Invariant observables in quantum gravity and graviton loop corrections to the Newtonian potential

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QG 2023, Nijmegen, Nederlands
14. July 2023

Perturbative quantum gravity

- Study gravity $S = S_\mathrm{G} + S_\mathrm{M}$ with $S_\mathrm{G} = \frac{1}{\kappa^2} \int (R - 2\Lambda) \sqrt{-g} \, \mathrm{d}^n x$ and S_M matter action perturbatively around a given background
- Metric decomposition $g_{\mu\nu} = g^{(0)}_{\mu\nu} + \kappa h_{\mu\nu}$, quantize $h_{\mu\nu}$
- Perturbation parameter: $\kappa = \sqrt{16\pi G_\mathrm{N}} \sim \ell_\mathrm{Pl}$
- Infinitesimal coordinate transformation (diffeomorphism) $x^\mu \to \tilde{x}^\mu = x^\mu + \kappa \xi^\mu$ $(\delta_\xi x^\mu = \kappa \xi^\mu)$ gives gauge transformation $\delta T = \kappa \mathcal{L}_\xi T$
- Gauge transformations of $h_{\mu\nu}$: $\delta_\xi h_{\mu\nu} = \nabla^{(0)}_\mu \xi_\nu + \nabla^{(0)}_\nu \xi_\mu + \kappa \mathcal{L}_\xi h_{\mu\nu}$
- Usual methods of QFTCS and gauge theories work, pQG is a well-defined theory according to the effective field theory paradigm that makes testable predictions at length scales above the fundamental scale $\kappa \sim \ell_\mathrm{Pl}$ Burgess, Living Rev. Rel. 7 (2004) 5

Perturbative quantum gravity

- Extremely efficient perturbation theory since ℓ_{Pl} is so small: only tree-level predictions can at present be experimentally verified (CMB)
- Main issue: construction of observables (gauge-invariant quantities)
- For gauge theories of Yang–Mills type, gauge symmetry is internal (acts on field variables at given point), such that e.g. $(\mathrm{tr} F^2)(x) = F^a_{\mu\nu}(x) F_a^{\mu\nu}(x)$ is gauge-invariant
- In (p)QG, gauge symmetry are (infinitesimal) diffeomorphisms, which move points, i.e. an external symmetry $\Rightarrow$ fields at fixed point cannot be gauge-invariant
- $\Rightarrow$ there are no local observables in (p)QG (with small exceptions)

Gauge-invariant observables

- Exception to the rule: gauge-invariant observables at *linear* order via IDEAL characterization of background spacetime Canepa/Dappiaggi/Khavkine 1704.05542, Fröb/Hack/Khavkine 1801.02632, Aksteiner/Andersson/Bäckdahl/Khavkine/Whiting 1910.08756
- At higher orders, straightforward non-local observables are manifold averages such as $\int R \sqrt{-g}\, \mathrm{d}^n x$
- Less straightforward non-local observables: averaged correlation functions at fixed geodesic distance such as $\iint R(x) R(y)\, \delta\left(d_g(x,y)^2 - D^2 \right) \sqrt{-g(x)} \sqrt{-g(y)}\, \mathrm{d}^n x\, \mathrm{d}^n y$
- Other well-known non-local observable: S-matrix in flat space
- Q: Nevertheless, we make local measurements including gravity — how can this be reconciled with the non-locality of observables?
- A: We actually make relational measurements, namely the state of one field (gravity) with respect to another (matter)

Relational observables

- Construct dynamical coordinate system: take n fields $X^{(\mu)}[g, \phi, \ldots]$ depending on field content, transforming under diffeo's as scalars: $\delta_\xi X^{(\mu)} = \kappa \xi^\rho \partial_\rho X^{(\mu)}$, and their background value $X_0^{(\mu)}$
- Expand $X^{(\mu)} = X_0^{(\mu)} + \kappa X_1^{(\mu)} + \ldots$ in perturbation theory and invert to obtain $X_0^{(\mu)}[X]$ $\Rightarrow$ transforms inversely to a scalar
- Invariant observable $\mathcal{A}(\chi)$ is given by evaluating a field A at the position $X_0^{(\mu)}$, holding $X^{(\mu)}$ fixed
- $\Rightarrow$ Relational observables: the $X^{(\mu)}$ are field-dependent coordinates, $\mathcal{A}(\chi)$ is the value of A provided that $\chi^\mu = X^{(\mu)}$, and by evaluating at $X_0^{(\mu)}$ we interpret $\mathcal{A}$ as field on background
- Relational observables can be used in all formalisms: canonical/loop (Dittrich/Giesel/Thiemann), asymptotic safety (Falls/Ferrero), … $\Rightarrow$ talk by Höhn

Gauge-invariant observables

- Choices for $X^{(\mu)}$: curvature scalars, additional scalar fields (e.g., Brown–Kuchař dust)
- First choice does not discriminate points on highly symmetric background, second choice alters physical content of theory
- Essential idea: obtain $X^{(\mu)}$ as solutions of scalar differential equation
  Brunetti/Fredenhagen/Hack/Pinamonti/Rejzner 1605.02573
- Original proposal for inflationary (FLRW) background with inflaton ϕ as time coordinate, $\triangle X^{(i)} = 0$ has unphysical action-at-a-distance effects and problems with renormalizability
- Use hyperbolic equation, e.g., $\nabla^2 X^{(\mu)} = 0$ (generalized harmonic coordinates) and invert using retarded Green's function to obtain causal evolution Fröb 1710.00839, Fröb/Lima 1711.08470
- Resulting invariant observables are non-local in accordance with general result, but non-locality restricted to past light cone

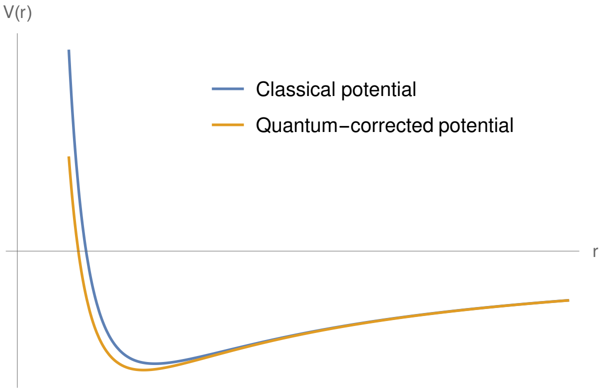
Graviton corrections to the Newtonian potential in flat space

- Invariant metric perturbation $\mathcal{H}_{\mu\nu} = h_{\mu\nu} - 2\partial_{(\mu} X^{(1)}_{\nu)} + \mathcal{O}(\kappa)$
- $\delta_\xi \mathcal{H}_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu - 2\partial_{(\mu}\xi_{\nu)} + \mathcal{O}(\kappa) = 0$
- Invariant Newtonian gravitational potential $V \equiv -\frac{1}{2} \langle \mathcal{H}_{00} \rangle$
- Matter action = Point particle action: $S_{\text{PP}} = \frac{1}{2} \int \left[\frac{1}{e(\tau)} g_{\mu\nu}(z(\tau)) \dot{z}^\mu(\tau) \dot{z}^\nu(\tau) - m^2 e(\tau) \right] \mathrm{d}\tau$
- e: einbein, τ: affine parameter, z^μ: particle position, m: mass
- Reparametrisation invariance: $\delta\tau = -\epsilon(\tau)$, $\delta z^\mu = \epsilon \dot{z}^\mu$, $\delta e = \dot{\epsilon} e + \epsilon \dot{e}$ with $\dot{\circ} = \mathrm{d}\circ/\mathrm{d}\tau$
- Large mass expansion to supress fluctuations of the particle itself: $z^\mu(\tau) = v^\mu \tau + m^{-1/2} y^\mu(\tau)$, $v^\mu = \delta^\mu_0$, $e(\tau) = m^{-1} + m^{-3/2} f(\tau)$
- Extend BRST formalism to include reparametrisations

Graviton corrections to the Newtonian potential in flat space

- Four types of diagrams to order G_N^2:
  1. Usual graviton & ghost loops
  2. Worldline corrections involving y^μ in the loop
  3. Classical corrections (tree diagrams)
  4. Corrections from field-dependent coordinates $X^{(\mu)}$
- Suitable choice of gauge kills over half of diagrams
- Classical correction $\langle \mathcal{H}_{\mu\nu}(x) \rangle^{\text{class}}$ agrees with expansion of Schwarzschild metric in harmonic coordinates
- Quantum-corrected potential $V(r) = -\frac{G_N m}{r}\left(1 - \frac{G_N m}{r} + \frac{41}{10\pi}\frac{\hbar G_N}{r^2} - \frac{43}{12\pi}\frac{\hbar G_N}{r^2}\right)$ has two contributions:
  1. agrees with inverse scattering method Khriplovich/Kirilin gr-qc/0402018, Bjerrum-Bohr/Donoghue/Holstein hep-th/0211072
  2. is remnant of particle fluctuations (“Zitterbewegung”) Fröb/Rein/Verch 2109.09753

Graviton corrections to the Newtonian potential in flat space



- Gravitational force strengthened at small distances
- Decrease of black hole horizon radius: $r_H = r_S - \left(\frac{41}{10\pi} - \frac{43}{12\pi} \right) \frac{\hbar}{m}$ with $r_S = 2 G_N m$

Matter loop corrections to the Newtonian potential in de Sitter space

- Compute corrections due to (possibly self-interacting) conformal matter Fröb/Verdaguer 1601.0356
- $V(r) = -\frac{G_\text{N} m}{\hat{r}} \left[1 - \frac{128\pi b}{3} \frac{\ell_\text{Pl}^2}{\hat{r}^2} - 32\pi \ell_\text{Pl}^2 H^2 \left(\beta - 4b - 3b' + 2c(\bar{\mu}) + 2b \ln \left(\bar{\mu} \hat{r} \right) \right) \right]$
- H: Hubble constant, $\hat{r} = ar$: physical distance on equal-time hypersurface, b, b': conformal central charges, β: R^2 coupling constant, $\bar{\mu}$: renormalisation scale
- Generalisation to spinning particles gives corrections to gravitomagnetic potential $V_i = h_{0i}$
- $\mathbf{V} = -2G \frac{\mathbf{S} \times \hat{\mathbf{r}}}{\hat{r}^3} \left[1 + 96\pi b \frac{\ell_\text{Pl}^2}{\hat{r}^2} + 32\pi \ell_\text{Pl}^2 H^2 \left(\beta - 5b - b' + 2b \ln \left(\bar{\mu} \hat{r} \right) \right) \right]$ Fröb/Verdaguer 1701.06576

Conclusions

- Issue of gauge-invariant observables connects pQG with fundamental QG
- Observables are non-local, relational observables can be constructed to all orders
- Results:
  - Strengthening of gravitational force at small distances Fröb/Rein/Verch 2109.09753
  - Logarithmically growing correction to Newtonian and gravitomagnetic potential in de Sitter Fröb/Verdaguer 1601.0356, Fröb/Verdaguer 1701.06576
  - Secular screening of the Hubble rate in cosmology Fröb 1806.11124, Lima 2007.04995
  - Your favourite effect can be computed now!

Funded by

- The European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 702750 "QLO-QG"
- Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) — project nos. 415803368 and 406116891 within the Research Training Group RTG 2522/1