Decoupling Limits in Quadratic Gravity

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Aim of this talk

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Strictly renormalizable QFT of gravity:

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$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left(M_p^2 (R - 2\Lambda) + \frac{\alpha}{6} R^2 - \frac{\beta}{2} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right)$$

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Question:

$$\lim_{\beta\to\infty} S = \ ?$$

Additional spin-0 field

1. Auxiliary scalar field
2. Conformal transformation
3. Canonical normalization

$$S[g,\phi] = \frac{\bar{M}_p^2}{2}\int d^4x\sqrt{-g}(R-2\Lambda) - \frac{\beta}{4}\int d^4x\sqrt{-g}\,C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + S_0[g,\phi],$$

$$S_0[g,\phi] = \int d^4x\sqrt{-g}\left[-\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - \frac{m_0^2}{2}\frac{3\bar{M}_p^2}{2}\left(1 - e^{\sqrt{\frac{2}{3}}\frac{\phi}{\bar{M}_p}}\right)^2\right]$$

$$\bar{M}_p^2 \equiv M_p^2 + \frac{4}{3}\alpha\Lambda$$

$$m_0^2 \equiv \frac{M_p^2}{\alpha}$$

Rescaled Planck Mass when $\Lambda \neq 0$

Mass of the scalar field

Additional spin-2 field

1. Auxiliary spin-2 field $\varphi_{\mu\nu}$ [Kaku et al. (1977); Hindawi et al. (1996); Anselmi & Piva (2018)]

$$S[g,\phi,\varphi] = \frac{\widetilde{M}_p^2}{2}\int d^4x\sqrt{-g}(R-2\Lambda) + S_0[g,\phi]$$
$$-\frac{\beta}{2}\int d^4x\sqrt{-g}\left[\bar{M}_p^2(G_{\mu\nu}+\Lambda g_{\mu\nu})\varphi^{\mu\nu} - \frac{\bar{M}_p^4}{4}(\varphi_{\mu\nu}\varphi^{\mu\nu}-\varphi^2)\right]$$

2. Metric transformation: $g_{\mu\nu} \to g_{\mu\nu} - \beta\frac{\bar{M}_p^2}{\widetilde{M}_p^2}\varphi_{\mu\nu}$

3. Canonical normalization: $f_{\mu\nu} = \frac{\beta}{2}\frac{\bar{M}_p^2}{\widetilde{M}_p}\varphi_{\mu\nu}$

$$S[g,\phi,f] = \frac{\widetilde{M}_p^2}{2}\int d^4x\sqrt{-g}(R-2\Lambda) + S_0\left[g-2f/\widetilde{M}_p^2,\phi\right] + S_2[g,f],$$

$$\boxed{\widetilde{M}_p^2 \equiv \bar{M}_p^2 + \frac{2}{3}\beta\Lambda = M_p^2 + \frac{2}{3}(2\alpha+\beta)\Lambda}$$

Rescaled Planck Mass when $\Lambda \neq 0$

Additional spin-2 field

Action for the massive spin-2:

$$S_2[g,f] = -S_{PF}[g,f] - \int d^4x\sqrt{-g}\left[\left(2f_\mu^\rho f_{\rho\nu} - f f_{\mu\nu}\right)R^{\mu\nu} + \left(\Lambda - \frac{R}{2}\right)\left(f_{\mu\nu}f^{\mu\nu} - \frac{1}{2}f^2\right)\right]$$

$$-\frac{1}{2}\frac{m_2^2}{\widetilde{M}_p}\int d^4x\sqrt{-g}\left[5f_{\mu\nu}f^{\mu\nu}f - 4f^{\mu\nu}f_\mu^\rho f_{\rho\nu} - f^3\right]$$

$$+\frac{8}{3}\frac{1}{M_p^2}\frac{1}{\widetilde{M}_p}\int d^4x d^4y d^4z \frac{\delta^{(3)}S_{EH}}{\delta g_{\mu\nu}(x)\delta g_{\rho\sigma}(y)\delta g_{\alpha\beta}(z)} f_{\mu\nu}(x) f_{\rho\sigma}(y) f_{\alpha\beta}(z)$$

$$+O(f^4)$$

$S_{PF}[g,f]$ is the covariant Fierz-Pauli action for $f_{\mu\nu}$ with mass m_2^2

$$m_2^2 = \frac{\widetilde{M}_p^2}{\beta} = \frac{\bar{M}_p^2}{\beta} + \frac{2}{3}\Lambda = \frac{M_p^2}{\beta} + \frac{2}{3}\left(2\frac{\alpha}{\beta} + 1\right)\Lambda$$

$$m_2^2 \geq \frac{2}{3}\Lambda$$

$$(\Lambda \geq 0, \beta > 0)$$

spin-2 ghost mass depends on Λ !

Additional spin-2 field: couplings

- n-point interaction couplings from variations of S_{EH}

$$\sim \left(\frac{1}{\widetilde{M}_p}\right)^{n-2} = \left(\frac{1}{M_p}\right)^{n-2} \left(\frac{1}{1+2\Lambda(2\alpha+\beta)/3M_p^2}\right)^{\frac{n-2}{2}}$$

- n-point interaction couplings from variations of the mass term

$$\sim m_2^2 \left(\frac{1}{\widetilde{M}_p}\right)^{n-2} = \frac{1}{\beta} \left(\frac{1}{M_p}\right)^{n-4} \left(\frac{1}{1+2\Lambda(2\alpha+\beta)/3M_p^2}\right)^{\frac{n-2}{2}}$$

Interaction couplings depend on Λ $\Longrightarrow$ additional dependences on β !

Limit $\beta \to \infty$ with $\Lambda = 0$

When $\Lambda = 0$, the limit $\beta \to \infty$ gives

- S_{EH}-term couplings $\sim \left(\frac{1}{M_p}\right)^{n-2} \to$ finite
- Mass-term couplings $\sim \frac{1}{\beta}\left(\frac{1}{M_p}\right)^{n-4} \to 0$
- It is a massless limit: $m_2^2 = \frac{M_p^2}{\beta} \to 0$

NB: typically, the massless limit in theories of Massive Gravity can lead to strong coupling even below M_p. [Reviews by Hinterbichler (2011) and de Rham (2014)]

Limit $\beta \to \infty$ with $\Lambda \neq 0$

When $\Lambda \neq 0$, the limit $\beta \to \infty$ gives

- S_{EH}-term couplings $\sim \left(\frac{1}{M_p}\right)^{n-2} \left(\frac{1}{1+2\Lambda(2\alpha+\beta)/3M_p^2}\right)^{\frac{n-2}{2}} \to 0$
- Mass-term couplings $\sim \frac{1}{\beta}\left(\frac{1}{M_p}\right)^{n-4} \left(\frac{1}{1+2\Lambda(2\alpha+\beta)/3M_p^2}\right)^{\frac{n-2}{2}} \to 0$
- It is NOT a massless limit: $m_2^2 = \frac{\bar{M}_p^2}{\beta} + \frac{2}{3}\Lambda \to \frac{2}{3}\Lambda$

NB: In Massive Gravity theories this limit is known as *partially massless limit* and in general may lead to strong coupling! [de Rham et al. (2018)]

$\beta \to \infty$ with $\Lambda = 0$: resulting theory

Stückelberg formalism:

$$f_{\mu\nu} = f'_{\mu\nu} + \frac{1}{m_2}\left(\nabla_\mu A_\nu + \nabla_\nu A_\mu\right) + \frac{2}{m_2^2}\nabla_\mu \nabla_\nu \chi$$

The limit is regular:

$$\lim_{\beta\to\infty} S = \frac{M_p^2}{2}\int d^4x\sqrt{-g}(R-2\Lambda) - M_p \int d^4x\sqrt{-g}G_{\mu\nu}f'^{\mu\nu} + \frac{1}{4}\int d^4x\sqrt{-g}F^{\mu\nu}F_{\mu\nu} + S_{00}[g,\phi,\chi]$$

$$S_{00}[g,\phi,\chi] = \int d^4x\sqrt{-g}\left[\frac{1}{2}\nabla_\mu\chi\nabla^\mu\chi - \frac{1}{2}\frac{\chi^2}{6M_p^2}\nabla_\mu\phi\nabla^\mu\phi - \frac{m_0^2}{2}\frac{3M_p^2}{2}\frac{\chi^4}{36M_p^4}\left(1-e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_p}}\right)^2\right]$$

- The massive spin-2 splits into 5 massless ghost-like dofs (± 2, ± 1, 0)
- All interactions couplings are proportional to powers of $\frac{1}{M_p}$
- The resulting theory is <u>NOT</u> conformal gravity!

$\beta \to \infty$ with $\Lambda \neq 0$: resulting theory

Stückelberg formalism:

$$f_{\mu\nu} = f'_{\mu\nu} + \frac{1}{\sqrt{\Lambda}\sqrt{m_2^2 - 2\Lambda/3}}\left(\nabla_\mu \nabla_\nu \chi + \frac{\Lambda}{3} g_{\mu\nu}\chi\right)$$

The limit is regular:

$$\lim_{\beta\to\infty} S = \frac{\widetilde{M}_p^2}{2}\int d^4x\sqrt{-g}(R - 2\Lambda) + \int d^4x\sqrt{-g}\left[-\widetilde{M}_p\left(G_{\mu\nu} + \Lambda g_{\mu\nu}\right)f'^{\mu\nu} + \frac{\Lambda}{3}\left(f'_{\mu\nu}f'^{\mu\nu} - f'^2\right)\right] + S_{00}$$

$$S_{00} = \int d^4x\sqrt{-g}\left[\frac{1}{2}\nabla_\mu\chi\nabla^\mu\chi - \bar{M}_p^2\Lambda\left(\frac{\chi^2}{6\bar{M}_p^2} - 1\right)^2 - \frac{1}{2}\frac{\chi^2}{6\bar{M}_p^2}\nabla_\mu\phi\nabla^\mu\phi - \frac{m_0^2}{2}\frac{3\bar{M}_p^2}{2}\frac{\chi^4}{36\bar{M}_p^4}\left(1 - e^{\sqrt{\frac{2}{3}}\frac{\phi}{\bar{M}_p}}\right)^2\right]$$

- The massive spin-2 splits into 1 partially massless graviton (4 dof) + 1 scalar dof
- Additional scalar gauge symmetry:

$$\delta_\zeta f'_{\mu\nu} = \left(\nabla_\mu\nabla_\nu\zeta + \frac{\Lambda}{3}g_{\mu\nu}\zeta\right)$$

$\beta \to \infty$ with $\Lambda \neq 0$: resulting theory

Interactions couplings: $\frac{1}{\widetilde{M}_p}$ for spin-2 sector & $\frac{1}{\bar{M}_p}$ for spin-0 sector

- Diagonalize $g_{\mu\nu}$ and $f'_{\mu\nu}$
- Expand in $f'_{\mu\nu}$ and $g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{2}{\widetilde{M}_p} h_{\mu\nu}$

$$\lim_{\beta\to\infty} S = S_{EH}^{(2)}[\bar{g}, h] - S_{PF}[\bar{g}, f'] \,|_{m_2^2 = \frac{2}{3}\Lambda}$$
$$- \int d^4x \sqrt{-\bar{g}} \left[\left(2 f_\mu^{\ \rho} f_{\rho\nu} - f f_{\mu\nu}\right) \bar{R}^{\mu\nu} + \left(\Lambda - \frac{\bar{R}}{2}\right)\left(f_{\mu\nu} f^{\mu\nu} - \frac{1}{2} f^2\right)\right] + S_{00}[\bar{g}, \phi, \chi]$$

Scalar gauge symmetry constrains the background metric:

$$\beta \to \infty \qquad \Longrightarrow \qquad \bar{g}_{\mu\nu}: \quad \bar{R}_{\mu\nu} = \Lambda \bar{g}_{\mu\nu}$$

Summary and discussion

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left(M_p^2(R-2\Lambda)+\frac{\alpha}{6}R^2-\frac{\beta}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right)$$

- Renormalizability + ghost-like nature of spin-2 $\Longrightarrow$ limit $\beta\to\infty$ is regular
- The limit $\beta\to\infty$ depends non-trivially on Λ
- When $\Lambda\neq 0$: structure of degrees of freedom is different (but same number); spin-2 sector decouples; helicity-zero acquires a self-potential
- Can the limit $\beta\to\infty$ help to understand the high-energy behavior of the spin-2 ghost?
- Current approaches to the ghost problem are mainly formulated in Minkowski background. Are they still valid when $\Lambda\neq 0$?
- Role of the cosmological constant in quantum gravity?

…Extra Slides…

Renormalizable Quantum Gravity

Quadratic gravity is 'strictly' renormalizable (in $D = 4$):

[Stelle, PRD (1977)]

$$S = \frac{1}{2}\int d^4x\,\sqrt{-g}\left(M_p^2(R - 2\Lambda) + \frac{\alpha}{6}R^2 - \frac{\beta}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right)$$

<u>Usual story</u>: assume $\Lambda \simeq 0$, expand $g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_p}h_{\mu\nu}$, the propagator is

$$\Pi(k) = \Pi_{GR} + \frac{1}{2}\frac{\mathcal{P}^{(0)}}{k^2 + m_0^2} - \frac{\mathcal{P}^{(2)}}{k^2 + m_2^2}, \qquad m_0^2 = \frac{{M_p}^2}{\alpha}, \qquad m_2^2 = \frac{{M_p}^2}{\beta}$$

Spin-2 massive ghost

Reason why such a QFT of gravity is usually claimed to be pathological when quantized with <u>conventional methods</u>!

Motivations

General Relativity (GR): $S_{GR} = \frac{M_p^2}{2} \int d^4x \sqrt{-g}\, R$

Physics beyond GR:

1. Current accelerated expansion of the Universe (*late time)*
2. CMB anisotropies (*early time*)

Motivations

General Relativity (GR): $S_{GR} = \frac{M_p^2}{2} \int d^4x \sqrt{-g}\, R$

Physics beyond GR:

1. Current accelerated expansion of the Universe (*late time*)
2. CMB anisotropies (*early time*)

Simplest phenomenological model:

$$S = \frac{M_p^2}{2} \int d^4x \sqrt{-g} R + S_\Lambda + S_\phi + \cdots ,$$

$$S_\Lambda \equiv \frac{1}{\kappa^2} \int d^4x \sqrt{-g}\, \Lambda,$$

$S_\phi \equiv$ inflaton action

Motivations

How to select fundamental theories?

1. Guidance from Nature: experiments!

2. ‘Guiding principles’: perturbative QFT (renormalizability criterion!)

Motivations

How to select fundamental theories?

1. Guidance from Nature: experiments!

2. ‘Guiding principles’: perturbative QFT (renormalizability criterion!)

The criterion of (strict) renormalizability gives: [Stelle, PRD (1977)]

$$S = \frac{1}{2}\int d^4x\,\sqrt{-g}\left(M_p^2(R-2\Lambda)+\frac{\alpha}{6}R^2-\frac{\beta}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right)$$

$\Lambda \sim 10^{-122} M_p^2$

$\alpha \sim 10^{10}$

Natural explanation for inflation!

[Starobinsky, 1980+]

Quadratic Gravity as Quantum Gravity

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left(M_p^2(R-2\Lambda) + \frac{\alpha}{6}R^2 - \frac{\beta}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right)$$

Cosmological constant: $\Lambda \sim 10^{-122} M_p^2$

Natural candidate for inflaton: $\alpha \sim 10^{10}$

[Starobinsky, 1980+]

In my opinion, if we accept these facts very <u>important implications</u> follow:

1. The framework of perturbative QFT and the criterion of renormalizability (as a tool to select theories) are quite successful also when applied to gravity!

2. CMB observations have provided for the first time a <u>test</u> of higher-curvature gravity and an 'indirect' proof of quantized gravity (the scalar field is a gravitational dof)!!

3. Contrary to some beliefs, Starobinsky inflation is <u>not</u> just a model!

Motivations

Obvious question: What about the spin-2 massive ghost?

1. Throw the entire theory away just because maybe we don't know how to deal with the spin-2 ghost?

2. Or, instead, after appreciating the achievements described before, should we feel very motivated to understand the role of the ghost at a deeper level?

I opt for the 2nd option!

Recent proposals to recover unitarity with ghost

S-matrix unitarity and optical theorem:

$$S^{+}S = 1, \qquad S = 1 + iT,$$
$$1 = \sum_{\{n\}} c_n \, |n\rangle\langle n| \qquad \Longrightarrow \qquad 2Im\{\langle a|T|a\rangle\} = \sum_{\{n\}} c_n |\langle n|T|a\rangle|^2$$

Interesting approaches

- Quantize the ghost with negative norms ($c_n < 0$ for ghost states but positive energies) [Salvio, Strumia (2014+); Holdom (2021+); etc]
- Loop corrections make the ghost decay after times of order $\tau \sim M_p^2/m_2^3$: treat the ghost as an unstable particle, unitarity restored for $t > \tau$ [Donoghue, Menezes (2018+)]
- Replace the Feynman $i\epsilon$ with the *Fakeon* prescription and convert the ghost into a purely virtual particle (LHS=0 for ghost cuts and $c_n = 0$ for ghost states) [Anselmi & Piva 2017+]

Additional spin-0 field

1. Auxiliary scalar field φ

$$S[g,\varphi]=\frac{\bar{M}_p^2}{2}\int d^4x\sqrt{-g}(R-2\Lambda)+S_W[g]+\frac{\alpha}{12}\int d^4x\sqrt{-g}(2R-8\Lambda-\varphi)\varphi$$

2. Conformal transformation: $g_{\mu\nu}\to\frac{1}{1+\alpha\varphi/3\bar{M}_p}g_{\mu\nu}$

3. Canonical normalization: $\phi=\sqrt{\frac{3}{2}\frac{\alpha}{3\bar{M}_p}}\varphi$

$$S[g,\phi]=\frac{\bar{M}_p^2}{2}\int d^4x\sqrt{-g}(R-2\Lambda)+S_W[g]+S_0[g,\phi],$$

$$S_0[g,\phi]=\int d^4x\sqrt{-g}\frac{1}{\left(1+\sqrt{2/3}\phi/\bar{M}_p\right)^2}\left(-\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi-\frac{m_0^2}{2}\phi^2\right)$$

$$\bar{M}_p^2\equiv M_p^2+\frac{4}{3}\alpha\Lambda$$

Rescaled Planck Mass when $\Lambda\neq 0$

$$m_0^2\equiv\frac{M_p^2}{\alpha}$$

Mass of the scalar field

Additional spin-0 field: couplings

$$S_0[g,\phi] = \int d^4x\,\sqrt{-g}\left(1 - 2\sqrt{\frac{2}{3}}\frac{1}{\bar{M}_p}\phi + \cdots + (n-1)(-1)^{n-2}\left(\sqrt{\frac{2}{3}}\frac{1}{\bar{M}_p}\right)^{n-2}\phi^{n-2} + \cdots\right)$$
$$\times\left(-\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - \frac{m_0^2}{2}\phi^2\right)$$

Derivative interaction couplings $\sim \left(\frac{1}{\bar{M}_p}\right)^{n-2} = \left(\frac{1}{M_p}\right)^{n-2}\left(\frac{1}{1+4\alpha\Lambda/3M_p^2}\right)^{\frac{n-2}{2}}$

Non-derivative interaction couplings $\sim m_0^2\left(\frac{1}{\bar{M}_p}\right)^{n-2} = \frac{1}{\alpha}\left(\frac{1}{M_p}\right)^{n-4}\left(\frac{1}{1+4\alpha\Lambda/3M_p^2}\right)^{\frac{n-2}{2}}$

Couplings for the massless spin-2

When $\Lambda \neq 0$ the couplings of the massless spin-2 also change:

$$\frac{\widetilde{M}_p^2}{2}\int d^4x\,\sqrt{-g}(R-2\Lambda)\,, \qquad\qquad g_{\mu\nu}=\bar{g}_{\mu\nu}+\frac{2}{\widetilde{M}_p}h_{\mu\nu},$$

Self-interaction couplings

$$\sim\left(\frac{1}{\widetilde{M}_p}\right)^{n-2}=\left(\frac{1}{M_p}\right)^{n-2}\left(\frac{1}{1+2\Lambda(2\alpha+\beta)/3M_p^2}\right)^{\frac{n-2}{2}}$$

When $\Lambda \neq 0 \quad \Longrightarrow \quad$ additional dependences on Λ, α, β !

Brief digression on Massive Gravity with $\Lambda = 0$

Massive Gravity:

$$S_{MG} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}\nabla_\rho f_{\mu\nu}\nabla^\rho f^{\mu\nu} + \nabla_\rho f_{\mu\nu}\nabla^\mu f^{\rho\nu} - \nabla_\mu f\nabla_\nu f^{\mu\nu} + \frac{1}{2}\nabla_\rho f\nabla^\rho f - \frac{m_2^2}{2}\left(f_{\mu\nu}f^{\mu\nu} - f^2\right)\right] + O(f^3)$$

Naively, the limit $m_2^2 \to 0$ seems to give a massless spin-2 with 2 dofs…

But, the number of degrees of freedom must be preserved: Stückelberg trick

$$f_{\mu\nu} = f'_{\mu\nu} + \frac{1}{m_2}\left(\nabla_\mu A_\nu + \nabla_\nu A_\mu\right) + \frac{2}{m_2^2}\nabla_\mu\nabla_\nu\varphi,$$

Gauge symmetries:

$$\delta f'_{\mu\nu} = \nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu,$$
$$\delta A_\mu = -m_2\xi_\mu + \nabla_\mu\xi,$$
$$\delta\varphi = -m_2\,\xi,$$

Massless limit (2+2+1 = 5 dofs):

$$S_{MG}[f', A, \varphi] = S_{FP}^{(m_2=0)}[f'] + \int d^4x\sqrt{-g}\left(-\frac{1}{2}F^{\mu\nu}F_{\mu\nu} - 3\nabla_\rho\varphi\nabla^\rho\varphi\right) + O(f^3)$$

Possible strong coupling from helicity-0 interactions: $O(f^3) \sim \frac{1}{m_2} O(\varphi^3) \;\to\; \infty$

[Reviews by Hinterbichler (2011) and de Rham (2014)]

Brief digression on Massive Gravity with $\Lambda > 0$

Massive Gravity around de Sitter:

$$S_{MG} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2}\nabla_\rho f_{\mu\nu}\nabla^\rho f^{\mu\nu} + \nabla_\rho f_{\mu\nu}\nabla^\mu f^{\rho\nu} - \nabla_\mu f \nabla_\nu f^{\mu\nu} + \frac{1}{2}\nabla_\rho f \nabla^\rho f$$
$$+\Lambda\left(f_{\mu\nu}f^{\mu\nu} - \tfrac{1}{2}f^2\right) - \tfrac{m_2^2}{2}\left(f_{\mu\nu}f^{\mu\nu} - f^2\right)\Big] + O(f^3)$$

Naively, the limit $m_2^2 \to \frac{2}{3}\Lambda$ seems to give a *partially massless* spin-2 with 4 dofs…

The quadratic part of the action becomes:

$$S_{FP} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2}\nabla_\rho f_{\mu\nu}\nabla^\rho f^{\mu\nu} + \nabla_\rho f_{\mu\nu}\nabla^\mu f^{\rho\nu} - \nabla_\mu f \nabla_\nu f^{\mu\nu} + \frac{1}{2}\nabla_\rho f \nabla^\rho f$$
$$+\Lambda\left(f_{\mu\nu}f^{\mu\nu} - \tfrac{1}{2}f^2\right) - \tfrac{\Lambda}{3}\left(f_{\mu\nu}f^{\mu\nu} - f^2\right)\Big]$$

Scalar gauge symmetry gives in total: (5-1=4 dofs)

$$\delta f_{\mu\nu}(x) = \nabla_\mu\nabla_\nu\zeta(x) + \frac{\Lambda}{3}g_{\mu\nu}\zeta(x),$$

This is known as partially massless gravity where the graviton propagates 4 dofs

[Deser, Nepomechie (1984); Deser, Waldron (2001+); etc]

Brief digression on Massive Gravity with $\Lambda > 0$

Massive Gravity around de Sitter:

$$S_{MG} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2}\nabla_\rho f_{\mu\nu}\nabla^\rho f^{\mu\nu} + \nabla_\rho f_{\mu\nu}\nabla^\mu f^{\rho\nu} - \nabla_\mu f\nabla_\nu f^{\mu\nu} + \frac{1}{2}\nabla_\rho f\nabla^\rho f$$
$$+\Lambda\left(f_{\mu\nu}f^{\mu\nu} - \frac{1}{2}f^2\right) - \frac{m_2^2}{2}\left(f_{\mu\nu}f^{\mu\nu} - f^2\right)\Big] + O(f^3)$$

Naively, the limit $m_2^2 \to \frac{2}{3}\Lambda$ seems to give a *partially massless* spin-2 with 4 dofs…

The number of degrees of freedom must be preserved: Stückelberg trick

$$f_{\mu\nu} = f'_{\mu\nu} + \sqrt{\frac{3}{\Lambda}}\frac{1}{\Delta}\left(\nabla_\mu\nabla_\nu\varphi + g_{\mu\nu}\frac{\Lambda}{3}\varphi\right)$$

$$\Delta \equiv m_2^2 - \frac{2}{3}\Lambda$$

Gauge symmetries:

$$\delta f'_{\mu\nu} = \nabla_\mu\nabla_\nu\zeta + \frac{\Lambda}{3}g_{\mu\nu}\zeta,$$

$$\delta\varphi = -\sqrt{\frac{\Lambda}{3}}\,\Delta\,\zeta,$$

Partially massless limit $\Delta \to 0$ after substitution (4+1=5 dofs):

$$S_{MG}[f',\varphi] = S_{FP}^{(\Delta=0)}[f'] + 3\int d^4x\sqrt{-g}\left(-\frac{1}{2}\nabla_\rho\varphi\nabla^\rho\varphi - \frac{m_\varphi^2}{2}\varphi^2\right) + O(f^3), \qquad m_\varphi^2 \equiv -\frac{4}{3}\Lambda$$

Possible strong coupling from φ interactions: $O(f^3) \sim \frac{1}{\Delta}O(\varphi^3) \;\to\; \infty$

[de Rham, Hinterbichler, Johnson (2018)]

Limit $\beta \to \infty$ with $\Lambda = 0$: massless limit

Does a strong coupling (below M_p) arise in quadratic gravity?

[first asked by Hinterbichler & Saravani (2016)]

A strong coupling in the limit $m_2^2 \to 0$ (i.e., $\beta \to \infty$) can be avoided *only* in $D = 4$!

Stückelberg decomposition for $\Lambda = 0$ and in D dimensions:

$$f_{\mu\nu} = \tilde{f}_{\mu\nu} + \frac{1}{m_2}(\nabla_\mu \tilde{A}_\nu + \nabla_\nu \tilde{A}_\mu), \qquad \tilde{A}_\mu = A_\mu + \frac{1}{m_2}\nabla_\mu \chi$$

$$\Longrightarrow S_2'[g,f] = \frac{M_p^{D-2}}{2}\int d^D x\sqrt{-g}R + \int d^D x\sqrt{-g}\left[-M_p^{\frac{D-2}{2}} G_{\mu\nu} f^{\mu\nu} + \frac{m_2^2}{2}(f_{\mu\nu}f^{\mu\nu} - f^2)\right]$$

$$= \frac{M_p^{D-2}}{2}\int d^D x\sqrt{-g}R + \int d^D x\sqrt{-g}\left[-M_p^{\frac{D-2}{2}} G_{\mu\nu}\tilde{f}^{\mu\nu} + \frac{m_2^2}{2}(\tilde{f}_{\mu\nu}\tilde{f}^{\mu\nu} - \tilde{f}^2) + \frac{1}{2}F^{\mu\nu}F_{\mu\nu} + 2m_2 \tilde{f}^{\mu\nu}(\nabla_\mu \tilde{A}_\nu - g_{\mu\nu}\nabla^\rho \tilde{A}_\rho) - 2R^{\mu\nu}\tilde{A}_\mu \tilde{A}_\nu\right],$$

Possible strong coupling from $R^{\mu\nu}\tilde{A}_\mu \tilde{A}_\nu \sim \frac{1}{m_2^2} R^{\mu\nu}\nabla_\mu \chi \nabla_\nu \chi$???

Limit $\beta \to \infty$ with $\Lambda = 0$: massless limit

Make a field redefinition:

$$\tilde{f}_{\mu\nu} \to \tilde{f}_{\mu\nu} + a\, \tilde{A}_\mu \tilde{A}_\nu + b\, g_{\mu\nu}\, \tilde{A}_\rho \tilde{A}^\rho$$

In the massless limit $m_2^2 \to 0$ $(\beta \to \infty,\ \Lambda = 0)$ we get

$$\Longrightarrow \quad S_2'[g, \tilde{f}, \tilde{A}] = \frac{M_p^{D-2}}{2} \int d^D x \sqrt{-g} R + \int d^D x \sqrt{-g} \Big[-M_p^{\frac{D-2}{2}} G_{\mu\nu} \tilde{f}^{\mu\nu} + \frac{m_2^2}{2} (\tilde{f}_{\mu\nu} \tilde{f}^{\mu\nu} - \tilde{f}^2) + \frac{1}{2} F^{\mu\nu} F_{\mu\nu}$$

$$+ 2 m_2 \tilde{f}^{\mu\nu} \left(\nabla_\mu \tilde{A}_\nu - g_{\mu\nu} \nabla^\rho \tilde{A}_\rho \right) + m_2^2 a\, \tilde{f}^{\mu\nu} \tilde{A}_\mu \tilde{A}_\nu + m_2^2 [b(1-D) - a] \tilde{f} \tilde{A}_\rho \tilde{A}^\rho$$

$$- \left(a M_p^{\frac{D-2}{2}} + 2 \right) R^{\mu\nu} \tilde{A}_\mu \tilde{A}_\nu + M_p^{\frac{D-2}{2}} \left(\left(1 - \frac{D}{2}\right) b - \frac{a}{2} \right) R \tilde{A}_\rho \tilde{A}^\rho$$

$$- m_2 (2b(1-D) - 3a) \tilde{A}_\mu \tilde{A}_\nu \nabla^\mu \tilde{A}^\nu - \frac{m_2^2}{2} (b^2 D(1-D) + 2ab(1-D)) \left(\tilde{A}_\rho \tilde{A}^\rho \right)^2 \Big]$$

4 conditions to avoid strong coupling in the massless limit:

$$a M_p^{\frac{D-2}{2}} + 2 = 0, \qquad 2b(1-D) - 3a = 0,$$

$$\left(1 - \frac{D}{2}\right) b - \frac{a}{2} = 0, \qquad b^2 D(1-D) + 2ab(1-D) = 0$$

can be simultaneously satisfied *only* in $D = 4$!!!

$$a = -\frac{2}{M_p} = -2b,$$