# Testing the Cosmological Principle in a Quantum Universe

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Based on: "MEASURING THE HOMOGENEITY OF THE QUANTUM UNIVERSE", *by R. Loll and A. Silva, Physical Review D 107 (2023) 086013*

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# Presentation Overview

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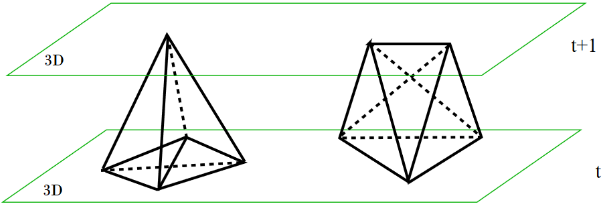
# Causal Dynamical Triangulations (CDT)




(4,1) (3,2)

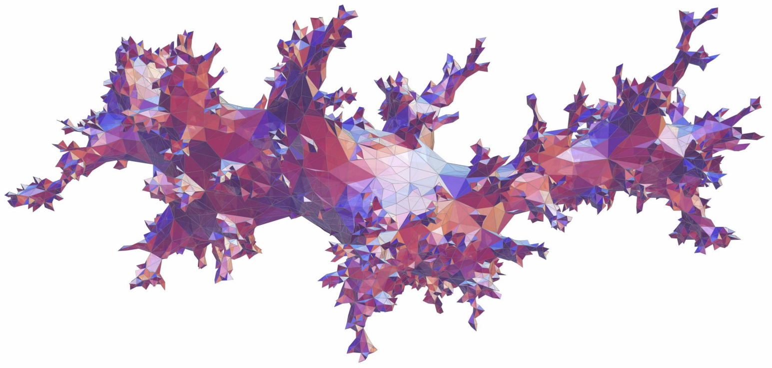
$$\langle \mathcal{O}[g] \rangle = \frac{1}{Z} \int \mathcal{D}[g]\, \mathcal{O}[g]\, e^{-S[g]} \rightarrow \langle \mathcal{O}[\mathcal{T}] \rangle \doteq \frac{1}{Z} \sum_{\mathcal{T}} \mathcal{O}[\mathcal{T}]\, e^{-S[\mathcal{T}]}$$

$$S[\mathcal{T}] = -K_0 N_0 + K_4 N_4 + \Delta(N_{14} - 6N_0)$$

Simple action for path integral formulation of Quantum Gravity without coordinates!

[1] Ambjorn, J., J. Jurkiewicz, and R. Loll. Approaches to Quantum Gravity, Cambridge University Press, Cambridge (2009): 341-359.

# Path integral Geometry of QG in Causal Dynamical Triangulations

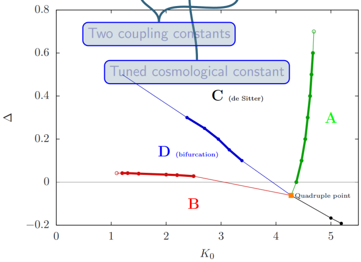


Due to the very non-smooth nature of the path integral configurations, there is no hope to use tensor calculus in the resulting space-times. All the tools we have to study space-time, are distances and volumes!

[1] https://www.imo.universite-paris-saclay.fr/ nicolas.curien/simulation.html

# Emergence of de Sitter universe

$$S[\mathcal{T}] = -K_0 N_0 + K_4 N_4 + \Delta(N_{14} - 6N_0)$$



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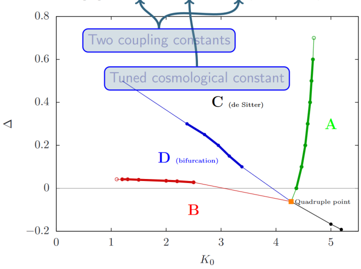


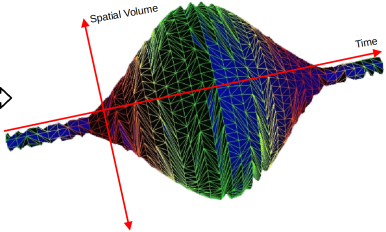


[1] Ambjørn, J., Gizbert-Studnicki, J., Görlich, A., Jurkiewicz, J., Németh, D. Journal of High Energy Physics, 2018(6), 1-23

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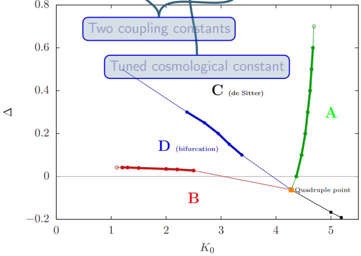


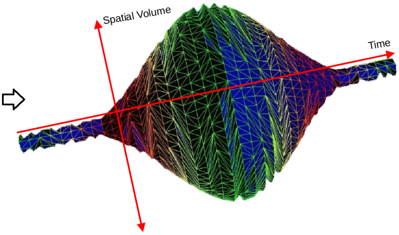


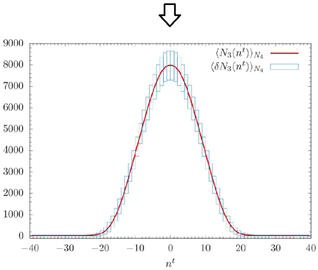

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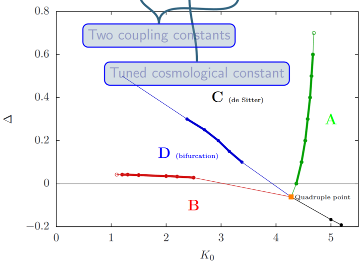


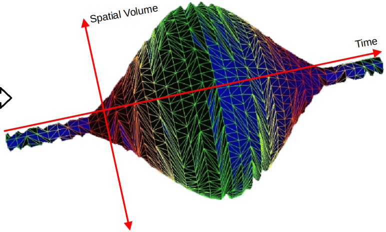


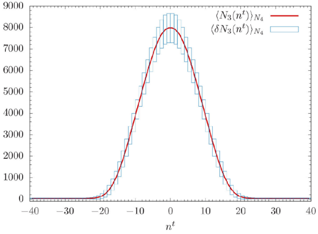


- emergent de Sitter-like universe with no symmetries assumed at the outset!
- We want to construct observables capable of providing geometrical information of this emergent space-time. We want to study its properties!

[1] Klitgaard, N., and R. Loll. Physical Review D 97.10 (2018): 106017.

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# Motivation for the work

*Can we, using this emergent de Sitter-like Universe, somehow justify the usually assumed initial condition of our universe as an isotropic and homogeneous exponentially expanding universe?*

$\Downarrow$

**DOES IT SATISFY THE BOUNDS IN INHOMOGENEITIES AND ANISOTROPIES FOR INFLATION TO HAPPEN?**

$\Downarrow$

To answer this question, we need **quantum gravitational observables** capable of measuring **homogeneity** and **isotropy** that we can compute in these quantum geometries!

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# Definition of Homogeneity

*H*omogeneous Space $\iff$ *T*ranslation Symmetry of all its properties

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<u>Difficulties:</u>

- There is no notion of group action and neither of tensor calculus in these quantum geometries.
- Too restrictive definition. In physics, symmetries, like homogeneity, can emerge at certain scales.

<u>Solutions:</u>

- We will use a statistical definition of homogeneity, needing only a distance measure in the space.
- We will define observables capable of testing this symmetry at different scales.

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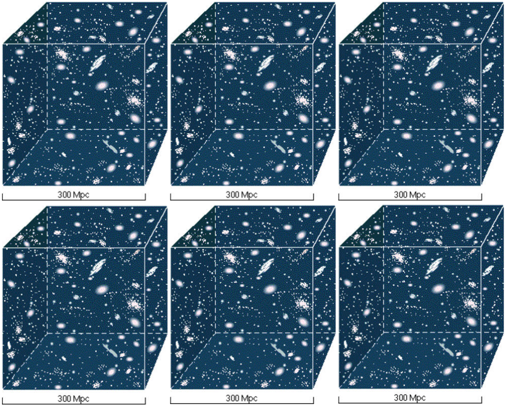
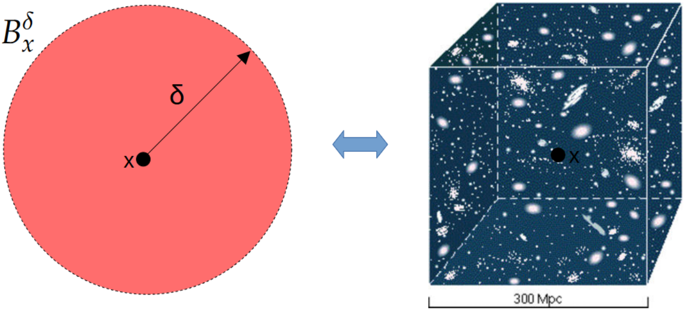
# Scale Dependent Homogeneity




*Homogeneity appears only at a certain scale in our universe. It is an "emergent" symmetry, as the result of a statistical study of its properties at different scales. Let's try to do the same in a quantum universe!*

[1] Luca Bombelli, Astronomy Lecture Notes, University of Mississippi (2013)

# Coarse-graining for Homogeneity




We use geodesic spheres as chunks of the universe, to average properties $Q(x)$. In a flat space, this coarse-graining imposes a momentum cut-off $\Lambda \simeq \frac{1}{\delta}$. In this sense, we are covariantly "washing out" high energy details, and keeping only a coarse-grained value of property at a certain scale $\delta$.

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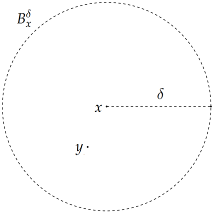
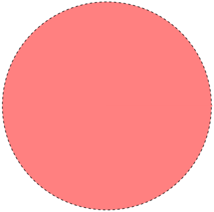
# Coarse-graining for Homogeneity

*CG only using distances and volumes!*




$\Downarrow$



Property of the space$= Q(x)$

$$B_x^\delta = \{y \in M \,/\, d(x,y) = \delta\}$$

$$||B_x^\delta|| = \int_{B_x^\delta} d^D y \sqrt{det(g)}$$

$$\Downarrow$$

Coarse-Grained Property $= Q(x,\delta)$

$$Q(x,\delta) = \frac{1}{||B_x^\delta||} \int_{B_x^\delta} d^D y \sqrt{det(g)} Q(y)$$

# Homogeneity with no coordinates

Reference value for $Q(x, \delta)$:

$$\bar{Q}(\delta) = \frac{1}{||M||} \int d^D x \sqrt{det(g)} Q(x, \delta)$$

$$\Downarrow$$

Absolute Homogeneity Observable:

$$\mathcal{H}_Q[g](\delta) = \sqrt{\frac{1}{||M||} \int d^D x \sqrt{det(g)} (Q(x, \delta) - \bar{Q}(\delta))^2}$$

$$\Downarrow$$

Relative Homogeneity Observable (% **Inhomogeneity**):

$$\boxed{\mathcal{H}_Q^{rel}[g](\delta) = \frac{\mathcal{H}_Q[g](\delta)}{|\bar{Q}(\delta)|}}$$

# Homogeneity of a Quantum Universe

**Quantum Gravitational Observable**

$$\langle \mathcal{H}_Q^{rel}[g](\delta)\rangle = \frac{1}{Z}\int \mathcal{D}[g]\,\mathcal{H}_Q^{rel}[g](\delta)\,e^{-S[g]}$$

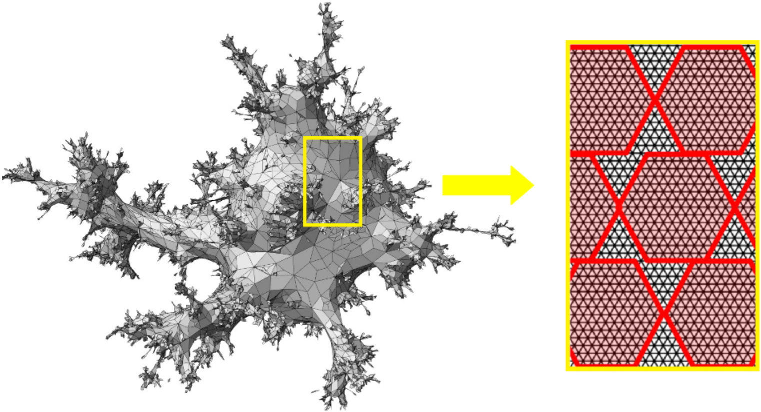
$$\Downarrow$$

**Definition in CDT of Quantum Gravitational Observable**

$$\boxed{\langle \mathcal{H}_Q^{rel}[\mathcal{T}](\delta)\rangle \doteq \frac{1}{Z}\sum_{\mathcal{T}} \mathcal{H}_Q^{rel}[\mathcal{T}](\delta)\,e^{-S[\mathcal{T}]}}$$

*This can be evaluated numerically using Monte Carlo Simulations!*

# 2D Quantum Universe from CDT



We can apply the whole formalism developed to these discrete geometries, using as distance and volume measures, the link distance and the Hausdorff measure.

[1] https://hef.ru.nl/ tbudd/gallery/

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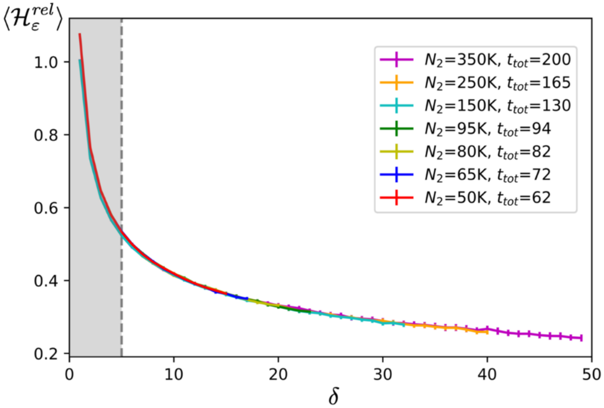
# Homogeneity in 2D CDT

$$Q(x) = R(x) = \textit{Scalar Curvature at a point}$$
$$\textit{(Regge discretized form)}$$




$\sim$ %20 size of inhomogeneities in the continuum

# Definition of Isotropy

*I*sotropic Space $\Longleftrightarrow$ *S*pherical Symmetry of all its properties at all points

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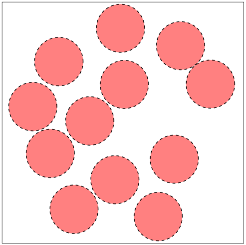
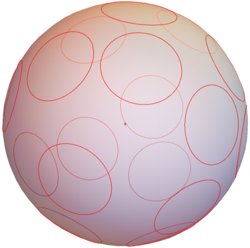
| Translation Symmetry | Spherical Symmetry |
|---|---|
|  |  |
| Translations: Compares properties in different points of the entire space | Rotations: Compares properties in different points of the surface of a sphere |

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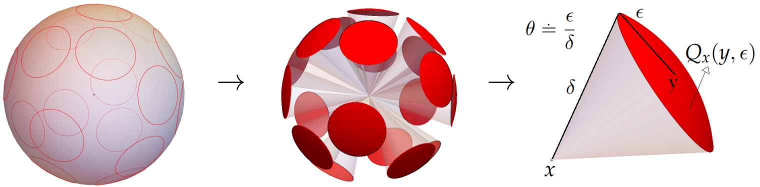
# How the story goes for Isotropy




Spherical Symmetry Test:

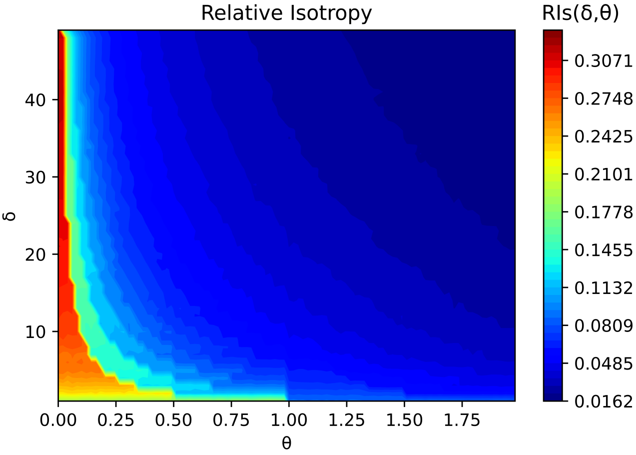
$$\mathcal{S}_Q(\delta,\epsilon,x) = \sqrt{\frac{1}{||S_x^\delta||}\int_{S_x^\delta} d^{D-1}y\sqrt{det(h)}\left(\frac{Q_x(y,\epsilon)-\bar{Q}_x(\epsilon)}{\bar{Q}_x(\epsilon)}\right)^2}$$

Relative Isotropy Observable (% **Anisotropy**):

$$\boxed{\mathcal{I}_Q^{rel}[g](\delta,\epsilon) = \frac{1}{||M||}\int d^Dx\sqrt{det(g)}\mathcal{S}_Q(\delta,\epsilon,x)}$$

# Isotropy in 2D CDT

$$Q(x) = c(x) = \textit{Coordination number of a point}$$




$\sim$ %1 size of anisotropies in the continuum

# Conclusions and forthcomings

- We defined quantum gravitational observables capable of determining % deviations from perfect homogeneity and isotropy of spacetime.
- The observables are capable of being implemented in highly fluctuating spacetimes, and do not require smoothness of the space nor the existence of tensor calculus, but only need the existence of a distance measure and a volume measure.
- We implemented these observables in the quantum geometries obtained in 2D quantum gravity using Causal Dynamical Triangulations, obtaining results compatible with inhomogeneities and anisotropies in the continuum limit of the theory.
- These observables will be implemented in 4D Causal Dynamical Triangulations, to characterize the inhomogeneities and anisotropies of its De Sitter like emergent universe.
- A positive outcome for the implementation in 4D CDT would be that our observables indicate a universe compatible with the initial conditions of inflation. Such results could be used to justify the usually assumed initial conditions of the universe in inflation.

# Thank you for your attention :)

Check our paper about this!