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Loop quantization as a continuum limit: QG

Quantum Gravity 2023, Nijmegen, Netherlands

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Point of view

LQG as Loop Quantization (LQ) applied to G
(using a variant of Ashtekar-Barbero variables)

with LQ as a quantization framework applicable to other gauge theories e.g. Yang-Mills
(and sigma models).

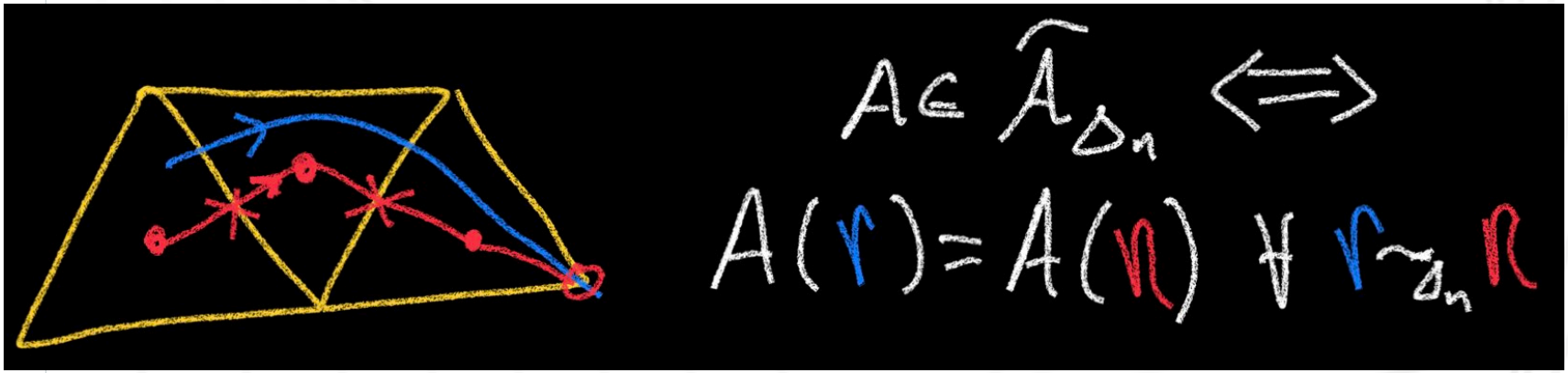
Take away idea

LQ (together with a regularization map)
for gauge theory
is the continuum limit of
Lattice Gauge Theory.

Results: space of fields

$\varinjlim_{\Delta(n)} \tilde{\mathcal{A}}_{\Delta(n)} = \bar{\mathcal{A}}_M$

The direct limit of spaces of states of lattice gauge fields $(\tilde{\mathcal{A}}_{\Delta(n)} = R_n(\mathcal{A}_{\Delta(n)}) \subset \bar{\mathcal{A}}_M)$ *is dense in* $\bar{\mathcal{A}}_M$ *see [1, 2].*



Results: physical measure

(Physical) measures in $\bar{\mathcal{A}}_M$ can be constructed as the continuum limit of Wilsonian renormalization:

$$R_n(\tilde{\mathcal{A}}_{\Delta(n)}, \exp(-\beta_n S_n)\mu_{Haar}) \underset{n\to\infty}{\longrightarrow} (\bar{\mathcal{A}}_M, \mu_{phys})$$

When the lattice theories converge in usual lattice sense the corresponding measures converge [2].

Example: Cont. lim. of the 2d Ising model

The continuum limit of the 2d Ising model induces a LOOP QUANTIZED interacting relativistic quantum field theory [2].

(The example is based on previously known analytic results [3].)

Example: Polymer Quantum Mechanics

$$(\mathcal{H}_{\mathrm{Poly}_n}, H_n) \xrightarrow[n\to\infty]{} (\mathcal{H}_{\mathrm{Schr}_n}, H = T + V_{\mathrm{convex}_n})$$

The same ideas applied to
"polymer quantum mechanics"
show that its continuum limit is
standard quantum mechanics
(when the potential is convex) [4].

Lesson: Scale

$M = \sqcup_\alpha c_\alpha$ induces a notion of scale for LQ on M.

$\Leftrightarrow$

PART. ORD. DIRECTED COLL. OF ALGS. $Cyl_C = Cyl_M / \sim_C$.

- (Key ingredient to prove that:)

Loop Quantization is the continuum limit of Lattice Quantization of gauge theories.

Lesson: Physical observables

Physical observables
must survive the continuum/macroscopic limit.
$\Rightarrow$ they must be “extensive” in some sense.

Questions: Phases in LQG

Are there several phases in LQG?
(Some other approaches have found them already.)

Is $W(C)_\beta =< Tr(PT(C)) >_\beta$
studied WITH RESPECT TO THE QUANTUM GEOMETRY a good order parameter for LQG?
Be aware: Quantum geometry of timelike curves and surfaces needed.

Challenges: Renormalization prescription

Wilson's renormalization procedure needs a renormalization prescription to flow from coarse to fine, as needed for the continuum limit.

We need to find one for LQG.

Challenges: Diffeomorphism invariance

This point of view sacrifices diffeo. invariance at intermediate cutoff scales and expects to recover it in the continuum limit.
The hope is that if at intermediate cutoff scales regularization does not essentially rely on foreign structures the limit will regain the sacrificed symmetry, as is the case for the 2d Ising model.

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Thank you !

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