Modern (discrete) 3rd quantization and emergent cosmology

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QFT of spacetime: what does it mean?

- spacetime = events and their geometric (& causal) relations

neglect fact that events =/= manifold points (due to diffeo invariance, have to be defined wrt dynamical fields)

- QFT on spacetime = QFT of physical entities for given spacetime

(including perturbative QG, partially QFT of spacetime if backreaction is considered)

→ **QFT of spacetime = spacetime is fully dynamical**

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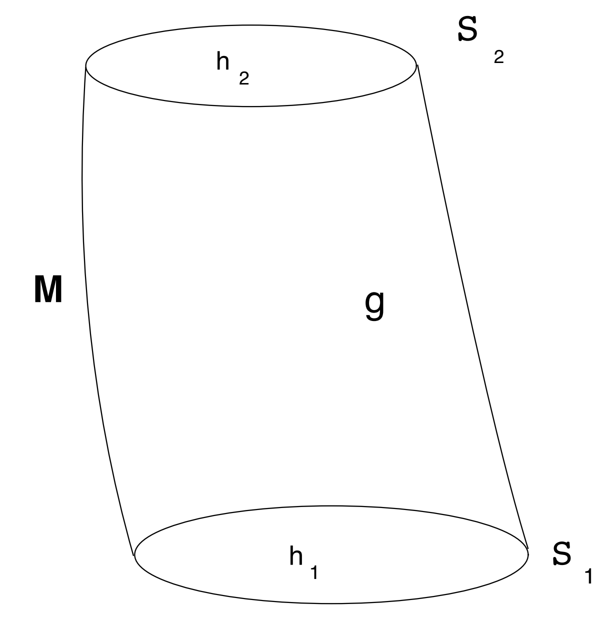
- non-perturbative QG = fully dynamical geometry = background-independence = no spacetime is fixed
- any non-perturbative QG theory is a QFT of spacetime, by definition

example: QG path integral

$$G({}^3g, {}^3g') = \int_{{}^3g}^{{}^3g'} \mathcal{D}\, {}^4g\, e^{-S}$$



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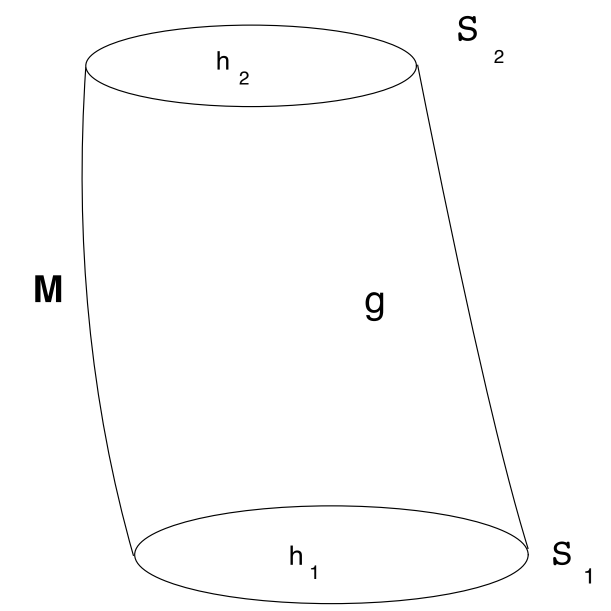

but spacetime topology is fixed, thus possible geometries are constrained

→ **QFT of spacetime = both spacetime geometry and spacetime topology are dynamical**

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- available/allowed background structures in GR:
  - spatial topology
  - spacetime topology
  - space of metrics (up to diffeos) + matterfield configurations = superspace
  - signature
  - local gage group (Lorentz)

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plus, can have in mind "emergent spacetime/gravity" scenarios, with continuum gravitational field and spacetime replaced by more abstract non-spatiotemporal (possibly discrete) entities

Quantum gravity = quantum theory of atomic constituents of emergent spacetime

quantum theory of "new" non-spatiotemporal entities

continuum spacetime and geometric quantum observables reconstructed from collective quantum dynamics of "atoms of space"

quantum spacetime as a (background-independent) quantum many-body system

extraction of spacetime and cosmology similar to typical problem in condensed matter theory (from atoms to macroscopic effective continuum physics)

- GR from "hydrodynamic" approximation of fundamental "atomic" quantum theory
- all GR structures and dynamics are to be approximately obtained (in relational language) at effective level
- not just emergent gravity; flat spacetime itself would be emergent, highly excited, collective state of "QG atoms"

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QFT of spacetime with "standard" QFT language and dynamical topology alongside dynamical geometry has been proposed long time ago .....

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Coleman, Banks, Giddings, Strominger, Caderni, Martellini, Rubakov, McGuigan, Klebanov, Susskind,.....,

- canonical quantum geometrodynamics

see talk by Kiefer

S. Giddings, A. Strominger, '88

globally hyperbolic topology, with given (e.g. spherical) spatial topology $S^3 \times R$

$$S = \int_0^T dt(\pi \cdot \dot{g} - NH)$$

proper time gauge

- Wheeler-DeWitt operator $N\Box\Psi({}^3g) = 0$

analogous to Dalambertian on superspace, with DeWitt supermetric

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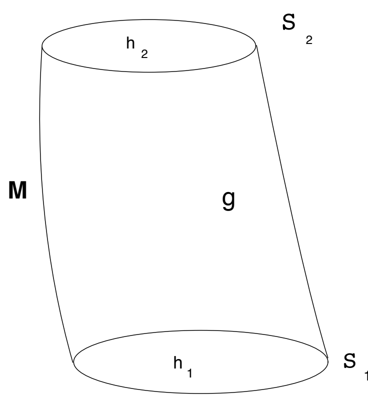

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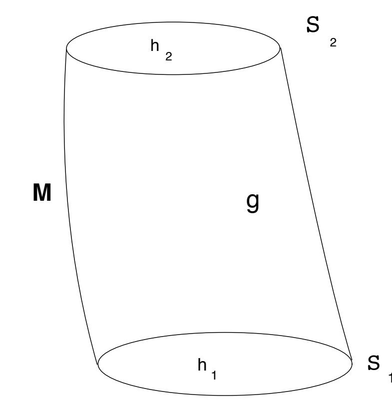

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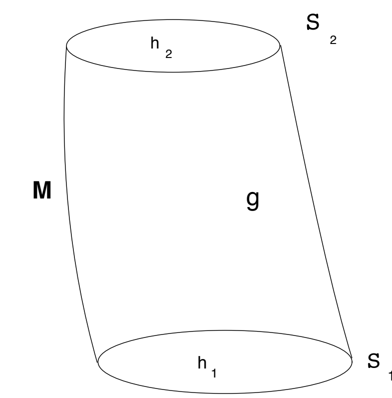

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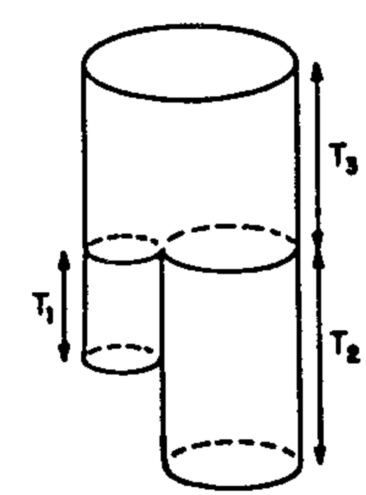
- path integral can be defined for manifolds with spatial topology change $G({}^3g_1, {}^3g_2, {}^3g_3)$

specifying matching conditions at junctions (ensuring that 4-geometries are counted only once)



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- action

$$S = S_2 + V[\Phi] = S_2(\Phi) + \lambda_3 S_3(\Phi) + \dots$$

S. Giddings, A. Strominger, '88

$$S_2 = -\frac{1}{2}\int \mathcal{D}\, {}^3g \Phi \Box \Phi$$

kinetic term = WdW operator

$$S_3 = \int \mathcal{D}\, {}^3g_1 \mathcal{D}\, {}^3g_2 \mathcal{D}\, {}^3g_3 \Phi[{}^3g_1]\Phi[{}^3g_2]\Phi[{}^3g_3]\delta({}^3g_2, {}^3g_1^-)\delta({}^3g_3, {}^3g_1^+)$$

topology-changing process

encoding matching conditions at junctions

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non-linear and non-local (on superspace) correction to WdW eqn

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$$\frac{\delta \Gamma}{\delta \Phi_B} = 0$$

quantum corrected non-linear WdW eqn, including topology change

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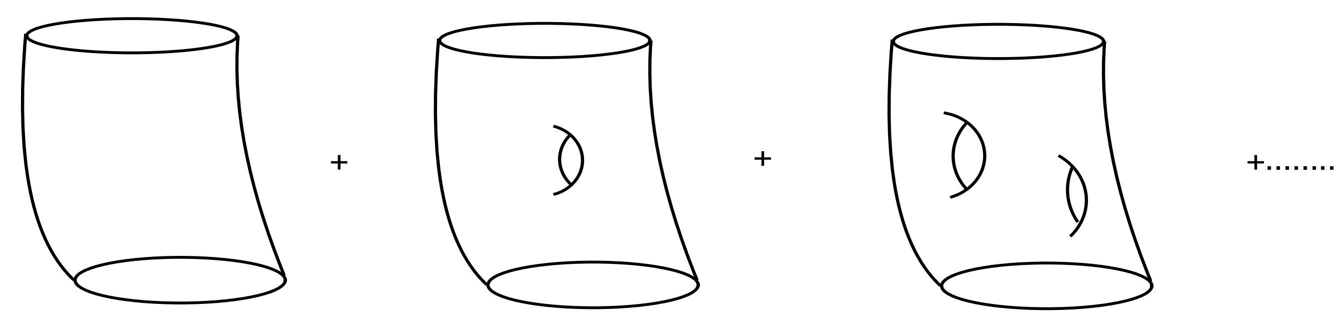

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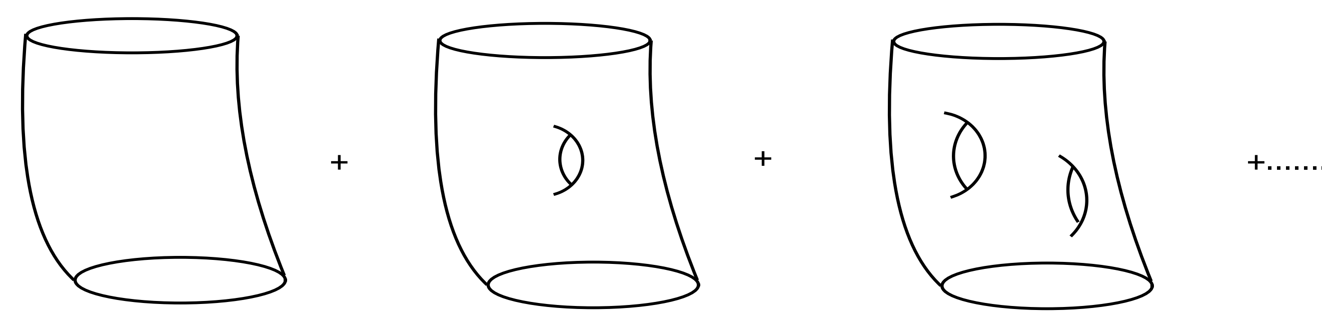

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- Hilbert space:
  - canonical Hilbert space (solutions of QG constraints) ⟶ "timeless Fock space" of "many universes"
  - "deparametrized" many-universes Fock space wrt to "clock field" appearing in 3rd quantized action

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- enormous (mathematical) difficulties - entirely formal   very limited results

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e.g. $$\left(\frac{\partial^2}{\partial a^2} - \frac{1}{a^2}\frac{\partial}{\partial \phi_i}\frac{\partial}{\partial \phi_i} - a^2 + a^4[\lambda + V(\phi_i)]\right)\Phi(a,\phi_i) = 0 \qquad a \to t\,, \qquad \phi_i \to x_i$$

  - action (only indicating dependence on scale factor)

W. Fischler, I. Klebanov, J. Polchinski, L. Susskind, '89

$$S_{\text{cubic}} = \tfrac{1}{2}\int_0^\infty \mathrm{d}t\,\left[\dot{\Phi}^2(t) + (t^2 - \lambda t^4)\Phi^2(t)\right] + \frac{g}{2}\int_0^\infty \mathrm{d}t\,\mathrm{d}t'\,\mathrm{d}t''\,\Phi(t)\Phi(t')\Phi(t'')\rho(t,t',t'')$$

classical eqns of motion = non-linear quantum cosmology

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- LQC-minisuperspace version

G. Calcagni, S. Gielen, DO, '12; M. Bojowald et al., '12;

$$\hat{\mathcal{K}}\,\psi(\nu, \phi) := -B(\nu)\left(\Theta + \partial_\phi^2\right)\psi(\nu, \phi) = 0$$

difference eqn wrt to volume eigenvalues + (massless) scalar field

$$S_{\rm i}[\Psi] = \frac{1}{2}\sum_\nu \int d\phi\; \Psi(\nu, \phi)\hat{\mathcal{K}}\Psi(\nu, \phi) + \sum_{j=2}^n \frac{\lambda_j}{j!}\sum_{\nu_1\ldots\nu_j}\int d\phi_1 \ldots d\phi_j\; f_j(\nu_i, \phi_i)\prod_{k=1}^j \Psi(\nu_k, \phi_k)$$

different choices of interaction terms (conserved quantities and matching quantities) are possible

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$$S_{\text{cubic}} = \tfrac{1}{2}\int_0^\infty \mathrm{d}t\,\left[\dot{\Phi}^2(t) + (t^2 - \lambda t^4)\Phi^2(t)\right] + \frac{g}{2}\int_0^\infty \mathrm{d}t\,\mathrm{d}t'\,\mathrm{d}t''\,\Phi(t)\Phi(t')\Phi(t'')\rho(t,t',t'')$$

classical eqns of motion = non-linear quantum cosmology

- LQC-minisuperspace version

G. Calcagni, S. Gielen, DO, '12; M. Bojowald et al., '12;

$$\hat{\mathcal{K}}\,\psi(\nu,\phi) := -B(\nu)\left(\Theta + \partial_\phi^2\right)\psi(\nu,\phi) = 0$$

difference eqn wrt to volume eigenvalues
+ (massless) scalar field

$$S_{\rm i}[\Psi] = \frac{1}{2}\sum_\nu \int d\phi\; \Psi(\nu,\phi)\hat{\mathcal{K}}\Psi(\nu,\phi) + \sum_{j=2}^{n}\frac{\lambda_j}{j!}\sum_{\nu_1\ldots\nu_j}\int d\phi_1\ldots d\phi_j\; f_j(\nu_i,\phi_i)\prod_{k=1}^{j}\Psi(\nu_k,\phi_k)$$

different choices of interaction terms (conserved quantities and matching quantities) are possible

- in fact, two possible interpretations of 3rd quantized minisuperpace QG:
  - spatial topology change - universe creation/annihilation - wormholes
  - merging/splitting of homogeneous patches of inhomogeneous universe ("separate universe" cosmology)

classical eqns of motion = non-linear quantum cosmology

3rd quantization of gravity, a QFT of universes

Coleman, Banks, Giddings, Strominger, Caderni, Martellini, Rubakov, McGuigan, Klebanov, Susskind,....., Marolf, Maxfield, ...

- enormous (mathematical) difficulties - entirely formal very limited results

- minisuperspace toy versions

e.g. $$\left(\frac{\partial^2}{\partial a^2} - \frac{1}{a^2} \frac{\partial}{\partial \phi_i} \frac{\partial}{\partial \phi_i} - a^2 + a^4 [\lambda + V(\phi_i)] \right) \Phi(a, \phi_i) = 0 \qquad a \to t, \qquad \phi_i \to x_i$$

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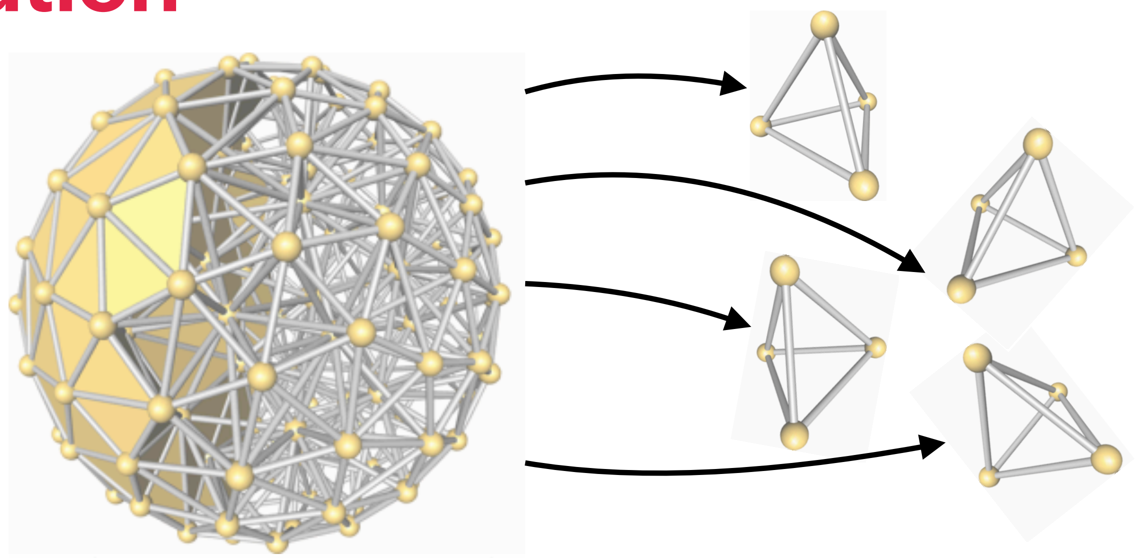
way forward? going discrete

general idea of discrete 3rd quantization

- chop universe into building blocks



result: discrete gravity path integral replacing continuum one in Feynman expansion

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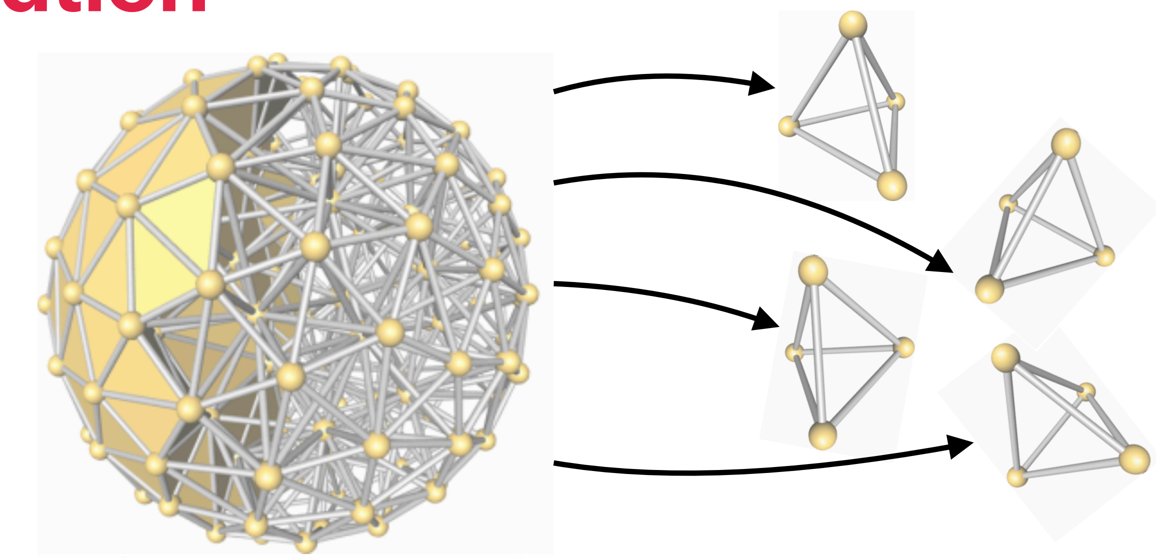

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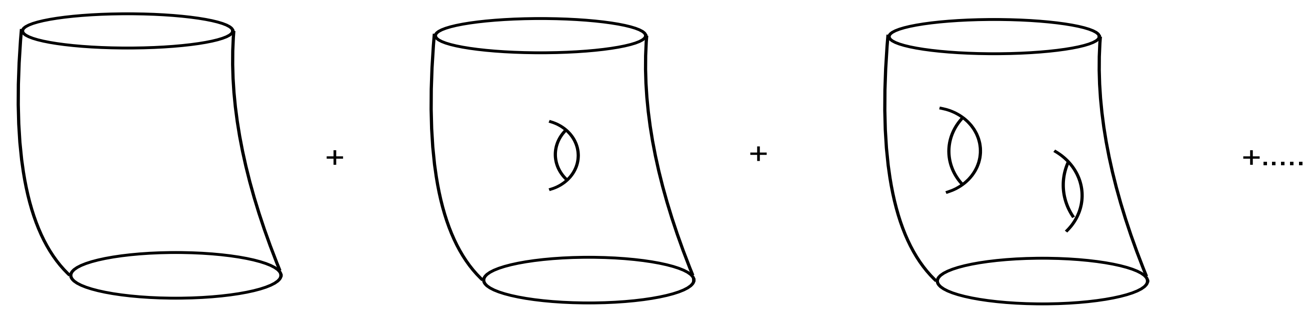

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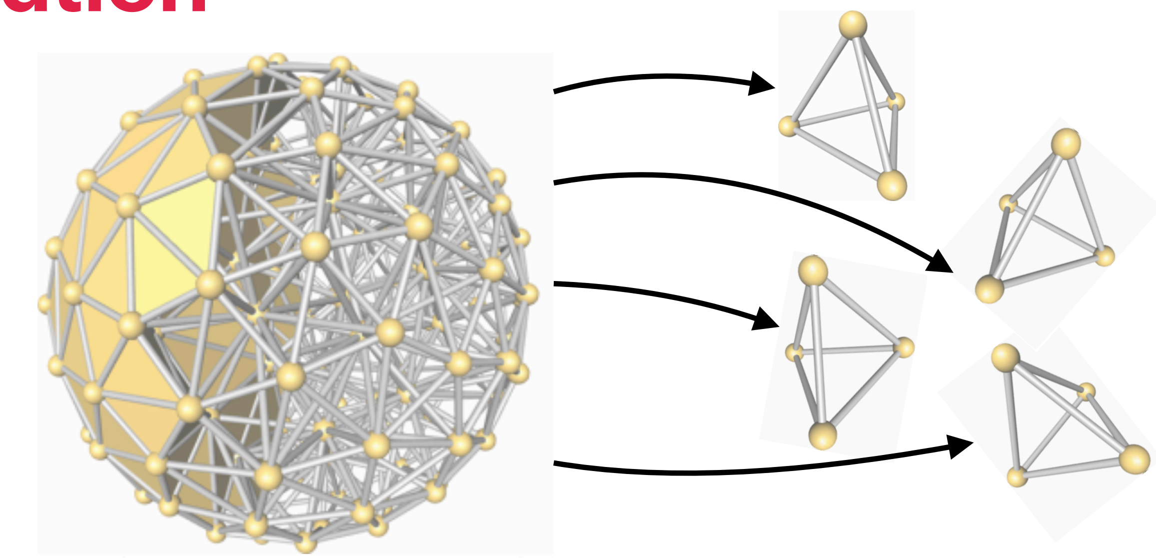

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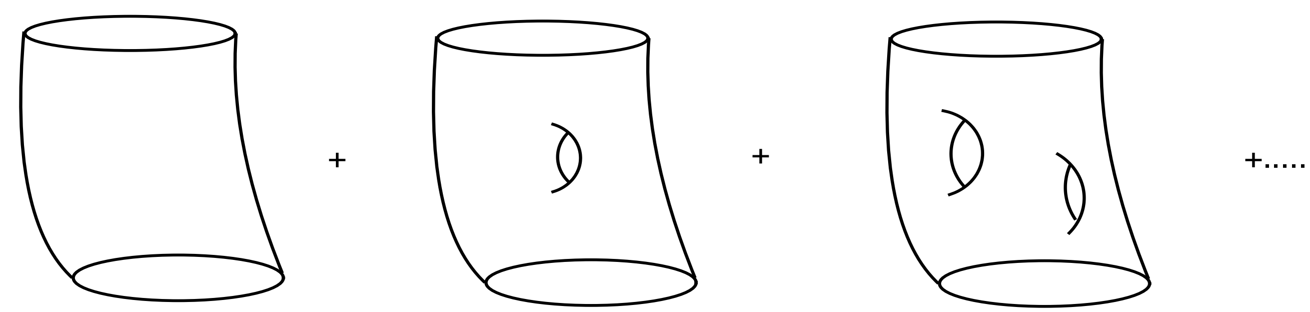

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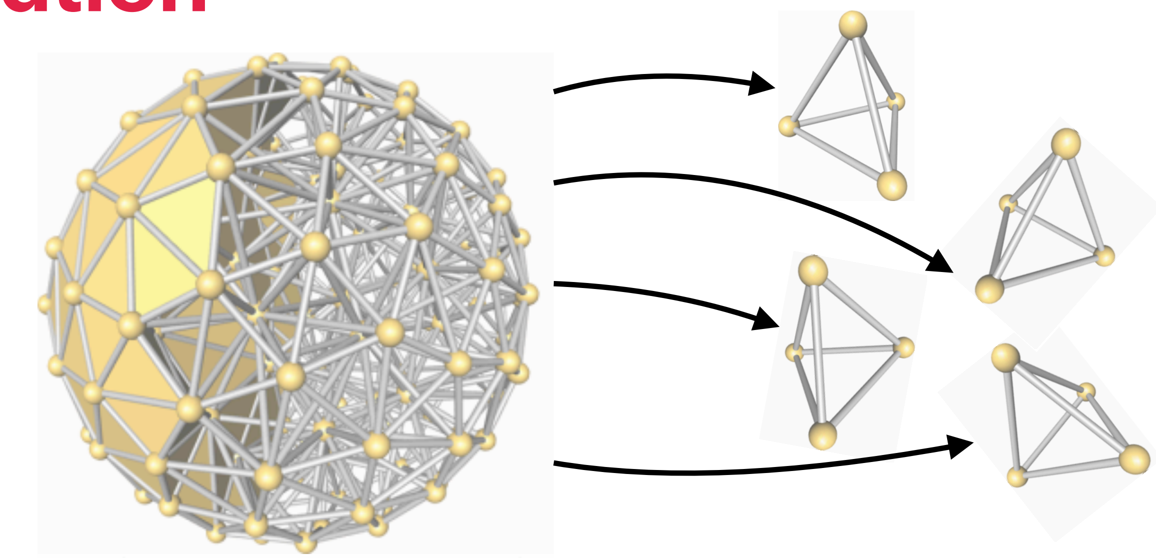

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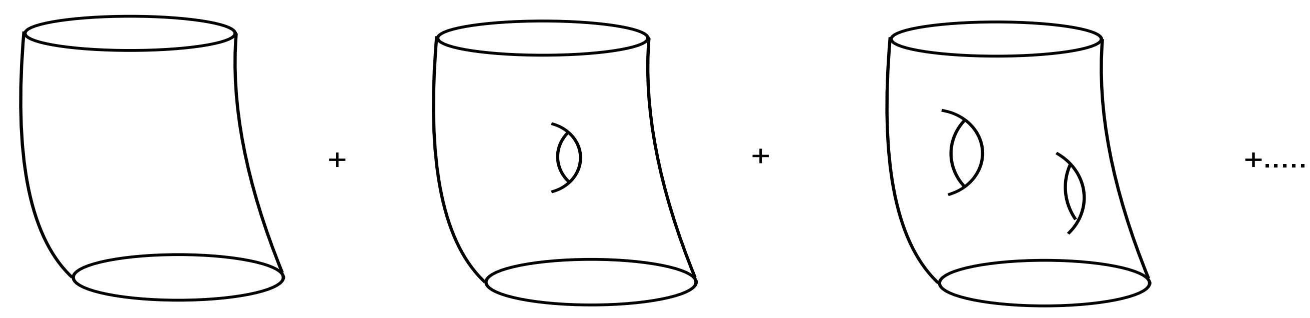

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J. Ambjorn et al, '09, '15, '21

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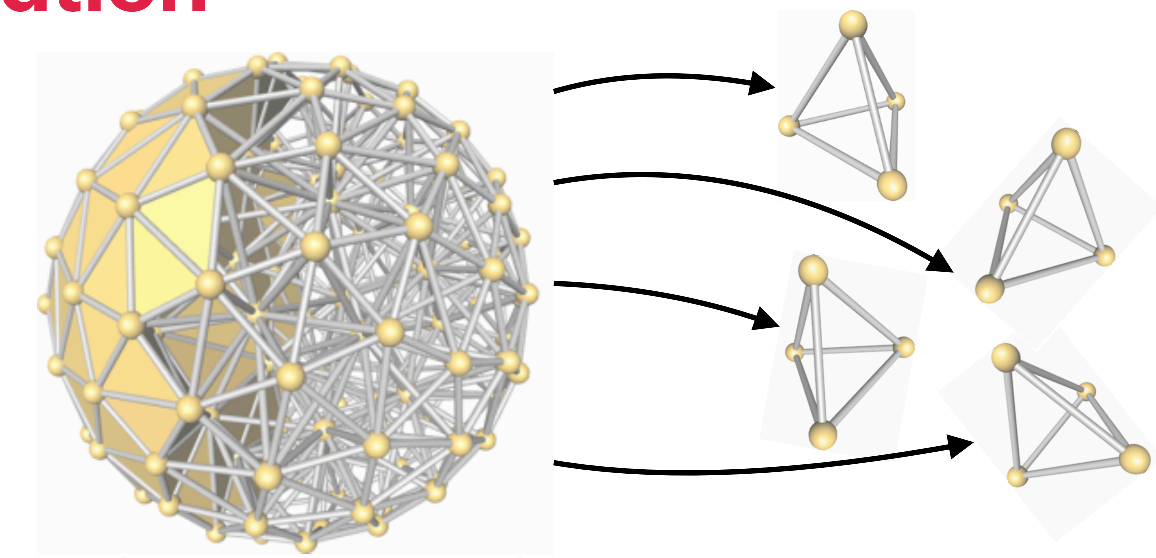

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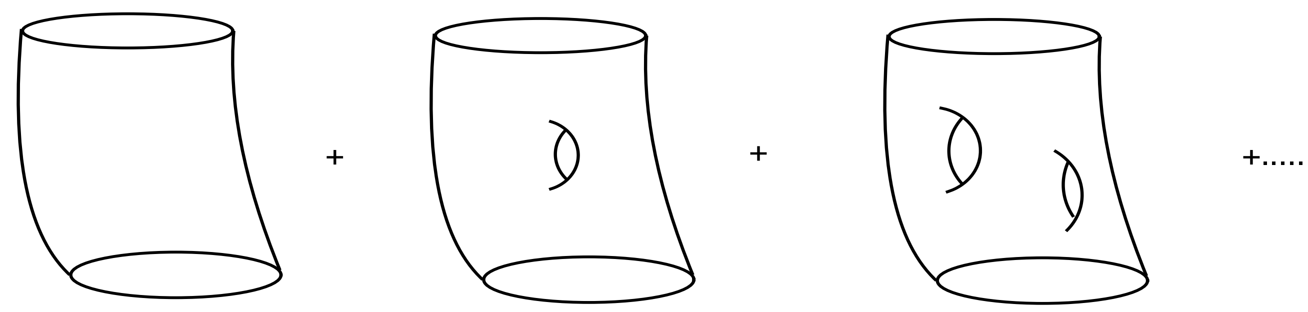

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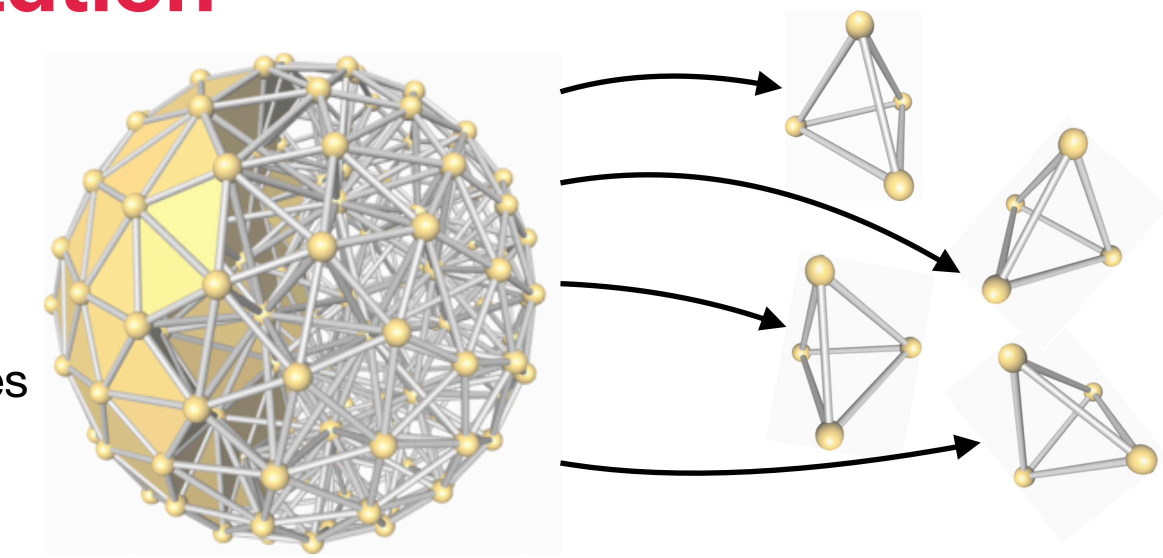

- write field theory for building blocks

states = generic assemblies of building blocks, including glued ones

interactions = discrete spacetime structures

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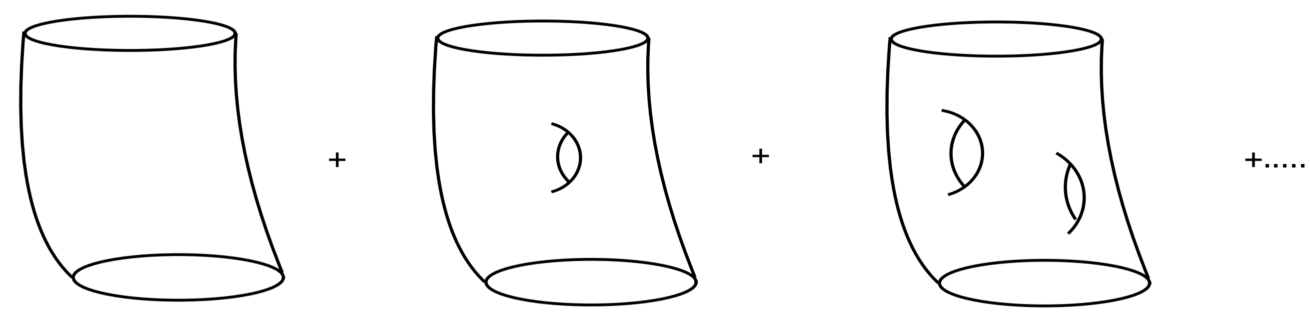

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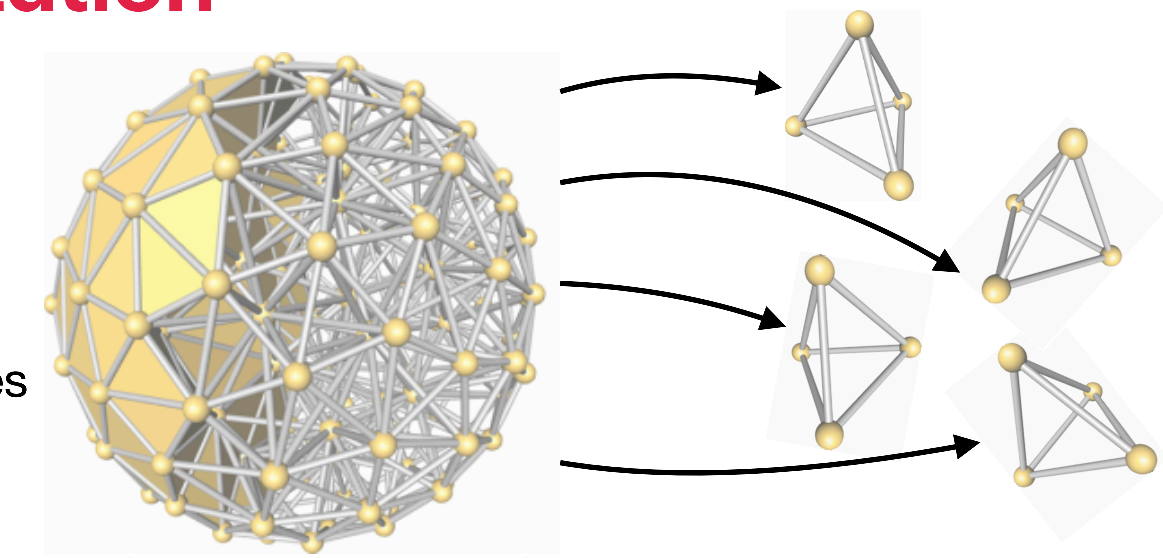

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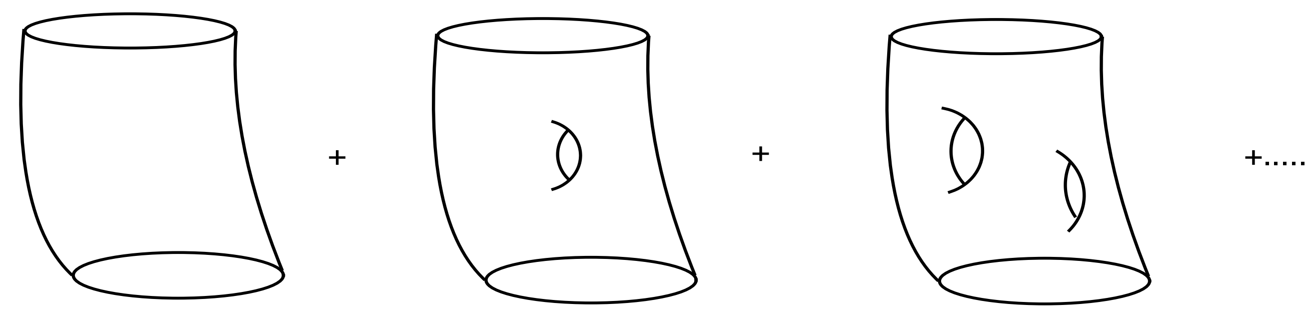

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more in spirit of emergent spacetime scenarios

Quantum gravity = quantum theory of atomic constituents of emergent spacetime

quantum theory of "new" non-spatiotemporal entities

continuum spacetime and geometric quantum observables reconstructed from collective quantum dynamics of "atoms of space"

quantum spacetime as a (background-independent) quantum many-body system

extraction of spacetime and cosmology similar to typical problem in condensed matter theory (from atoms to macroscopic effective continuum physics)

- GR from "hydrodynamic" approximation of fundamental "atomic" quantum theory
- all GR structures and dynamics are to be approximately obtained (in relational language) at effective level
- not just emergent gravity; flat spacetime itself would be emergent, highly excited, collective state of "QG atoms"

general idea of discrete 3rd quantization

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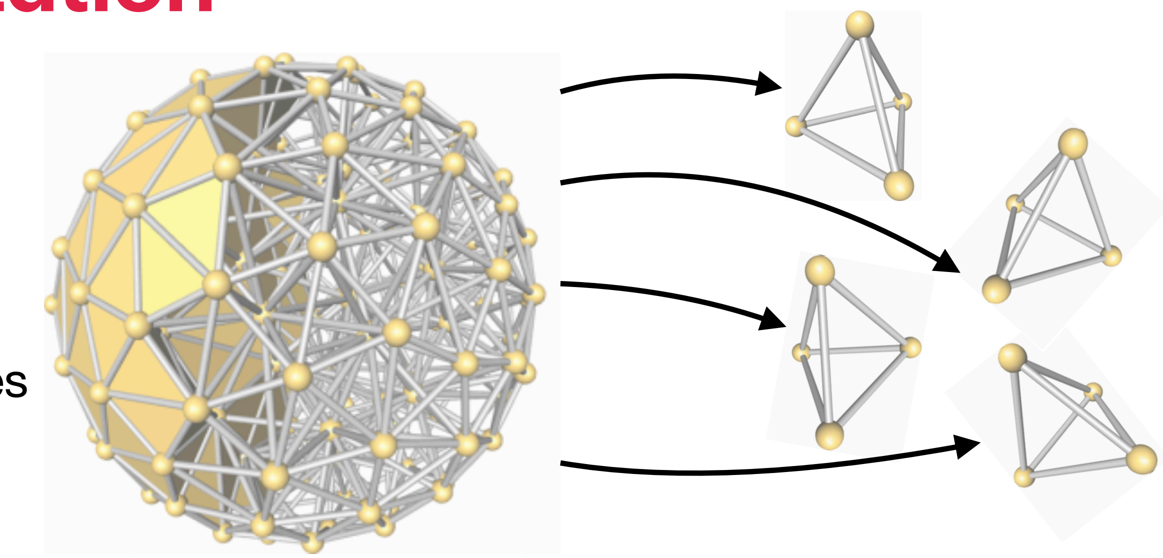

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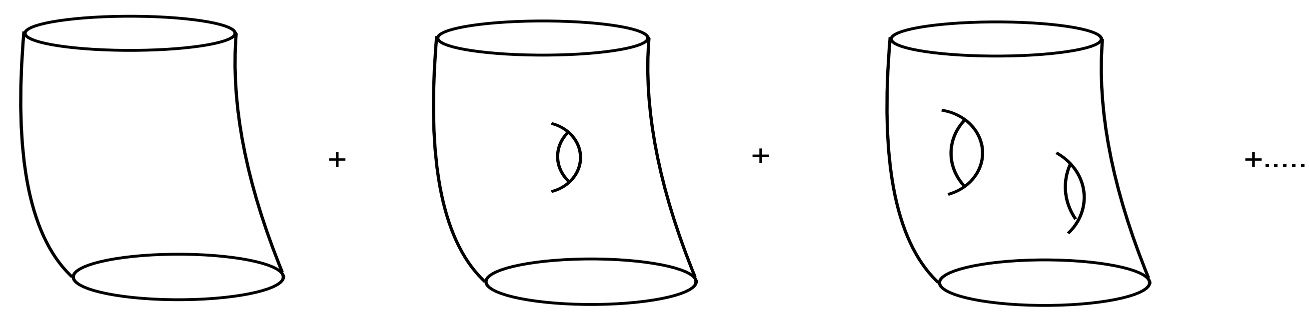

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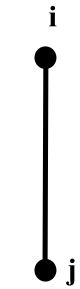
we have successful examples and promising generalizations of it

random matrix models for 2d (euclidean) QG

- matrices ~ 1d simplices (building blocks of 1d space)

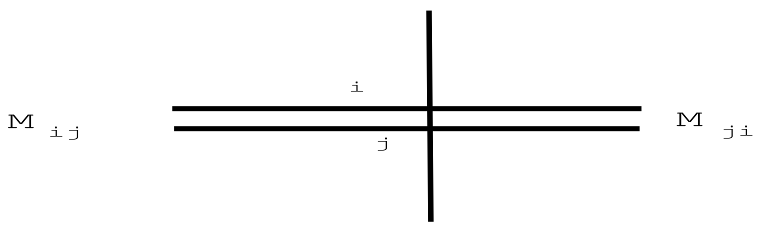
links/matrices $M^i{}_j \quad i,j = 1,\ldots,N$

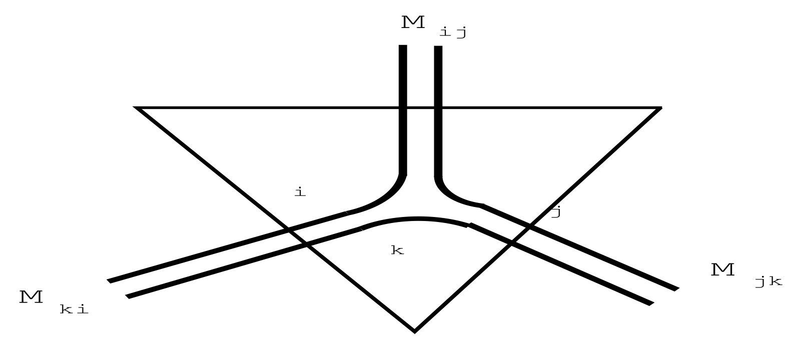


- action

example: $$S(M) = \frac{1}{2} trM^2 - \frac{g}{\sqrt{N}} trM^3 = \frac{1}{2} M^i{}_j K^{jl}{}_{ki} M^k{}_l - \frac{g}{\sqrt{N}} M^i{}_j M^m{}_n M^k{}_l V^{jnl}{}_{mki}$$

$$K^{jl}{}_{ki} = \delta^j{}_k \delta^l{}_i \qquad V^{jnl}{}_{mki} = \delta^j{}_m \delta^n{}_k \delta^l{}_i$$





- partition function:

$$Z = \sum_\Gamma \left(\frac{g}{\sqrt{N}}\right)^{V_\Gamma} Z_\Gamma = \sum_\Gamma g^{V_\Gamma} N^{F_\Gamma - \frac{1}{2} V_\Gamma} = \sum_\Gamma g^V N^\chi = \sum_\Delta e^{+\frac{4\pi}{G}\chi(\Delta) - \frac{a\Lambda}{G} t_\Delta}$$

  - Feynman diagrams = (ribbon graphs dual to) 2d cellular complexes (here, simplicial) of arbitrary topology

  - Feynman amplitudes = 2d discrete gravity path integral on equilateral lattice

discrete "locally generated" 3rd quantization: sum over (discrete) geometries + sum over topologies

topologies ~wormholes (2d setting is crucial)

control over random matrix models

$$S(M) = \frac{1}{2} trM^2 - \frac{g}{\sqrt{N}} trM^3 = \frac{1}{2} M^i{}_j K^{jl}{}_{ki} M^k{}_l - \frac{g}{\sqrt{N}} M^i{}_j M^m{}_n M^k{}_l V^{jnl}{}_{mki}$$

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- in large-N limit, planar (spherical) diagrams dominate, i.e. trivial topology

- continuum limit = phase transition (condensation) to theory of large continuum surfaces

expectation value for the total area of surface, for large number of vertices, is:

$$\langle A \rangle = a \langle t_\Delta \rangle = \langle V_\Gamma \rangle = a \frac{\partial}{\partial g} \ln Z_0(g) \simeq \frac{a}{g - g_c}$$

- which continuum theory does it correspond to? 2d quantum Liouville gravity

- double scaling limit:

defining: $\kappa^{-1} = N(g - g_c)^{\frac{(2-\beta)}{2}}$ we get: $Z \simeq \sum_h \kappa^{2h-2} f_h = \kappa^{-2} f_0 + f_1 + \kappa^2 f_2 + \ldots\ldots$

can take combined limit $N \to \infty$ and $g \to g_c$ holding κ fixed $\Rightarrow$ continuum limit to which all topologies contribute!

⟶ 3rd quantization of 2d Liouville (euclidean) QG

Tensor models

(Ambjorn, Durhuus, Sasakura, …, Gurau, Rivasseau, Bonzom, Ryan, Benedetti, Ben Geloun, Tanasa, …..)

Construction generalized to D dimensions (tensor models generating D-dimensional simplicial complexes)

$$T_{i_1 \ldots i_D}$$ corresponding to a (D-1)-simplex

real rank-D tensor

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← pattern of gluing of D+1 (D-1)-simplices to form boundary of D-simplex

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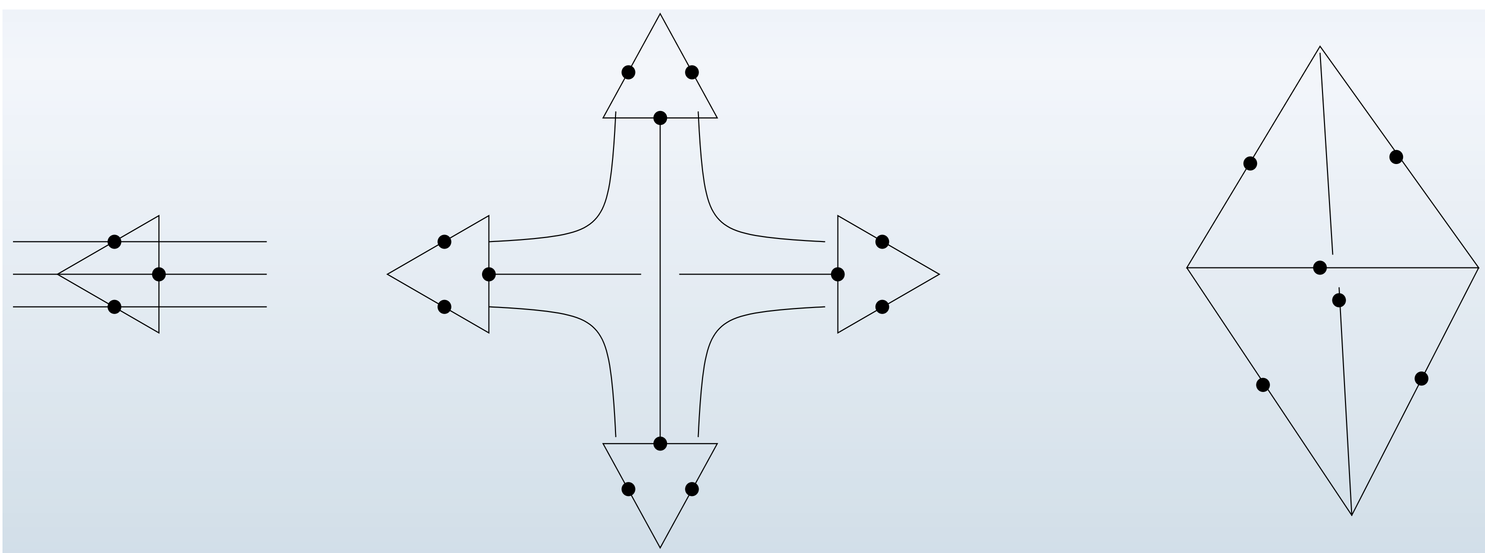

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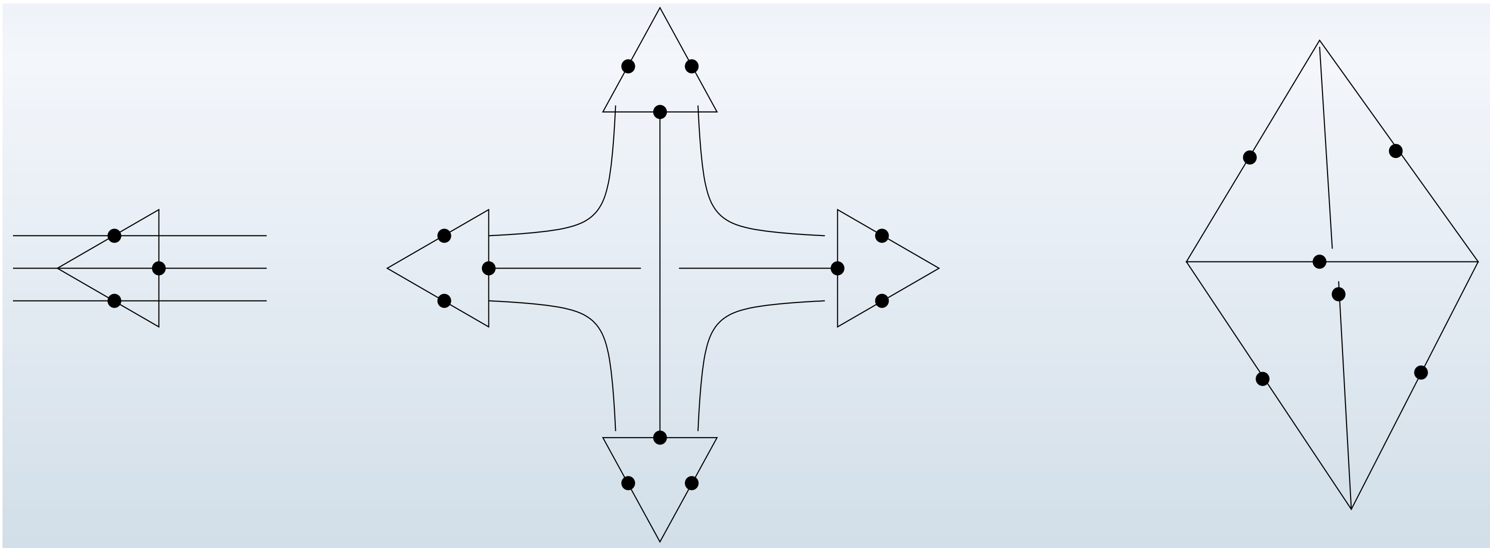

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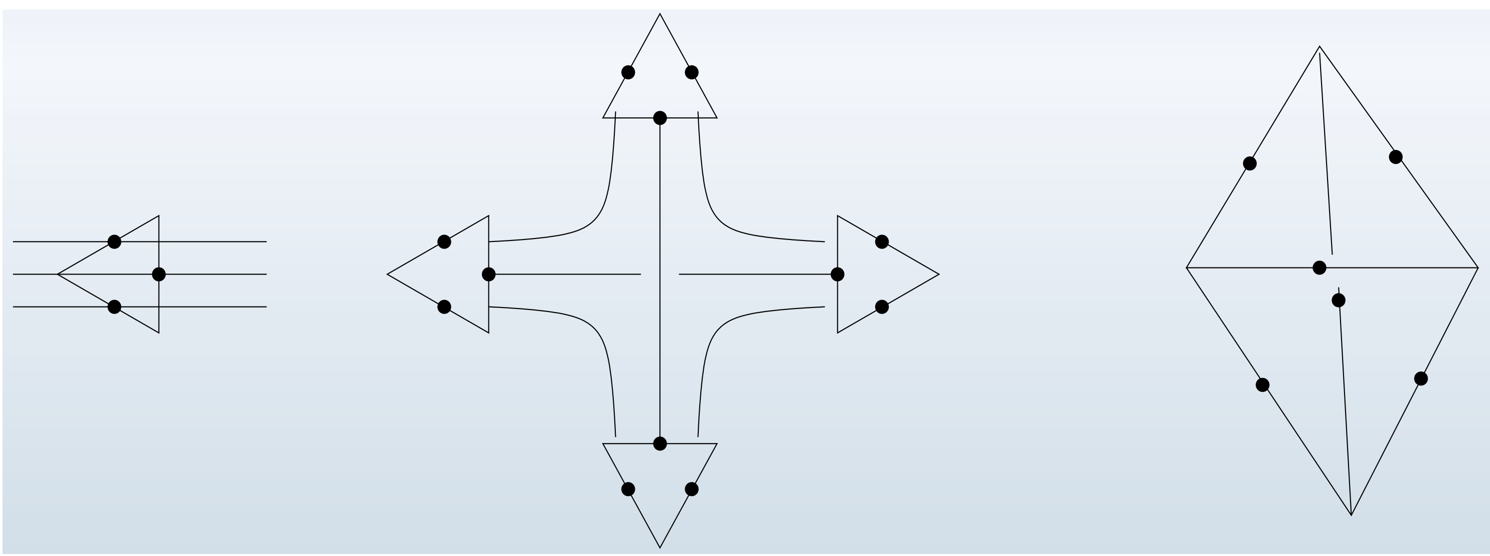

- Feynman amplitudes = discrete gravity path integral on equilateral lattice
- purely combinatorial 3rd quantization
- all topologies (not just wormholes) included in perturbative sum
- also spatial topologies can be dynamical
- finite system - correspondence to gravity to be looked for in continuum large-N limit

adding data to the tensors: tensorial group field theories

toward a full 3rd quantization picture (i.e. richer field domain & quantum geometry)

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G = Lie group

$$T_{i_1,\ldots,I_D} \longrightarrow \varphi(g_1, \ldots, g_D) \qquad \varphi : G^D \to \mathbb{C}$$

(can extend to quantum groups)

domain can be extended to include local directions $\varphi(g_1, \ldots, g_D; \vec{\chi})$ $\qquad \varphi : G^D \times \mathbb{R}^d \to \mathbb{C}$

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- field theory action with non-local interactions, describing how simplices connect to form higher-cells

details depend on (class of) models

$$S(\varphi, \overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia}, \bar{g}_{iD}) \quad + \quad c.c.$$

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in pairing of field arguments

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- Feynman diagrams are dual to cellular complexes of any topology

- perturbative expansion of quantum dynamics gives sum over cellular complexes of all topologies

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proper QFT (on group manifold)

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proper QFT (on group manifold)

which data? which dynamics (action, Feynman amplitudes)? ------> quantum geometric models

TGFTs: well defined, controllable QFTs?

fully quantum geometric TGFTs --> more involved quantum amplitudes

simpler TGFT models --> more mathematical control

- control over topology/combinatorics of TGFT diagrams
  - techniques from crystallization theory (colored graphs, …) are crucial

Gurau, Rivasseau, Bonzom, Ben Geloun, Tanasa, Riello, Carrozza, Kaminski, Ryan, ……

- large-N limit and melonic regime (N ~size of tensors ~ cut-off in irrep labels)

- perturbative renormalizability

Benedetti, Ben Geloun, Carrozza, Tanasa, DO, Rivasseau, Gurau, Lahoche, Ousmane-Samary, ……

  - many renormalizable TGFT models
    - different dimensions (rank), abelian & non-abelian groups, various conditions (e.g. gauge invariance)
  - quantum geometric 4d TGFT models (GFTs) more challenging
    - results on scaling of amplitudes (for some diagrams) ~ radiative corrections
    - (including all those obtained from spin foam perspective)

T. Krajewski et al., ’10; A. Riello, ’13; V. Bonzom, B. Dittrich, ’15; P. Dona’, ‘17; P. Dona et al, '19; M. Finocchiaro, DO, '20; P. Dona et al. '22

- constructive aspects

Benedetti, Gurau, Rivasseau, …..

TGFTs: well defined, controllable QFTs?

fully quantum geometric TGFTs --> more involved quantum amplitudes

simpler TGFT models --> more mathematical control

- Functional Renormalization Group analysis
  - different dimensions (rank), abelian & non-abelian groups, various conditions (e.g. gauge invariance)
  - flows beyond melonic sector, studies of asymptotic safety/freedom

Ben Geloun, Carrozza, Tanasa, Toriumi, Krajewski, Martini, DO, Rivasseau, Gurau, Lahoche, Ousmane-Samary, Benedetti, Pithis, Thürigen, ..

- critical behaviour
  - under analytic control for tensor models and simple TGFTs
  - analysis of critical behaviour and phase transitions in IR, via FRG, for TGFTs

- Landau-Ginzburg mean field analysis

Marchetti, DO, Pithis, Thurigen, ...

  - also for fully quantum geometric models
  - mean field approx appears more reliable for more physical GFTs

see talk by A. Pithis

GFTs: basics

4d case - specific class of models

Barrett, Crane, De Pietri, Freidel, Krasnov, Rovelli, Perez, DO, Livine, ...

see talk by H. Haggard

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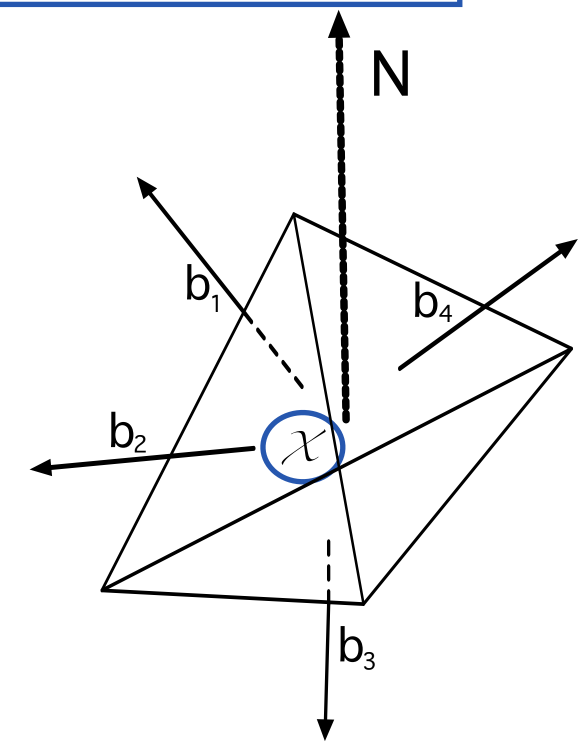

- geometric variables: triangle vectors ~ su(2) Lie algebra elements

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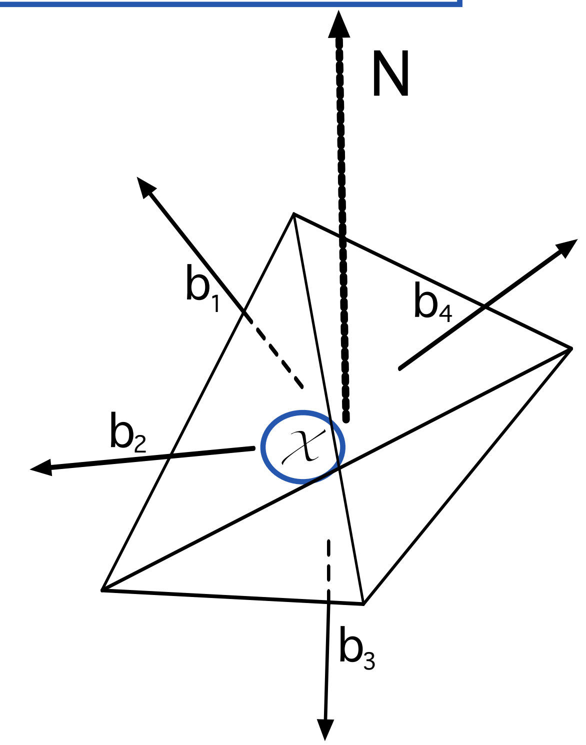

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- observables: e.g. triangle areas, volume

$$A_i = |b_i| \qquad V = \frac{1}{6}\sqrt{\vec{b_1} \cdot \vec{b_2} \times \vec{b_3}}$$

become operators: $\vec{b_i} \to \hat{\vec{J}}_i$

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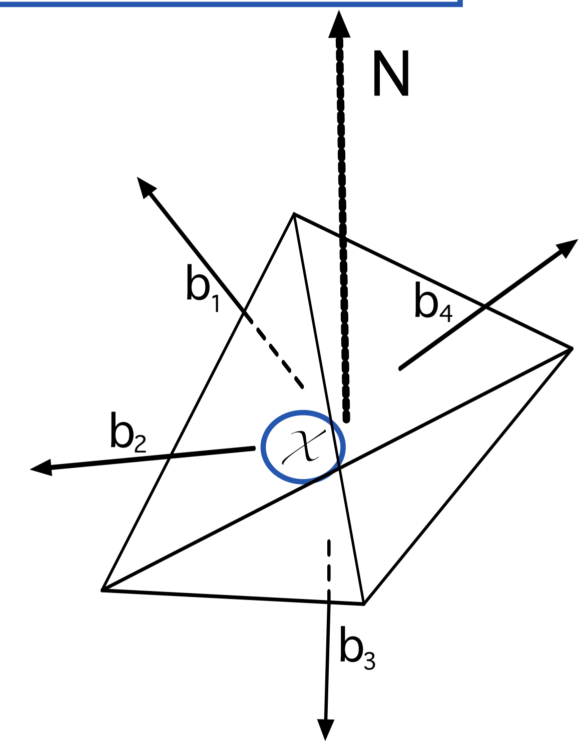

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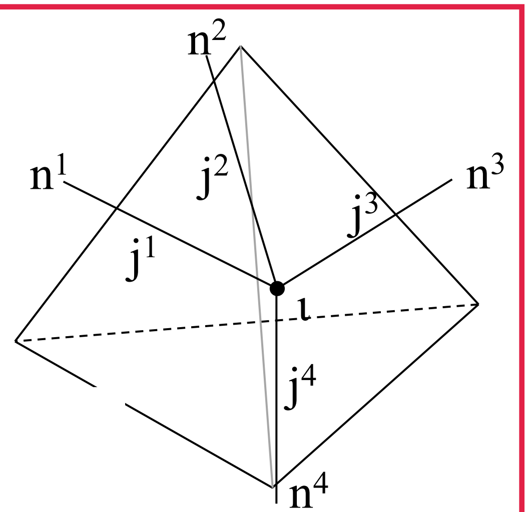
Hilbert space of quantum tetrahedron

(in terms of SU(2) irreps)

spin network vertex ~ quantum tetrahedron

$$\mathcal{H}_v = \bigoplus_{\vec{j}_v}\left(\bigotimes_{i=1}^{d} \underbrace{V^{j_v^i}}_{\text{repr. space}} \otimes \underbrace{\mathcal{I}^{\vec{j}_v}}_{\text{intertwiner space}}\right)$$



quantum geometric operators act on this Hilbert space:

$|j^i n^i\rangle \in V^{j^i}$ diagonalises area operator

$|\iota^{\vec{j}}\rangle \in \mathcal{I}^{\vec{j}} = \mathrm{Inv}_G\left[V^{j^1}\otimes ... \otimes V^{j^d}\right]$ diagonalises volume operator

+ scalar dofs $\otimes L^2(\mathbb{R}\times ... \times\mathbb{R})$

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Barrett, Crane, De Pietri, Freidel, Krasnov, Rovelli, Perez, DO, Livine, ...

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  thus

  $L^2(SU(2)^4 / SU(2))$ (quantum geometry dofs)

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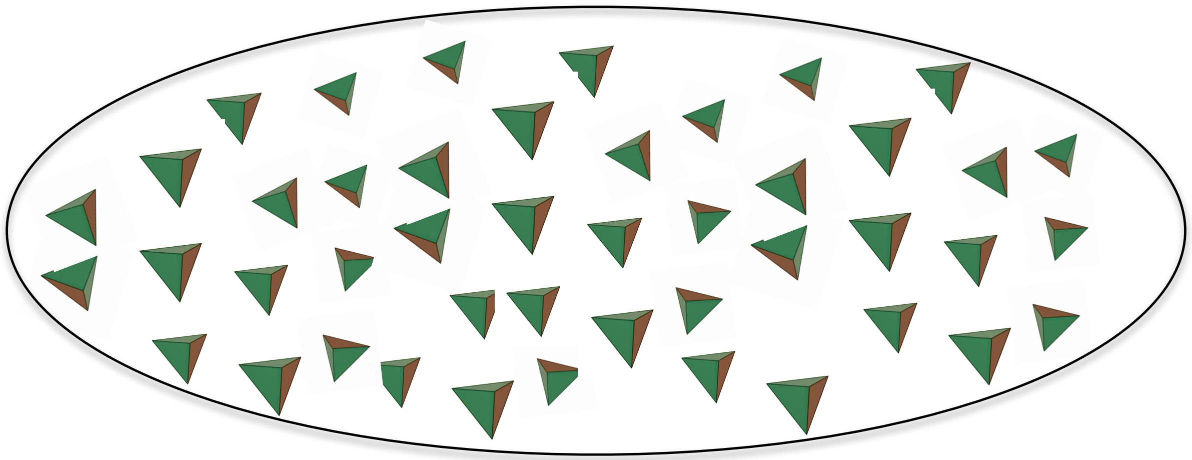

- Fock space

$$\mathcal{F}(\mathcal{H}_v) = \bigoplus_{V=0}^{\infty} sym\left\{\left(\mathcal{H}_v^{(1)} \otimes \mathcal{H}_v^{(2)} \otimes \cdots \otimes \mathcal{H}_v^{(V)}\right)\right\}$$

- GFT field operators (creating/annihilating tetrahedra):

$$\hat{\varphi}(g_I, \chi^a) \equiv \hat{\varphi}(g_I, \chi^1, \ldots, \chi^n) \qquad \left[\hat{\varphi}(\vec{g}), \hat{\varphi}^\dagger(\vec{g}')\right] = \mathbb{I}_G(\vec{g}, \vec{g}') \qquad \left[\hat{\varphi}(\vec{g}), \hat{\varphi}(\vec{g}')\right] = \left[\hat{\varphi}^\dagger(\vec{g}), \hat{\varphi}^\dagger(\vec{g}')\right] = 0$$

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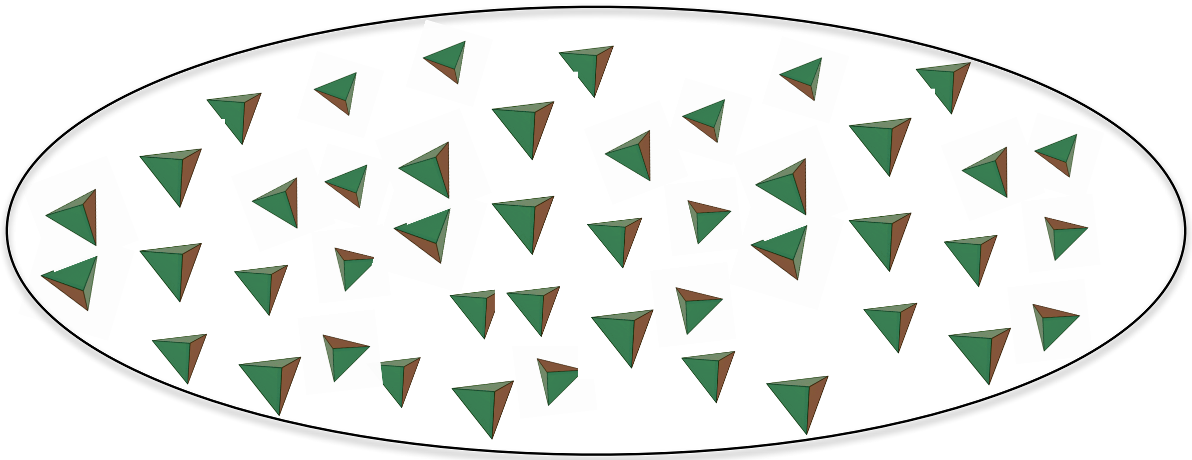

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  e.g. volume $\hat{V}_{tot} \; = \int [dg_i][dg'_j] \hat{\varphi}^\dagger(g_i) \, V(g_i, g'_j) \, \hat{\varphi}(g'_j) \; = \sum_{J_i} \hat{\varphi}^\dagger(J_i) \, V(J_i) \, \hat{\varphi}(J_j)$

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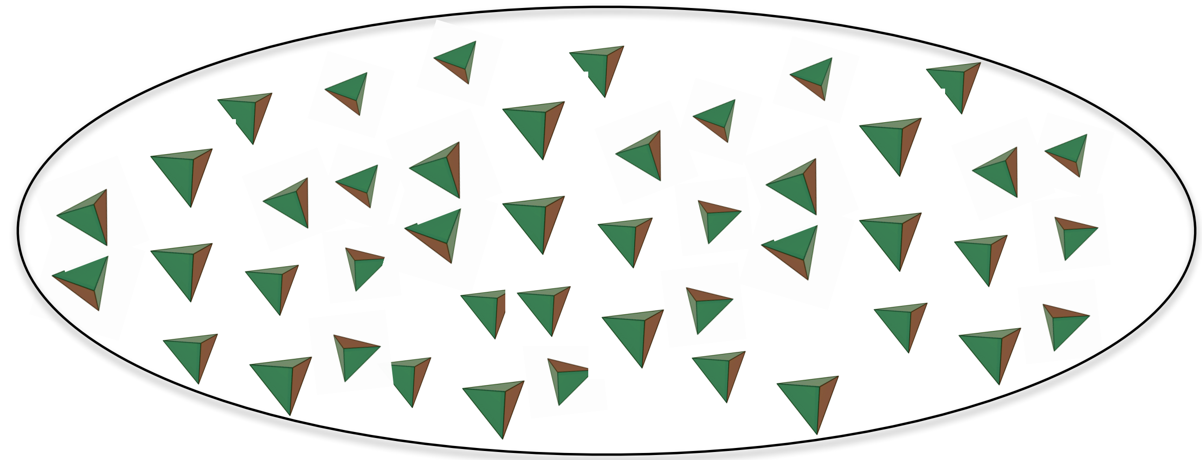

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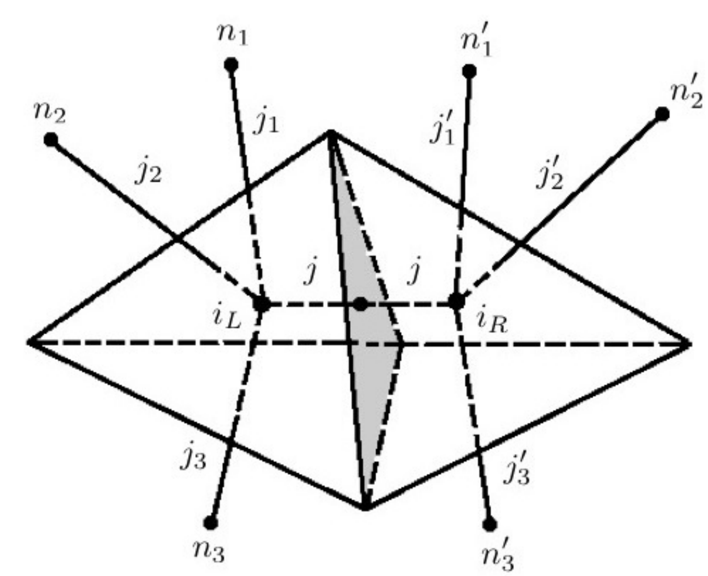
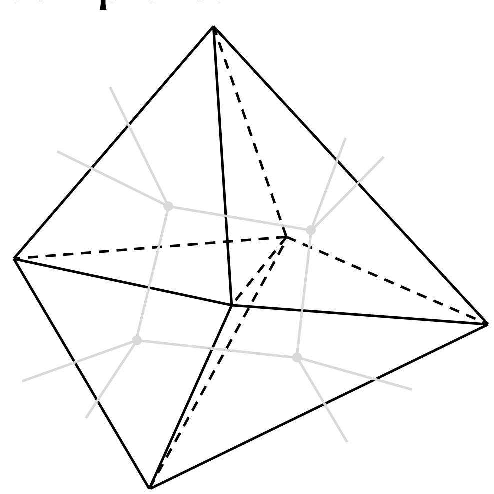
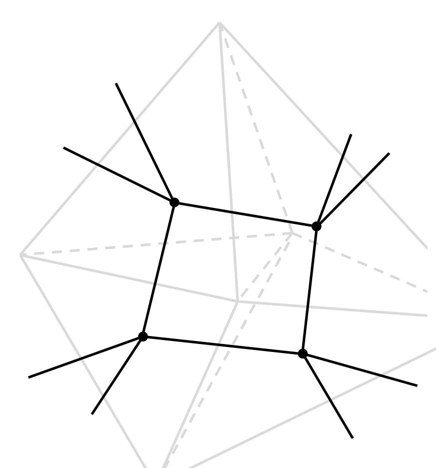
- maximal entanglement of "triangle dofs" ~ gluing of tetrahedra across triangle



entangled states ~ extended simplicial complexes

see talks by S. Langenscheidt & G. Chirco





GFTs: basics

4d case - specific class of models

Barrett, Crane, De Pietri, Freidel, Krasnov, Rovelli, Perez, DO, Livine, ...

dynamics of quantum atomic geometry

GFT action = prescription for weights associated to building blocks of 4d lattice in sum over discrete geometries

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De Pietri, Petronio, '00; R. Gurau, '10; ...

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basic guideline for choosing action: quantum geometric input from canonical LQG, simplicial geometry

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Reisenberger,Rovelli, '00

A. Baratin, DO, '11

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fully discrete and quantum geometric 3rd quantization: QFT for quantum "atoms of space"

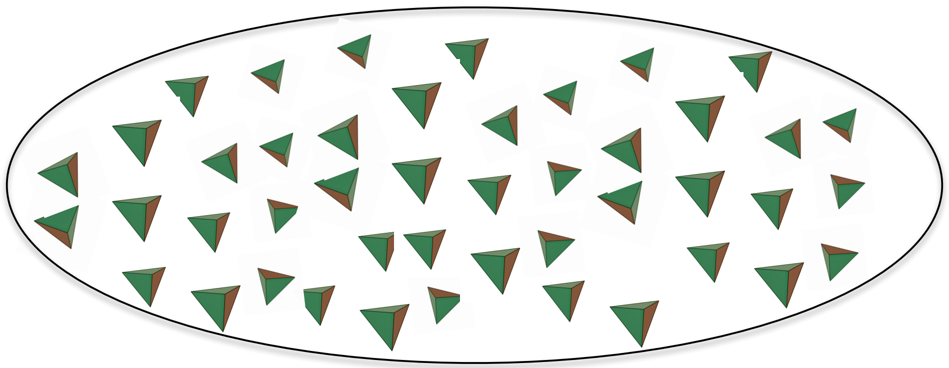
- GFT quanta ~ quantum tetrahedra ~ spin network vertices
- entangled GFT many-body states ~ (2nd quantized) spin networks
- GFT (perturbative) amplitudes = spin foam amplitudes ~ simplicial gravity path integrals

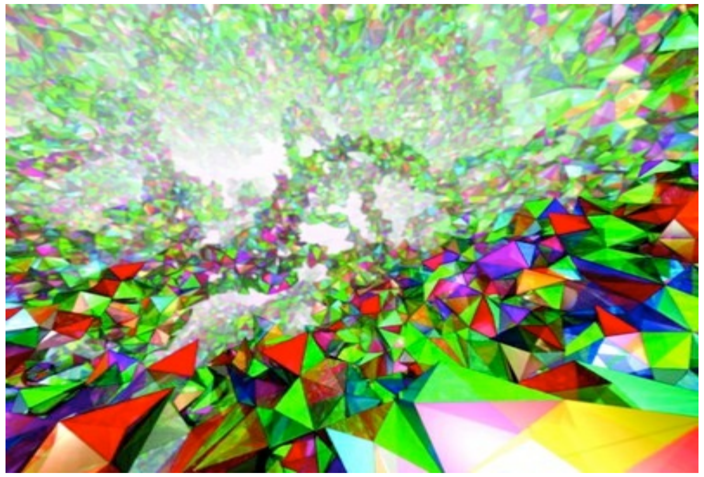
GFT (condensate) cosmology: general strategy

from perspective of fundamental QG atoms of space:

continuum geometry = coarse-grained description of discrete geometry of many (infinite) QG atoms

GR dynamics = approximate description of collective quantum dynamics of many (infinite) QG atoms





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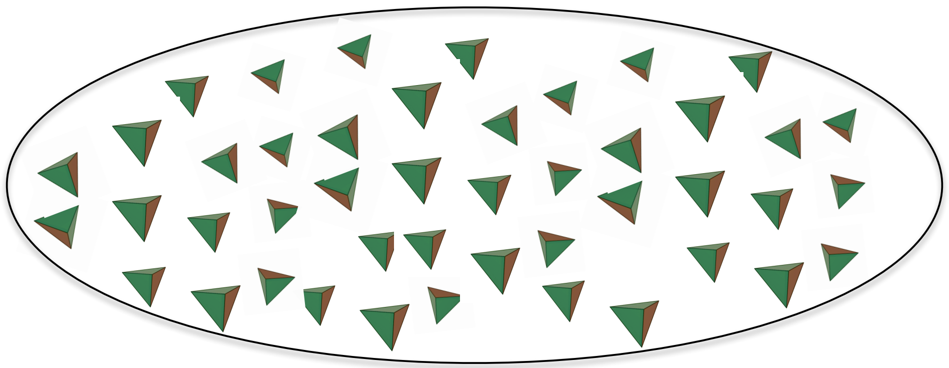

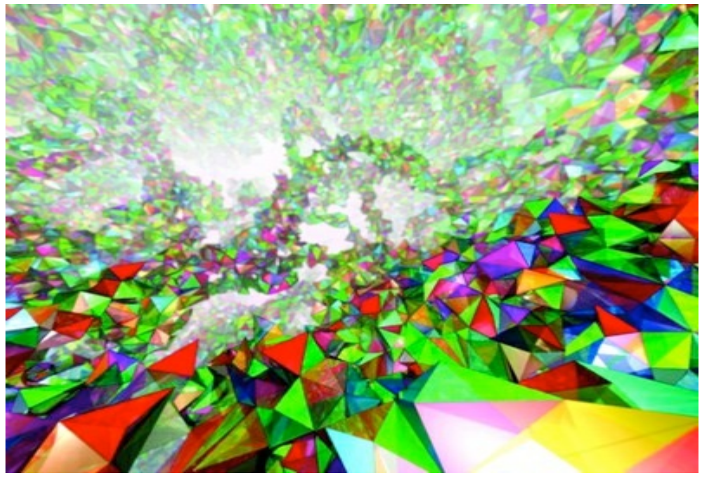

extracting effective continuum dynamics from QG ~ typical problem of quantum many-body physics

GFT: similar QFT language and tools as in quantum many-body physics

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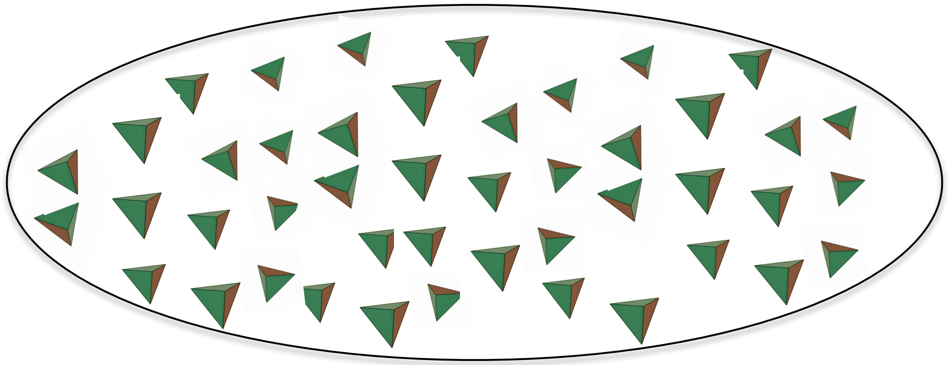

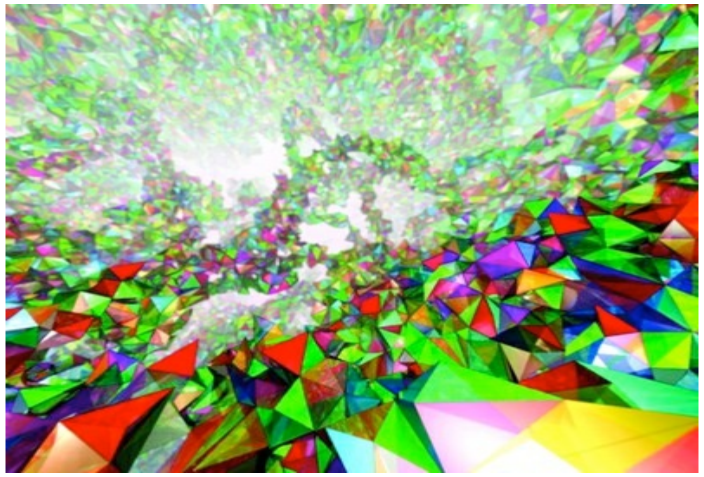

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GFT: similar QFT language and tools as in quantum many-body physics

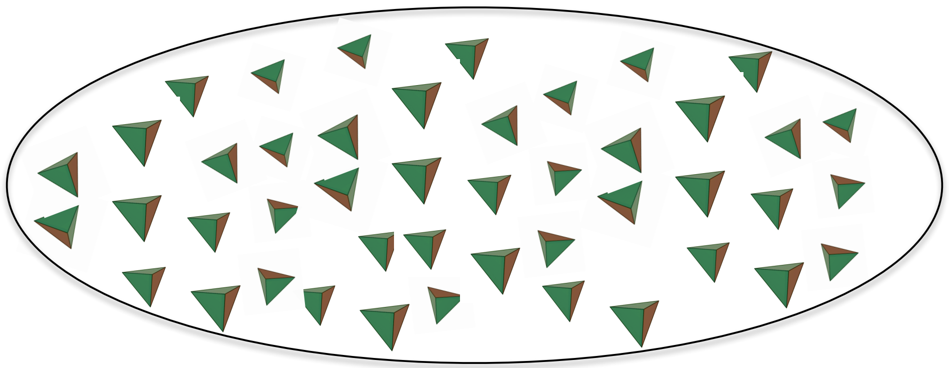
note: this is main outstanding issue of all non-perturbative QG approaches

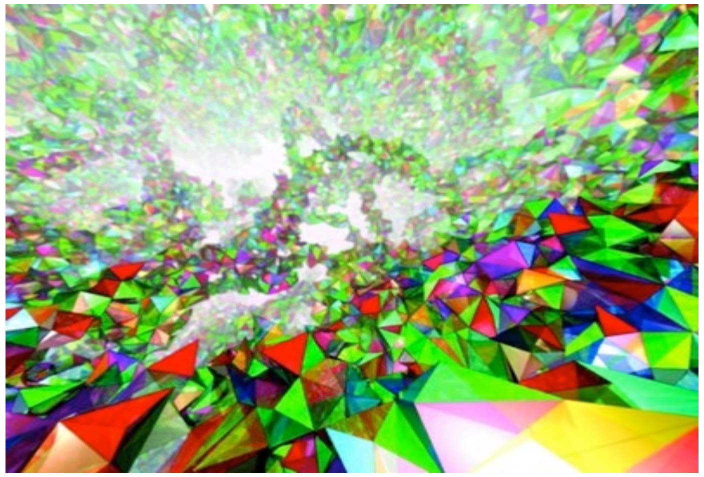
GFT (condensate) cosmology: general strategy

from perspective of fundamental QG atoms of space:

continuum geometry = coarse-grained description of discrete geometry of many (infinite) QG atoms

GR dynamics = approximate description of collective quantum dynamics of many (infinite) QG atoms





extracting effective continuum dynamics from QG ~ typical problem of quantum many-body physics

GFT: similar QFT language and tools as in quantum many-body physics

note: this is main outstanding issue of all non-perturbative QG approaches

cosmology expected to correspond to "most coarse-grained" dynamics

→ in other words: effective dynamics of special (global) observables of full theory → QG hydrodynamics

GFT (condensate) cosmology: general strategy

- hypothesis: universe as QG quantum fluid (condensate)
- extract approximate hydrodynamic eqns for QG fluid (density and phase)
- compute relational cosmological observables in hydrodynamic approximation
- translate hydrodynamic eqns into eqns for cosmological observables

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$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\overline{\varphi}\; e^{i\,S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\,\mathcal{A}_\Gamma$$

$$F_\lambda(J) \;=\; \ln Z_\lambda[J] \qquad \Gamma[\phi] \;=\; sup_J\,(J\cdot\phi - F(J)) \qquad \langle\varphi\rangle = \phi$$

✳ simplest approximation: mean field hydrodynamics

$$\Gamma[\phi] \;\approx\; S_\lambda(\phi)$$

mean field ~ condensate wavefunction

- corresponding quantum states:

(simplest): GFT field coherent state

$$|\sigma\rangle := \exp\left(\hat{\sigma}\right)|0\rangle$$

$$\hat{\sigma} := \int d^4g\;\sigma(g_I)\hat{\varphi}^\dagger(g_I) \qquad \sigma(g_I k) \;=\; \sigma(g_I)$$

superposition of infinitely many spin networks dofs, “gas”of tetrahedra, all associated with same state

condensate wavefunction (also incl. scalar dofs)

GFT (condensate) cosmology: general features

- immediate cosmological interpretation of (domain of) condensate wavefunction:

isomorphism between domain of TGFT condensate wavefunction and minisuperpsace

$$\sigma\left(\mathcal{D}\right) \qquad \mathcal{D} \simeq \{\text{geometries of tetrahedron}\} \simeq$$
$$\simeq \{\text{continuum spatial geometries at a point}\} \simeq$$
$$\simeq \text{minisuperspace of homogeneous geometries}$$

S. Gielen, DO, L. Sindoni, '13

S. Gielen, '15

A. Jercher, DO, A. Pithis, '21

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- general form of resulting (Gross-Pitaevskii) equations of motion for condensate wavefunction (mean field):

$$\int [dg'] d\chi' \mathcal{K}(g,\chi;g',\chi')\sigma(g',\chi') + \lambda \frac{\delta}{\delta\varphi}\mathcal{V}(\varphi)|_{\varphi\equiv\sigma} = 0$$

Gielen, DO, Sindoni, '13; DO, Sindoni, Wilson-Ewing, '16

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that is, in isotropic restriction and with just one matter field:

$\sigma(a,\phi)$	"wavefunction" on minisuperspace
$\mathcal{K}(a,\partial_a,\phi,\partial_\phi)\sigma(a,\phi) + \mathcal{V}'[\sigma(a,\phi)] = 0$	hydrodynamic (non-linear, possibly non-local) eqn on minisuperspace

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like in minisuperspace 3rd quantization, but:

- kinetic term is not WdW operator
- interaction term dictated by simplicial quantum geometry, not continuum topology change or separate universe cosmology

Derivation of effective cosmological dynamics: main steps

quantum geometric EPRL model with 4 scalar dofs (1 clock+ 3 rods + 1 matter scalar field)

$$S_{\text{GFT}} = K + U + U^*$$

$$K = \int \mathrm{d}g_I \, \mathrm{d}h_I \int \mathrm{d}^d\chi \, \mathrm{d}^d\chi' \, \mathrm{d}\phi \, \mathrm{d}\phi' \, \bar{\varphi}(g_I, \chi) \mathcal{K}(g_I, h_I; (\chi - \chi')^2_\lambda, (\phi - \phi')^2) \varphi(h_I, (\chi')^\mu, \phi')$$

$$U = \int \mathrm{d}^d\chi \, \mathrm{d}\phi \int \left(\prod_{a=1}^{5} \mathrm{d}g_I^a \right) \mathcal{U}(g_I^1, \ldots, g_I^5) \prod_{\ell=1}^{5} \varphi(g_I^\ell, \chi^\mu, \phi)$$

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restriction to "good clock+rods" condensate states - peakedness properties on clock/rod values

peaked functions (e.g. Gaussians)

$$\sigma_{\epsilon,\delta,\pi_0,\pi_x;x^\mu}(g_I, \chi^\mu, \phi) = \eta_\epsilon(\chi^0 - x^0; \pi_0) \eta_\delta(|\boldsymbol{\chi} - \mathbf{x}|; \pi_x) \tilde{\sigma}(g_I, \chi^\mu, \phi)$$

L. Marchetti, DO, '20, '21

$|\boldsymbol{\chi} - \mathbf{x}|^2 = \sum_{i=1}^d (\chi^i - x^i)^2$ $\quad \mathbb{C} \ni \delta = \delta_r + i\delta_i$ $\quad \delta_r > 0$ $\quad \epsilon, |\delta| \ll 1$ $\quad z_0 \equiv \epsilon \pi_0^2/2$ $\quad z \equiv \delta \pi_x^2/2$

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simplifying assumptions:

- subdominant GFT interactions: U << K (consistent with LQG/spin foam and discrete gravity interpretation)
- isotropy: condensate wavefunction depends on single j (plus clock/rods/matter)

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resulting (free) mean field hydrodynamics eqn:

Fourier mode of matter field variable

$$\partial_0^2\tilde{\sigma}_j(x,\pi_\phi) - i\gamma\partial_0\tilde{\sigma}_j(x,\pi_\phi) - {}^{(\lambda)}E_j^2(\pi_\phi)\tilde{\sigma}_j(x,\pi_\phi) + \alpha^2\nabla^2\tilde{\sigma}_j(x,\pi_\phi) = 0$$

dependence on parameters of model and state

$$\gamma \equiv \frac{\sqrt{2\epsilon z_0}}{\epsilon z_0^2} \qquad {}^{(\lambda)}E_j^2 \equiv \frac{1}{\epsilon z_0^2} - r_{j;2}(\pi_\phi)\left(1+3\lambda\alpha^2\right) \qquad \alpha^2 \equiv \frac{1}{3}\frac{\delta z^2}{\epsilon z_0^2} \qquad r_s^{(\lambda)} \equiv \frac{\tilde{K}_\lambda^{(s)}}{\tilde{K}_\lambda^{(0)}}$$

Derivation of effective cosmological dynamics: main steps

quantum geometric EPRL model with 4 scalar dofs (1 clock+ 3 rods + 1 matter scalar field)

using: $\tilde{\sigma}_j \equiv \rho_j \exp[i\theta_j]$ rewrite in standard hydrodynamic form (fluid density, phase)

homogeneous background + inhomogeneous perturbations (spacetime localization defined in relational terms)

$$\rho_j = \bar{\rho}_j + \delta\rho_j \qquad \theta_j \equiv \bar{\theta}_j + \delta\theta_j \qquad \bar{\rho} = \bar{\rho}(x^0, \pi_\phi) \qquad \bar{\theta} = \bar{\theta}(x^0, \pi_\phi)$$

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background eqns:

$$\bar{\rho}_j''(x^0,\pi_\phi) - \left[\left(\bar{\theta}_j'(x^0,\pi_\phi)\right)^2 + {}^{(\lambda)}\eta_j^2(\pi_\phi) - \gamma\bar{\theta}_j'(x^0,\pi_\phi)\right]\bar{\rho}_j(x^0,\pi_\phi) = 0$$

$$\bar{\theta}_j''(x^0,\pi_\phi) + (\bar{\theta}_j'(x^0,\pi_\phi) - \gamma/2)\frac{(\bar{\rho}_j^2)'(x^0,\pi_\phi)}{\bar{\rho}_j^2(x^0,\pi_\phi)} - {}^{(\lambda)}\beta_j^2 = 0$$

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now, need to obtain equations for physical observables

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now, need to obtain equations for physical observables

- universe volume $$\hat{V} = \int \mathrm{d}^n\chi \int \mathrm{d}g_I\, \mathrm{d}g_I'\, \hat{\varphi}^\dagger(g_I,\chi^a) V(g_I,g_I') \hat{\varphi}(g_I',\chi^a)$$
- value of clock/rods scalar fields $$\hat{X}^b \equiv \int \mathrm{d}^n\chi \int \mathrm{d}g_I\, \chi^b \hat{\varphi}^\dagger(g_I,\chi^a)\hat{\varphi}(g_I,\chi^a)$$
- momentum of clock/rods scalar fields $$\hat{\Pi}_b = \frac{1}{i}\int \mathrm{d}^n\chi \int \mathrm{d}g_I \left[\hat{\varphi}^\dagger(g_I,\chi^a)\left(\frac{\partial}{\partial\chi^b}\hat{\varphi}(g_I,\chi^a)\right)\right]$$
- value of matter scalar field $$\hat{\Phi} = \frac{1}{i}\int \mathrm{d}g_I \int \mathrm{d}^4\chi \int \mathrm{d}\pi_\phi\, \hat{\varphi}^\dagger(g_I,\chi^\mu,\pi_\phi)\partial_{\pi_\phi}\hat{\varphi}(g_I,\chi^\mu,\pi_\phi)$$
- momentum of matter scalar field $$\hat{\Pi}_\phi = \int \mathrm{d}g_I \int \mathrm{d}^4\chi \int \mathrm{d}\pi_\phi\, \pi_\phi \hat{\varphi}^\dagger(g_I,\chi^\mu,\pi_\phi)\hat{\varphi}(g_I,\chi^\mu,\pi_\phi)$$

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2nd quantized operators acting on fundamental GFT Fock space

Derivation of effective cosmological dynamics: main steps

- expectation values of fundamental observables in peaked states: relational spacetime-localized interpretation

$$N(x^0,x^i) \equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \hat{N} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle \qquad V(x^0,x^i) \equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \hat{V} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle$$

$$X^\mu(x^0,x^i) \equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \hat{V} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle \simeq x^\mu \qquad \Pi(x^0,x^i) \equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \widehat{\Pi_\nu} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle$$

$$\phi(x^0,x^i) \equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \hat{\Phi} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle \qquad \Pi_\phi(x^0,x^i) \equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \widehat{\Pi}_\phi | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle$$

observables of effective continuum gravitational physics = collective observables, averages in suitable QG states

Derivation of effective cosmological dynamics: main steps

- expectation values of fundamental observables in peaked states: relational spacetime-localized interpretation

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- can now turn GFT hydrodynamic eqns into equations for cosmological observables

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background volume dynamics: L. Marchetti, DO, '21 A. Jercher, DO, A. Pithis, 21

$$\left(\frac{V'}{3V}\right)^2 \simeq \left(\frac{2\sum_j \int d\pi_\phi V_j \,\mathrm{sgn}(\rho')\rho_j \sqrt{\mathcal{E}_j - Q_j^2/\rho_j^2 + \mu_j^2\rho_j^2}}{3\sum_j \int d\pi_\phi V_j \rho_j^2}\right)^2$$

$$\frac{V''}{V} \simeq \frac{2\sum_j \int d\pi_\phi V_j \, \left[\mathcal{E}_j + 2\mu_j^2\rho_j^2\right]}{\sum_j \int d\pi_\phi V_j \rho_j^2}$$

- derivatives with respect to "clock time" = expectation value of "clock scalar field"
- depend on conserved quantities associated to choice of condensate state

Derivation of effective cosmological dynamics: main steps

- expectation values of fundamental observables in peaked states: relational spacetime-localized interpretation

$$N(x^0,x^i) \equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \hat{N} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle \qquad V(x^0,x^i) \equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \hat{V} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle$$

$$X^\mu(x^0,x^i) \equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \hat{V} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle \simeq x^\mu \qquad \Pi(x^0,x^i) \equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \widehat{\Pi_\nu} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle$$

$$\phi(x^0,x^i) \equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \hat{\Phi} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle \qquad \Pi_\phi(x^0,x^i) \equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \widehat{\Pi}_\phi | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle$$

observables of effective continuum gravitational physics = collective observables, averages in suitable QG states

- can now turn GFT hydrodynamic eqns into equations for cosmological observables

background volume dynamics: L. Marchetti, DO, '21 A. Jercher, DO, A. Pithis, 21

$$\left(\frac{V'}{3V}\right)^2 \simeq \left(\frac{2\sum_j \int d\pi_\phi V_j \,\mathrm{sgn}(\rho')\rho_j \sqrt{\mathcal{E}_j - Q_j^2/\rho_j^2 + \mu_j^2\rho_j^2}}{3\sum_j \int d\pi_\phi V_j \rho_j^2}\right)^2 \qquad \frac{V''}{V} \simeq \frac{2\sum_j \int d\pi_\phi V_j \left[\mathcal{E}_j + 2\mu_j^2\rho_j^2\right]}{\sum_j \int d\pi_\phi V_j \rho_j^2}$$

- derivatives with respect to "clock time" = expectation value of "clock scalar field"
- depend on conserved quantities associated to choice of condensate state

- now we can analyse the emergent cosmological dynamics in different regimes

GFT cosmology

some results (among many....)

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- very early times: very small volume - QG interactions subdominant

for large class of states:

$$\exists j \;/\; \rho_j(\chi) \neq 0 \;\forall \chi$$

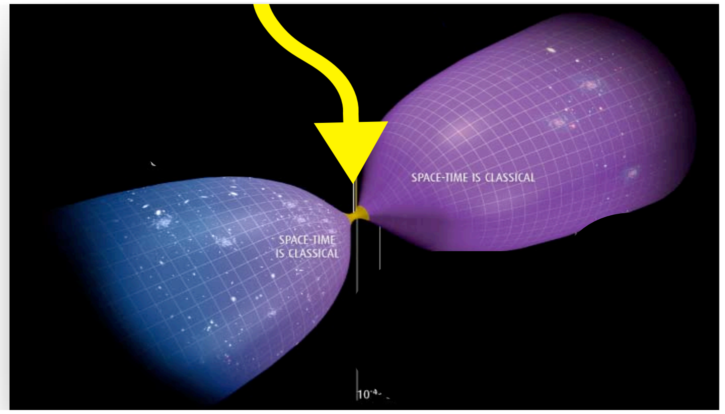
→

$$V = \sum_j V_j \rho_j^2$$

remains positive at all times (with single turning point)

DO, L. Sindoni, E. Wilson-Ewing, '16;
L. Marchetti, DO, '20, '21

quantum bounce (no big bang singularity)!



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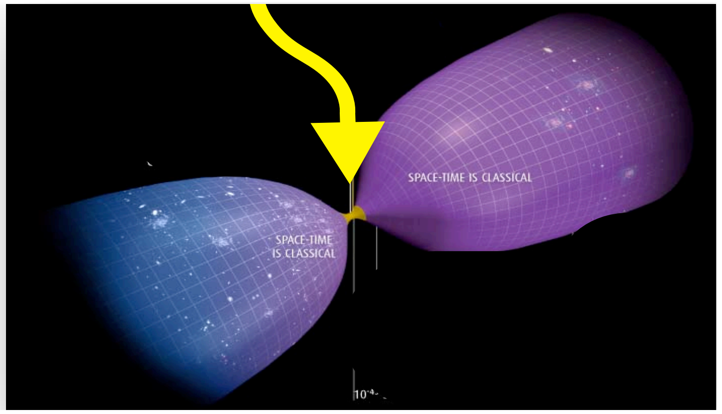

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(here written neglecting matter contribution)

$$\left(\frac{V'}{V}\right)^2 = \frac{V''}{V} = 12\pi\tilde{G}$$

classical Friedmann dynamics in GR (wrt relational clock, with effective Newton constant) - flat FRW

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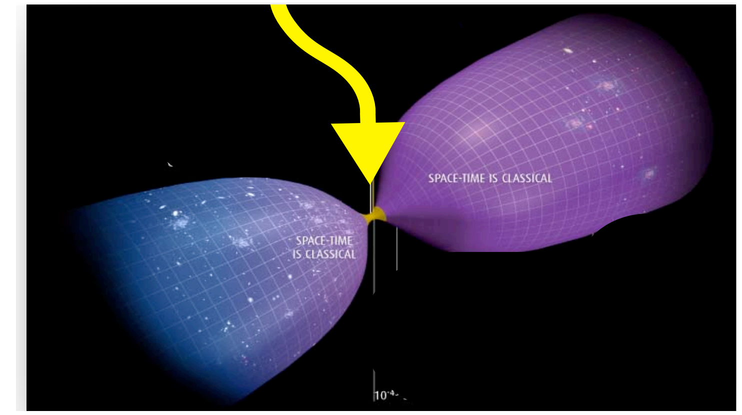

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$\longrightarrow$ accelerated cosmological expansion

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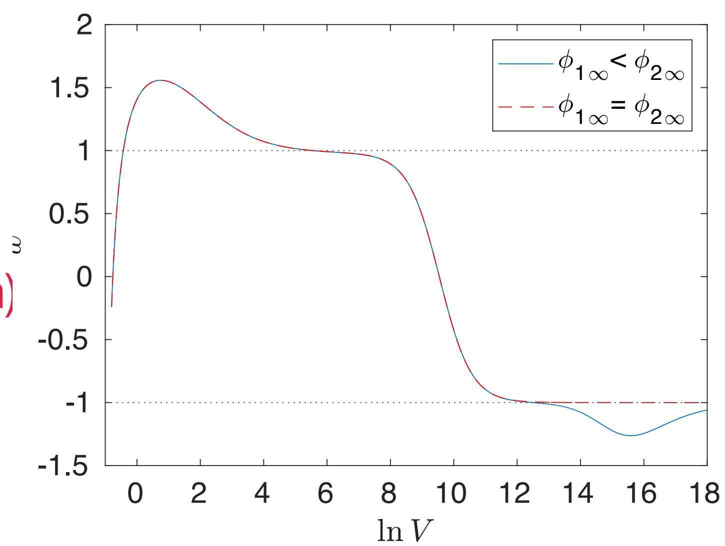

order-6 interactions

2 modes $\longrightarrow$ effective phantom-like dark energy (of pure QG origin)

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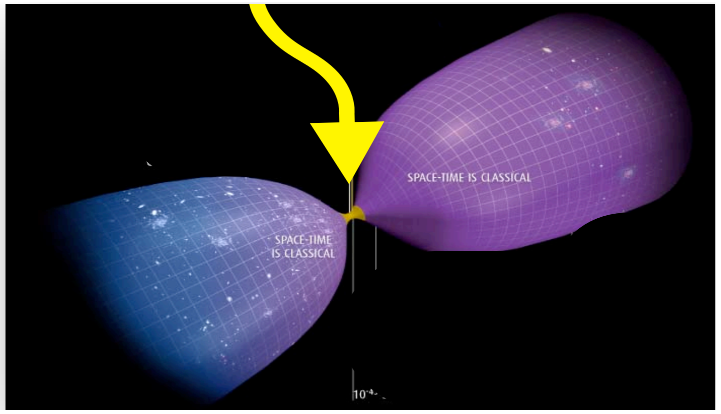

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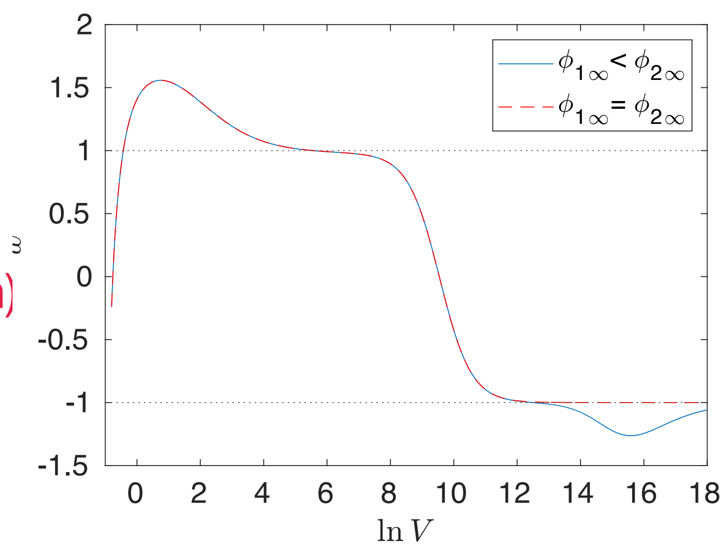

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- value of cosmological constant linked to value of critical density at quantum bounce (both depending on volume eigenvalue of dominant mode and state-dependent constant) $\Lambda = \frac{4Q_1^2}{3V_1^2}(-\lambda_1)$

DO, X. Pang, to appear

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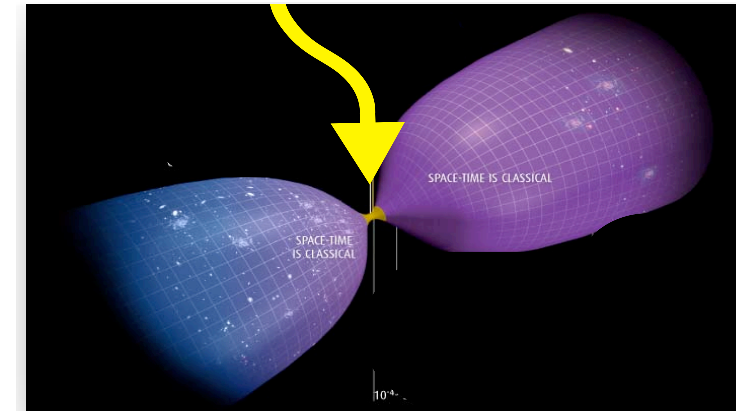

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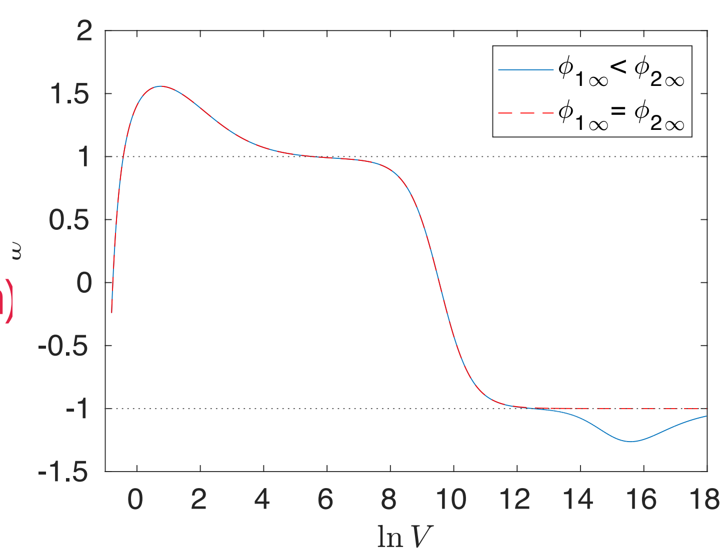

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DO, X. Pang, to appear

- QG-produced early-time acceleration possible

M. De Cesare, A. Pithis, M. Sakellariadou, '17;
T. Landstätter, L. Marchetti, DO, to appear; P. Fischer, L. Marchetti, DO, to appear

GFT cosmology many other results

- GFT (deparametrized) quantization wrt scalar field clock — E. Wilson-Ewing, '18; S. Gielen, A. Polaczek, E. Wilson-Ewing, '19
- relation between "frozen" and deparametrized formalism — S. Gielen, '21
- cosmological perturbations

  see talk by L. Marchetti

  - localization fully relational, analysis still in mean field approx.
  - dynamics of cosmological perturbations — L. Marchetti, DO, '21
  - cosmological perturbations in GFT models including timelike tetrahedra — A. Jercher, L. Marchetti, A. Pithis, to appear
  - effective field theory for scalar matter (QG signatures?) — R. Dekhil, S. Liberati, DO, to appear
- other approaches to cosmological perturbations — S. Gielen, DO, '17; S. Gielen, '18; F. Gerhardt, DO, E. Wilson-Ewing, '18
- reduction to LQC (as special sector of GFT cosmology) — DO, L. Sindoni, E. Wilson-Ewing, '16; S. Gielen, '17; L. Marchetti, DO, '20, '21; G. Calcagni, ...
- anisotropies — A. Pithis, M. Sakellariadou, '16; M. De Cesare, DO, A. Pithis, M. Sakellariadou, '17; A. Calcinari, S. Gielen, '22; Y. Wang, DO, in prog
- thermal fluctuations (of QG observables) during cosmological evolution — M. Assanioussi, I. Kotecha, '19,'20
  - requires extension of GFT formalism to thermal states - concrete proposal for covariant quantum statistical mechanics — I. Kotecha, '20; I. Kotecha, DO, '18; G. Chirco, I. Kotecha, DO, '18
- cosmological dynamics from generalised (squeezed) GFT states — S. Gielen, A. Polaczek, '19
- analysis of quantum fluctuations of observables during cosmic evolution — S. Gielen, A. Polaczek, '19; L. Marchetti, DO, '21
- many free scalar fields — S. Gielen, A. Polaczek, '20
- ....

Main messages

- modern discrete version of 3rd quantization formalism for QG, incorporating topology change, exist
- tensorial group field theory as combinatorial generalization and quantum geometric enrichment of 2d matrix models
- candidate definition of simplicial gravity path integrals, including their continuum limit
- candidate definition of spin foam models, including their continuum limit
- can be controlled (sum over topologies, renormalizability, etc) - level of control depends on complexity of model
- continuum cosmological dynamics can be extracted from their (mean field) hydrodynamics
- emergent cosmological dynamics shows quantum bounce (and late-time acceleration)

Thank you for your attention