Search for a UV fixed point in 4d CDT

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Talk at QG 2023,
Radboud University, July 10-14th , 2023

4d (lattice) QG

Main goal (at least in 80ties) for QG

- Obtain the background geometry $\langle g_{\mu\nu} \rangle$ we observe
- Study the fluctuations around the background geometry

What 4d CDT offers:

- A non-perturbative QFT definition of QG
- A background independent formulation
- An emergent background geometry $\langle g_{\mu\nu} \rangle$
- The possibility to study the quantum fluctuations around this emergent background geometry.

Problems to confront for a (lattice) theory of QG

(1) How to define the quantum theory

(2) How to face the non-renormalizability of quantum gravity

(3) Provide evidence of a continuum limit (where the continuum field theory has the desired properties)

(4) If it turns out that there exists no continuum quantum field theory of gravity definable at all scales, can a lattice theory be of any use?

(1) How to define the quantum theory

The classical action in 3+1 dimensions:

$$S[g] = \frac{1}{16\pi G} \int d^4x \sqrt{-g(x)} \Big(R(x) - 2\Lambda \Big)$$

One may define the quantum theory via the path integral

$$Z[G, \Lambda] = \int \mathcal{D}[g] \, e^{iS[g]/\hbar}$$

But what precisely is meant by $\int \mathcal{D}[g]$?

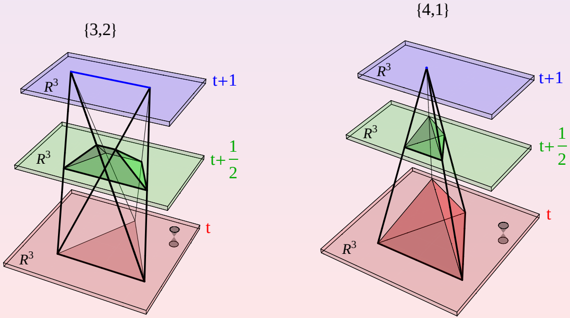
Classical GR refers to a manifold and a metric $g_{\mu\nu}$ with Lorentzian signature. Consider instead metrics with Euclidean signature. Then GR refers to smooth Riemannian manifolds.

The path integral in QM integrates not only over classical smooth path. Should we integrate over continuous geometries and should we include an integration over different manifolds ?

CDT takes a minimalistic view on these problems: one fixes the manifold. Here I will discuss only the situation where $\mathcal{M} = R \times S^3$. We use a subset of continuous geometries, in fact a subset of so-called piecewise linear geometries, that can be constructed by gluing together identical building blocks (4-simplices). They are hopefully dense in the set of continuous geometries.

Inspired by canonical quantization we assume a Lorentzian signature, a hyperbolic spacetime and a time foliation. We triangulate it using the building blocks and use Regge's action for piecewise linear manifolds in the path integral.

The somewhat remarkable aspect is that one can actually rotate each such Lorentzian geometry to an Euclidean geometry, where each building block is then a 4-simplex where all links have the same length a. This length then plays the role of a UV cut-off, precisely as in ordinary lattice field theories.



Another remarkable feature is that the Regge action becomes very simple when using identical building blocks. For a given 4d triangulation T, denote by $N_4(T)$, $N_2(T)$ and $N_0(T)$ the number of 4-simplices, 2-simplices (triangles) and 0-simplices (vertices). Then

$$S[T] = -k_2 N_2(T) + k_4 N_4(T) = -k_0 N_0(T) + \tilde{k}_4 N_4(T)$$

where

$$k_2 = c_1 \frac{a^2}{G}, \quad k_4 = c_2 \frac{a^2}{G} + c_3 \frac{a^4 \Lambda}{G}.$$

$$k_0 = c_1' \frac{a^2}{G}, \quad \tilde{k}_4 = c_2' \frac{a^2}{G} + c_3' \frac{a^4 \Lambda}{G}.$$

$$\begin{aligned} S[T] &= -k_0 N_0(T) + k_{32} N_{32}(T) + k_{41} N_{41}(T) \\ &= -(k_0 + 6\Delta) N_0(T) + k_4 N_4(T) + \Delta N_{41}(T) \end{aligned}$$

$$Z_L(G,\Lambda) = \int \mathcal{D}[g_L]\, e^{iS_L[g_L]} \to Z^L_{CDT}(k_2,k_4) \to Z^E_{CDT}(k_2,k_4)$$

$$Z^E_{CDT}(k_2,k_4) = \sum_T \frac{1}{C_T}\, e^{-S[T]} = \sum_{N_4,N_2} e^{k_2 N_2 - k_4 N_4} \mathcal{N}(N_2,N_4),$$

$$\mathcal{N}(N_2,N_4) = \sum_{T(N_2,N_4)} \frac{1}{C_T}$$

The partition function for QG is the generating function for the number of abstract triangulations with N_4 4–simplices and N_2 2–simplices. QG is pure combinatoric!

(2) Facing the non-renormalizability of QG

It is known how to make 4d QG renormalizable: add an R^2 term to the action (Stelle, 1977). It makes the theory asymptotically free. Problem with unitarity.

Another route is via the asymptotic safety scenario (ASS) (Weinberg 1979), implemented via the functional renormalization group (FRG). Here one investigates if there exists a non-perturbative UV fixed point in a Wilsonian formulated theory of QG. So far FRG results have provided support for this idea.

4d CDT has a reflection positive transfer matrix. Such lattice theories result in unitary theories if a continuum limit exists. Thus unitarity is probably not an issue in CDT.

Lattice field theories are well suited to investigate fixed points and the corresponding continuum limits. Thus 4d CDT seems ideally suited to study ASS.

(3) How to define the continuum limit

Recall standard lattice field theory (LFT)

(1) Asymptotic free theories (Gaussian fixed points)

Prime example: YM theories in 4d. One (bare) coupling constant g_0. From perturbation theory we know that the fixed point is UV (the β-function is negative).

$$\beta(g_0) = -a\frac{dg_0}{da} = -\beta_1 g_0^3 - \cdots, \qquad a(g_0) = \frac{1}{\Lambda_{YM}} e^{-1/2\beta_1 g_0^2},$$

For a physical mass m_{ph} (from stringtension , glueball mass...)

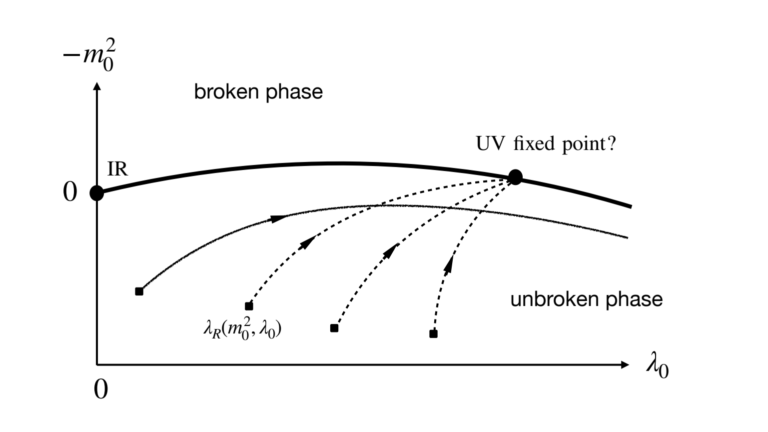
$$m_0(g_0) = m_{ph}\, a(g_0) = \frac{m_{ph}}{\Lambda_{YM}} e^{-1/2\beta_1 g_0^2}$$

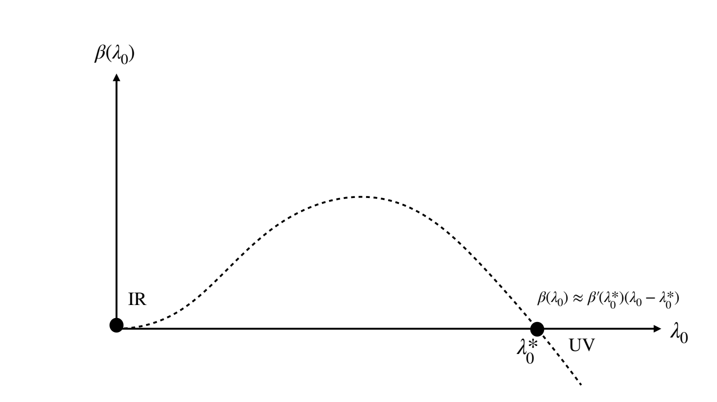
We can measure $m_0(g_0)$ on the lattice and thus reconstruct the β-function, even if we could not calculate it perturbatively.

(2) Non-Gaussian UV fixed points

ϕ^4 theory in 4d. Two dimensionless coupling constants m_0^2, λ_0

$$\mathcal{L} = (\partial\phi_0)^2 + m_0^2\phi_0^2 + \lambda_0\phi_0^4, \quad \lambda_R \propto \Gamma_4(p_i = 0, m_0^2, \lambda_0)$$



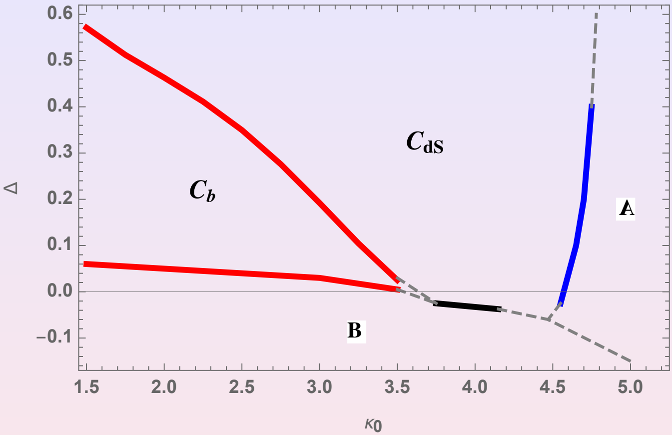


If there had been a UV fixed point:

$$-a\frac{d\lambda_0}{da} = \beta(\lambda_0) \approx \beta'(\lambda_0^*)(\lambda_0 - \lambda_0^*), \qquad a(\lambda_0) \propto |\lambda_0 - \lambda_0^*|^{-1/\beta'(\lambda_0^*)}$$

For $\beta'(\lambda_0^*) < 0$ we can define a continuum limit for $\lambda_0 \to \lambda_0^*$.

The CDT phase diagram (red lines second order transition)

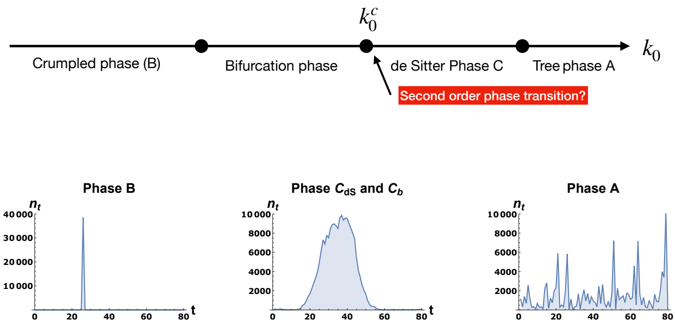


Only coupling constants k_0, Δ are shown, the reason being that we in the computer simulations keep N_4 fixed. Thus k_4 plays no role as the coupling constant of N_4.

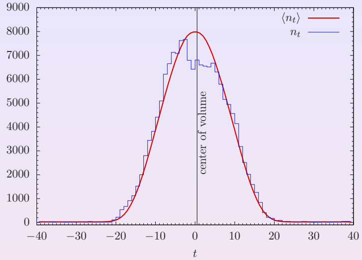
The C_b-C_{dS} transition line has the interpretation as breaking of homogeneity and isotropy of space. A UV fixed point on this line would imply that the short distance physics is related to this symmetry breaking and could have implication for cosmology. Of course one has to quantify what it means that the quantum spacetime is homogeneous and isotropic (R. Loll and A. Silva, PRD 107 (2023)).

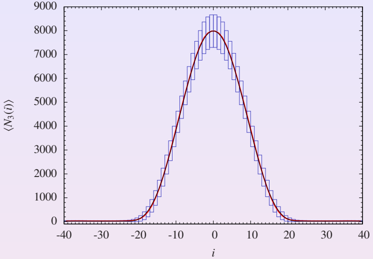
Let us look closer at how the structure of spacetime could change at the phase transition and whether these changes can be used to define a continuum limit of the lattice theory.

For each time-slice t we have a spatial volume $V_3(t) \propto N_3(t)a^3$.



The de Sitter phase





$$\langle N_3(t) \rangle \propto N_4 \frac{1}{\omega(k_0) N_4^{1/4}} \cos^3 \left(\frac{t}{\omega(k_0) N_4^{1/4}} \right),$$

This is exactly the spatial volume profile of a (elongated) four-sphere of volume N_4 if we use a metric

$$ds^2 = dt^2 + \ell^2(t) d\Omega_3, \quad V_3(t) \propto \ell^3(t).$$

The fluctuations behave like

$$\Delta N_3(t) = C(k_0)\sqrt{N_4}\, F\Big(\frac{t}{\omega(k_0) N_4^{1/4}}\Big), \quad F(0) = 1$$

Thus we have seemingly obtained some of the goals declared in the beginning: obtaining a $\langle g_{\mu\nu} \rangle$ and being able to study the fluctuations around this configuration (at least in the limiting sense of studying the spatial volume).

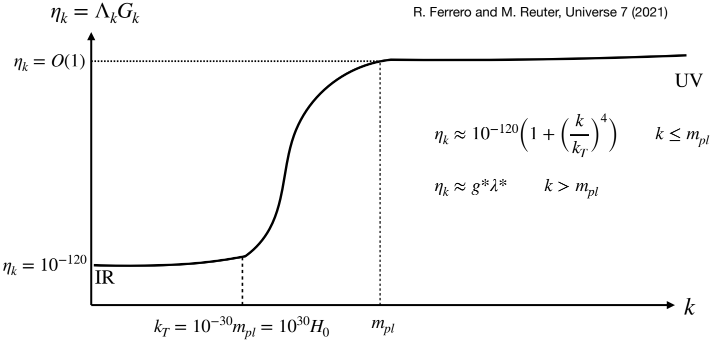
In fact we can do more: we can obtain the effective minisuperspace action for $\langle N_3(t) \rangle$ from the study of correlation functions $\langle N_3(t) N_3(t') \rangle$ and show that the fluctuations around $\langle N_3(t) \rangle$ are well described expanding this minisuperspace action to quadratic order in the fluctuations.

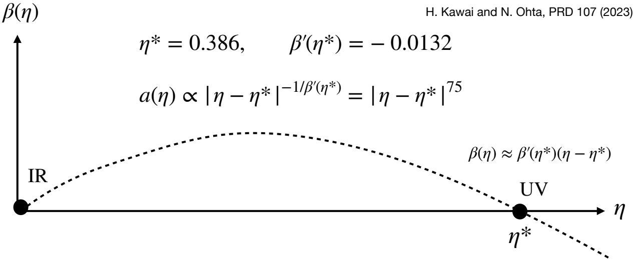
But this is still on a lattice. Can we take a controlled limit $a \to 0$, as described above? It would be convenient to have a correlation length that diverges when we approach the critical surface and an observable that characterizes the UV and distinguishes it from a IR fixed point. We will introduce a length scale via the size of the system (the Universe !) and an "observable" via the dimensionless coupling ΛG. Using the simplest FRG truncation:

$$\Gamma_k = \frac{1}{8\pi G_k} \int d^4x \sqrt{g} \left(\frac{1}{2} R - \Lambda_k\right)$$

$$\frac{\Delta V_3(t)}{V_3(t)} \propto \sqrt{G_k \Lambda_k} = \sqrt{g_k \lambda_k}, \qquad G_k = \frac{g(k)}{k^2}, \;\; \Lambda_k = \lambda(k) k^2.$$

We will consider k as a "momentum" cut-off and and our lattice cut-off as $\boxed{a \propto 1/k}$.


R. Ferrero and M. Reuter, Universe 7 (2021)
$\eta_k = \Lambda_k G_k$
$\eta_k = O(1)$
UV
$\eta_k \approx 10^{-120}\left(1 + \left(\frac{k}{k_T}\right)^4\right) \qquad k \leq m_{pl}$
$\eta_k \approx g^* \lambda^* \qquad k > m_{pl}$
$\eta_k = 10^{-120}$
IR
k
$k_T = 10^{-30} m_{pl} = 10^{30} H_0$
m_{pl}


$\beta(\eta)$
H. Kawai and N. Ohta, PRD 107 (2023)
$\eta^* = 0.386, \qquad \beta'(\eta^*) = -0.0132$
$a(\eta) \propto |\eta - \eta^*|^{-1/\beta'(\eta^*)} = |\eta - \eta^*|^{75}$
$\beta(\eta) \approx \beta'(\eta^*)(\eta - \eta^*)$
IR
UV
η
η^*

Returning to the lattice system we have for fixed k_0 in the de Sitter phase

$$V_4 \propto N_4 a^4, \qquad V_3(t) \propto N_3(t) a^3$$

What happens when $N_4 \to \infty$? Will $V_4 \to \infty$ and a stay fixed or will V_4 stay fixed and $a \to 0$.

$$\frac{\Delta V_3(t)}{V_3(t)} = \frac{\Delta N_3(t)}{N_3(t)} \propto \frac{C(k_0)}{N_4^{1/4}} = \frac{C(k_0)\, a}{V_4^{1/4}}$$

The most natural interpretation is that for fixed k_0 and $N_4 \to \infty$ we have $V_4 \to \infty$ while a stays fixed.

Suppose there exists (fixed) point k_0^* such that

$$C(k_0) = c(k_0 - k_0^*)^{-\nu}$$

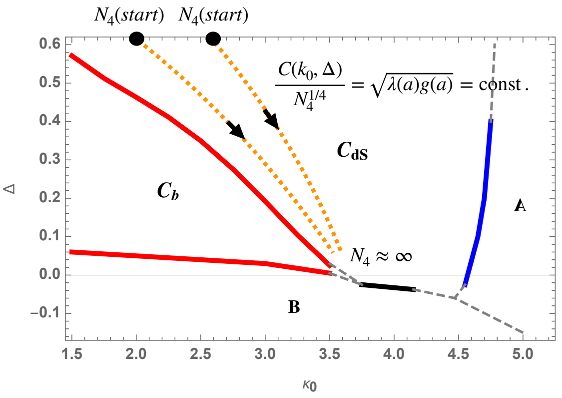
Then

$$\frac{\Delta V_3(t)}{V_3(t)} = \frac{\Delta N_3(k_0,t)}{N_3(k_0,t)} \propto \frac{C(k_0)}{N_4^{1/4}(k_0)} = \mathrm{const} \quad \text{if} \quad N_4(k_0) \propto \frac{1}{|k_0 - k_0^*|^{\nu}}$$

This can be checked directly by MC simulations. Results still somewhat inconclusive (requires very large volumes)

In the case of a UV fixed point

$$a(k_0) \propto \frac{1}{N_4^{1/4}} \propto (k_0 - k_0^*)^{\nu}, \qquad \nu = -\frac{1}{\beta'(k_0^*)}.$$



$$V_4 = N_4 a^4$$

$$a \propto \frac{1}{N^{1/4}}$$

In the MC simulations we see $C(k_0, \Delta)$ increases by at least a factor 10 when moving towards the tentative UV fixed point. Still strong finite size effects, but whenever we have been able to increase N_4 we have obtained more convincing sign of an UV fixed point, where we can define the continuum limit, i.e. remove the cut-off a.