Towards new numerical methods in spin foams

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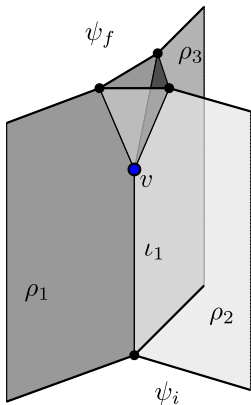Quantum Gravity 2023 @ Radboud University Nijmegen
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1/9

Spin foam gravity [Rovelli, Reisenberger, Barrett, Crane, Freidel, Livine, Krasnov, Perez, Speziale, Engle, Pereira, Kaminski...]

- **Non-perturbative path integral of geometries**
- Regulator: **Discretization** / 2-complex
- **Quantum geometric** building blocks
    - (Constrained) **topological quantum field theory**
    - **Discrete** area spectrum
- Physical content: **Transition amplitudes**
    - **Single building block** $\sim$ **discrete gravity** [Conrady, Freidel '08, Barrett, Dowdall, Fairbairn, Gomes, Hellmann '09, Kaminski, Kisielowski, Sahlmann '17, Liu, Han '18, Simão, S.St. '21]
   - Quantum amplitudes (not Wick-rotated)

Derived from **general relativity**

No reference to background geometry

Aim to implement **diffeomorphism symmetry**

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Sebastian Steinhaus

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Computational challenges

$$Z(\Delta^*) = \sum_{\rho_f, \iota_e} \prod_{f \in \Delta^*} \mathcal{A}_f(\rho_f) \prod_{e \in \Delta^*} \mathcal{A}_e(\iota_e) \prod_{v \in \Delta^*}$$
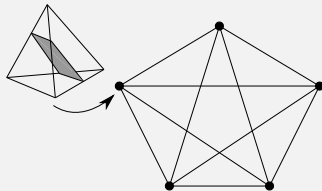
- **Explicit** calculation of **vertex amplitude** (amplitude of a 4-simplex)
- **Sum over representations** and intertwiners

Amplitudes not positive (semi-)definite and **oscillate**
Cannot be used for **Monte Carlo importance sampling**

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Recent (numerical) progress

- Explicit calculation of **vertex amplitude**:
  - `sl2cfoam` for EPRL/FK model [Donà, Fanizza, Sarno, Speziale '19, Gozzini '21]
  - SU(2) BF theory [Donà, Fanizza, Sarno, Speziale, '17, Asante, Simão, S.St. w.i.p.]
- Utilizing **semi-classical** insights
  - **Effective spin foams** [Asanta, Dittrich, Haggard PRL '20, Asante, Dittrich, Padua-Argüelles '21] → Talk(s) by Hal Haggard and Seth Asante
  - Acceleration operators for **series convergence** [Dittrich, Padua-Argüelles '23]
  - **Hybrid representation** of spin foams [Asante, Simão, S.St. '22]
  - Complex critical points [Han, Huang, Liu, Qu '21, Han, Liu, Qu '23]
  - Restricted spin foam models [Bahr, S.St. '15, Bahr, Rabuffo, Klöser '16, Assanioussi, Bahr '20]
- **Monte Carlo methods**
  - Markov chain Monte Carlo on Lefshetz thimbles [Han, Huang, Liu, Qu, Wan '20]
  - Random sampling Lorentzian EPRL model [Donà, Frisoni '23]

This talk: **idea for importance sampling** in spin foams.

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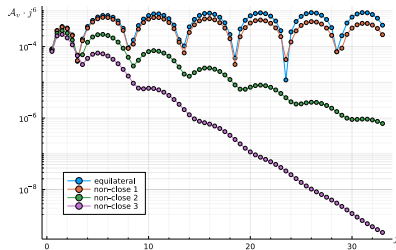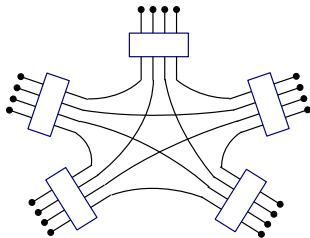July 10th 2023
4/9

Semi-classical properties of spin foams

- **Critical points** dominate vertex amplitude for large representations [Conrady, Freidel '08, Barrett, Dowdall, Fairbarn, Gomes, Hellmann '09, '10, Kaminski, Kisielowski, Sahlmann '17, Han, Liu '18, Simão, S.St. '21]
  - **Coherent** boundary data [Livine, Speziale '07]

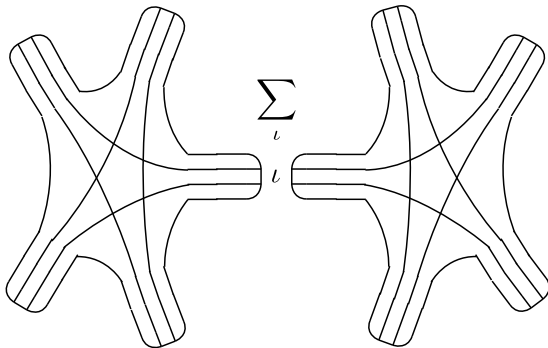## Critical points of spin foam vertex amplitude

- **Geometric 4-simplices**
  - Oscillate with **Regge action**
- Vector geometries (degenerate)

- **Exponential suppression** away from critical points

What can we learn from this for **larger triangulations**?

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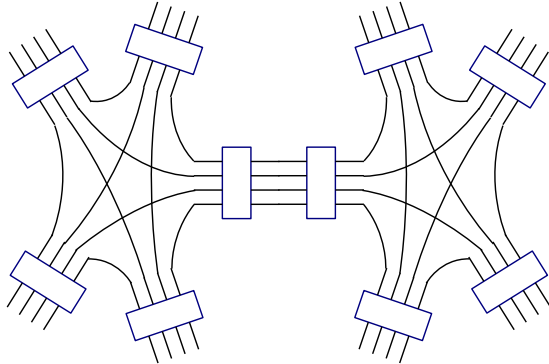Quantum Gravity 2023 @ Radboud University Nijmegen ,
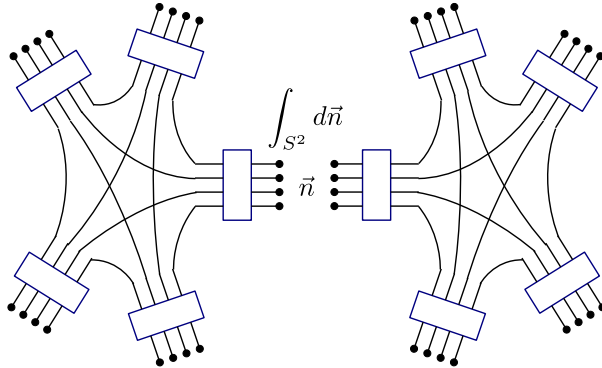July 10th 2023
5/9

Spin foam as coherent amplitudes

- Equip each vertex with **independent set of coherent data**

Spin foam as coherent amplitudes

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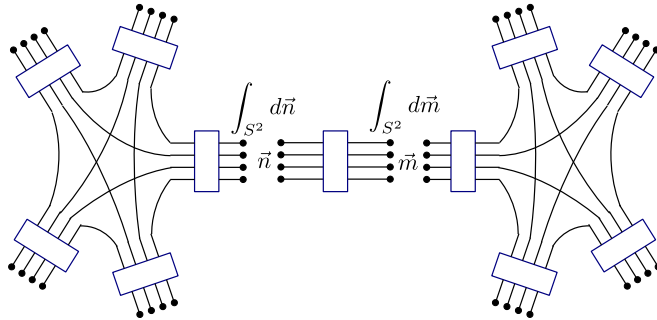6/9

Spin foam as coherent amplitudes

- Equip each vertex with **independent set of coherent data**

Spin foam as coherent amplitudes

- Equip each vertex with **independent set of coherent data**

Interpolate between vertices by **gluing constraints**.

Hybrid representation of spin foams [Asante, Simão, S.St. '22]

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Gluing constraints [Asante, Simão, S.St. '22]

- **Gluing constraints** peaked on **matching tetrahedra**
  - Compare to effective spin foams [Asante, Dittrich, Haggard '20]

$|G(\Phi_{equi}, \Phi)|$

Non-matching, semi-classical vertices

Non-metricity: **Torsion** degrees of freedom [Asante, Dittrich, Haggard '20]

- Expect **major contribution** from **matching critical points**
  - Define a **probability distribution** for intertwiners from coherent states
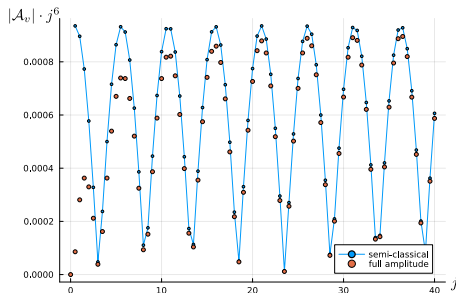   - Absolute value of **change of basis** from coherent to ONB **intertwiners**
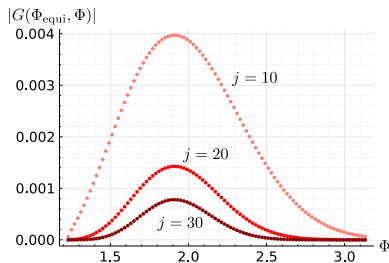
Sample intertwiners from coherent states.

Can we **test sampling** with this **probability distribution**?

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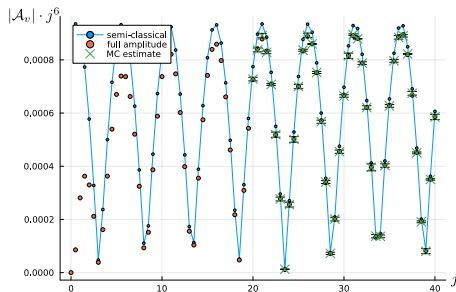Quantum Gravity 2023 @ Radboud University Nijmegen ,
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First test: SU(2) coherent vertex amplitude

- Coherent vertex amplitude: **importance sampling from boundary data**
  - Equilateral boundary data: **sum over** $(2j+1)^5$ **combinations**
  - 10 runs with 10^6 **samples** for $j = 20, \ldots, 80$.

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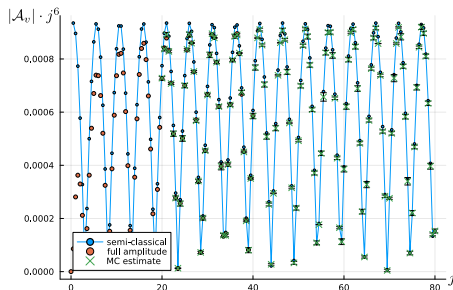Quantum Gravity 2023 @ Radboud University Nijmegen ,
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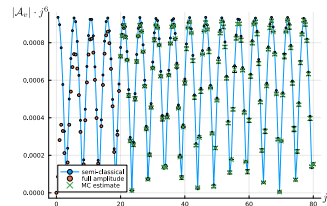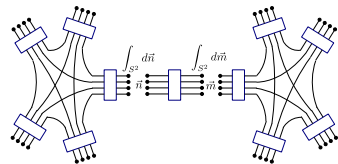 - 10 runs with 10^6 **samples** for $j = 20, \ldots, 80$.

Good agreement and fast convergence!

Sign problem present: **global phase** of amplitude

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Summary and Outlook

- **Monte Carlo importance sampling** via **critical points**
  - **Exponential suppression** away from critical points
  - Decompose spin foam: (non-matching) vertices
  - Sample intertwiners w.r.t. critical points
- Test of **coherent vertex amplitude** encouraging!
  - Test for larger triangulations: **Convergence?**
  - How to sample representation labels?
- Study **observables** in **simplified models** first
  - Spectral dimension [Jercher, S.St., Thürigen '23]
  - Matter coupled to spin foams [Ali, S.St. '22]

Measure sign problem to estimate potential of Monte Carlo methods!

Thank you for your attention!

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9/9