HOW TO CONSTRAIN THE UNIVERSE ANISOTROPIES ACROSS A BIG BOUNCE

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Based on:
"The role of spatial curvature in constraining the Universe anisotropies across a Big Bounce"
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OUTLINE

- Polymer Quantum Mechanics representation
- The Vilenkin proposal
- Vilenkin meets polymer
- Dynamics of the semiclassical sector
- Quantum behaviour of the anisotropies
- Conclusions

POLYMER QUANTUM MECHANICS REPRESENTATION

PQM is an alternative representation of quantum mechanics not equivalent to the Schrödinger one

that can be introduced through the following steps:

- definition of abstract kets $|\mu\rangle$ in the Hilbert space $\mathcal{H}_{poly}$ with $\mu \in \mathbb{R}$;
- definition of the inner product $\langle\mu|\nu\rangle = \delta_{\mu\nu}$;
- definition of two main operators on $\mathcal{H}_{poly}$:

the *label operator* $\hat{\epsilon}|\mu\rangle := \mu|\mu\rangle$ that is discrete,

the *shift operator* $\hat{s}(\lambda)|\mu\rangle := |\mu + \lambda\rangle$ that is discontinuous.

PQM is able to implement quantum gravity effects
through a simpler mathematical framework with respect to Loop Quantum Cosmology.

In the p polarization:

$$\psi_\mu(p) := \langle p|\mu\rangle = e^{i\mu p}$$

$$\longrightarrow \begin{cases} \hat{q}\psi_\mu(p) = -i\dfrac{\partial}{\partial p}\psi_\mu(p) = \mu\psi_\mu(p) \\ \hat{V}(\lambda)\psi_\mu(p) = e^{i\lambda p}e^{i\mu p} = e^{i(\mu+\lambda)p} = \psi_{\mu+\lambda}(p) \end{cases}$$

$\hat{q}$ is discrete then $\hat{p}$ does not exist

- polymer lattice: $\gamma_\mu = \{q \in \mathbb{R} : q = n\mu, \forall n \in \mathbb{Z}\}$
- polymer approximation: $\boxed{p \sim p_\mu = \dfrac{1}{\mu}\sin(\mu p) = \dfrac{1}{2i\mu}(e^{i\mu p} - e^{-i\mu p})}$ $(\mu p \ll 1)$

$$\hat{\mathcal{H}} = \frac{\hat{p}^2}{2m} + \hat{V}(q) \longrightarrow \boxed{\hat{\mathcal{H}}_\mu := \frac{\hat{p}_\mu^2}{2m} + \hat{V}(q)}$$

THE **VILENKIN** PROPOSAL

WAVEFUNCTION INTERPRETATION ⟶ the phase space is divided into a **semiclassical** (s) and a **quantum** (q) region.

HYPOTHESES

1. $< H_q > / < H_s >= o(\hbar)$ ⟶ the quantum region is small and the effects on the semiclassical region are negligible
2. $\mathcal{A}(s)e^{-i/\hbar\mathcal{S}(s)}\chi(q, s)$ ⟶ the Universe wave function is the product of a semiclassical contribution (WKB expandable in $\hbar$) and a quantum one;
3. $\partial_s\chi(q, s)| \ll ||\partial_s\mathcal{A}(s)|$ ⟶ the dependence of χ on s is parametric (Born-Oppenheimer approximation).

From the Wheeler-DeWitt equation we obtain

- the *HAMILTON-JACOBI EQUATION* (phase $\mathcal{S}$) at the zero order in $\hbar$;
- the *CONTINUITY EQUATION* (amplitude $\mathcal{A}$)

and the *SCHRÖDINGER EQUATION* (quantum wave function χ) at the first order in $\hbar$;

⟶ **semiclassical clock for the quantum system and semipositive-defined probability!**

VILENKIN MEETS POLYMER

Bianchi IX with a quadratic (Taylor-expanded) potential

polymerized Universe area

$$\mathcal{H} = \frac{A^{-3/2}\sqrt{2}}{2}\left[-\frac{2}{3\sqrt{2}(4\pi)^2}A^2\frac{\sin^2(\mu p_A)}{\mu^2} + p_\phi^2 + p_+^2 + p_-^2 + A^2(\beta_+^2 + \beta_-^2)\right] = 0\,, \quad A = e^{2\alpha}$$

Vilenkin approach in the momentum representation → CLASSICAL PHASE SPACE (A, ϕ) + SMALL QUANTUM D.O.F. (β_+, β_-)

$$-\frac{2}{3\sqrt{2}(4\pi)^2}\frac{\sin^2(\mu p_A)}{\mu^2}\left(\frac{\partial \mathcal{S}}{\partial p_A}\right)^2 + p_\phi^2 = 0$$

HAMILTON-JACOBI EQUATION

$$i\frac{\partial}{\partial p_A}\left(\mathcal{A}\frac{\partial \mathcal{S}}{\partial p_A}\right) = 0$$

CONTINUITY EQUATION

$$\left[p_+^2 + p_-^2 - A^2(\tau)\left(\frac{\partial^2}{\partial p_+^2} + \frac{\partial^2}{\partial p_-^2}\right)\right]\chi(p_\pm, \tau) = i\frac{\partial \chi(p_\pm, \tau)}{\partial \tau}$$

SCHRÖDINGER EQUATION

DYNAMICS OF THE **SEMICLASSICAL** SECTOR

HAMILTONIAN OF THE SEMICLASSICAL BACKGROUND WITH POLYMER MODIFICATIONS

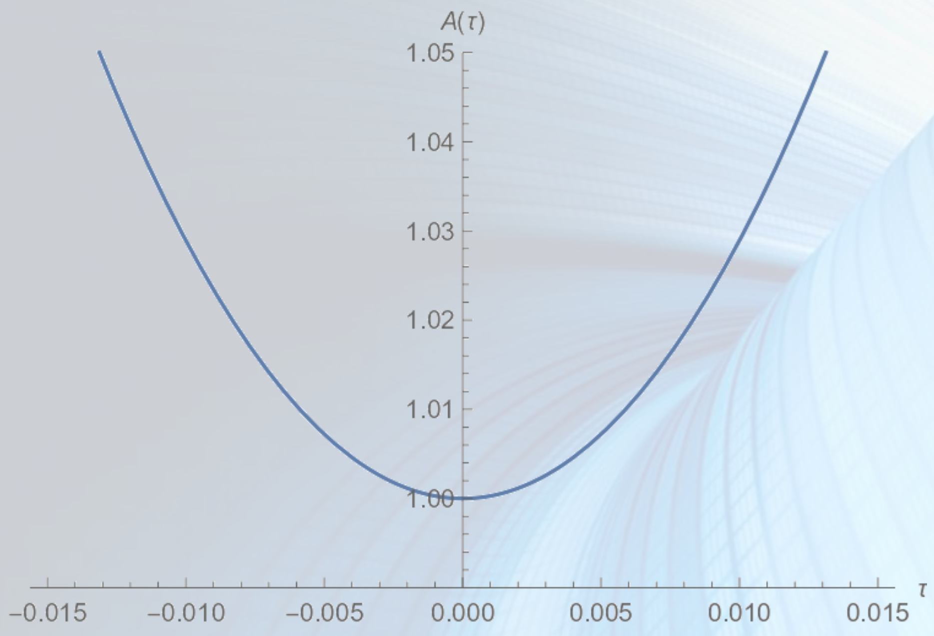
$$\mathcal{H}_{class}=\frac{A^{-3/2}\sqrt{2}}{2}\left[-\frac{2}{3\sqrt{2}(4\pi)^2}\frac{A^2\sin^2(\mu p_A)}{\mu^2}+p_\phi^2\right]$$

$$\mathrm{N}=2A^{3/2}/\sqrt{2}$$ *time gauge*

$$\begin{cases}\dot{A}=\mathrm{N}\dfrac{\partial H_0^{pol}}{\partial p_A}=-\dfrac{1}{\sqrt{288}\pi^2}\dfrac{A^2}{\mu}\sin(\mu p_A)\cos(\mu p_A)\\ \dot{p_A}=-\mathrm{N}\dfrac{\partial H_0^{pol}}{\partial A}=\dfrac{1}{\sqrt{288}\pi^2}\dfrac{A\sin^2(\mu p_A)}{\mu^2}\end{cases}$$

$$A(0)=1,\ p_A(0)=3\pi/2$$ *initial conditions*

$$\begin{cases}A(\tau)=\cosh\left(\dfrac{\sin(\frac{3\mu\pi}{2})\tau}{\sqrt{288}\pi^2\mu}\right)-\cos\left(\dfrac{3\mu\pi}{2}\right)\sinh\left(\dfrac{\sin(\frac{3\mu\pi}{2})\tau}{\sqrt{288}\pi^2\mu}\right)\\ p_A(\tau)=\dfrac{2}{\mu}\mathrm{arccot}\left[\exp\left(-\dfrac{\sin(\frac{3\mu\pi}{2})\tau}{\sqrt{288}\pi^2\mu}\right)\cot\left(\dfrac{3\mu\pi}{4}\right)\right]\end{cases}$$



The polymerization of the Universe area removes the singularity and reproduces a Big Bounce.

QUANTUM BEHAVIOUR OF THE **ANISOTROPIES**

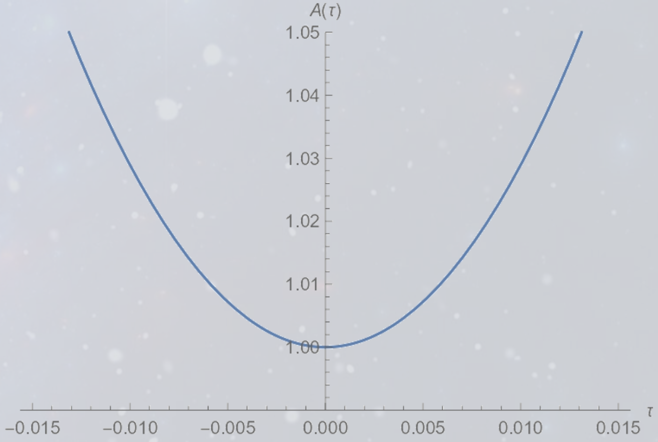
TIME-DEPENDENT HARMONIC OSCILLATOR

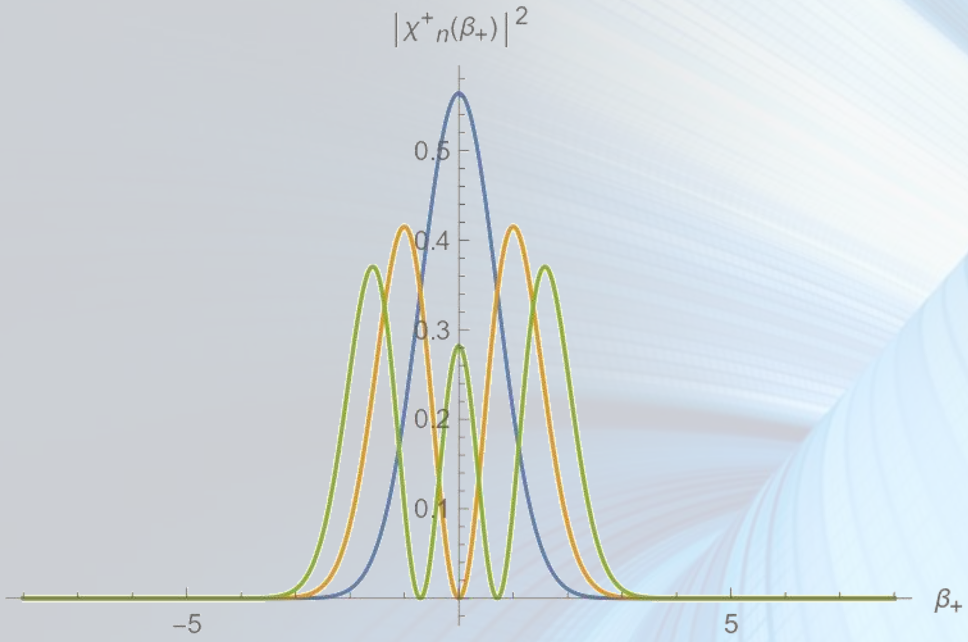
$$\left[p_+^2 + p_-^2 - A^2(\tau)\left(\frac{\partial^2}{\partial p_+^2} + \frac{\partial^2}{\partial p_-^2}\right)\right]\chi(p_\pm,\tau) = i\frac{\partial \chi(p_\pm,\tau)}{\partial \tau}$$

solution

$$\chi_n^\pm(\beta_\pm,\tau) = \frac{e^{i\gamma_n(\tau)}}{\sqrt{\sqrt{\pi}n!2^n\rho}}\mathfrak{H}\left(\frac{\beta_\pm}{\rho}\right)e^{\frac{i}{4}\left(\frac{\dot\rho}{\rho}+\frac{2i}{\rho^2}\right)\beta_\pm^2}$$

$$\ddot\rho + 4A(\tau)^2\rho = 0$$ the time-dependent frequency is the Universe area!





Thanks to the presence of the potential the polymer effects enter in the quantum dynamics of the anisotropies by means of the bouncing solution of the Universe area, i.e. the time-dependent frequency.

QUANTUM BEHAVIOUR OF THE **ANISOTROPIES**

Exact computation of the mean value and variance of the **anisotropies** on the time-dependent harmonic oscillator eigenstates

$$\hat{\beta}_\pm = \frac{\rho}{\sqrt{2}}(\hat{a}_\pm + \hat{a}^\dagger_\pm)$$

creation and annihilation operators

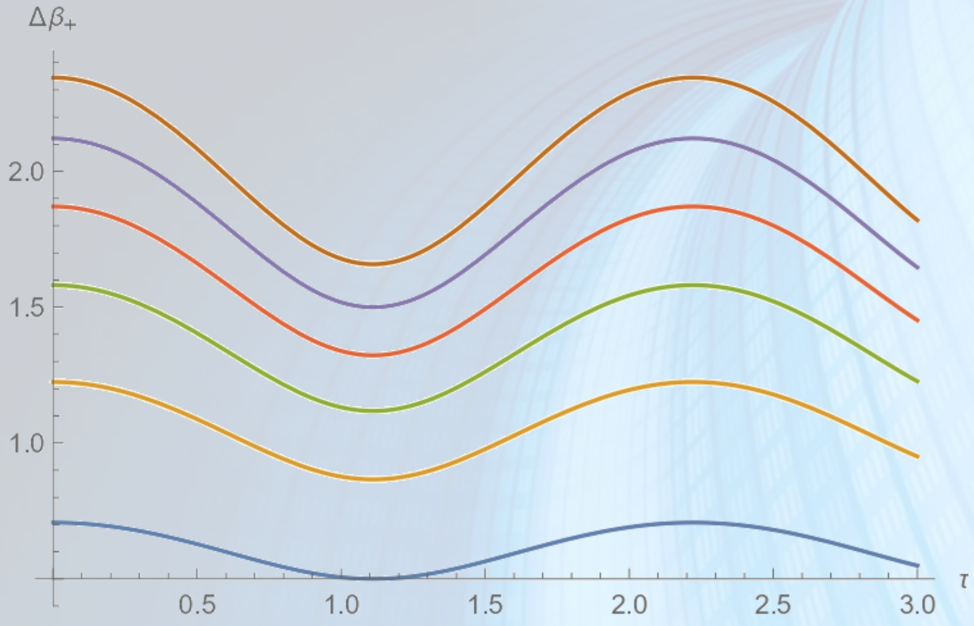
MEAN VALUE

$$< n|\hat{\beta}_\pm|n > = < n|\frac{\rho}{\sqrt{2}}(\sqrt{n}|n-1> + \sqrt{n}|n+1>) = 0$$

→ the eigenstates are quasi-isotropic states!

VARIANCE

$$\Delta\beta_\pm = \sqrt{< \hat{\beta}^2_\pm > - < \hat{\beta}_\pm >^2} = \left(n + \frac{1}{2}\right)\rho^2$$



The variance of the anisotropies has an oscillatory and confined character.

QUANTUM BEHAVIOUR OF THE **ANISOTROPIES**

Exact computation of the mean value and variance of the **shear** on the time-dependent harmonic oscillator eigenstates

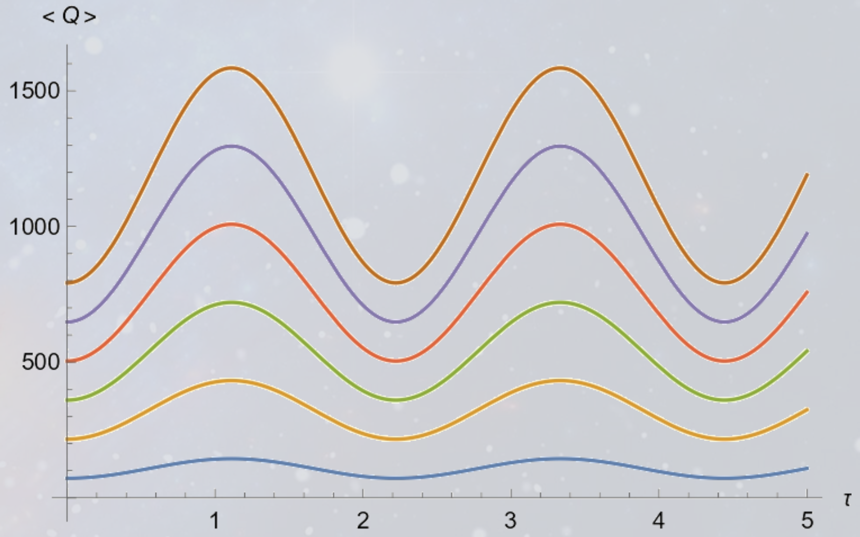
$$Q = (H_1 - H_2)^2 + (H_2 - H_3)^2 + (H_3 - H_1)^2 = 18(\dot{\beta}_+^2 + \dot{\beta}_-^2) = 72(p_+^2 + p_-^2)$$

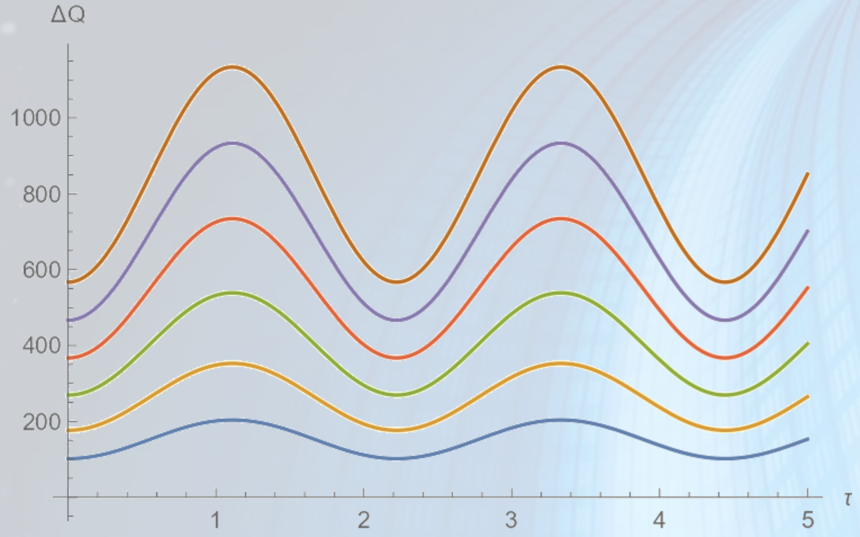
MEAN VALUE

$$< \hat{Q} >= 144\left(\frac{1}{\rho^2} + \dot{\rho}^2\right)$$

VARIANCE

$$\Delta\hat{Q} = \sqrt{< \hat{Q}^2 > - < \hat{Q} >^2} = 144\sqrt{\frac{n^2+n+1}{2}}\left(\frac{1}{\rho^2} + \dot{\rho}^2\right)$$





QUANTUM BEHAVIOUR OF THE **ANISOTROPIES**

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The presence of a harmonic potential is a NECESSARY CONDITION
in order to maintain the Universe in a quasi-isotropic state.

IS IT ALSO A SUFFICIENT CONDITION?

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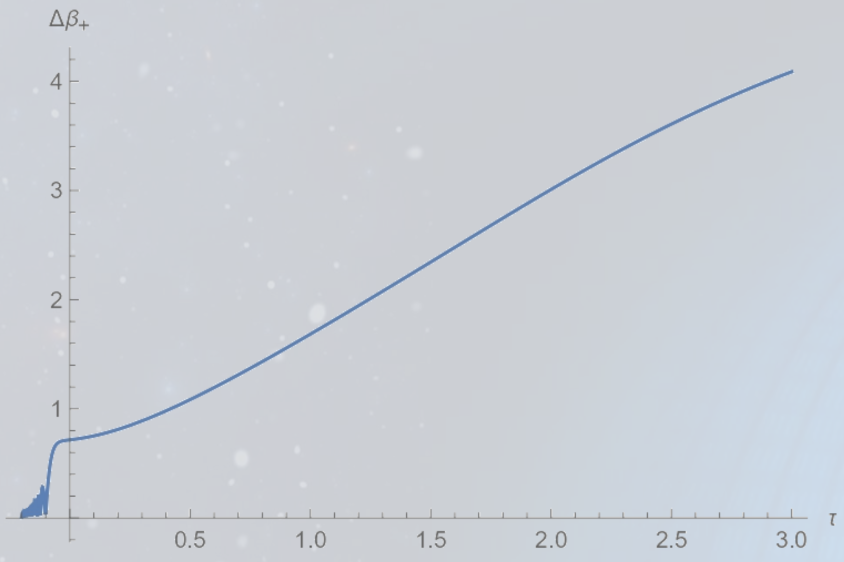

Behaviour of anisotropy variance in time along the collapsing (singular) branch.

CONCLUSIONS

- The Vilenkin approach can be reformulated in the momentum representation to make it compatible with the polymer formulation.
- A Big Bounce emerges and a Schrödinger evolution is recovered.
- The Bianchi IX quadratic potential is able to maintain the Universe in a quasi-isotropic state across a Big Bounce.

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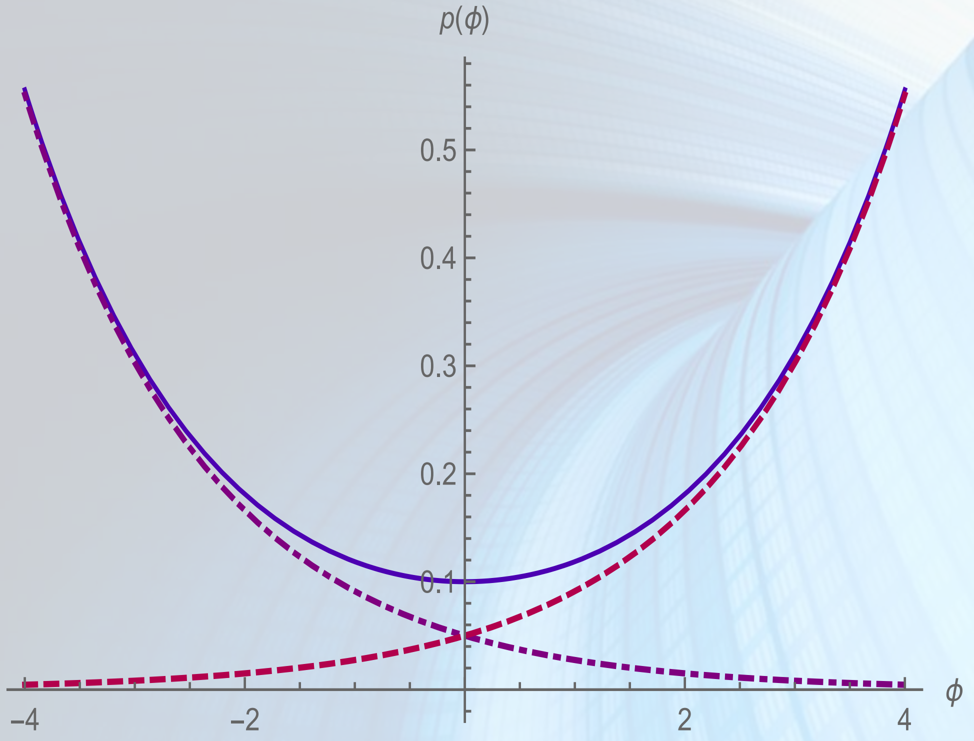
THANK YOU
FOR THE ATTENTION

WHAT IS THE **BIG BOUNCE?**

Best attempt to solve the Big Bang singularity in Loop Quantum Cosmology thanks to the discrete spectrum of geometrical operators like area and volume.

1. The Big Bounce has been characterized mainly at a semiclassical level but in the Planckian era the quantum effects are not negligible.

2. Anisotropic degrees of freedom can arise approaching the singularity. Then, a FLRW model could turn into a Bianchi one and hence the hypothesis of localized wave-packets would be violated.



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THE **BIANCHI IX** MODEL

Bianchi IX metrics in the Misner variables

$$ds^2 = \mathrm{N}(t)^2 dt^2 - \frac{1}{4} e^{2\alpha} \left(e^{2\beta}\right)_{ij} \sigma^i \sigma^j$$

Bianchi IX Hamiltonian

$$\mathcal{H} = \frac{\mathrm{N} e^{-3\alpha}}{3(8\pi)^2} \left[-p_\alpha^2 + p_+^2 + p_-^2 + 3(4\pi)^4 e^{4\alpha} U(\beta_\pm) \right]$$

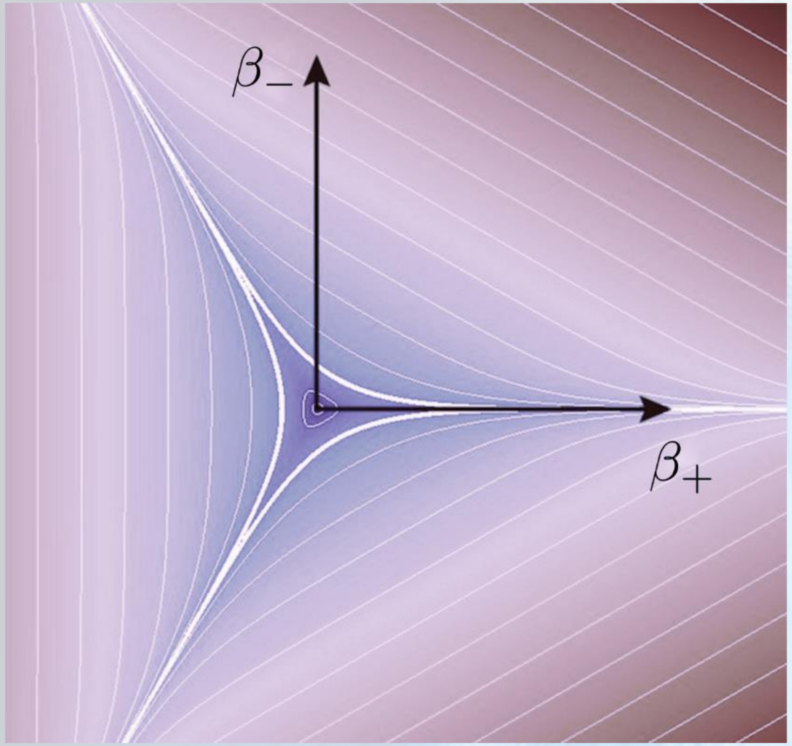
Arnowitt-Deser-Misner reduction of the Hamiltonian

$$\mathcal{H}_\alpha := -p_\alpha = \sqrt{p_+^2 + p_-^2 + 3(4\pi)^4 e^{4\alpha} U(\beta_\pm)}$$

The classical dynamics is singular and chaotic.

The motion of the point-Universe in the potential well is described by the following reflection law:

$$\frac{1}{2}\sin(\theta_i + \theta_f) = \sin\theta_i - \sin\theta_f$$



Equipotential lines of the Bianchi IX potential

$$U(\beta_\pm) = \frac{2}{3} e^{4\beta_+} \left(\cosh(4\sqrt{3}\beta_-) - 1\right) + 1 + \\ - \frac{4}{3} e^{2\beta_+} \cosh(2\sqrt{3}\beta_-) + \frac{1}{3} e^{-8\beta_+}$$

QUANTUM BEHAVIOUR OF THE **ANISOTROPIES**

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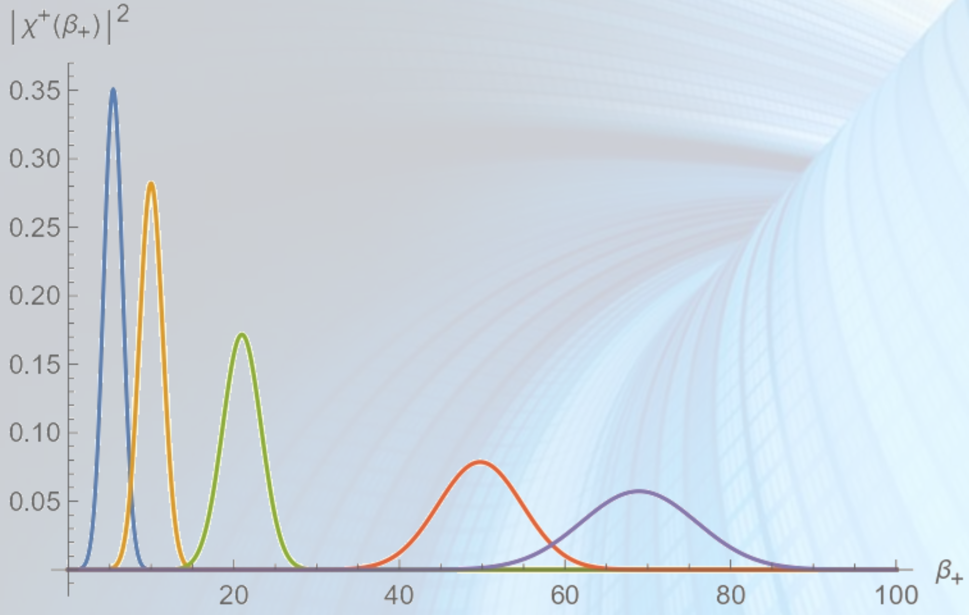
$$\left[p_+^2 + p_-^2 - A^2(\tau)\left(\frac{\partial^2}{\partial p_+^2} + \frac{\partial^2}{\partial p_-^2}\right)\right]\chi(p_\pm, \tau) = i\frac{\partial \chi(p_\pm, \tau)}{\partial \tau}$$

1 **BIANCHI I** approximation (the potential is neglected)

$$\left[p_+^2 + p_-^2 - \cancel{A^2(\tau)\left(\frac{\partial^2}{\partial p_+^2} + \frac{\partial^2}{\partial p_-^2}\right)}\right]\chi(p_\pm, \tau) = i\frac{\partial \chi(p_\pm, \tau)}{\partial \tau}$$

$$\frac{\partial}{\partial \tau} = \mathrm{N}\frac{\partial}{\partial t} = \frac{2A^{3/2}}{\sqrt{2}}\frac{\partial}{\partial t}$$

The polymer effects enter in the time variable by means of the bouncing solution of the Universe area and so they can affect the anisotropies quantum dynamics.



In the Bianchi I model the anisotropies are NOT under control across a Big Bounce.