4D Geometry Generated by Combinatorially Non-local Field Theory

Johannes Thürigen, WWU MÜNSTER

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Mathematics Münster Cluster of Excellence

The 4D challenge of Quantum Gravity

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- Matrix Models: generating function of discrete surfaces (comb. maps)

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- Tensor Models with rank r generate discrete r-dim. (pseudo) manifolds, but only branched polymere / 2D gravity at criticality
- $D_s > 2$ from additional combinatorial structure (as causal one in CDT)?

Combinatorially Non-Local Field Theory

Idea here: add geom. d.o.f. $\boldsymbol{q}$ generalizing tensors $T_{a_1 \dots a_r}$ to fields $\Phi(q_1, \dots, q_r)$:

$$S[\Phi] = \int \mathrm{d}\boldsymbol{q}\, \bar{\Phi}(\boldsymbol{q})(\boldsymbol{q}^2 + \mu)\Phi(\boldsymbol{q}) + \sum_{\gamma} \lambda_\gamma \int \prod_{i=1}^{k} \mathrm{d}\boldsymbol{q}_i \prod_{(ia,jb)} \delta(q_i^a - q_j^b) \prod_{i=1}^{k} \Phi(\boldsymbol{q}_i)$$

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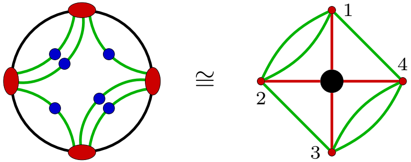
pairwise convolution of $q^a \in \mathbb{R}^d$, $a = 1, \dots, r$, given by graphs γ



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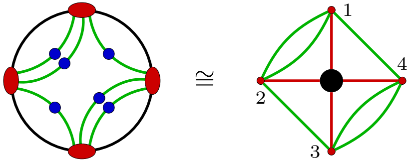

cNLFT provides sum over comb. manifolds and geometries thereon!

$$Z_{\mathrm{NLFT}} = \sum_{\Gamma} \boldsymbol{\lambda}^{V_\Gamma} A_\Gamma(\{q_f\}),\; A_\Gamma(\{q_f\}) = \int I_\Gamma = \prod_f v_d \int q_f^{d-1} \mathrm{d}q_f \prod_e \frac{1}{\sum_{f>e} \boldsymbol{q}_f^2 + \mu}$$

Feynman diagrams Γ with $\boldsymbol{q}_f$ on faces f $\to$ geometry on complex dual to Γ

Quantum spectral dimension

Spectral dimension $D_s(\tau) := -2\tau\partial_\tau \log P(\tau)$ from scaling of heat kernel $\mathrm{e}^{\tau\Delta}$,

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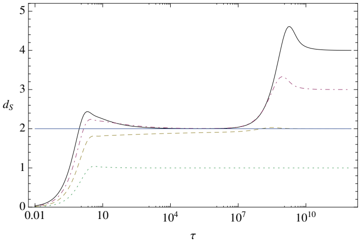
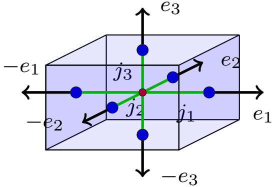
Result: Flow to intermediate $\boxed{D_s = \frac{\gamma - 1}{\beta}}$



Spectral dimension in cuboid / frustum spin foams

Result applies to the restricted spin-foam model of cuboids [Bahr/Steinhaus 1508]:



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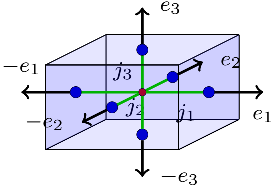

- Euclidean $\mathrm{Spin}(4) \cong \mathrm{SU}(2) \times \mathrm{SU}(2)$ model
- face weights $\mathcal{A}_f^{(\alpha)} = (\dim j_f^+ \cdot \dim j_f^-)^\alpha$
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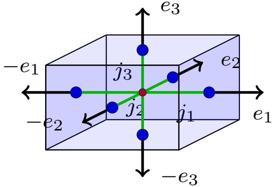

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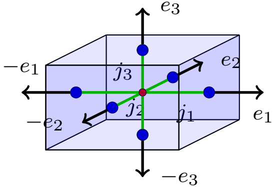

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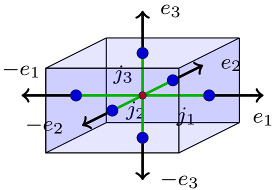

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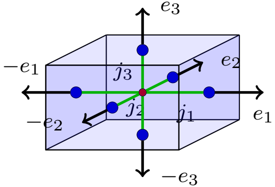

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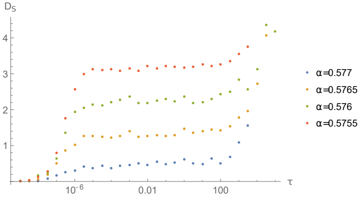

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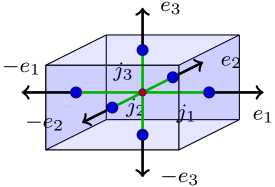

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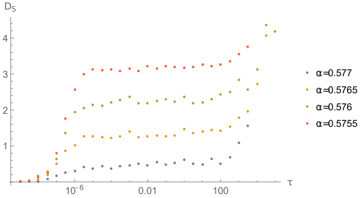

Similar results for frustum geometries [Jercher/Steinhaus/JT 2304]

New Analytic Explanation

If convergent, switch integrals

$$\langle P(\tau)\rangle = \int \mathrm{d}q\, q^{-\gamma} \int \mathrm{d}p\, \mathrm{e}^{-\frac{\tau}{q^{2\beta}}\Delta(p)} = \int \mathrm{d}p \int \mathrm{d}q\, q^{-\gamma} \mathrm{e}^{-\frac{\tau}{q^{2\beta}}\Delta(p)}$$

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- flow to $D_s = 0$ effect of $q_{\text{min}} > 0$, flow to $D_s = D$ effect of $q_{\text{max}} < \infty$.

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$$\langle P(\tau)\rangle = \int \mathrm{d}q\, q^{-\gamma} \int \mathrm{d}p\, \mathrm{e}^{-\frac{\tau}{q^{2\beta}}\Delta(p)} = \int \mathrm{d}p \int \mathrm{d}q\, q^{-\gamma} \mathrm{e}^{-\frac{\tau}{q^{2\beta}}\Delta(p)}$$

Change of variables $t = \frac{\tau\Delta}{q^{2\beta}}$ for $\tau\Delta \neq 0$ yields definition of Γ function:

$$\int_0^\infty \mathrm{d}q\, q^{-\gamma} \mathrm{e}^{-\frac{\tau}{q^{2\beta}}\Delta} = \frac{1}{2\beta}(\tau\Delta)^{\frac{1-\gamma}{2\beta}} \int_0^\infty \mathrm{d}t\, t^{\frac{\gamma-n}{2\beta}-1} \mathrm{e}^{-t} = \frac{1}{2\beta}\,(\tau\Delta)^{-\frac{\gamma-1}{2\beta}}\,\Gamma\left(\frac{\gamma-1}{2\beta}\right)$$

Universal result $\langle P(\tau)\rangle \propto \tau^{-\frac{D_s}{2}} = \tau^{-\frac{\gamma-1}{2\beta}}$ indep. of Δ/manifold !

- flow to $D_s = 0$ effect of $q_{\text{min}} > 0$, flow to $D_s = D$ effect of $q_{\text{max}} < \infty$.
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  - discrete spectrum: isolated $p_0 = 0$ needs regularization as $\int_0^\infty \mathrm{d}q\, q^{-\gamma} = \infty$

Spectral dimension in cNLFT

Global observables defined on individual diagrams Γ/ discrete geometries:

$$\langle P(\tau)\rangle = \sum_{\Gamma} \boldsymbol{\lambda}^{V_\Gamma} \langle P(\tau)\rangle_\Gamma = \sum_{\Gamma} \boldsymbol{\lambda}^{V_\Gamma} \int I_\Gamma(\{q_f\}) \,\mathrm{Tr}(\mathrm{e}^{\tau \Delta_\Gamma(\{q_f\})})$$

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Power $dF_\Gamma - 2E_\Gamma$ is just the superficial degree of divergence $\omega^{\text{s.d.}}$, indep. of F:

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Thanks for your attention!