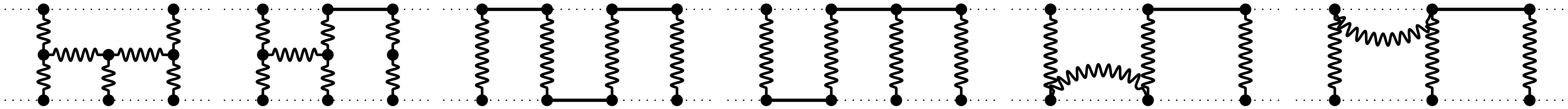
$$P^\mu_{\rm rad} = \frac{G^3 m_1^2 m_2^2 \pi}{|b|^3}\frac{v_1^\mu + v_2^\mu}{\gamma + 1}\left[e_1 + e_2 \log\left(\frac{\gamma+1}{2}\right) + e_3 \frac{\text{arccosh}\gamma}{\sqrt{\gamma^2-1}}\right]$$

$$e_1 = \frac{210\gamma^6 - 552\gamma^5 + 339\gamma^4 - 912\gamma^3 + 3148\gamma^2 - 3336\gamma + 1151}{48(\gamma^2-1)^{3/2}}$$

$$e_2 = -\frac{35\gamma^4 + 60\gamma^3 - 150\gamma^2 + 76\gamma - 5}{8\sqrt{\gamma^2-1}},$$

$$e_3 = \frac{\gamma(2\gamma^2-3)(35\gamma^4 - 30\gamma^2 + 11)}{16(\gamma^2-1)^{3/2}}.$$

4PM IMPULSE:



[Jakobsen,Mogull,JP,Sauer]

- Two integral families:

$$I^{[i](\sigma_1,\sigma_2,\ldots,\sigma_8)}_{n_1,n_2,\ldots,n_{12}} = \int_{l_1,l_2,l_3} \frac{\delta(l_1\cdot v_1)\delta(l_2\cdot v_1)\delta(l_3\cdot v_i)}{D_1^{n_1} D_2^{n_2} \ldots D_{12}^{n_{12}}}$$

$$D_j = l_j \cdot v_{i_j} + i0^+ \sigma_j$$

$$D_4 = -(l_1 + l_2 + l_3 + q)^2 - i0^+ \sigma_4 v \cdot (l_1 + l_2 + l_3)$$

$$D_5 = -(l_1 + l_2 + q)^2 - i0^+ \sigma_5 v \cdot (l_1 + l_2)$$

$$D_{5+k} = -(l_k + l_3)^2 - i0^+ \sigma_{6+k} v \cdot (l_k + l_3)$$

$$D_{7+j} = -l_j^2 \,, \qquad D_{10+k} = -(l_k + q)^2 \,.$$

$$J^{(\sigma_1,\sigma_2,\ldots,\sigma_5)}_{n_1,n_2,\ldots,n_{12}} := \int_{\ell_1,\ell_2,\ell_3} \frac{\delta(\ell_1\cdot v_1)\delta(\ell_2\cdot v_1)\delta(\ell_3\cdot v_2)}{D_1^{n_1} D_2^{n_2} \ldots D_{12}^{n_{12}}}$$

$$D_j = \ell_j \cdot v_i + i0^+ \sigma_j$$

$$D_4 = -(\ell_1 - \ell_3)^2 - i0^+ \sigma_4 v \cdot (\ell_1 - \ell_3)$$

$$D_5 = -(\ell_2 - \ell_3)^2 - i0^+ \sigma_5 v \cdot (\ell_2 - \ell_3)$$

$$D_6 = -(\ell_1 - \ell_2)^2 \,, \quad D_{6+j} = -\ell_j^2 \,, \quad D_{9+j} = -(\ell_j + q)^2$$

- Number of master integrals: I-type 23+23, J-type 64+66

- Function space:

$$F^{(b)}_\alpha(\gamma) = \{1, \mathrm{arccosh}[\gamma], \log[\gamma], \log\left[\frac{\gamma_\pm}{2}\right], \mathrm{arccosh}^2[\gamma], \mathrm{arccosh}[\gamma]\log\left[\frac{\gamma_\pm}{2}\right], \log\left[\frac{\gamma_+}{2}\right]\log\left[\frac{\gamma_-}{2}\right], \log^2\left[\frac{\gamma_+}{2}\right],$$
$$\mathrm{Li}_2\left[\pm\frac{\gamma_-}{\gamma_+}\right], \mathrm{Li}_2\left[\sqrt{\frac{\gamma_-}{\gamma_+}}\right], \mathrm{K}^2\left[\frac{\gamma_-}{\gamma_+}\right], \mathrm{E}^2\left[\frac{\gamma_-}{\gamma_+}\right], \mathrm{K}\left[\frac{\gamma_-}{\gamma_+}\right]\mathrm{E}\left[\frac{\gamma_-}{\gamma_+}\right]\}$$

Elliptic functions appear!

$$\gamma = v_1 \cdot v_2 \qquad (\gamma_\pm := \gamma \pm 1)$$

RESULT SPINNING IMPULSE @ 4PM PRECISION: CONSERVATIVE SECTOR

[Jakobsen,Mogull,JP,Sauer]

- Scattering angle:

$$\boxed{\theta^{(4,1)}_{\text{cons}} = \sum_{\alpha=1}^{16} \pi\nu\left(s_+ h^{(+)}_\alpha(\gamma) + \delta\, s_- h^{(-)}_\alpha(\gamma)\right) F^{(b)}_\alpha(\gamma) \; - \frac{21\pi\gamma\left(33\gamma^4 - 30\gamma^2 + 5\right)\left(13 s_+ - 3\delta s_-\right)}{32\left(\gamma^2-1\right)^{5/2}}}$$

$$\gamma = v_1 \cdot v_2$$

$$\delta = (m_2 - m_1)/M$$

$$\nu = m_1 m_2/M^2$$

Spin-orbit coupling: $s_\pm = -(a_1 \pm a_2)\cdot\hat{L}$

Function basis:

$$F^{(b)}_\alpha(\gamma) = \Big\{1, \text{arccosh}[\gamma], \log[\gamma], \log\left[\frac{\gamma_+}{2}\right], \log\left[\frac{\gamma_-}{2}\right], \text{arccosh}^2[\gamma], \text{arccosh}[\gamma]\log\left[\frac{\gamma_+}{2}\right], \text{arccosh}[\gamma]\log\left[\frac{\gamma_-}{2}\right],$$
$$\log\left[\frac{\gamma_+}{2}\right]\log\left[\frac{\gamma_-}{2}\right], \log^2\left[\frac{\gamma_+}{2}\right], \text{Li}_2\left[\frac{\gamma_-}{\gamma_+}\right], \text{Li}_2\left[-\frac{\gamma_-}{\gamma_+}\right], \text{Li}_2\left[\sqrt{\frac{\gamma_-}{\gamma_+}}\right], \text{K}^2\left[\frac{\gamma_-}{\gamma_+}\right], \text{E}^2\left[\frac{\gamma_-}{\gamma_+}\right], \text{K}\left[\frac{\gamma_-}{\gamma_+}\right]\text{E}\left[\frac{\gamma_-}{\gamma_+}\right]$$

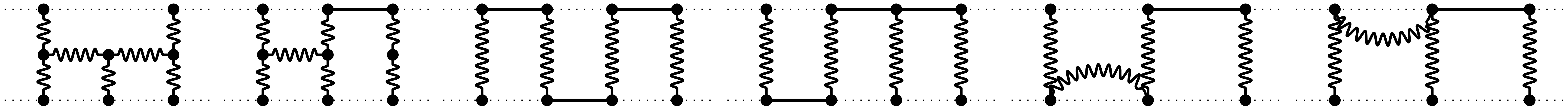
Coefficients:

$$h^{(+)}_1 = \frac{3\pi^2(\gamma+1)^2(1225\gamma^8 + 1225\gamma^7 - 1875\gamma^6 - 1875\gamma^5 + 795\gamma^4 + 3035\gamma^3 - 3601\gamma^2 + 1775\gamma - 448)}{192(\gamma+1)^3\sqrt{\gamma^2-1}} - \frac{1}{192\gamma^8(\gamma+1)(\gamma^2-1)^{5/2}}\Big(22050\gamma^{19}$$
$$+ 33075\gamma^{18} - 71725\gamma^{17} - 123397\gamma^{16} + 186555\gamma^{15} + 67503\gamma^{14} - 89885\gamma^{13} - 190167\gamma^{12} + 181103\gamma^{11} + 137042\gamma^{10} - 506830\gamma^9$$
$$+ 407004\gamma^8 - 33671\gamma^7 - 33671\gamma^6 + 8501\gamma^5 + 8501\gamma^4 - 1885\gamma^3 - 1885\gamma^2 + 315\gamma + 315\Big)$$

$$h^{(-)}_1 = \frac{3\pi^2(-1225\gamma^8 - 1225\gamma^7 + 1875\gamma^6 + 1875\gamma^5 - 795\gamma^4 - 1115\gamma^3 + 401\gamma^2 - 111\gamma + 64)(\gamma+1)^2}{192(\gamma+1)^3\sqrt{\gamma^2-1}} + \frac{1}{192\gamma^8(\gamma+1)(\gamma^2-1)^{5/2}}\Big(22050\gamma^{19}$$
$$+ 33075\gamma^{18} - 71725\gamma^{17} - 115333\gamma^{16} + 96699\gamma^{15} + 140871\gamma^{14} - 56261\gamma^{13} - 73191\gamma^{12} - 6593\gamma^{11} + 27498\gamma^{10} - 3718\gamma^9$$
$$+ 9004\gamma^8 - 1491\gamma^7 - 1491\gamma^6 + 313\gamma^5 + 313\gamma^4 + 95\gamma^3 + 95\gamma^2 - 105\gamma - 105\Big)$$

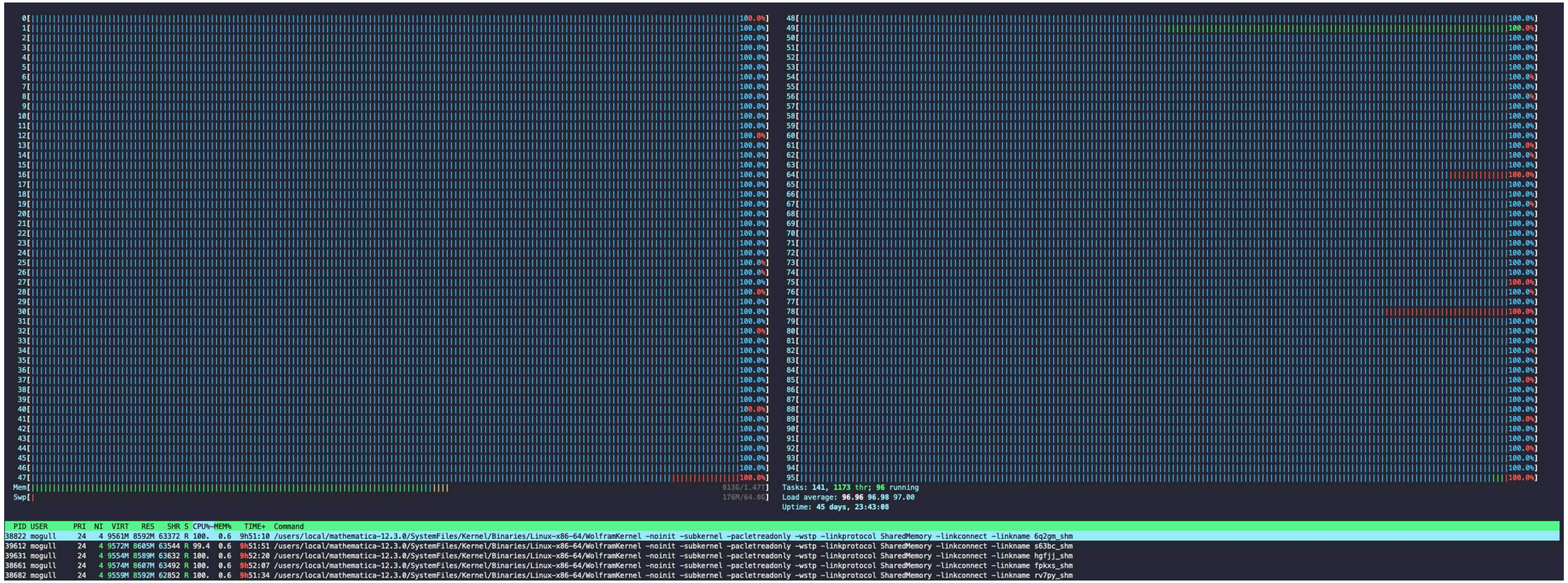
....

STATS FOR 4PM:



[Jakobsen,Mogull,JP,Sauer]

- High Performance Computing Needed:



```
Mem[                                   813G/1.47T]    Tasks: 141, 1173 thr; 96 running
Swp[                                   176M/64.0G]    Load average: 96.96 96.98 97.00
                                                      Uptime: 45 days, 23:43:08

  PID USER      PRI  NI  VIRT   RES   SHR S CPU%-MEM%   TIME+  Command
38822 mogull     24   4 9561M 8592M 63372 R 100.  0.6  9h51:10 /users/local/mathematica-12.3.0/SystemFiles/Kernel/Binaries/Linux-x86-64/WolframKernel -noinit -subkernel -pacletreadonly -wstp -linkprotocol SharedMemory -linkconnect -linkname 6q2gm_shm
39612 mogull     24   4 9572M 8605M 63544 R 99.4  0.6  9h51:51 /users/local/mathematica-12.3.0/SystemFiles/Kernel/Binaries/Linux-x86-64/WolframKernel -noinit -subkernel -pacletreadonly -wstp -linkprotocol SharedMemory -linkconnect -linkname s63bc_shm
39631 mogull     24   4 9554M 8589M 63632 R 100.  0.6  9h52:20 /users/local/mathematica-12.3.0/SystemFiles/Kernel/Binaries/Linux-x86-64/WolframKernel -noinit -subkernel -pacletreadonly -wstp -linkprotocol SharedMemory -linkconnect -linkname hgfjj_shm
38661 mogull     24   4 9574M 8607M 63492 R 100.  0.6  9h52:07 /users/local/mathematica-12.3.0/SystemFiles/Kernel/Binaries/Linux-x86-64/WolframKernel -noinit -subkernel -pacletreadonly -wstp -linkprotocol SharedMemory -linkconnect -linkname fpkxs_shm
38682 mogull     24   4 9559M 8592M 62852 R 100.  0.6  9h51:34 /users/local/mathematica-12.3.0/SystemFiles/Kernel/Binaries/Linux-x86-64/WolframKernel -noinit -subkernel -pacletreadonly -wstp -linkprotocol SharedMemory -linkconnect -linkname rv7py_shm
```

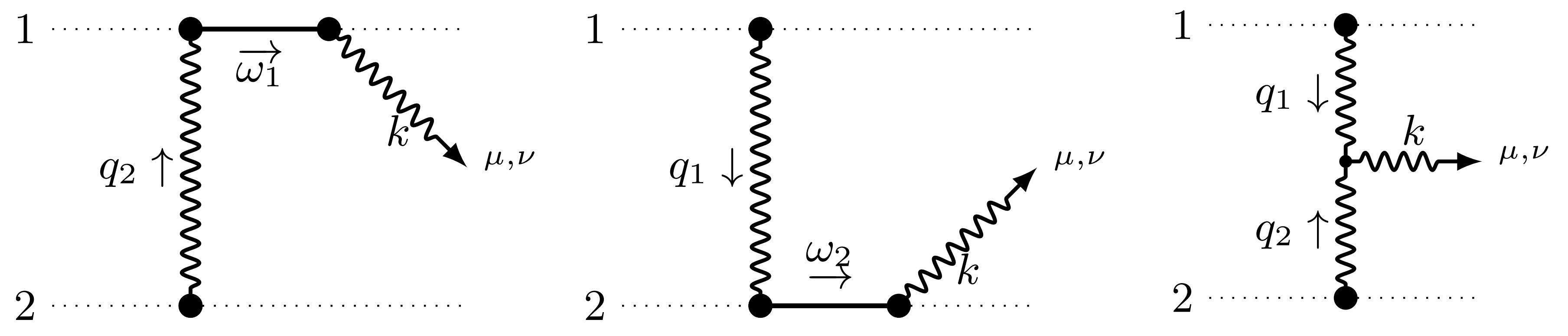
- Number of diagrams: 201 Non-spinning, 529 spin-orbit
- Some 10.000 integrals need to be reduced to master integrals
- Number of master integrals: I-type 23+23, J-type 64+66
- Analytic! Using Mathematica, FORM, Fire & KIRA

FAR FIELD WAVEFORM @ NLO

[Jakobsen,Mogull,JP,Steinhoff]

Sum of diagrams with outgoing graviton:

$$\langle h_{\mu\nu}(k)\rangle =$$



For time-domain waveform needs to integrate over outgoing energy : Ω

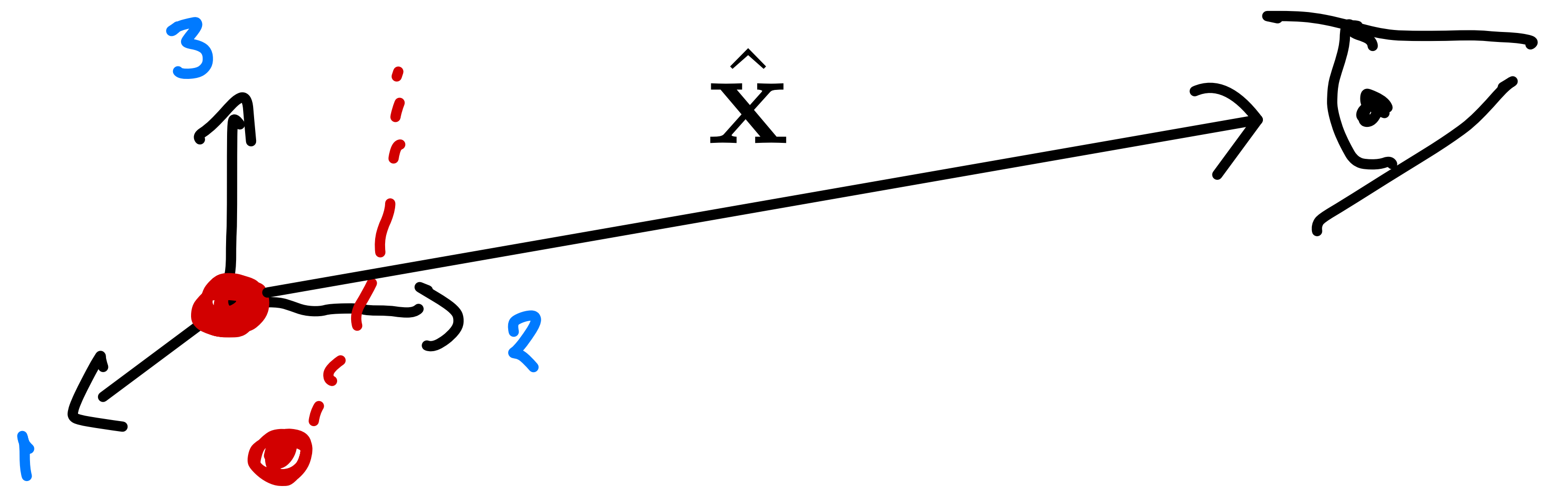
$$\frac{f_{+,\times}(t-r,\hat{\mathbf{x}})}{r} = \frac{4G}{r}\int d\Omega\, e^{-i\Omega(t-r)}\,\varepsilon^{\mu\nu}_{+,\times}\langle h_{\mu\nu}(k=\Omega(1,\hat{\mathbf{x}}))\rangle$$

where unit vector $\hat{\mathbf{x}}$ points towards the observer

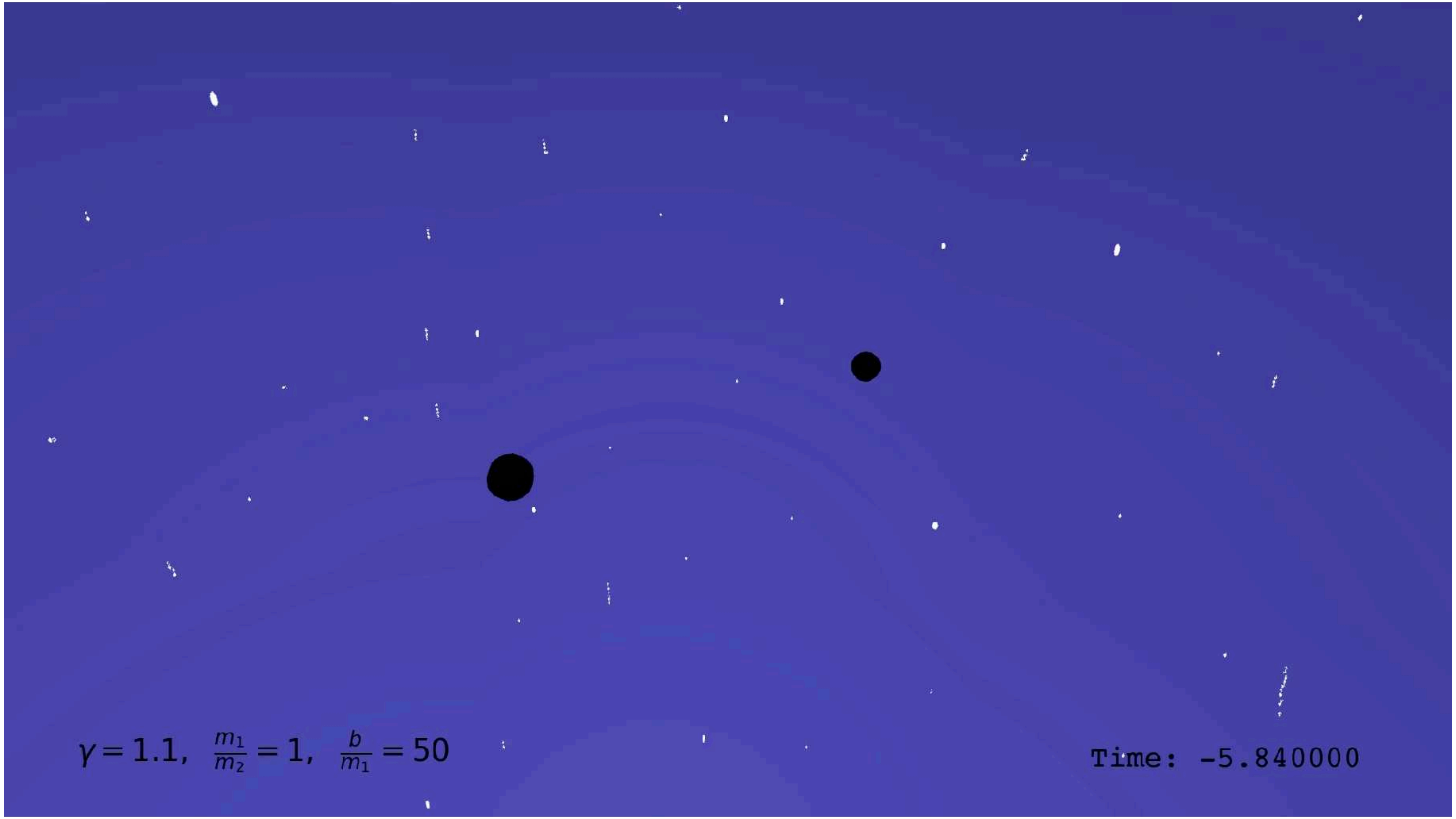
The waveform has two polarizations

$$f_{+,\times}(\underbrace{t-r}_{u},\underbrace{\theta,\phi}_{\hat{\mathbf{x}}};v,|b|,m_1,m_2)$$

retarded time



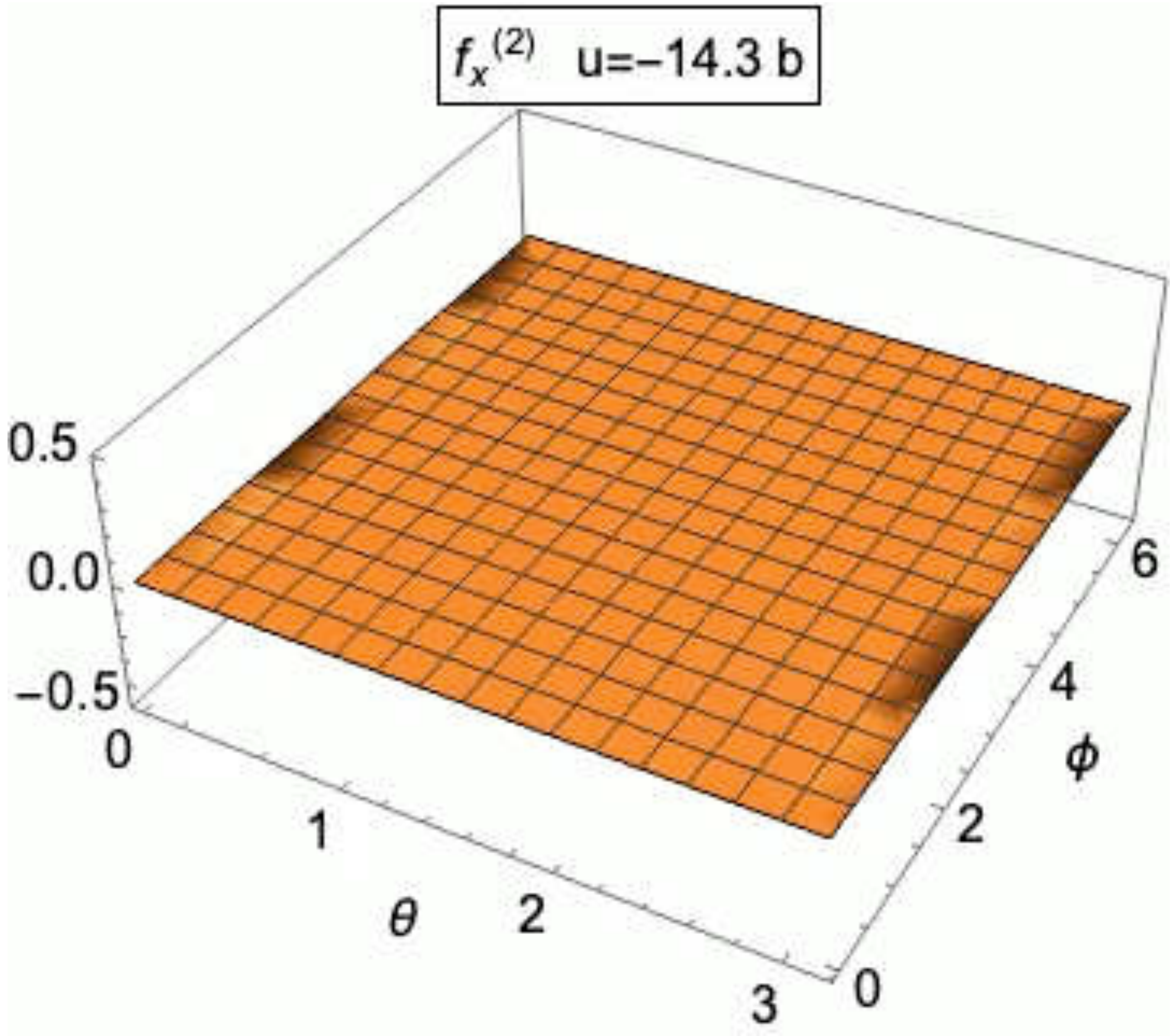
Visualization: Plus-Polarization $f_+^{(2)}$

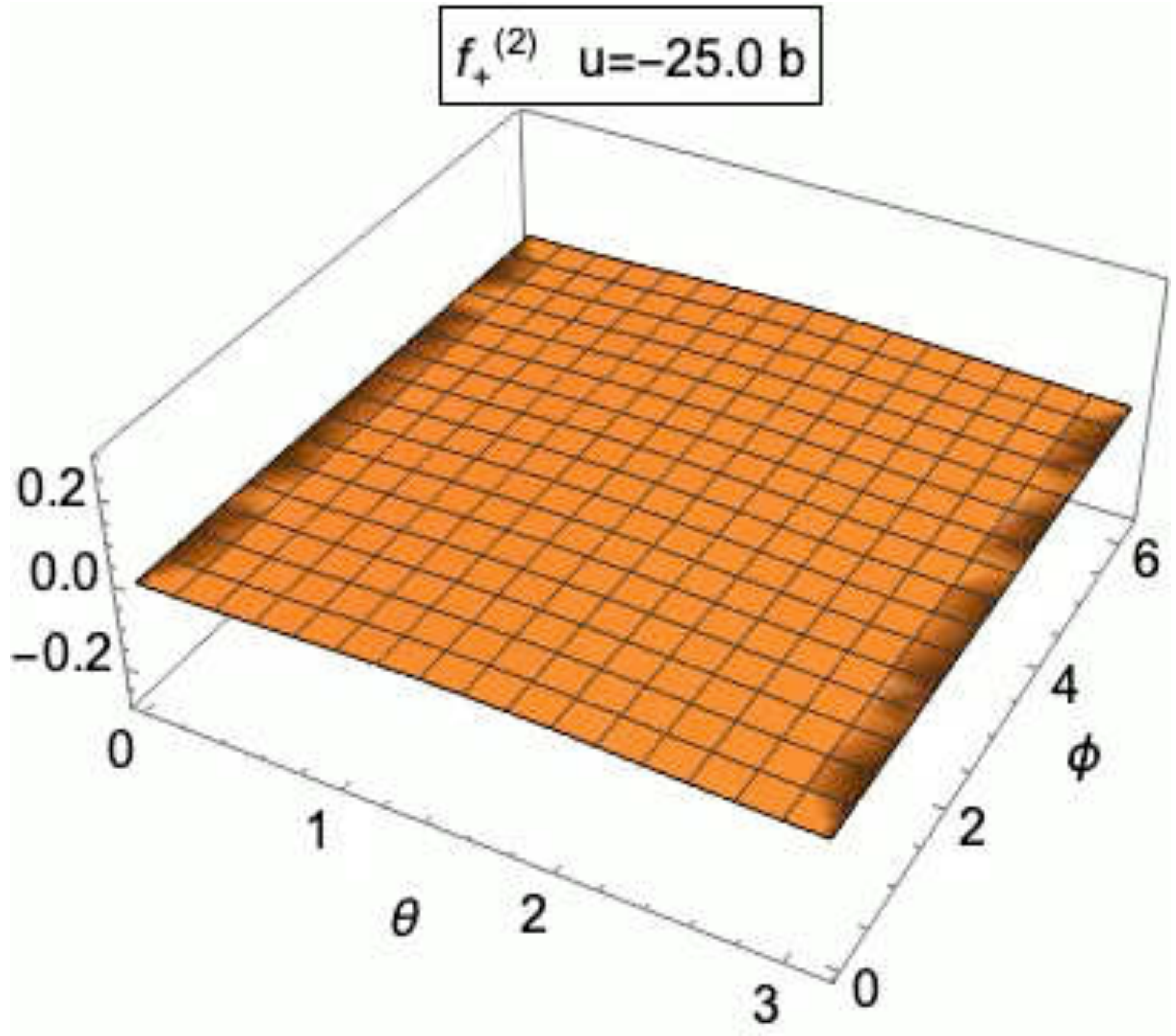


$v = 0.4$

[HU/AEI BSc thesis Babayemi]

Memory effect



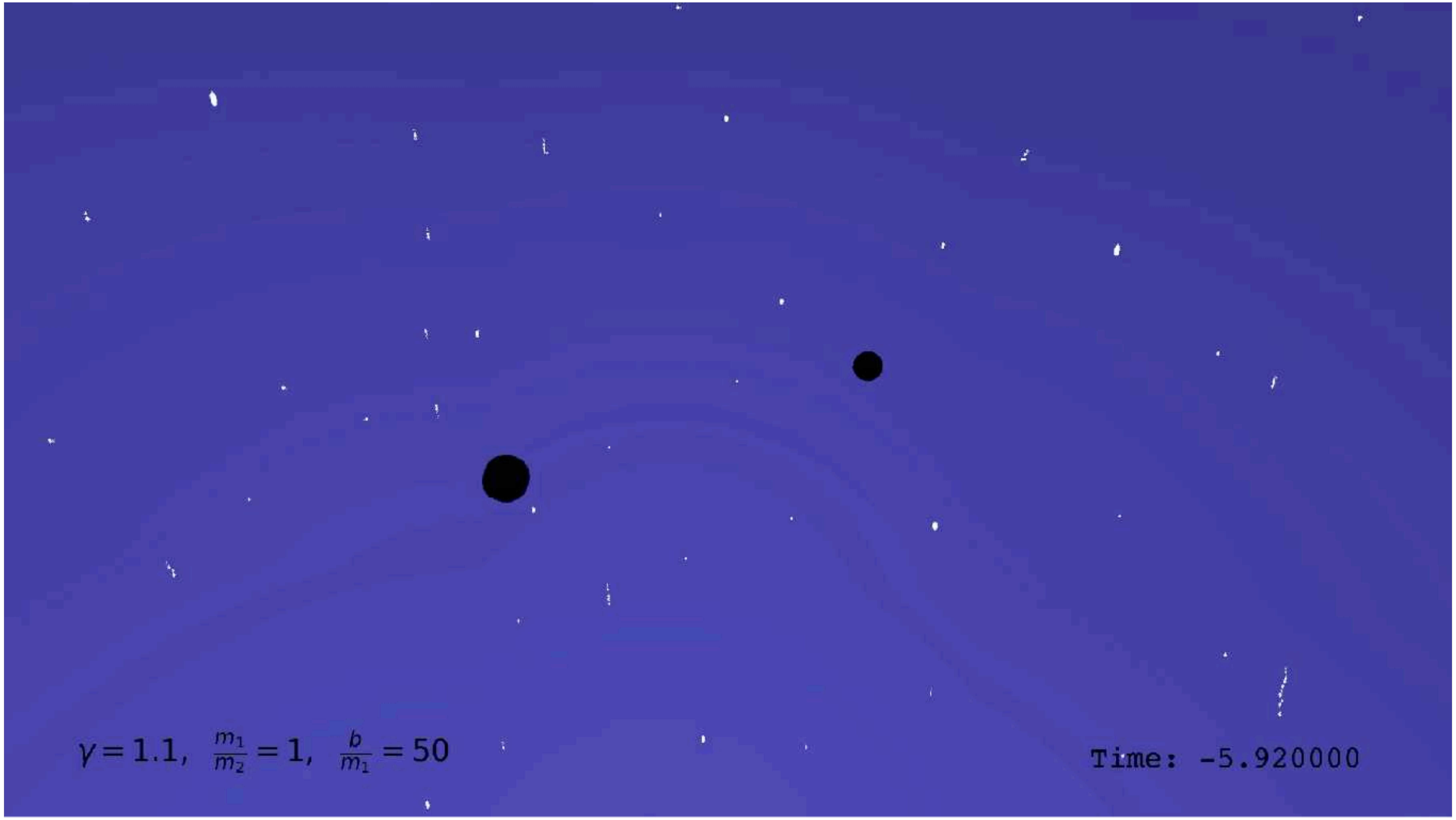


$$\frac{\Delta f^{(2)}_{\mathcal{S}=0}}{m_1 m_2} = \frac{4(2\gamma^2-1)\epsilon\cdot v_1(2b\cdot\epsilon\,\rho\cdot v_1 - b\cdot\rho\,\epsilon\cdot v_1)}{|b|^2\sqrt{\gamma^2-1}(\rho\cdot v_1)^2}$$

$$\rho = (1,\hat{\mathbf{x}})$$

$$\gamma = v_1\cdot v_2$$

Visualization: Plus-Polarization curvature



$$v = 0.4$$

More visualisations at: https://www.youtube.com/channel/UC5UVcydoMoG7ILkJo9bik1w

PM STATE-OF-THE-ART

WQFT
[us]

WEFT **Worldline effective theory**
[Källin,Porto,Dlapa,Cho,Liu,..]
[Riva,Vernizzi,Mougiakakos..]

HEFT **Heavy BH effective theory**
[Aoude,Haddad,Helset,Damgaard]
[Brandhuber,Travaglini,Chen]

Amps **Scattering amplitudes**
[Bern,Roiban,Shen,Parra-Martinez,Ruf,Zeng..]
[Bjerrum-Bohr,Damgaard,Plante,Vanhove,..]
[Di Vecchia,Veneziano,Heissenberg,Russo]
[Solon,Cheung,..][Huang,..]
[Guevera,Ochirov,Vines,…]
[Johansson,Pichini[Kosower,O'Connell,Maybee,
Cristofoli, Gonzo…]

order	deflection & spin kick					waveform			
	plain	spin-orbit	spin-spin	spin$^{>2}$	tidal	plain	spin-orbit spin-spin	tidal	Integration complexity
1PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps HEFT	X	trivial	trivial	trivial	~ tree-level
2PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps HEFT	WQFT WEFT Amps	WQFT WEFT Amps HEFT	WQFT WEFT	WQFT WEFT	~ 1-loop
3PM w/o r-r	WQFT WEFT Amps HEFT	WQFT Amps	WQFT (Amps)		WQFT WEFT	Amps HEFT			~ 2-loop
3PM r-r	WQFT WEFT Amps HEFT	WQFT	WQFT		WQFT WEFT				
4PM w/o r-r	WQFT WEFT Amps	WQFT							~ 3-loop
4PM w r-r	WQFT WEFT Amps	WQFT							

r-r: Radiation-reaction (…) : partial results

SUMMARY

WQFT: Highly efficient quantum field theory technology for classical scattering in GR

- „Quantize“ world-line degrees of freedom
- One-point functions = observables
- Classical theory = tree-level diagrams
- IN-IN Formalism: All propagators retarded.
- Include spin degrees of freedom through world-line supersymmetry

OUTLOOK

WQFT still needs to be extended:

- Higher precision (complete 4PM, 5PM)
- Higher spin (beyond Spin-Spin)
- Bound orbits? Relation to Effective-one-body Formalism
- Relation to self force expansion

WE ARE HIRING!

- Fall 23: Long Term Postdoc (5y), ~~2 PhD~~
- Fall 24: Postdoc (4y), 1 PhD

European Research Council
Established by the European Commission

Thank you for your attention!

BACKUP

EXTRACT GRAVITATIONAL POTENTIAL IN PM EXPANSION

[Jakobsen,Mogull]

- Make ansatz for 2-body Hamiltonian

$$H = \sqrt{\vec{p}^{\,2}+m_1^2} + \sqrt{\vec{p}^{\,2}+m_2^2} + V(\vec{r},\vec{p},\vec{S}_i) \qquad \vec{L} = \vec{r}\times\vec{p}$$

$$V = \sum_A O^A V^A \qquad O^A = \left\{1, \frac{\vec{L}\cdot\vec{S}_i}{r^2}, \frac{\vec{r}\cdot\vec{S}_i\,\vec{r}\cdot\vec{S}_j}{r^4}, \frac{\vec{S}_i\cdot\vec{S}_j}{r^2}, \frac{\vec{p}\cdot\vec{S}_i\,\vec{p}\cdot\vec{S}_j}{r^2}\right\}$$

$$V^A = \sum_n \left(\frac{G}{r}\right)^n c_n^A(|\vec{p}|)$$

Fix free coefficients c_n^A by matching to scattering data Δp_1^μ and $\Delta s_1^{\mu\nu}$ by solving Hamilton's eqs.

$$\dot{\vec{r}} = \frac{\partial H}{\partial \vec{p}}, \quad \dot{\vec{p}} = -\frac{\partial H}{\partial \vec{r}}, \quad \dot{\vec{S}}_i = -\vec{S}_i \times \frac{\partial H}{\partial \vec{S}_i}$$

Can make contact to bound problem

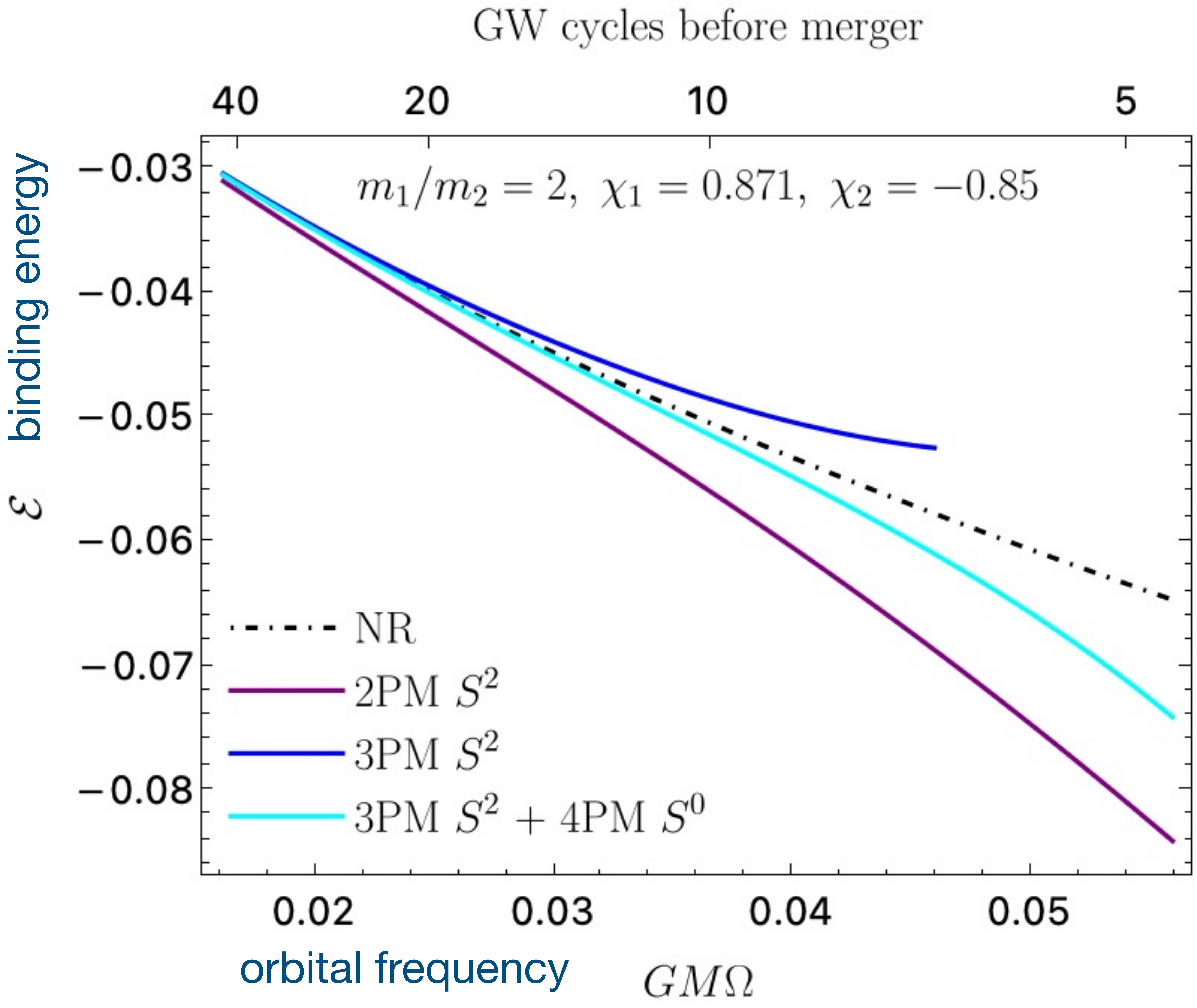
BINDING ENERGY: COMPARISON TO NUMERICAL RELATIVITY

Use 2-body Hamiltonian to compute binding energy $\mathscr{E} = \frac{E - M}{\mu}$ for circular orbits and and aligned spins to compare to numerical relativity

$$E = H(p_r, r, J; \chi_1, \chi_2)$$

$$\chi_1 = \frac{a_1}{m_1}, \chi_2 = \frac{a_2}{m_2}$$

$$\Omega(J, r) = \partial H / \partial J$$



Solve masters by Diff. Eq. technique

Single scale integrals: $x = \gamma - \sqrt{\gamma^2 - 1}$. Group master integrals in vector $\vec{I}(x, \epsilon)$

[Gehrmann Remiddi; Henn]

$$\frac{d}{dx}\vec{I}(x,\epsilon) = M(x,\epsilon)\,\vec{I}(x,\epsilon) \xrightarrow[\text{similarity transform}]{\vec{I}(x,\epsilon) = T(\epsilon,x)\,\vec{I}'(x,\epsilon)} \frac{d}{dx}\vec{I}'(x,\epsilon) = \epsilon\,A(x)\,\vec{I}'(x,\epsilon)$$

Canonical DE. Straightforward solution for masters: $\vec{I}'(x,\epsilon) = P\exp[\epsilon\int_C dA(x)]\,\vec{I}'_0$

- Finding $T(\epsilon, x)$ is a major challenge. 4PM Diff. eq. matrices $A(x)$ have poles in $\{x, 1+x, 1-x, 1+x^2\}$: Symbol alphabet [Jakobsen,Mogull,JP,Sauer]

- Integration yields Logs & Dilogs, Elliptic functions K & E appear in transformation matrix $T(\epsilon, x)$. Resulting function space:

$$F_\alpha^{(b)}(\gamma) = \{1, \mathrm{arccosh}[\gamma], \log[\gamma], \log\left[\frac{\gamma_\pm}{2}\right], \mathrm{arccosh}^2[\gamma], \mathrm{arccosh}[\gamma]\log\left[\frac{\gamma_\pm}{2}\right], \log\left[\frac{\gamma_+}{2}\right]\log\left[\frac{\gamma_-}{2}\right], \log^2\left[\frac{\gamma_+}{2}\right],$$
$$\mathrm{Li}_2\left[\pm\frac{\gamma_-}{\gamma_+}\right], \mathrm{Li}_2\left[\sqrt{\frac{\gamma_-}{\gamma_+}}\right], \mathrm{K}^2\left[\frac{\gamma_-}{\gamma_+}\right], \mathrm{E}^2\left[\frac{\gamma_-}{\gamma_+}\right], \mathrm{K}\left[\frac{\gamma_-}{\gamma_+}\right]\mathrm{E}\left[\frac{\gamma_-}{\gamma_+}\right]\}$$

CLAIM:

$$\langle \mathcal{O} \rangle_{\mathrm{WQFT}} = \int D[h, z] \mathcal{O} \, e^{-\frac{i}{\hbar} S[z,h]} \xrightarrow{\hbar \to 0}$$

Tree-level one-point functions $\langle h_{\mu\nu} \rangle$ and $\langle z^\mu \rangle$ solve classical equations of motion

CLASSICAL DYNAMICS FROM ONE-POINT FUNCTIONS

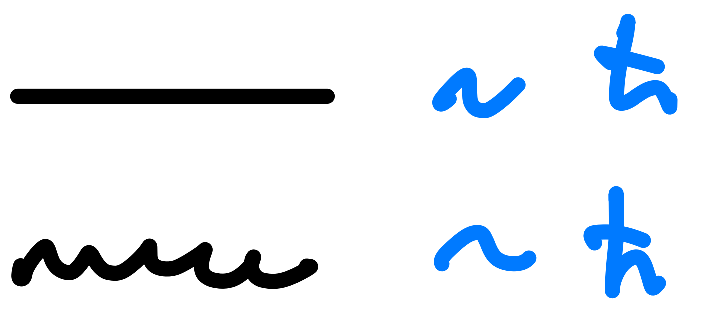
[Jakobsen]

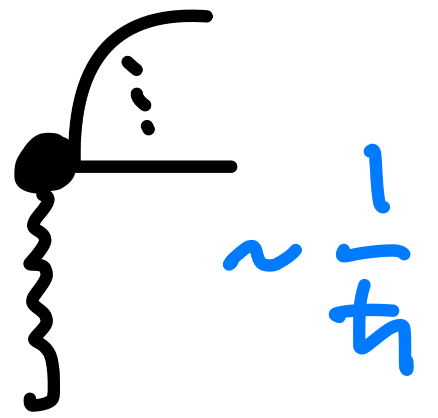
- Action: $S[\Phi_A]$ with fields $\Phi_A(x_A) = \{h_{\mu\nu}(x), z^\mu(\tau)\}$ and coordinates $x_A = \{x^\mu, \tau\}$

- Partition function in the presence of sources

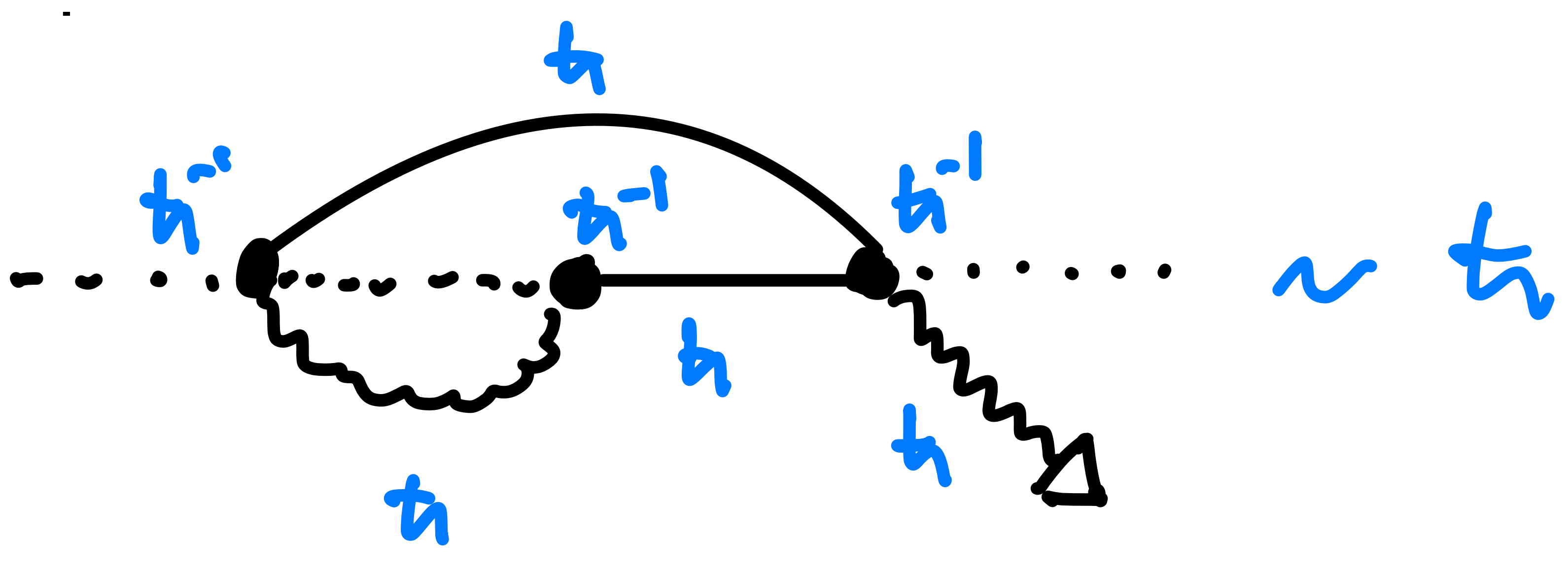
$$Z[J_A] = \int D[\Phi_A] \exp\left[\frac{i}{\hbar}\left(S[\Phi_A] + \sum_A \int dx_A J_A(x_A)\, \Phi_A(x_A)\right)\right]$$

- $\hbar$ counting:





E.g:



⇒ Loops are quantum effects!

- Scalings of **connected** n-point functions:

$$\langle \Phi_{A_1} \ldots \Phi_{A_n} \rangle_{\text{conn}} \sim \sum_L \hbar^{-1+n+L}$$ (L-loop connected n-point diagrams)

Well defined classical limit only for n=1 and L=0: **Tree-level one-point functions**

CLASSICAL DYNAMICS FROM ONE-POINT FUNCTIONS

[Jakobsen]

- Factorization $$\lim_{\hbar\to 0} \langle \Phi_{A_1}\Phi_{A_2}\ldots\Phi_{A_n}\rangle_{\text{discon}} = \langle\Phi_{A_1}\rangle^{\text{tree}}_{\text{con}}\langle\Phi_{A_2}\rangle^{\text{tree}}_{\text{con}}\ldots\langle\Phi_{A_n}\rangle^{\text{tree}}_{\text{con}}$$

- Consequence for Schwinger-Dyson equations: ⇔ Ehrenfest theorem in QM

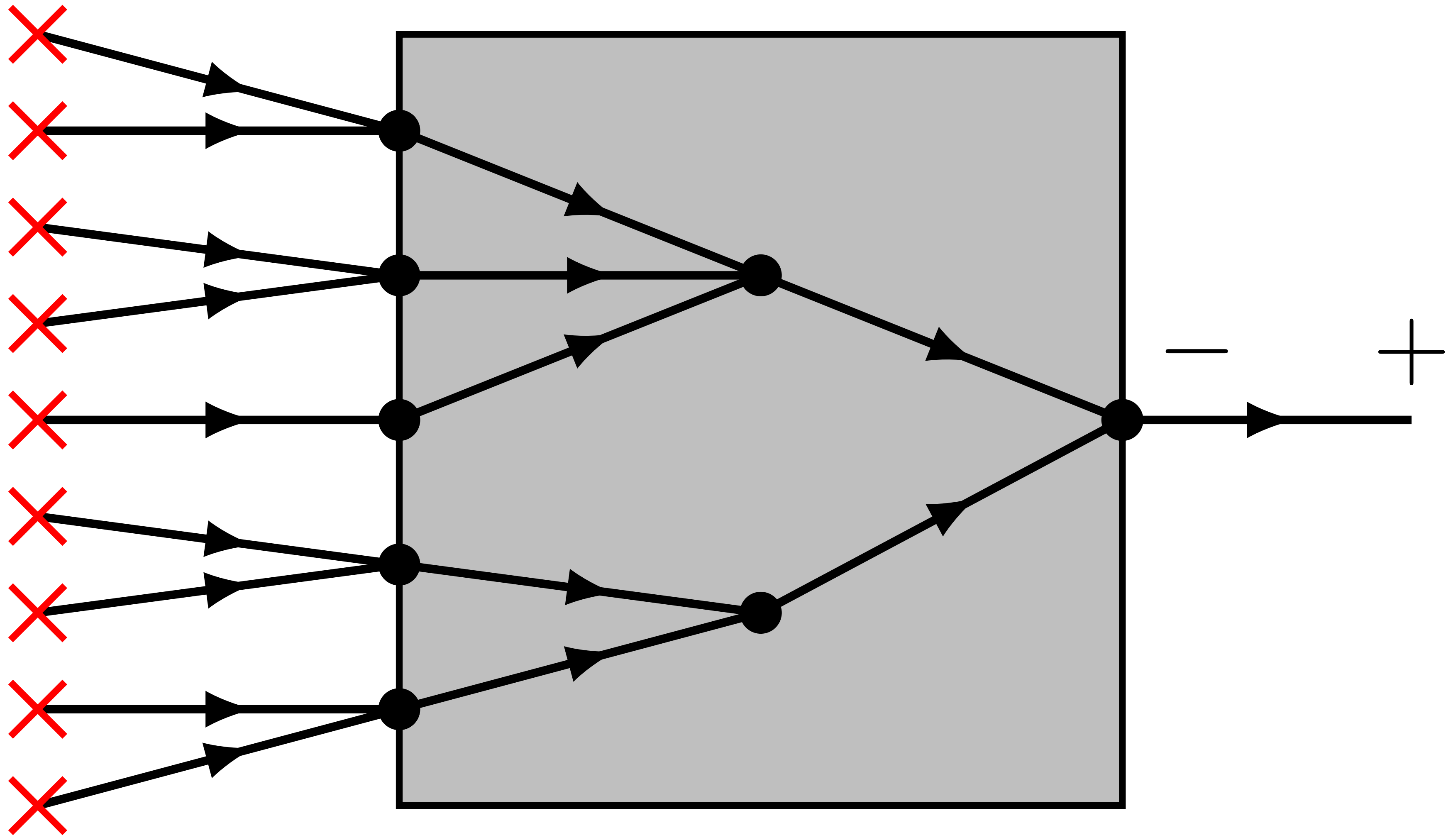
$$\left\langle \frac{\delta S[\Phi_A]}{\delta\Phi_A}\right\rangle = 0 \quad \xrightarrow{\hbar\to 0} \quad \boxed{\frac{\delta S[\langle\Phi_A\rangle_{\text{tree}}]}{\delta\Phi_A} = 0}$$

Tree-level one-point functions solve classical equations of motion

- Importantly $S[\Phi_A]$ must be independent of $\hbar$ (not the case in amplitudes approach - massive field!) -> Key advantage of WQFT approach (no „super classical" terms)

- Need non-trivial background field configurations for non-vanishing one-point functions

THE IN-IN (SCHWINGER-KELDYSH) FORMALISM FOR WQFT



OR WHY RETARDED PROPAGATORS?

[Jakobsen,Mogull,JP,Sauer]

STANDARD PATH INTEGRAL: IN-OUT FORMALISM

[Galley,Tiglio] [Jordan]

- Hamiltonian formalism: Time evolution operator $\hat{H} = \hat{H}_0 + \hat{H}_{\rm int}$

Background

$$U_{J}(T,T') = \mathcal{T}\exp\left[\frac{i}{\hbar}\int_{T'}^{T} dt \int d^3x \left\{ \hat{H}_{\rm int}[\phi_0(\mathbf{x},t), Q(\mathbf{x},t)] + J(x)\,\phi_0(\mathbf{x},t)\right\}\right]$$

Free fields

- Path integral representation:

$$\langle 0|U_{J}(\infty,-\infty)|0\rangle = \int [D\phi]\exp\left[\frac{i}{\hbar}\left(S[\phi,Q] + \int d^4x\, J(x)\,\phi(x)\right)\right] = \exp\left[\frac{i}{\hbar}W[J]\right]$$

- Heisenberg picture: $\phi_H(\mathbf{x},t) = U_0(-\infty,t)\,\phi_0(\mathbf{x},t)U_0(-t,\infty)$

- One-point function:

$$= U_0(t,-\infty)\,\phi_H(\mathbf{x},t)U_0(-\infty,t)$$

$$\langle\phi_H(\mathbf{x},t)\rangle_{\text{in-out}} = \frac{\delta W[J]}{\delta J(\mathbf{x},t)}\bigg|_{J=0} = \langle 0|\,U_0(\infty,t)\,\phi_0(\mathbf{x},t)\,U_0(t,-\infty)\,|0\rangle$$

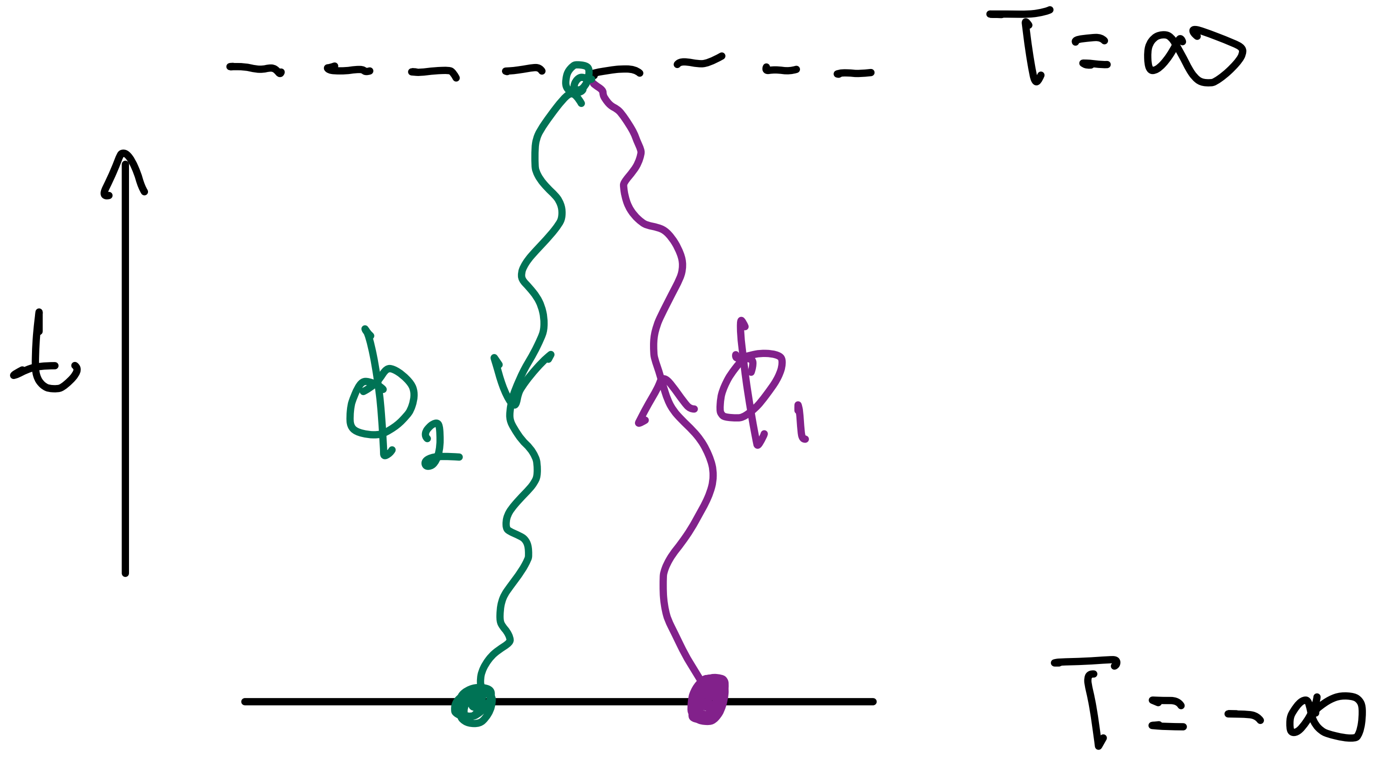
$$= \langle 0|\,U_0(\infty,-\infty)\,\phi_H(\mathbf{x},t)\,|0\rangle_{\text{in-out}} = {}_{\rm out}\langle 0|\phi_H(\mathbf{x},t)|0\rangle_{\rm in}$$

IN-IN (SCHWINGER-KELDYSH) FORMALISM

[Galley,Tiglio] [Jordan]

- Standard path integral yields $\langle \phi_H(x)\rangle_{\text{in}-\text{out}} = {}_{\text{out}}\langle 0|\phi_H(x)|0\rangle_{\text{in}}$ but want

$$\boxed{\langle \phi_H(x)\rangle_{\text{in}-\text{in}} = {}_{\text{in}}\langle 0|\phi_H(x)|0\rangle_{\text{in}} = \langle 0|U(-\infty,t)\,\phi_0(t,\mathbf{x})\,U(t,-\infty)\,|0\rangle}$$



⇒ need two evolution operators: Double the fields!

$$Z[J_1,J_2] = \langle 0|\,U_{J_1}(-\infty,\infty)\,U_{J_1}(\infty,-\infty)\,|0\rangle$$

$$= \int D[\phi_1,\phi_2]\exp\left[\frac{i}{\hbar}\left(S[\phi_1]-S[\phi_2]+\int_x J_1(x)\,\phi_1(x) - J_2(x)\,\phi_2(x)\right)\right]$$

Boundary conditions:

$$\phi_1(t=+\infty,\mathbf{x}) = \phi_2(t=+\infty,\mathbf{x})$$
$$\phi_1(t=-\infty,\mathbf{x}) = \phi_2(t=-\infty,\mathbf{x}) = 0$$

$$\boxed{\frac{1}{Z[0,0]}\frac{\delta Z[J_1,J_2]}{\delta J_1(x)}\bigg|_{J_i=0} = \langle \Phi_H(x)\rangle_{\text{in}-\text{in}}}$$

KELDYSH BASIS: $\phi_+ = \frac{1}{2}(\phi_1 + \phi_2)$ $\phi_- = \phi_1 - \phi_2$

- This yields

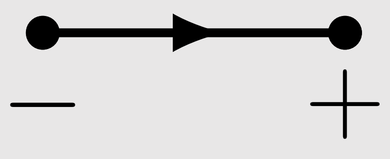
$$Z[J_\pm] = \int D[\phi_+,\phi_-]\,\exp\Bigg[\frac{i}{\hbar}\Bigg(S[\phi_+ + \frac{1}{2}\phi_-] - S[\phi_+ - \frac{1}{2}\phi_-] + \int d^4x(J_+\phi_- + J_-\phi_+)\Bigg]$$

- Propagator matrix from free part:

$$\langle\phi_a(x)\,\phi_b(y)\rangle = \begin{array}{cc} \begin{array}{cc} + & - \end{array} & \\ \begin{pmatrix} \frac{1}{2}D_H(x,y) & D_{\text{ret}}(x,y) \\ -D_{\text{adv}}(x,y) & 0 \end{pmatrix} & \begin{array}{c} + \\ - \end{array} \end{array}$$

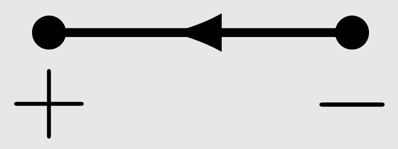
irrelevant @ tree-level

$\widetilde{D}_{\text{ret}}(k) =$



$$= \frac{-i}{(k^0 + i0)^2 - \mathbf{k}^2},$$

$\widetilde{D}_{\text{adv}}(k) =$



$$= \frac{-i}{(k^0 - i0)^2 - \mathbf{k}^2},$$

- Vertices from:

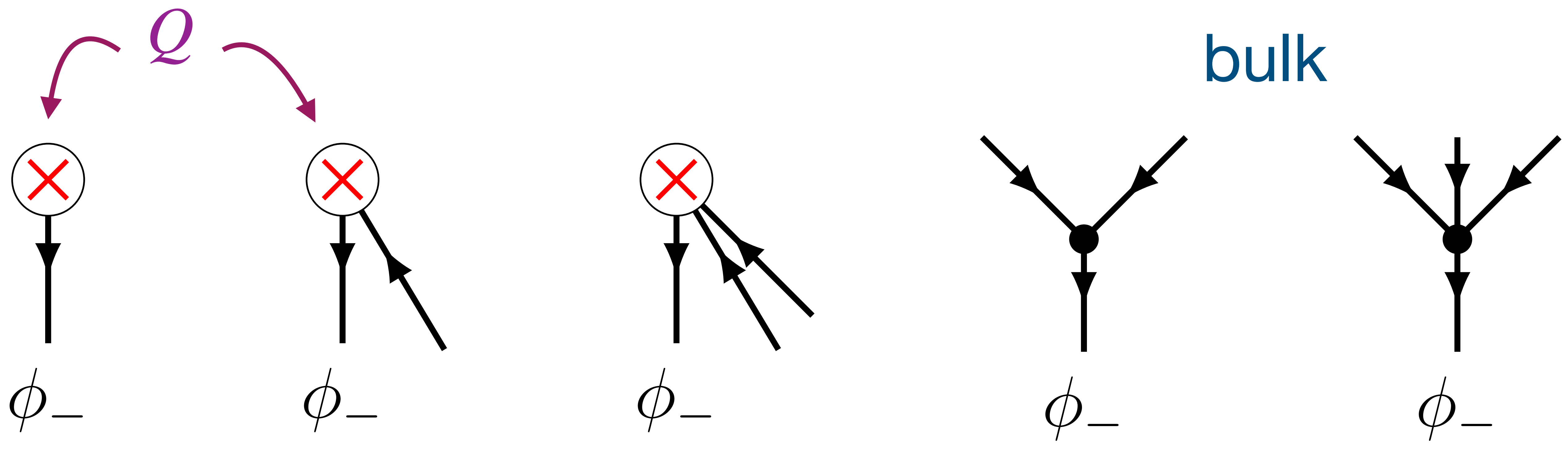
$$S_{\text{int}}[\phi_+ + \frac{1}{2}\phi_-] - S_{\text{int}}[\phi_+ - \frac{1}{2}\phi_-] = \phi_- \left(\frac{\delta S_{\text{int}}[\phi]}{\delta\phi}\right)\Bigg|_{\phi\to\phi_+} + \mathcal{O}(\phi_-^3)$$

⇒ only **odd** number of ϕ_- legs!

ONE POINT FUNCTIONS @ TREE-LEVEL:

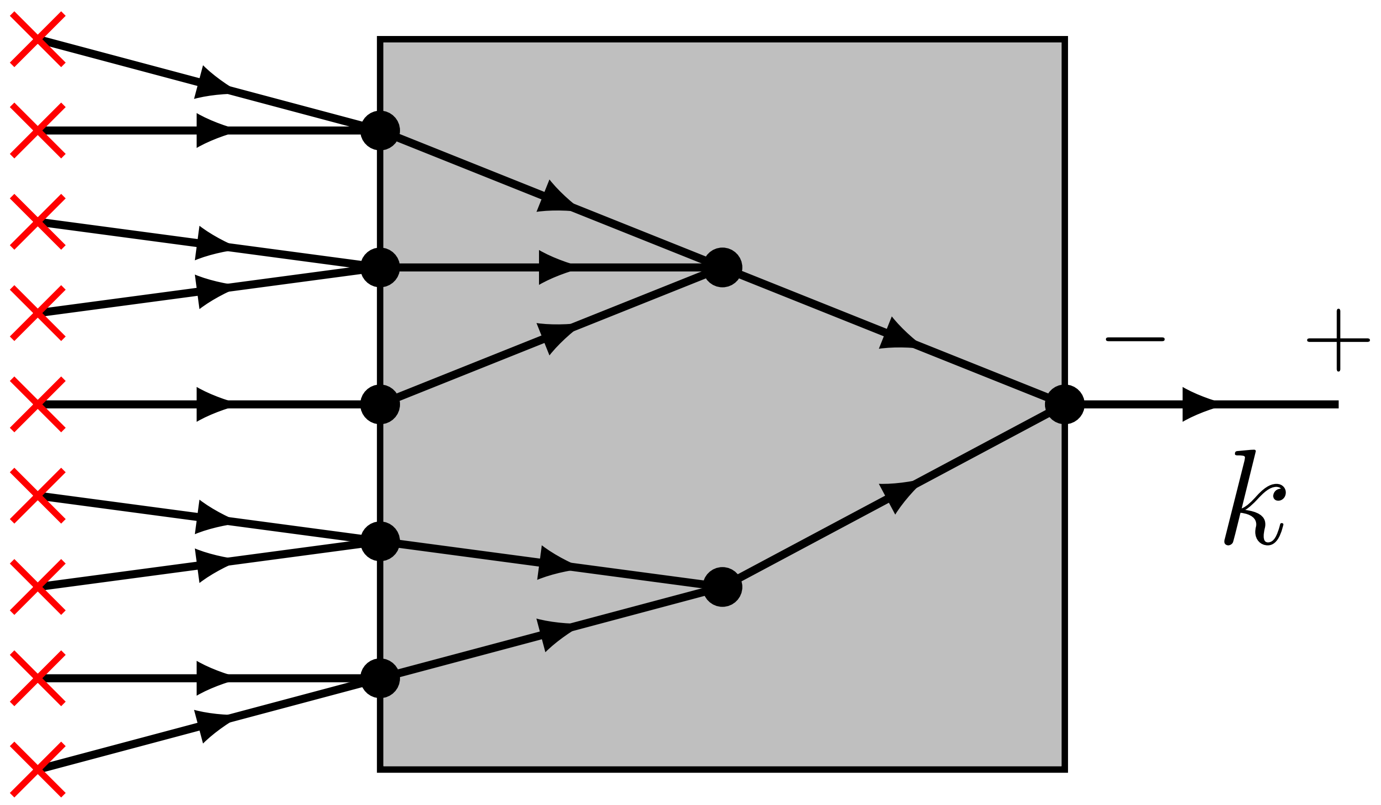
- Vertices including background field Q

$$S_{\rm int}[\phi; Q] \to \phi_- \left(\frac{\delta S_{\rm int}[\phi, Q]}{\delta \phi}\right)\Bigg|_{\phi\to\phi_+} + \mathcal{O}(\phi_-^3)$$



- One point function:

$$\langle \phi(k) \rangle_{\rm in\text{-}in} =$$



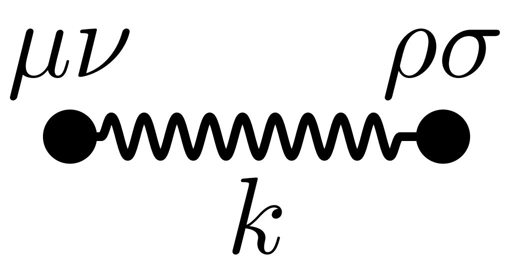
Only retarded propagators contribute!

CONSERVATIVE SECTOR: NO DISSIPATION

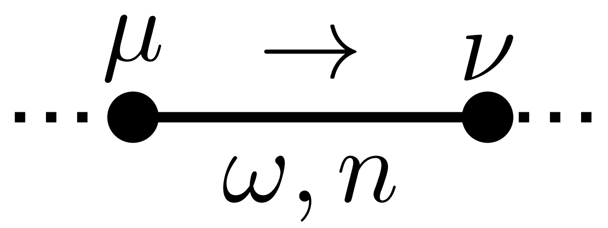
- For application of scattering results to the bound case scenario necessary to separate conservative dynamics from disspative one.

- Goal: Construct effective (one-body) Hamiltonian applicable to bound case scenarios by matching

- Useful definition of conservative sector: Take graviton fields in-out, worldline fluctuations in-in



$$= \frac{iP_{\mu\nu;\rho\sigma}}{k^2 + i0^+}$$



$$= \frac{-i\eta^{\mu\nu}}{m_i(\omega + i0^+)^n} \begin{cases} n = 2 \text{ for } z_i^{\mu}\,, \\ n = 1 \text{ for } \psi_i'^{\mu}\,. \end{cases}$$

POST-NEWTONIAN VS POST-MINKOWSKIAN EXPANSIONS

Conservative non-spinning 2-body dynamics:

		0PN	**1PN**	**2PN**	**3PN**	**4PN**	**5PN**		Integration complexity
0PM [Einstein]	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	…	
1PM [Westpfahl]		G/r [Newton]	$G\ v^2/r$ [EIH]	$G\ v^4/r$	$G\ v^6/r$	$G\ v^8/r$	$G\ v^{10}/r$	…	~ tree-level
2PM [many]			$G^2\ 1/r^2$	$G^2\ v^2/r^2$	$G^2\ v^4/r^2$	$G^2\ v^6/r^2$	$G^2\ v^8/r^2$	…	~ 1-loop
3PM				$G^3\ 1/r^3$	$G^3\ v^2/r^3$	$G^3\ v^6/r^3$	$G^3\ v^8/r^3$	…	~ 2-loop

[Bern,Cheung,Roiban,Shen, Solon,Zeng][Kälin, Liu, Porto][Di Vecchia, Heissenberg, Russo,Veneziano][Bjerrum-Bohr,Vanhove,Damgaard][Brandhuber,Chen,Travaglini,Wen][Jakobsen,Mogull,JP,Sauer]

(4PM)	PM state-of-the-art			→	$G^4\ 1/r^4$	$G^4\ v^2/r^4$	$G^4\ v^6/r^4$	…	~ 3-loop

[Bern,Parra-Martinez,Roiban,Ruf,Shen,Solon,Zeng][Dlapa,Källin,Liu,Porto]

....						$G^5\ 1/r^5$	↑ :		

PN state-of-the-art