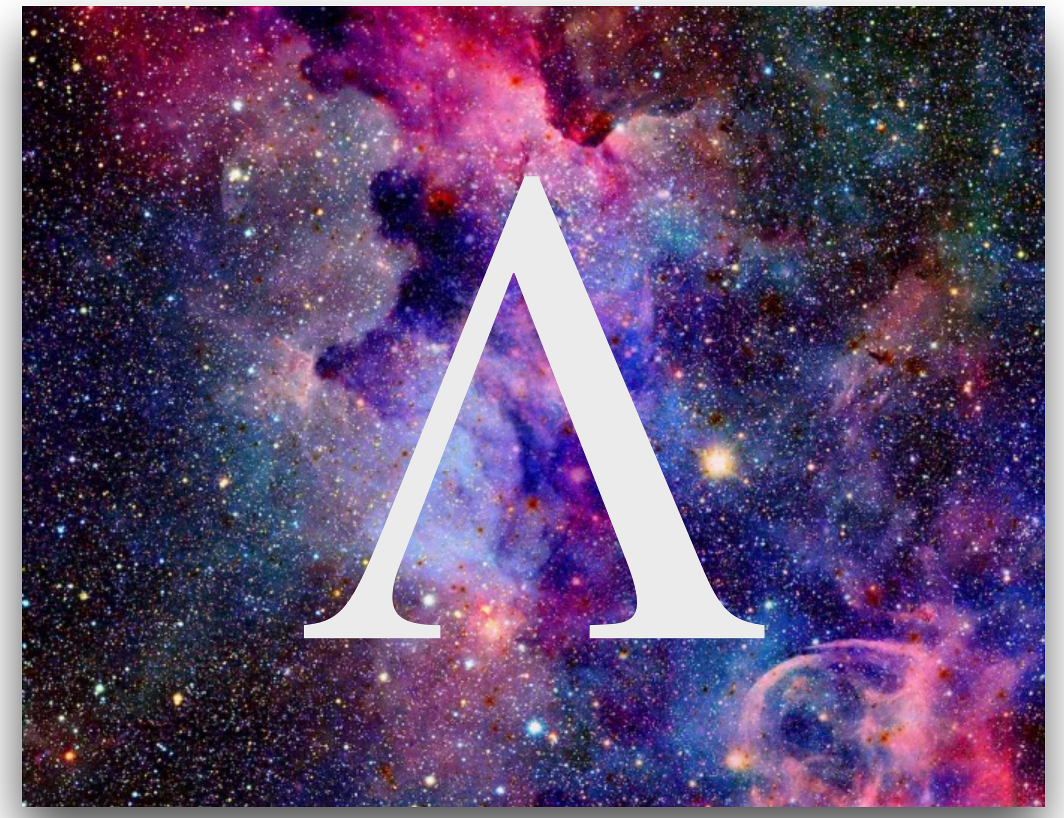

Melonic Radiative Correction in 4D Spinfoam with Cosmological Constant

Qiaoyin Pan

Based on collaboration with Muxin Han, [arXiv: 2307.xxxx]

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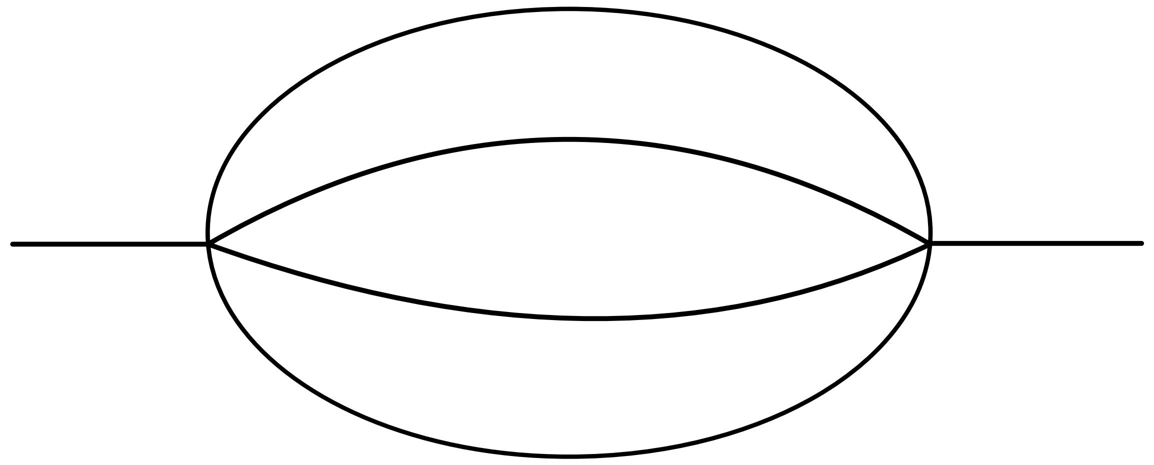
Spinfoam model of quantum gravity

- **Loop quantum gravity (LQG)** — a background-independent, non-perturbative approach to quantum gravity $\Longrightarrow$ quantize spacetime geometry

- A promising research area in LQG — inclusion of cosmological constant $\Lambda \longrightarrow$ completeness, apply to cosmology
- **Spinfoam** — a covariant LQG approach: to construct the transition amplitude of quantum gravity $\longrightarrow$ regularized path integral/sum over history
- **Divergence** in spinfoam — have *no* ultraviolet divergence due to fundamental "area gaps" $\propto \ell_{\mathrm{pl}}^2$
  — but have **infrared divergence** when $\Lambda = 0$
- **Self-energy/radiative correction** in spinfoam [Perini, Rovelli, Speziale '08]



melon graph

Melonic radiative correction $\longrightarrow$ first-order correction of propagator

Melonic radiative correction in spinfoam

- $\Lambda = 0 \longrightarrow$ the Engle-Pereira-Rovelli-Livine-Freidel-Krasnov (EPRL-FK) spinfoam model [Engle, Livine, Roveilli, Pereira '07, Freidel, Krasnov '08]
  - Introduce a cut-off for representation label **by hand** $\sum_{j=0}^{\infty} \to \sum_{j=0}^{k}$ and consider large k
  - Melonic radiative correction: $\mathcal{A}_{\Gamma_{\text{melon}}} \sim k$ [Frisoni, Gozzini, Vidotto '22] consistent with $\begin{cases} \text{the lower bound (analytic):} & \mathcal{A}_{\Gamma_{\text{melon}}} \sim \ln k \quad \text{[Riello '13]} \\ \text{the upper bound (numerical):} & \mathcal{A}_{\Gamma_{\text{melon}}} \sim k^9 \quad \text{[Donà '18]} \end{cases}$

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- Spinfoam model based on $\mathrm{SL}(2,\mathbb{C})$ Chern-Simons theory on the boundary [Han '21]
  - The amplitude is **finite by construction** — cut-off given by $|\Lambda|$: $\sum_{j=0}^{(k-1)/2}$ $k = \frac{12\pi}{|\Lambda|\ell_{\text{pl}}^2\gamma} \in \mathbb{Z}_+$; $\gamma \in \mathbb{R}$: Barbero-Immirzi parameter
  - Correct semi-classical limit $\longrightarrow$ 4D Regge calculus with $\Lambda > 0$ or $\Lambda < 0$ [Haggard, Han, Kaminski, Riello '14-15]

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Analytic Result for the melonic radiative correction — power law $\mathcal{A}_{\text{melon}} \sim k^p$

In this talk

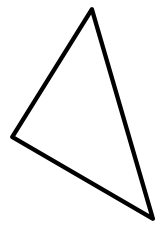
- Review on the spinfoam model with $\Lambda \neq 0$
- Spinfoam amplitude for melonic graph
- Melonic radiative correction

4D spinfoam with $\Lambda \neq 0$

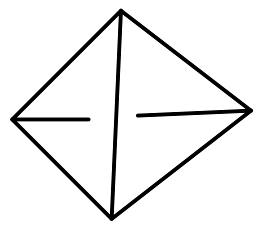
- ***Local amplitude ansatz*** defined on a spinfoam 2-complex:

$$\mathcal{A}[\mathbf{T}_4, \psi_{\partial \mathbf{T}_4}] = \sum_{j_f, \iota_e} \prod_f \mathcal{A}_f(j_f) \prod_e \mathcal{A}_e(j_{f\ni e}, \iota_e) \prod_v \mathcal{A}_v(j_{f\ni v}, \iota_{e\ni v})$$

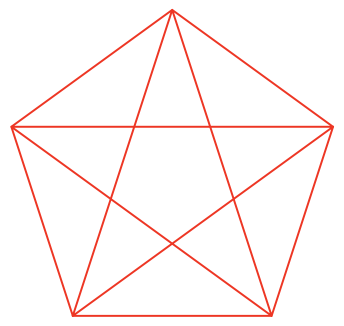
j : representation
ι : intertwiner $\quad \iota : \bigotimes_j V_j \to V_0$



face amplitude → $(\dim j_f)^{\mu}$



edge amplitude



vertex amplitude → local LQG dynamics

4D triangulation $\mathbf{T}_4$	$\overset{\text{dual picture}}{\Longleftrightarrow}$	spinfoam 2-complex
4-simplex T		spinfoam vertex v
tetrahedron $\triangle$		spinfoam edge e
triangle t		spinfoam face f

- **Goal of the spinfoam program:** construct suitable and consistent $\mathcal{A}_f, \mathcal{A}_e, \mathcal{A}_v$ to implement quantum dynamics of LQG

4D spinfoam with $\Lambda \neq 0$ — cont.

- **Starting point — Plebanski formulation** of 4D gravity: BF theory + simplicity constraint

$$S_{\text{BF}} = -\frac{1}{2}\int_{M_4} \text{Tr}\left[\left(\star B + \frac{1}{\gamma}B\right)\wedge \mathcal{F}(\mathcal{A})\right] - \frac{|\Lambda|}{12}\int_{M_4}\text{Tr}\left[\left(\star B + \frac{1}{\gamma}B\right)\wedge B\right]$$

$B : \mathfrak{sl}(2,\mathbb{C})$ 2-form; $\mathcal{A} : \mathfrak{sl}(2,\mathbb{C})$ connection;
$\gamma \in \mathbb{R}$: Barbero-Immirzi parameter; $\star$: Hodge operator

simplicity constraint: $B = \pm e \wedge e$ (encode the sign of Λ)

- Step 1: **Construct the path integral:** integrating out B field $\xrightarrow{\text{Gaussian integral}}$ constraint $\mathcal{F}[\mathcal{A}] = \frac{|\Lambda|}{3}B$

$$\int \mathrm{d}B \int \mathrm{d}\mathcal{A}\, e^{iS_{\text{BF}}} = \int \mathrm{d}\mathcal{A}\, e^{\frac{3i}{4|\Lambda|}\int_{M_4}\text{Tr}\left[\left(\star+\frac{1}{\gamma}\right)\mathcal{F}\wedge\mathcal{F}\right]}$$

- $\longrightarrow$ $\text{SL}(2,\mathbb{C})$ **Chern-Simons theory on the boundary**

$$\frac{t}{8\pi}\int_{\partial M_4}\text{Tr}\left(A\wedge \mathrm{d}A + \frac{2}{3}A\wedge A\wedge A\right) + \frac{\bar{t}}{8\pi}\int_{\partial M_4}\text{Tr}\left(\bar{A}\wedge \mathrm{d}\bar{A} + \frac{2}{3}\bar{A}\wedge\bar{A}\wedge\bar{A}\right)$$

$t = k(1+i\gamma)$: complex coupling constant
$k = \frac{12\pi}{|\Lambda|\ell_P^2\gamma} \in \mathbb{Z}_+$

A: self-dual part of $\mathcal{A}$; $\bar{A}$: anti-self-dual part of $\mathcal{A}$

- Step 2: **Quantize the simplicity constraint** and impose it to the quantum theory

4D spinfoam with $\Lambda \neq 0$ — cont.

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4D quantum gravity with $\Lambda \neq 0$ = complex **Chern-Simons theory** on boundary\graph + **simplicity constraints** $\mathcal{F} = \frac{\Lambda}{3} e \wedge e$ on the graph

[Haggard, Han, Kaminski, Riello '14-15]

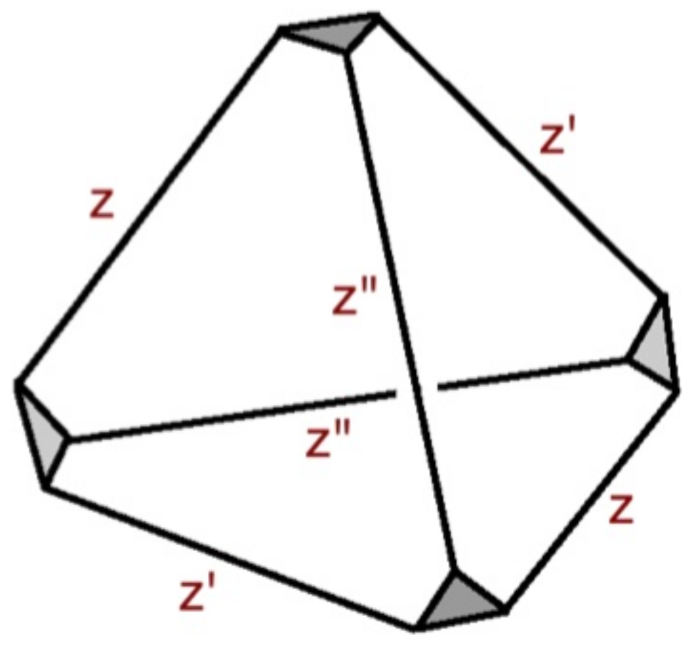
Vertex amplitude for $S^3\backslash\Gamma_5$

- CS partition function on $S^3\backslash\Gamma_5$ = **finite** sum of **convergent** state integral [Han '21]

$$\mathcal{Z}_{S^3\backslash\Gamma_5}(\vec{\mu}\mid\vec{m}) = \frac{4i}{k^{15}} \sum_{\vec{n}\in(\mathbb{Z}/k\mathbb{Z})^{15}} \int_{\mathcal{C}} \mathrm{d}^{15}\nu\, e^{S_0} \prod_{i=1}^{20} \Psi_{\triangle}(i)$$

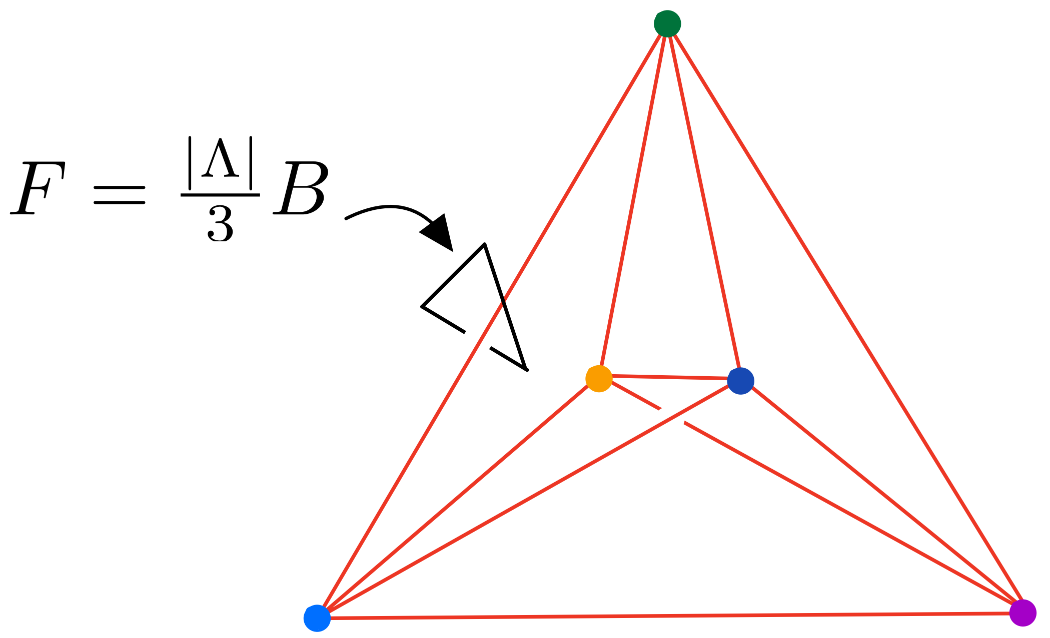
$\Psi_{\triangle}(i)$: quantum dilogarithm function

$$S_0 = \frac{\pi i}{k}\left[-2\left(\vec{\mu} - \frac{iQ}{2}\vec{t}\right)\cdot\vec{\nu} + 2\vec{m}\cdot\vec{n} - \vec{\nu}\cdot\mathbf{AB}^T\cdot\vec{\nu} + (k+1)\vec{n}\cdot\mathbf{AB}^T\cdot\vec{n}\right]$$

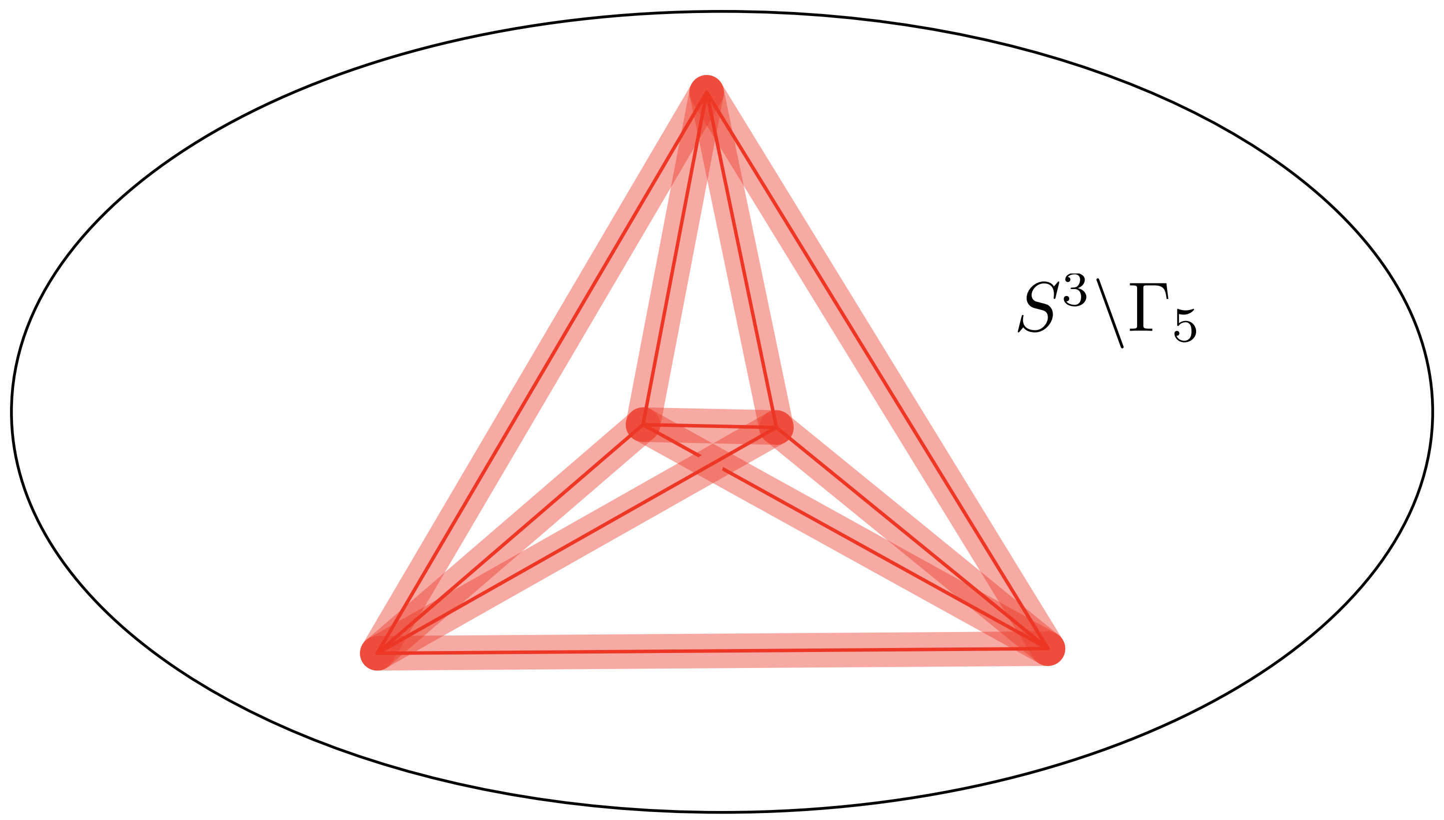


ideal tetrahedron $\triangle$ (vertices truncated)

[figure from: Han '21]



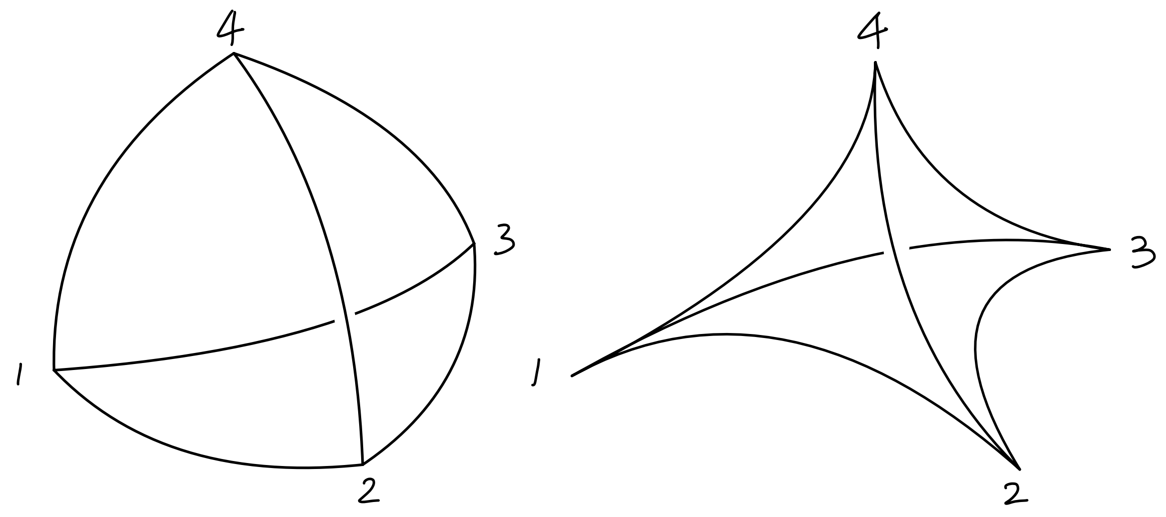
Γ_5 graph



- Impose the simplicity constraints $\longrightarrow$ The **vertex amplitude** is defined by the inner product of the CS partition function with 5 coherent states

$$\mathcal{A}_v(\{e^{i\frac{4\pi j}{k}}\},\{\hat{\rho}_a\}) = \left\langle \prod_{a=1}^{5} \overline{\Psi}_{\hat{\rho}_a} \mid \mathcal{Z}^{\vec{j}}_{S^3\backslash\Gamma_5}\right\rangle$$

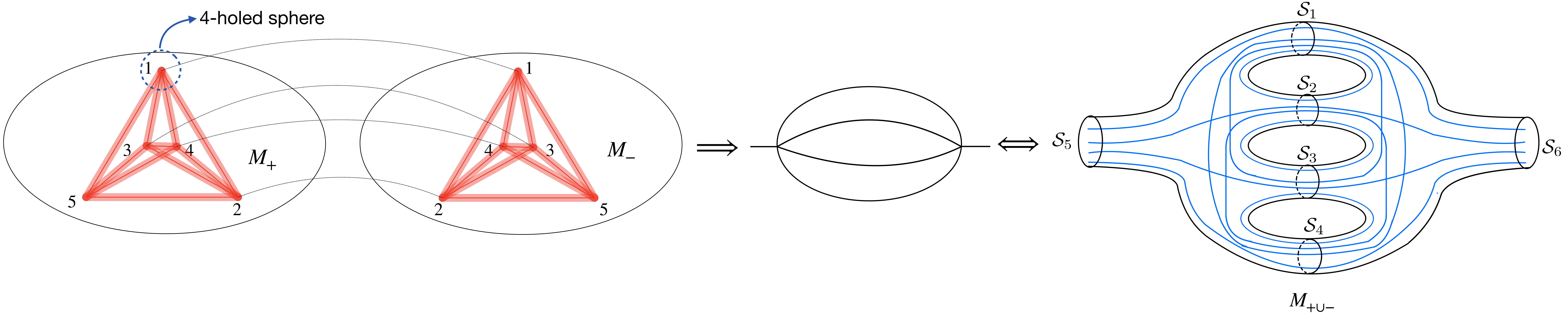
Melonic radiative correction — gluing two 3-manifold



- Consider two copies of $S^3\backslash\Gamma_5$'s and the CS partition function on the $\mathcal{Z}_{M_+}$, $\mathcal{Z}_{M_-}$
- Impose the "**gluing constraints**" on four of the 4-holed spheres $\xrightarrow{\text{discretization}} \begin{cases} \Lambda > 0 \rightarrow \text{spherical tetrahedron} \\ \Lambda < 0 \rightarrow \text{hyperbolic tetrahedron} \end{cases}$

  18 gluing constraints =**10** to match the (quantum) **areas** + **8** to match the (quantum) **shapes**

  $\Longrightarrow$ CS partition function for $M_{+\cup-}$: $\mathcal{Z}_{M_{+\cup-}}$



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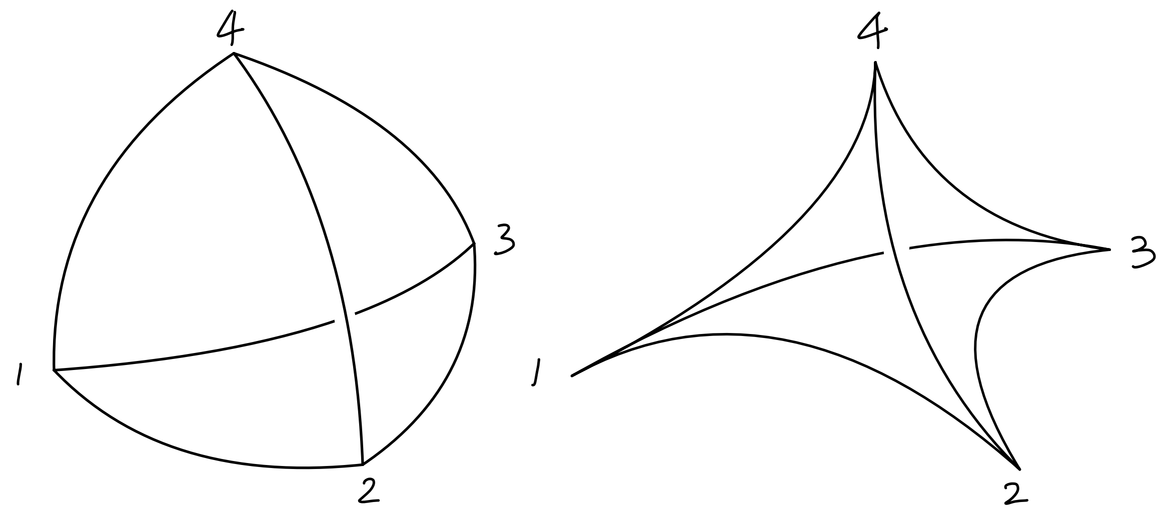

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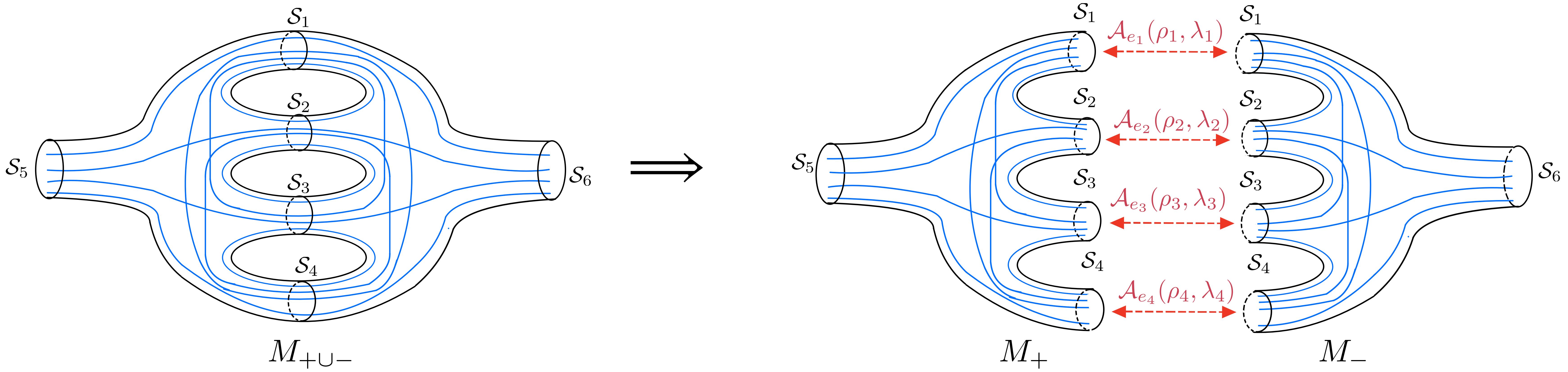
$\Longrightarrow$ CS partition function for $M_{+\cup-}$: $\mathcal{Z}_{M_{+\cup-}}$

- Impose the **simplicity constraints** — factorize the amplitude into **2 vertex amplitudes** and **4 edge amplitudes**

— use the over-completeness of coherent states

$$\sum_{m\in\mathbb{Z}/k\mathbb{Z}}\int \mathrm{d}\mu f(\mu\mid m)f'(\mu\mid m) = \sum_{m,n\in\mathbb{Z}/k\mathbb{Z}}\int \mathrm{d}\mu\mathrm{d}\nu f(\mu\mid m)[\delta(\mu,\nu)\delta_{m,n}]f'(\nu\mid n)$$

$$= \frac{k^2}{(2\pi)^4}\sum_{m,n\in\mathbb{Z}/k\mathbb{Z}}\int \mathrm{d}\mu\mathrm{d}\nu f(\mu\mid m)\left[\int \mathrm{d}\rho\,\overline{\Psi}_\rho(\mu\mid m)\Psi_\rho(\nu\mid n)\right]f'(\nu\mid n)$$



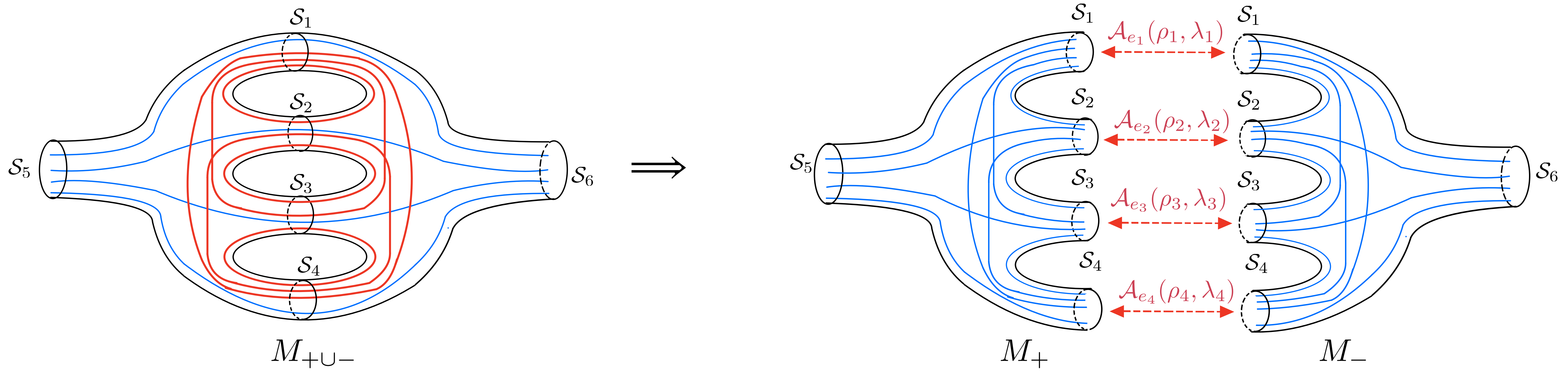
Spinfoam amplitude for $M_{+\cup-}$

- We derive the **edge amplitude** for each gluing sphere :

$$\mathcal{A}_e\left(\hat{\rho}_e, \hat{\eta}_e\right)=\frac{k^3}{(2\pi)^8} \int \mathrm{d}\mu_e\, e^{\frac{4\pi Q}{k}\mu_e}\, \overline{\Psi}_{\hat{\rho}_e}\left(\mu_e \mid m_e\right) \overline{\Psi}_{\hat{\eta}_e}\left(\mu_e \mid m_e\right)$$

- $M_{+\cup-}$ is of genus 6. For each internal loop, we associate a **face amplitude**

$$\mathcal{A}_f\left(j_f\right)=\left(2 j_f+1\right)^\mu, \quad \mu \text{ undetermined}, \quad j_f=0, \frac{1}{2}, \cdots, \frac{k-1}{2}$$



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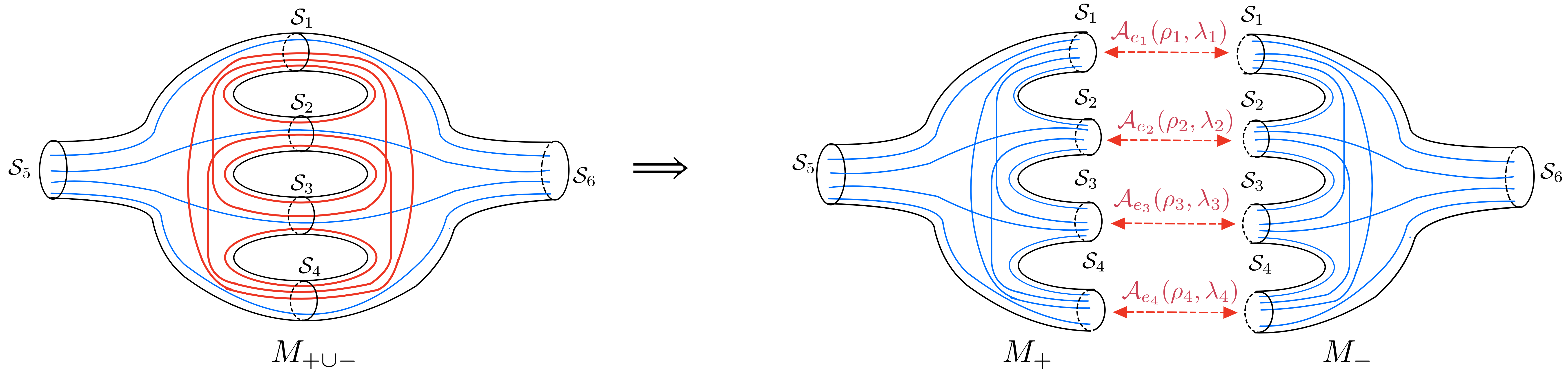
- The spinfoam amplitude for $M_{+\cup-}$:

$$\mathcal{A}_{\text{melon}}(\{j_b\}, \hat{\rho}_5, \hat{\eta}_6) = \sum_{\{j_f\}=0}^{(k-1)/2} \prod_{f=1}^{6} \mathcal{A}_f(j_f) \int [\mathbf{d}\hat{\rho}_e][\mathbf{d}\hat{\eta}_e] \prod_{e=1}^{4} \mathcal{A}_e\left(\hat{\rho}_e, \hat{\eta}_e\right) \mathcal{A}_{v,+}(\{j_f\}, \{j_b\}, \{\hat{\rho}_a\}) \mathcal{A}_{v,-}(\{j_f\}, \{j_b\}, \{\hat{\eta}_a\})$$

Theorem: The amplitude $\mathcal{A}_{\text{melon}}(\{j_b\}, \hat{\rho}_5, \hat{\eta}_6)$ for $M_{+\cup-}$ is **finite**.



Melonic radiative correction

- We fix the boundary data $(\{j_b\}, \hat{\rho}_5, \hat{\eta}_6)$ and consider the $\Lambda \to 0\,(k \to \infty)$ for the melonic amplitude $k = \frac{12\pi}{|\Lambda|\ell_{\text{pl}}^2\gamma} \in \mathbb{Z}_+$

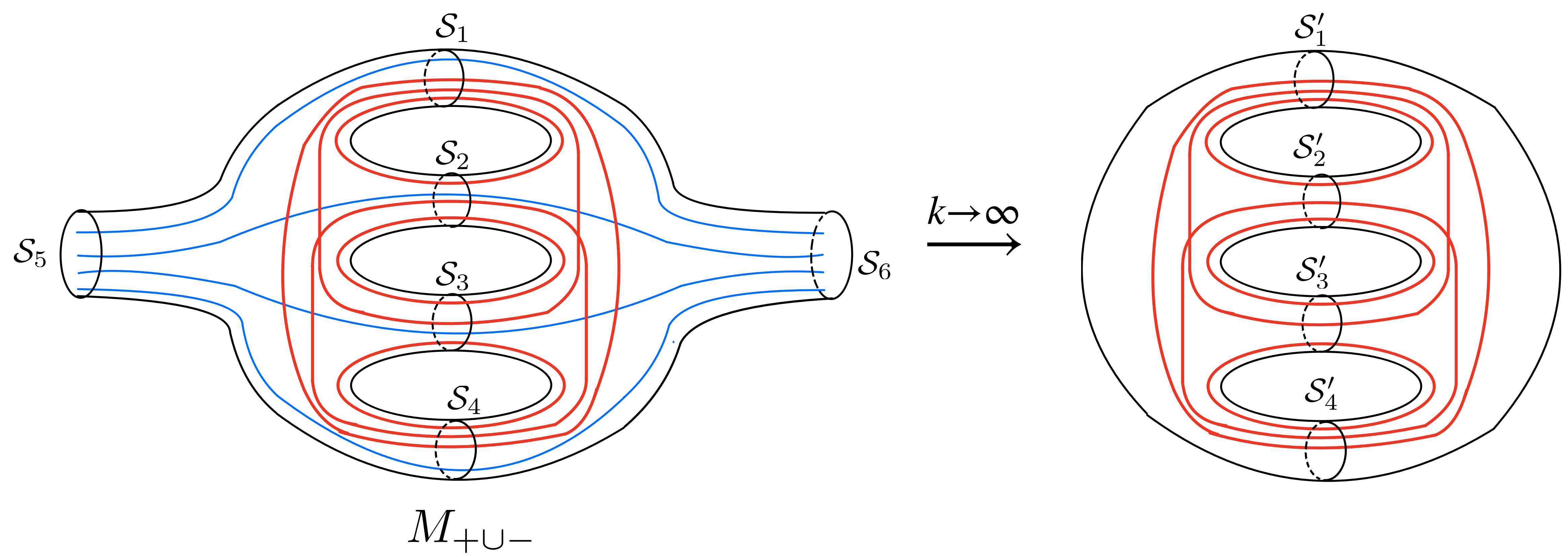
  - Use **Poisson summation** to change from sum to integral

$$\sum_{m=0}^{k-1} f(m) = \sum_{r\in\mathbb{Z}} \int_{-\delta}^{k-\delta} \mathrm{d}m\, f(m) e^{2\pi i r m}$$

  - **Change to coordinates** $(\vec{\mu}, \vec{m}) \to (\vec{\mathfrak{Q}}, \vec{\mathfrak{P}})$ which do not scale with k and take the large k approximation

$$\mathcal{A}_{\text{melon}}(\vec{\mathfrak{Q}}_{\text{ext}}) = \mathcal{N} \int [\mathrm{d}\vec{\mathfrak{Q}}_{\text{int}}][\mathrm{d}\vec{\mathfrak{P}}_{\text{int}}] \prod_{f=1}^{6} \mathcal{A}_f(\vec{\mathfrak{Q}}_{\text{int}}) \prod_{e=1}^{4} \mathcal{A}_e(\vec{\mathfrak{Q}}, \vec{\mathfrak{P}})\, \mathcal{A}_{v,+}(\vec{\mathfrak{Q}}, \vec{\mathfrak{P}}) \mathcal{A}_{v,-}(\vec{\mathfrak{Q}}, \vec{\mathfrak{P}})$$

$$\xrightarrow{k\to\infty} \mathcal{N} \int [\mathrm{d}\vec{\mathfrak{Q}}_{\text{int}}][\mathrm{d}\vec{\mathfrak{P}}_{\text{int}}]\, e^{S_{\text{tot}}}[1 + O(1/k)]$$



Result

- The **effective action** for the final amplitude at large k regime:

$$S_{\text{tot}} = \sum \text{Li}_2(e^{\text{Linear}\,(\vec{\mathfrak{Q}},\vec{\mathfrak{P}})}) + \sum \text{Poly}(\vec{\mathfrak{Q}},\vec{\mathfrak{P}}) + \sum \text{Poly}\left(\{\hat{\rho}_e\},\{\hat{\eta}_e\}\right)$$

$$\Psi_\triangle(z,\tilde{z}) = \exp\left[-\frac{ik}{2\pi(b^2+1)}\text{Li}_2\left(z^{-1}\right) - \frac{ik}{2\pi(b^{-2}+1)}\text{Li}_2\left(\tilde{z}^{-1}\right)\right][1+O(1/k)]$$

- **Stationary phase analysis**:

$$\mathcal{A}_{\text{melon}}(\vec{\mathfrak{Q}}_{\text{ext}}) \xrightarrow{k\to\infty} \mathcal{N}\int [\text{d}\vec{\mathfrak{Q}}_{\text{int}}][\text{d}\vec{\mathfrak{P}}_{\text{int}}]\, e^{S_{\text{tot}}}[1+O(1/k)]$$

$$\sim \sum_\alpha \frac{\mathcal{N}}{\sqrt{\det(H_\alpha/2\pi)}} e^{S_{\text{tot}}^{(\alpha)}}[1+O(1/k)]$$

stationary points: $\frac{\partial S_{\text{tot}}^{(\alpha)}}{\partial \vec{\mathfrak{Q}}_{\text{int}}} = \frac{\partial S_{\text{tot}}^{(\alpha)}}{\partial \vec{\mathfrak{P}}_{\text{int}}} = \frac{\partial S_{\text{tot}}^{(\alpha)}}{\partial \rho_e} = \frac{\partial S_{\text{tot}}^{(\alpha)}}{\partial \eta_e} = 0$

$$\mathcal{N} \propto k^{63+6\mu}, \quad \det\left(H_\alpha\right) \propto k^{102} \quad \longrightarrow \quad \boxed{\mathcal{A}_{\text{melon}} \sim k^{12+6\mu}}$$

$\mathcal{A}_{\text{melon}}$ diverges if $\mu > -2$

- It depends on the technical assumption that $\det(H_\alpha) \neq 0$
  - ☑ Pass some numerical experiments
  - Strictly speaking, $k^{12+6\mu}$ is a lower bound for the divergence

Conclusion and outlook

- The melonic radiative correction for the spinfoam model with $\Lambda \neq 0$ based on the SL$(2,\mathbb{C})$ CS theory on the boundary scales as $\frac{1}{|\Lambda|^{12+6\mu}}$ as the lower bound given the face amplitude $\mathcal{A}_f = (2j_f+1)^\mu$
- This result is promising as we start from a spinfoam model **truly** with a cosmological constant
- We proposed an **edge amplitude** in terms of CS coherent state consistent with the vertex amplitude
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- **Exploration**: improve the analysis for the $\det(H_\alpha) \longrightarrow$ numerical method [Han, Liu, Qu '21-23]

  connection with the EPRL-FK model $\longrightarrow$ relation between the coherent states

  relation to the canonical quantization of 4D gravity with $\Lambda \neq 0$ — combinatorial quantization

  — emergence of quantum groups

  etc.

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- The melonic radiative correction for the spinfoam model with $\Lambda \neq 0$ based on the SL$(2,\mathbb{C})$ CS theory on the boundary scales as $\frac{1}{|\Lambda|^{12+6\mu}}$ as the lower bound given the face amplitude $\mathcal{A}_f = (2j_f+1)^\mu$
- This result is promising as we start from a spinfoam model **truly** with a cosmological constant
- We proposed an **edge amplitude** in terms of CS coherent state consistent with the vertex amplitude
- As a state-integral formalism, this spinfoam model is as **computable** as the EPRL-FK model

- **Exploration**: improve the analysis for the $\det(H_\alpha) \longrightarrow$ numerical method [Han, Liu, Qu '21-23]

  connection with the EPRL-FK model $\longrightarrow$ relation between the coherent states

  relation to the canonical quantization of 4D gravity with $\Lambda \neq 0$ — combinatorial quantization

  — emergence of quantum groups

  etc.

Thank you for your attention!